FINANCIAL INTERMEDIATION VERSUS STOCK MARKETS IN A DYNAMIC INTERTEMPORAL MODEL

by

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Financial Intermediation Versus Stock Markets In a Dynamic Intertemporal Model

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ABSTRACT

We consider the transitions among intragenerational and alternative intergenerational financing and liquidity risk-sharing mechanisms, in an Overlapping Generations model with endogenous levels of long-lived investments. The existence and characterization of a Self-Sustaining Mechanism, stable across generations, are established. The long-run equilibrium outcome, in a Proposal Game across generations, is shown to depend on the risk-aversion and propensity for early liquidity needs of the agents.

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I. INTRODUCTION AND SUMMARY

Models of intertemporal liquidity risks, or "preference shocks" - developed in Bryant [1980] and further examined in the works of Diamond and Dybvig [1983], Bhattacharya and Gale [1987], and Jacklin [1987] among others - have recently been extended to the realm of ongoing dynamic overlapping generations (OLG) economies; see Bencivenga and Smith [1991], Qi [1994], Fulghieri and Rovelli [1993] and Dutta and Kapur [1994]. These models may be used to evaluate the essential tradeoffs which arise in two distinct areas of current debate. The first is on the evaluation of alternative (or possibly complementary) arrangements for the monetary system, for example the relative merits of a market-value-based vs. an intermediated banking system. The second interpretation, clearly requiring a considerably longer definition of the time unit, pertains to the debate on the viability and desirability of differing social security (pension) systems, and in particular the choice between Pay-As-You-Go (PAYG) versus Funded or Capital-reserve systems (e.g. Boldrin and Rustichini, 1995). Our paper considers the latter set of tradeoffs in an OLG model outside of long-run steady states.

In our paper, agents choose among intertemporal liquidity-sharing or insurance mechanisms, in an OLG economy in which the *capital stock is possibly longer-lived than agents' lives*. We focus on the comparisons between "Banking" or PAYG systems, of both an *intra-* and *inter*-generational nature, versus Stock Market based *inter*-generational financial systems. The essential tradeoffs across these institutional mechanisms arise from the need for financial contracting to provide agents with liquidity insurance, related to the uncertainty regarding their preferences for their intertemporal allocation of consumption[1]. Such preference shocks, coupled with real investment opportunities that are long-lived and technologically illiquid, create a demand for interim stock markets, or financial intermediaries and contracts such as deposits withdrawable (fully or partially) on demand.

In dynamic OLG economies, there is in addition the possibility of designing contracts (or mechanisms) for sharing such liquidity risk *across generations*, taking advantage of the "life cycle" structure to the demand for savings of each generation. With the long horizons envisioned, one may think of events such as uncertain workers' disability as a source of early withdrawal demand. In recent papers, by Qi [1994], Fulghieri and Rovelli [1993], and Dutta and Kapur [1994], alternative institutional arrangements for such intertemporal consumption-smoothing and liquidity risk-sharing mechanisms, within and across generations, have been examined and compared.

Under some assumptions, in particular the restriction of the analysis to steady states only, both Qi [1994] and Fulghieri and Rovelli [1993] show that there exist intergenerational financial intermediation contracts that can attain the Golden Rule (Phelps, 1961) levels of investment and consumption-smoothing. The paper of Fulghieri and Rovelli, and the later work of Bhattacharya and Padilla [1996], also focus on the comparison of intergenerational *Banks versus Stock Markets* for investments that are long-lived in nature. In contrast to the earlier models, which focus either on a static economy (Bhattacharya and Gale, 1987) or on *intra*-generational mechanisms only (Bencivenga and Smith, 1991), Fulghieri and Rovelli find that stock markets result (in steady state) in *underinvestment* in the long-lived technology, relative to the Golden Rule optimum, without any investment in short-term liquid technologies or early liquidation[2]. This occurs essentially owing to the incompleteness of contracts for resource transfers across generations, which in a stock market economy can take place only through real investments in long-lived technologies and bilateral trading at their interim market valuations.

In all of the above mentioned papers, the analysis of the functioning of intermediaries and stock markets is confined to steady states, in which the inheritance of a stationary level of investment by prior generations is taken as given. However, given the endowment structure in these models, this is a restrictive assumption, especially *vis à vis* the *first* generation which only has its initial endowment to start a resource-transfer *cum* liquidity-insurance scheme. While Qi [1994] briefly examines this issue (see his Propositions 5 and 6), his analysis is focused on PAYG systems only, rather than on the choices among *alternative risk sharing mechanisms*.

More specifically, the following questions remain unanswered: given initial conditions, is it possible to predict which allocational mechanisms would be chosen by the generations present, and would the same choice be repeated by the subsequent generations, i.e., be intergenerationally stable or Self-Sustaining? For example, consider the three institutional mechanisms which we shall analyze below: (i) *Intra*generational Bank (B), (ii) *Inter*generational Banking (IB) and (iii) *Inter*generational Stock Market (SM). Suppose these may be ranked in a particular way by the steady state welfare criterion. Then, is it necessarily true that, when the issues related to (a) transition to the steady state, and (b) non-cooperative intergenerational choices are taken into account, the most efficient steady state allocational mechanism would indeed be chosen and be intergenerationally self-sustaining? We seek to address these questions in this paper.

In defining our notions of inter-generational choices over mechanisms for financing of investment and consumption smoothing, we shall not be employing notions such as the Core, whose applicability to the OLG setting is dubious at best. As Esteban [1986] and Esteban and Millan [1990] have shown, even when the Golden Rule allocation *does* improve the welfare of the first generation (the Samuelson, 1958, case of Gale, 1973), it is *not* in the intergenerational Core (never for a single commodity setting)[3]. Fortunately, as Esteban and Sakovics [1993] have emphasized, negative results like those mentioned above do not imply that intergenerational transfer mechanisms become infeasible. Society may cope with such a conundrum, created by the "not in the Core" result, by *restricting the language* or *strategy space of intergenerational proposals* for choosing its Institutional Mechanisms for intergenerational reallocations. In the context of the Esteban and Sakovics model, of a pure-exchange OLG environment with agents' endowments tilted to the early period(s) of life, they argue that in a time-stationary model the appropriate notion of an Institutional proposal is that of a time-stationary transfer scheme, from *each* current and future generation when young to its preceding generation when old. Institutions survive if and only if they constitute Subgame Perfect Equilibria (SPE) (Selten, 1975) in such an intergenerational proposal game. A similar restriction is employed by Boldrin and Rustichini [1995], in their modelling of inter-generational social security schemes versus *intra*-generational accumulation of short-lived investments.

In our model below, we define the concept of a *Self-Sustaining* investment and consumption allocation *Mechanism*, using restrictions motivated by analogous considerations. The spirit of these is that proposals that make *later* generations strictly worse off than the proposing one are not allowed. Such a restriction may arise spontaneously for market-based mechanisms, or be a result of societal Norms embodied in a *legislated* Pay As You Go (PAYG) mechanism such as IB. In particular, we restrict the set of proposals by each generation to be one of the three Institutions that were indicated above: Transition to Steady-State of Intergenerational Banking (TIB), Intergenerational Stock Market (SM), and Intragenerational Banking (B) (The first of these takes into account the lack of inherited prior capital investments for the first generation). Some of these proposals may require that an older and a younger generation agree on the "sharing" of the inherited capital stock plus the endowment of the young. Hence, each stage game in our model is one of Proposals and Counter-Proposals by the two overlapping generations, unlike in Esteban and Sakovics [1993] where only young propose transfers to the currently old.

Our paper is organized as follows. In Section II, we set out the basic model and delineate the Golden Rule and other steady state allocations under the three alternative institutional arrangements of B, IB, and SM. The dynamics of transition to these steady states, and the ranking of welfare levels attainable under each alternative institutional mechanism (TIB, SM, B), taking transition into account, are the *foci* of Section III. Intergenerational Proposal Games over these (transition) Institutions and their subgame perfect equilibria, which we term intergenerationally *Self-Sustaining Mechanisms*, are described and characterized in Section IV. In Section V we conclude with a discussion of our results in the context of recent literature and with suggestions for further research.

The major results that emerge from the analysis in the paper are the following. *First*, we point out that there exists a feasible stationary consumption path (TIB) such that capital accumulation along that path will lead, within a finite horizon, to the Golden Rule steady state optimal allocation. Furthermore the consumption levels attained along this path ensure that the welfare of each generation involved in the transition *strictly* exceeds that attainable from *any* *intra*generational mechanism[4]. *Second,* we show that there exists a faster one-shot transition to the steady state *inter*generational stock market (SM) allocation (Fulghieri and Rovelli, 1993)[5]. *Third,* we prove that a transition to SM that benefits the proposing current young, which exists unless they are *extremely* risk averse, *must* involve strictly rationing the participation of the concurrent preceding generation in the initial market (only). *Fourth,* we prove that under reasonable parameter values, there exists an *intergenerational conflict* in any stage game. Specifically, at any time, the currently young may stand to gain by proposing a (rationed and immediate) transition to the stationary intergenerational stock market allocation (SM), whereas the currently middle-aged are made strictly worse off, relative to the "slowest turnpike" allocation (TIB), from accepting such a proposal. *Finally,* in the last step we delineate how this conflict may be resolved, in the sequence of equilibria of the intergenerational proposal games across overlapping generations.

The equilibrium outcome of the intergenerational proposal games depends *crucially* on the relative expected utility payoff(s) of the middle-aged generation(s) from their rationed transition to the stock market (SM), versus their *fallback option* of doing *intra*generational liquidity risk-sharing (B), based on their inherited endowment net of prior payout obligations. It turns out that the currently middle-aged generation(s) are better off from their fallback option (B), relative to the constrained (for them only) transition to SM proposed by their "children", if and only if *(i)* the proportion of the generation requiring *early liquidity* (withdrawal) is *small*, and *(ii)* agents in the generation are ex ante *highly risk-averse* (or averse to unsmoothed consumption profiles). If *either* of these conditions on agents' ex ante characteristics is *not* satisfied then, and only then, the stock market (SM) institutional arrangement (which is always suboptimal relative to IB in the steady state) becomes the unique subgame - and "trembling-hand" perfect (Selten, 1975) equilibrium of the intergenerational proposal games[6].

II. THE MODEL AND STATIONARY ALLOCATIONS

II.A Optimal Allocations and Intermediated Outcomes

We consider an extended version of the overlapping generation (OLG) model of Samuelson [1958], that was first advanced by Bryant [1980]. Time is indexed by $t = 0, 1, 2, \ldots, \infty$. Each generation consists of a continuum of agents of unit measure. An agent born at time point $t \geq 0$ is alive at time point (t+1), and possibly also at time point (t+2). Each generation is endowed with one unit of the single commodity, spread equally across its members, at birth; this may be thought of as labor income or prior savings. To simplify matters, we assume that an agent born at t only wishes to consume from his savings at time-point (t+1) if he is an *early dier*, or at time-point (t+2) if she is a *late dier*. Such simple ex post corner preferences, introduced by Diamond and Dybvig [1983], imply that the allocations produced by *intra*generational stock markets and banking systems coincide, provided market participation is suitably restricted; see Jacklin [1987], and Bhattacharya and Gale [1987].

Agents become early or late diers with probabilities ε and $(1-\varepsilon)$ respectively. These events are independent across the continuum of agents in each generation, so that a proportion $(1-\varepsilon)$ of agents born at time t is alive at time (t+2), when they die. Thus, the ex ante expected utility of a representative agent of generation t is given by:

$$V_t = \varepsilon U(C_{t,1}) + (1-\varepsilon)U(C_{t,2}) \quad (1)$$

where $\left\{C_{t,1}\ ,\ C_{t,2}\right\}$ are the (certain) allocations of consumption to early and late diers of generation t, respectively. The utility function U(.) is assumed to be strictly increasing and concave, with $U'(0) = \infty$.

There are two investment technologies in the model. The first is long-lived with constant returns to scale: investment of I_t at time t returns RI_t, $R > 1$, at time (t+2). The second technology is that of storage, without any loss or depreciation, between time points t to (t+1). The long-lived investment I_t can also be liquidated at time (t+1) to yield $Q\, L_{t+1}$, where $0 \leq Q \leq 1$, and $0 \leq L_{t+1} \leq I_t$; we assume $Q = 1$, so that early liquidation and storage technologies are equivalent[7].

The Golden Rule (Phelps, 1961) *stationary* optimal *intergenerational* investment and consumption plan thus solves the problem:

$$V^* \equiv \underset{\{ I_t\, ,\, L_t\, ,\, C_{t,1}\, ,\, C_{t,2} \}}{\text{Max}} \left[\varepsilon\, U(C_{t,1}) + (1-\varepsilon)\, U(C_{t,2})\right] \tag{2}$$

s.t.:

$$\varepsilon\, C_{t-1,1} + (1-\varepsilon)\, C_{t-2,2} \leq 1 + R\,(I_{t-2} - L_{t-1}) + L_t - I_t \ , \ \forall t \ , \tag{2.a}$$

$$0 \ \leq \ L_{t+1} \ \leq \ I_t \ \leq \ 1 \tag{2.b}$$

As shown, for example, in Fulghieri and Rovelli (1993, Proposition 1), the solution to this problem is characterised by full investment of the initial endowment of each new generation, no liquidation of invested capital, and a level of consumption for early and late diers which is constant and equal to R. Furthermore, Qi [1994], Fulghieri and Rovelli [1993], and Dutta and Kapur [1994] give the conditions under which the Golden Rule allocation may be implemented by an *inter*generational banking system. These results are summarised in the following proposition.

Proposition 1. *(a) (Fulghieri and Rovelli 1993).The Golden Rule (Stationary) Optimal Intergenerational Allocation Rule, which maximizes (2) subject to (2.a) and (2.b), is given by:*

$$I_t^* = 1\ ,\ \ L_t^* = 0\ ,\ \ C_{t,1}^* = C_{t,2}^* = R\ ,\ \ \forall t\ .$$

(b) (Qi 1994, Fulghieri and Rovelli 1993, and Dutta and Kapoor 1994). If agents/depositors can be subject to a No Redepositing Condition within (and across) banks, then a single welfare-maximizing bank (or equivalently, many representative competing banks) would, in equilibrium, offer each generation the deposit contract: I_t *in deposits leads to withdrawal rights* RI_t *at time (t+1)* ***or*** *time (t+2), at the choice of the depositor. Furthermore, all agents*

born at t would deposit all their initial endowment in banks, which would invest it ($I_t = 1$) in long-lived investments only, without early liquidation. Thus intergenerational banking (IB) would implement the Golden Rule optimum. Hence in particular:

$$C_{t,i}^{IB} = C_{t,i}^{*} = R \ , \qquad i \in \{1,2\} \ , \forall t$$

(c) If redepositing after early withdrawal by agents is not monitorable (indistinguishable from an initial deposit), especially if done across banks, but depositing at one bank by another can be prohibited, then ex ante Bertrand competition among banks would lead to the outcome: depositing $I_t = 1$ by each depositor of each generation t confers her with withdrawal rights C_1^U at time (t+1) or C_2^U at time (t+2), where thèse satisfy (suppressing the t-subscript):

$$C_1^U < R < C_2^U, \tag{4.a}$$

$$\left(C_1^U\right)^2 \leq C_2^U \tag{4.b}$$

$$\varepsilon C_1^U + (1-\varepsilon) C_2^U = R \tag{4.c}$$

Proof. See either Qi, Fulghieri and Rovelli, or Dutta and Kapur.

Remark. The Golden Rule optimum is, of course, strictly valid as a solution concept only when $t \in \{-\infty, ..., -1, 0, 1, ..., \infty\}$ since, for example, if $t \in \{0, 1, ..., \infty\}$, and $I_1 = 1$ and $L_1 = 0$, then the early diers of generation 0 will obtain consuimption $C_{0,1} = 0$. These issues raise questions about the *transition* to these stationary optimal allocations, which we discuss in the next section. The notion of a competitive banking system is also not unambiguous in an OLG setting, and one may think instead of the resulting allocations as arising from a legislated allocation mechanism.

In contrast, the constrained optimal *intragenerational* allocation, solves the problem:

$$\overline{V} \equiv \underset{\{I,L,C_1,C_2\}}{\text{Max}} \left[\varepsilon U(C_1) + (1-\varepsilon) U(C_2)\right] \tag{3}$$

s.t.:

$$0 \leq \varepsilon C_1 \leq L, \tag{3.a}$$

$$0 \leq (1-\varepsilon) C_2 \leq R(I-L), \tag{3.b}$$

$$0 \leq I \leq 1. \tag{3.c}$$

The solution to this problem is characterized by Diamond and Dybvig [1993], who also show that this allocation may be implemented with a single welfare-maximizing bank or several (representative) competing banks who are Bertrand Competitors and obtain clienteles of strictly positive measure. The latter result assumes *no interim trading of assets across competing banks* at the interim period; see Bhattacharya and Gale [1987].

Proposition 2. *(Diamond and Dybvig 1993). The Optimal Intragenerational Allocation Rule, which maximizes in (3) subject to (3.a) to (3.c), is given by:*

$$\bar{I} = 1, \; U'(\bar{C}_1) = RU'(\bar{C}_2), \; \bar{C}_1 = \bar{L}/\varepsilon \, ; \; \bar{C}_2 = R(1-\bar{L})/(1-\varepsilon)$$

Furthermore, in a competitive intragenerational banking system equilibrium, deposit contracts of the form { deposit I at time t implies withdrawal rights $C_1^B = \bar{C}_1 I$ *or* $C_2^B = \bar{C}_2 I$*, at times (t + 1) or (t + 2) respectively} would prevail, leading to expected utility* $V^B = \bar{V}$*, and each depositor would deposit* $I = 1$ *at one (or many) bank(s), and each bank would invest its deposits in short-term (early liquidation) and long-term investments in the proportions* $\bar{L}$ *and* $1 - \bar{L}$*, respectively.*

Proof. See Diamond and Dybvig [1983][8].

The following implications of Proposition 2 are straightforward.

Corollary 1. *For all* $\varepsilon > 0$ *, the expected utility* V^B *of an agent in the optimal intragenerational allocation is strictly less than its level* V^* *at the Golden Rule intergenerational optimum.*

Proof. Since $\left\{\bar{C}_1, \bar{C}_2\right\} > 0$ and hence $\bar{L} \in (0,1)$, given $U'(0) = \infty$, we have

$$1 < \varepsilon \bar{C}_1 + (1-\varepsilon)\bar{C}_2 = \bar{L} + R(1-\bar{L}) < R,$$

whereas $C_1^* = C_2^* = R$, from which the conclusion follows using Jensen's inequality. **QED.**

Corollary 2. *(Diamond and Dybvig, 1983) If* $U(C_i)$*,* $i \in \{1,2\}$*, has its Relative Risk Aversion Coefficient uniformly strictly greater than unity (that for logarithmic utility), then*

$$1 < \bar{C}_1 < \bar{C}_2 < R,$$

implying: $\varepsilon < \bar{L} < 1.$

We should briefly mention the *alternatives* to the existence of bank-type intermediaries (and "unconditional" withdrawal rights) in this setting. The extreme one is that of intra- and inter-generational Autarky, under which each agent would invest I=1 when young, and consume $C_1^A = 1$ if she is an early dier, and $C_2^A = R$ if she is a late dier, with expected utility:

$$V^A \equiv \varepsilon U(C_1^A) + (1-\varepsilon)U(C_2^A) \,, \tag{5}$$

where it is obvious that: $V^A \leq \overline{V} < V^*$, with the first inequality also strict unless U(C) = log(C), since $\{ L = \varepsilon, C_1 = 1, C_2 = R\}$ is always feasible under intragenerational banking.

A less extreme comparison is with an institution often analyzed in "standard" financial modelling: the Intragenerational Stock Market. If individuals still invest on their own, but can sell their long-term investments I_t at a linear price $p_{t+1}I_t$ at time (t+1), then it turns out that the *interim value-maximizing* consumption-investment outcome will be the *same* under intragenerational stock markets as under Autarky; see Bhattacharya and Gale [1987]. The reason is that, under the interim value/wealth maximization criterion used to choose liquidation L_{t+1} from I_t , subject to $0 \leq L_{t+1} \leq I_t$, $L_{t+1} \in (0 , I_t)$ will be chosen by individuals if and only if $p_{t+1} = 1$. With the ex post corner preferences assumed, this will be the case if and only if $\{C_{t,1} = 1 ; C_{t,2} = R \}$, as shown by Bhattacharya and Gale [1987].

Bencivenga and Smith [1991] and others have emphasized *this difference* between Banking and Stock Market allocations, suggesting that it should apply even when the setting is *intergenerational* in nature. However, as argued in Bhattacharya and Gale [1987] and Bhattacharya and Padilla [1996], with *intragenerational* stock markets investment decisions would be made by *firms*, or mutual funds, which consist of *coalitions* of agents of strictly positive measure (to pool their liquidity risks). Such a coalition, if it plans to invest I and liquidate L ≤ I next period, will issue a dividend stream {L ; R(I-L)} at the two consecutive points, and early diers would sell their ex-first-dividend shares to late diers of the *same* fund. However, the appropriate choice criterion for such mutual funds, if interim trading of ex-dividend shares is *restricted* to the members of a fund, would be to maximize the ex ante expected utility V of their members, and not the interim market value of the {I,L} tuple. Hence $L = \overline{L}$ would be chosen, leading to $C_1 = \overline{C_1}$, $C_2 = \overline{C}_2$, and expected utility $V^B = \overline{V}$.

As this is the <u>same</u> allocation which occurs under intragenerational (Diamond-Dybvig) banking, we will disregard *Intragenerational* Stock Markets in the analysis to follow.

We now proceed to consider alternative institutions in an intergenerational setting.

II.B Intergenerational Stock Markets (SM)

We consider a situation in which the generation born at time t invests $0 \leq I_t \leq 1$ in new real investments, creating I_t new firms, and $(1 - I_t)$ is invested by the new born in the shares of existing firms. The price of ongoing (one period old) firms per unit of real investment at any point of time t is given by p_t , with the price of completed technologies being R. These prices must satisfy the perfect foresight No Arbitrage Conditions, where q_t is the unit price of *new* investments at time t:

$$\frac{p_{t+1}}{q_t} = \frac{R}{p_t}, \tag{6.a}$$

$$\frac{p_{t+1}}{q_t}\frac{p_{t+2}}{q_{t+1}} = \frac{R}{q_t}. \tag{6.b}$$

The right-hand-side of equations (6.b) represents the gross rate of returns from the two-period invest and hold strategy, while the left-hand-side of (6.b) represents the rate of return from holding a share of a new firm for one period, selling it, and then reinvesting in a share of a new firm at (t+1). In contrast, the criterion in equation (6.a) results from equating the contemporaneous rates of return on holding the shares of new firms, with that on holding (one-period) old firms.

If agents can short-sell existing firms' shares to obtain additional resources (beyond their endowments) for new real investment, or they can sequentially create, sell, and then create new investments, value-maximizing investment equilibrium requires that for $I_t \in (0,1]$ to be optimal:

$$q_t = 1, \qquad \forall t \tag{6.c}$$

Also, substituting back from (6.c) into (6.b), we see that both (6.a) and (6.b) imply that, if the chosen investment levels are interior and finite, then for all $t \geq 1$ the following intertemporal equilibrium condition must hold:

$$p_t p_{t+1} = R, \ \forall t \geq 1 \qquad (6.d)$$

Let $\{I_t\} \in (0,1]$, $t = 0, 1, 2, ..., \infty$, with $(L_t) = 0$, be the levels of new real investments in the economy above, in an intergenerational stock market equilibrium (so that $q_t = 1, \ \forall t$). Denote $\theta_{t,t}$ and $\theta_{t-1,t}$, for $t \geq 1$, as the *per capita proportions* of one period old capital stock, I_{t-1}, held at time t by a representative agent of generation t and a late dier of generation (t-1), respectively. Then, agents' budget constraints at different points of their lives, and stock market clearing, together imply that:

$$I_t + \theta_{t,t} p_t I_{t-1} = 1, \quad \forall t \geq 1, \qquad (7.a)$$

$$\theta_{t,t} p_t I_{t-1} = \varepsilon p_t I_{t-1}, \quad t = 1, \qquad (7.b)$$

$$\theta_{t,t} p_t I_{t-1} + (1-\varepsilon)\theta_{t-1,t-1} R I_{t-2} = \varepsilon p_t I_{t-1}, \quad \forall t \geq 2, \qquad (7.c)$$

$$\theta_{t,t} + (1-\varepsilon)\theta_{t-1,t} = 1, \quad \forall t \geq 1, \qquad (7.d)$$

with : $$I_0 = 1 \quad \text{and} \quad \theta_{0,0} = 0. \qquad (7.e)$$

Equation (7.a) results from the fact that the newborn will invest all their endowment in either new real investments, or in buying the shares of one-period old firms, from "early dier" agents of the previous generation. Equation (7.b) states that the t=1 sales of early diers of the 0-th generation must be wholly absorbed by purchases by the members from the 1-st generation. Equation (7.c) states that at all time points $t \geq 2$, the dividends which survivors (late diers) from generation t-1 obtain from maturing technologies are used to purchase the difference between sales by early diers of the same generation minus purchases of one period old technologies by the new born. Finally, equation (7.d) represents the fact that total shareholdings of one-period-old investments must add up to unity.

These stock market equilibrium conditions also have implications for the consumption allocations of early and late diers. For early diers, it is straightforward to see, by updating (7.a), that:

$$C_{t,1} = p_{t+1} I_t + R\theta_{t,t} I_{t-1}, \ \forall t \geq 0. \qquad (7.f)$$

For late diers, multiply (7.d) by $p_t I_{t-1}$ and substitute in (7.c) to obtain, after dividing by (1-ε):

$$\theta_{t-1,t}\, p_t\, I_{t-1} = p_t I_{t-1} + \theta_{t-1,t-1}\, R\, I_{t-2} \tag{7.g}$$

In other words, their per capita portfolio value, after rebalancing at time t (L.H.S.), equals the value of their old real investment made at (t-1), plus the dividends currently received on maturing financial investments (in ongoing technology), which they had made at time (t-1). Updating the R.H.S. of (7.g) we obtain:

$$C_{t,2} = \theta_{t,t+1}\, R\, I_t \ , \qquad \forall t \geq 0 \tag{7.h}$$

In an *Intergenerational Stock Market Equilibrium* (SM), with perfect foresight, agents of generation t therefore maximize to solve:

$$V^{SM} \equiv \underset{\{I_t\,,\ \theta_{t,t},\ \theta_{t-1,t}\}}{\text{Max}} \left[\varepsilon\, U(C_{t,1}) + (1-\varepsilon)U(C_{t,2})\right] \tag{8}$$

subject to the constraints embodied in equations (7.a, f, g, h). Furthermore, prices and quantities satisfy the no-arbitrage (with perfect foresight) and market-clearing conditions of equations (6.d), (7.b, c, d, e). We show the existence of such equilibria *constructively* below.

Lemma 1. *The characteristic dynamic equilibrium equation of SM, relating interim-age (secondary market) real investment prices and levels over time, can be written as:*

$$(1 - I_{t+1}) = p_{t+1}[\varepsilon - (1 - I_t)] \ , \qquad \forall\ t \geq \tau \tag{9}$$

where $(\tau + 1)$ is the time of the first unconstrained transaction on the intergenerational stock-market (SM).

Proof.

To characterize the set of intergenerational stock market equilibria, it is useful to first simplify equation (7.g), by dividing both sides by p_{t+1}. Then using (6.d) and (7.a) successively, obtain:

$$\theta_{t,t+1} I_t = I_t + \theta_{t,t}\, p_t\, I_{t-1} = I_t + (1 - I_t) = 1 \ , \ \forall\ t \geq 1 \tag{9.a}$$

Next, letting $\tau \equiv (t-1)$ in equation (7.d) only, we obtain on multiplying by I_t that:

$$\theta_{\tau+1,\tau+1} I_\tau = (1-\varepsilon)\left[\theta_{\tau,\tau+1} I_\tau\right] = I_\tau \ , \ \forall\ \tau \geq 1$$

which, on using (9.a) implies:

$$1 - I_\tau = \varepsilon - \theta_{\tau+1,\tau+1} I_\tau \ , \ \forall\ \tau \geq 1 \tag{9.b}$$

Finally, using (7.a) again in (9.b) to substitute for $\theta_{\tau+1,\tau+1}$, we get our desired result that:

$$p_{\tau+1}(1-I_\tau) = \varepsilon p_{\tau+1} - \theta_{\tau+1,\tau+1} p_{\tau+1} I_\tau = \varepsilon p_{\tau+1} - (1 - I_{\tau+1})$$

from (7.a), or:

$$(1 - I_{\tau+1}) = p_{\tau+1}[\varepsilon - (1 - I_\tau)] \quad , \quad \forall \tau \geq 1 \tag{9.c}$$

which is equivalent to (9). Notice though that since $I_0 = 1$ and (from 7.a and 7.b) $I_1 = 1 - \varepsilon\, p_1\, I_0$, if the early diers of generation 0 can sell *all* their shares to the next young, then *in that eventuality*, equation (9.c) is also satisfied for $\tau = 0$. **QED.**

As we show in Proposition 3 below, the characteristic equation (9), reflecting perfect foresight, absence of intemporal arbitrage, and market clearing, has only *two* consistent solutions. The first, due to Fulghieri and Rovelli [1993], and Dutta and Kapur [1994], satisfies: $p_{\tau+\kappa} = \sqrt{R}$, $\forall\, \kappa \geq 0$. But, as we shall see in the next section, it also requires *rationing* of the very first generation to sell in such a market at τ. The second solution is a *two-periodic* one, with $p_{\tau+\kappa} = 1$, for $\kappa \geq 1$ and odd, and $p_{\tau+\kappa} = R$, $\forall\, \kappa \geq 2$ and even, where τ is the first generation to trade in such a market. To simplify notation (only) in the following 'steady-state' result, we assume $\tau = 0$, although that is inconsistent with the 0-th generation having no prior investments to buy in the secondary market.

Proposition 3. *The set of stationary or periodic intergenerational steady state stock market equilibria, satisfying equations (7a,b,c,d,e,f,g,h) and (8), are isolated, being one of the two following, assuming the anticipation of stock market trading commencing at some t=0:*

(a) $\{p_t\} = \sqrt{R}\ , \quad \forall\ t = 1, 2, \ldots, \infty,$ *(6.e)*

implying: $\{I_t\} = I^{SM}\ , \quad \forall\ t = 1, 2, \ldots, \infty,$

which, from equation (9) satisfies: $1 - I^{SM} = \sqrt{R}[\varepsilon - (1 - I^{SM})],$

that is: $$I^{SM} = 1 - \frac{\varepsilon\sqrt{R}}{1+\sqrt{R}}, \tag{10}$$

or:

(b) $\{p_t\} = 1$ *for t = 1, 3, 5, ..., odd ; and* *(6.f)*

$\{p_t\} = R$ *for* $t = 2, 4, 6, ...,$ *even* (6.g)

with: $I_t = 1$ *for* $t \geq 0$ *even, and* (11.a)

$I_t = 1 - \varepsilon$ *for* $t \geq 1$ *odd,* (11.b)

which clearly satisfies equation (9).

The consumption allocations at the two alternative stock market allocations are given, using equations (8.a,b), by:

(a) $C_{t,1} = \sqrt{R}$; $C_{t,2} = R$, $\forall\, t \geq 0,$ (12)

and:

(b) $C_{t,1} = 1$; $C_{t,2} = R$, $\forall\, t \geq 0$ *and even ;*

$C_{t,1} = R$; $C_{t,2} = R$, $\forall\, t \geq 0$ *and odd.* (13)

Proof. *For Part (a), see Fulghieri and Rovelli [1993] or Dutta and Kapur [1994]. For Part (b), it suffices to note that eqs. (6.f, g), (11.a, b) and (13) satisfy (9).*

Remarks. (i) If stock market equilibrium commences at time t = 0, with $I_{t-1} = 0$, then given (7.a) the *only* intergenerational stock market equilibrium consistent with the initial condition is the periodic stock market equilibrium (b).

(ii) Also note that, since iterating equation (6.d) implies that $p_t = p_{t+2}$, $\forall\, t \geq 1$, *no other* periodic stock market equilibrium with periodicity k > 2 exists.

We now move on to consider the *dynamics of transition* to the steady-state intergenerational allocation and risk-sharing mechanisms described above.

III. TRANSITIONS AND COMPARATIVE PAYOFFS

III.A The "Slowest Turnpike" Path to Intergenerational Banking

The first question we ask is the following: if the very first generation, born at $t = 0$, wishes to announce a feasible program of transition to the steady-state intergenerational banking (IB) outcome, then what is the maximum expected utility (consumption levels of early and late diers) that can be guaranteed on *a feasible and stationary* transition path to the steady-state level of full investment of all new endowments in real long-lived technologies? We have the following result, which identifies the "Slowest Turnpike" transition path to IB, hence TIB.

Proposition 4. *A stationary (along the path) and finite transition path of consumption and long-lived investment (possibly coupled with partial early liquidation), to the Golden Rule (Phelps, 1961) optimum allocation exists, and it can support any interim consumption pair* $\left\{C_1^{TIB}, C_2^{TIB}\right\}$ *for early diers and late diers satisfying:*

$$(1+R)\,\varepsilon\, C_1^{TIB} + 2\,(1-\varepsilon)\, C_2^{TIB} < 2R \tag{14}$$

Proof (Sketch). The proof is closely related to Qi [1994, Proposition 5]. Consider the transition path described in Table 1, which ensures a constant level of expected utility to all members of all generations alongside the path. Given $I_0 = 1$, it is easily shown that the investment sequence along this path is described by:

$$I_t = 1 + \delta \sum_{k=0}^{\frac{t}{2}-1} R^k = 1 + \delta \frac{R^{t/2} - 1}{R - 1} \tag{15.a}$$

for t even, and:

$$I_{t+1} = 1 + \delta \sum_{k=0}^{\frac{t}{2}-1} R^k - R^{t/2} \varepsilon\, C_1 = 1 + \delta \frac{R^{t/2} - 1}{R - 1} - R^{t/2} \varepsilon\, C_1 \tag{15.b}$$

for $(t+1)$ odd, and where:

$$\delta \equiv R - \varepsilon\, C_1 - (1-\varepsilon)\, C_2 > 0 \tag{15.c}$$

Table 1.

Intergenerational Banking: Transition to the Golden Rule

TIME	RESOURCES*	AGGREGATE CONSUMPTION	INVESTMENT
t = 0	1	0	1
1	1	εC_1	$1-\varepsilon\ C_1$
2	$1+R$	$\varepsilon C_1+(1-\varepsilon)C_2 \equiv R-\delta$	$1+\delta$
3	$1+R-R\varepsilon C_1$	$R-\delta$	$1+\delta-R\varepsilon C_1$
4	$1+R+\delta R$	$R-\delta$	$1+\delta(1+R)$
5	$1+R+\delta R-R^2\varepsilon C_1$	$R-\delta$	$1+\delta(1+R)-R^2\varepsilon C_1$
6	$1+R+\delta R+\delta R^2$	$R-\delta$	$1+\delta(1+R+R^2)$
7	$1+R+\delta(R+R^2)-R^3\varepsilon C_1$	$R-\delta$	$1+\delta(1+R+R^2)-R^3\varepsilon C_1$

* Resources = endowment + maturing investment

For this transition path to reach the steady state optimal allocation from some finite time point on, it is necessary that for some τ sufficiently large

$$I_\tau + I_{\tau+1} \geq 2 \tag{16}$$

for τ even, so that at $(\tau+1)$ a level of investment $(I_\tau - 1)$ can be liquidated and $(1 - I_{\tau+1})$ can be reinvested in the long term technology, thus having $I_t = 1$ for all $t \geq \tau + 1$. Using equations (15.a, b, c), condition (16) is equivalent, for τ sufficiently large, to:

$$R^{\tau/2}\left[2\ \delta - (R-1)\ \varepsilon\ C_1\right] \geq 2\ \delta \tag{17.a}$$

For $0 < \delta \leq R$, a τ sufficiently large satisfying (17.a) exists if

$$2\delta - (R\text{-}1)\ \varepsilon\ C_1 > 0\ , \tag{17.b}$$

which, using the definition (15.c) of δ, yields inequality (14).

Note that a feasible transition path to the steady state (Golden Rule) allocation must also ensure that all interim investment levels along the path are non-negative. To satisfy this non-negativity constraint it may become necessary to liquidate in odd periods some of the investment made during the previous even period. In Qi [1994] it is shown that, if condition (17.b) is satisfied, than these additional non-negativity constraints are also satisfied. **QED.**

Remarks. (i) Consider then the following program $\{C_1^{TIB}, C_2^{TIB}\}$, which a transitional generation may advance as a proposal:

$$V^{TIB} \equiv \underset{\{C_1,C_2\}}{\text{Max}} [\varepsilon\, U(C_1) + (1-\varepsilon)\, U(C_2)] \tag{18}$$

subject to a *weak* inequality in eq. (14), leading to $[I_t + I_{t+1} = 2 - \varepsilon C_1]$, for all t even. This is the maximum stationary sustainable utility profile of Qi [1994, Proposition 5]. With the slightest altruism towards future generations, or discreteness in consumption and/or investment choices, eq. (14) will hold *strictly* and hence the Golden Rule steady state of Proposition 1 above will be attained by generations born at time τ and afterwards.

(ii) If the transition path of Table 1 is continued past period τ (defined in equation 16) then, instead of moving on to the Golden Rule steady state, the accumulation of inherited capital could proceed *beyond* the steady state level of unity, eventually allowing even higher levels of consumption for some future generations. We do not consider such schemes, as the $\{C_1^{TIB}, C_2^{TIB}\}$ proposal is motivated by the concerns of <u>not</u> making future generations worse off than earlier ones, as in Esteban and Sakovic [1993], with generational self-concern applying subject to this constraint.

We can now prove the following:

Corollary 1. *The optimal consumption plan* $\{C_1^{TIB}, C_2^{TIB}\}$ *available on the slowest transition path to the Golden Rule steady state strictly dominates the consumption plan* $\{C_1^B, C_2^B\}$ *available with intra-generational banking, that is:*

$$V^{TIB} > V^B \tag{19}$$

Proof. From Propositions 2 and 4, the consumption plan offered by an *intragenerational* bank must satisfy the constraint, obtained by eliminating L between equations (3.a) and (3.b) above, that:

$$\{C_1^B, C_2^B\} \in S^B \equiv [\{C_1, C_2\} \mid R\,\varepsilon\, C_1 + (1-\varepsilon)\, C_2 \le R] \tag{3.a, b}$$

Conversely, consumption plans $\{C_1^{TIB}, C_2^{TIB}\}$ must satisfy:

$$\{C_1^{TIB}, C_2^{TIB}\} \in S^{TIB} \equiv \left[\{C_1, C_2\} \mid \frac{(1+R)}{2} \varepsilon\, C_1 \; + \; (1-\varepsilon)\, C_2 \; < \; R \right] \quad (14)$$

The desired result now follows from the fact that $S^B \subset S^{TIB}$, for $R > 1$. **QED.**

III.B Transitions to and Payoffs from Stock Markets

The next question we pose relates to the concern that a transition path to the intergenerational banking (TIB) steady state might be "upset" (see below for details) by an alternative proposal to establish an intergenerational stock market. We thus prove the following extension to the results in Proposition 3 above, delineating feasible transitions to the stationary intergenerational stock market equilibria (SM).

Proposition 5. *A proposed transition to an intergenerational stock market equilibrium at time t by the young must result in either:*

(a) if the generation prior to t is not allowed to trade in stock market at t:

$p_{t+i} = 1$, *for* $i \geq 1$ *and odd;* $\quad$ $p_{t+i} = R$, *for* $i \geq 2$ *and even;*

$I_{t+k} = 1$, *for* $k \geq 0$ *and even;* $\quad$ $I_{t+k} = 1 - \varepsilon$, *for* $k \geq 1$ *and odd; or:*

(b) if early diers of generation (t-1) are allowed to do a quantity-constrained trading with the newborn generation t of $1/(1+\sqrt{R})$ *of their one-period old investment per capita at time t:*

$p_{t+i} = \sqrt{R}$, *for all* $i \geq 0$, *and*

$I_{t+k} = 1 - \varepsilon\sqrt{R}/(1+\sqrt{R})$ *for all* $k \geq 0$,

with no rationing for subsequent generations.

No other transition path of (interim) investment prices and levels exists, that is consistent with perfect foresight, no arbitrage and stock market clearing at all times $\tau \geq t + 1$.

Proof. *See Appendix.*

Remarks. Since the transition to the periodic equilibrium in part (a) above leads to the consumption profile: $C_{t,1} = 1$; $C_{t,2} = R$ (as in Proposition 3, Part (b)) for the proposing generation t, it follows from Corollary 1 above that *no generation* when young can gain relatively to TIB (nor, for that matters, to B) from such a transition. Hence it will never be proposed. Thus, in what follows, we restrict attention to the immediate transition to the *non-periodic* SM equilibrium. In doing so, we assume that the strictly positive but quantity-constrained participation in this market by the middle-aged generation at time t is feasible to implement. In particular, only sell orders by the currently middle-aged generation will be acceptable, and rationed *pro rata.*

At this point it is useful to ask how the proposed quantity-constrained (for the middle-aged generation) transition to the stock market compares in expected utility, for the two transitional generations, with that resulting from the TIB transition to the steady-state Golden Rule Optimal allocation. We have the following result, applicable to the initial generation born at time t = 0.

Proposition 6. *If the currently middle-aged generation at time t have inherited one-period old real investments of unity, and no other (financial) maturing investments (as is the case with generation 0, for example) then the constrained transition to the non-periodic and stationary intergenerational stock market equilibrium path results in expected utilities of* $V^{SM}(MA)$ *for the currently middle-aged, and* $V^{SM}(NB)$ *for the new born starting the Stock Market, such that:*

(a) $$V^{SM}(MA) < V^{TIB};$$

and

(b) $$V^{SM}(NB) \overset{>}{\underset{<}{=}} V^{TIB};$$

with $V^{SM}(NB) > V^{TIB}$ *if the agents' (ex post corner) utility functions for consumptions* $U_i(C_i)$, *i = 1, 2, has globally a relative risk aversion coefficient* ρ *in some open neighborhood of the closed interval:* $1 \le \rho \le \dfrac{2\log((1+R)/2)}{\log(R)} \le 2$.

Proof. *(a)* From the characterization in Part (b) of Proposition 5 above, we know that the early diers of generation t-1 are able to sell at time t, per capita, $\dfrac{1}{1+\sqrt{R}}$ units of real investments made at (t-1) at price $\sqrt{R}$ to the new born of generation t. Hence, they must physically *liquidate* their remaining real investments, since late diers of their own generation have no dividends (from maturing investments) to buy shares with. Thus, the expected consumption level of the early diers of generation (t -1) is:

$$C_{t-1,t}^{SM(MA)} = \frac{\sqrt{R}}{1+\sqrt{R}} + \left[1 - \frac{1}{1+\sqrt{R}} \right] = \frac{2\sqrt{R}}{1+\sqrt{R}} \tag{20.a}$$

whereas the consumption level of its late diers is:

$$C_{t-1,2}^{SM(MA)} = R \tag{20.b}$$

Comparing the allocation-tuple in (20.a, b) with that resulting from the "slowest turnpike" transition path, TIB, we see from Proposition 4 and inequality (14) that, if C_2^{TIB} is set at R, then

$$C_1^{TIB} < \frac{2R}{1+R} \tag{21}$$

with the inequality holding with arbitrarily small slack (see *Remark (i)* to Proposition 4). The result then follows from noting that $\frac{2R}{1+R} > \frac{2\sqrt{R}}{1+\sqrt{R}}$ for $R > 1$, and that $U(C_i)$, $i = 1, 2$, is assumed to be strictly increasing.

(b) See Appendix.

Remarks. In ***Figure 1***, we have drawn a graph of numerical simulations of the inequality $V^{TIB} \gtreqless V^{SM}(NB)$, for ε in discrete grids of 0.1 over {0.1 to 0.9}, constant ρ in discrete grids of 0.5 over {0.6, 5.1} and R in discrete grids of 0.5 over {1, 5}, where R=5 corresponds to a real rate of return of 4.1% over 40 years, the half-life of a generation[9]. *On or above* the drawn surface, $V^{SM}(NB) > V^{TIB}$. The graph shows that $V^{SM}(NB) \le V^{TIB}$ is indeed possible, but only for relatively *high* levels of risk aversion. For instance, with $\varepsilon = 0.3$ and $R \ge 1.5$, $\rho > 3.6$ is necessary for $V^{SM}(NB) \le V^{TIB}$, so that the newborn are <u>not</u> tempted to propose a transition to SM away from TIB.

[INSERT FIGURE 1 ABOUT HERE]

III.C The Stand-Alone Option And Its Payoff

In this section we compare $V^{SM}(MA)$ with V^B, the expected utility arising from *intragenerational* banking (or stock market), the latter being also the *stand-alone option* of the currently middle-aged when faced with any proposal by the younger generation[10]. In the next section, we shall in fact analyze a class of *intergenerational proposal games* over liquidity-sharing mechanisms, across the infinite sequence of overlapping generations at times t = 1, 2, 3, ..., ∞. As a preliminary to that discussion, we have here the following result, which relies on comparisons analogous to those made in Proposition 6 above, and is applicable also to the initial generation born at t = 0.

***Proposition** 7. For a middle-aged generation with one unit of inherited real investments, their expected utility, $V^{SM}(MA)$, from a (quantity-constrained) transition to the intergenerational stock market proposed by the next generation, compares with their intragenerational optimum (Diamond and Dybvig, 1983) expected utility, V^B, as follows.*

$V^{SM}(MA) > V^B$ *if any of the following holds:*

(a) $$1 \le \rho \le \left[\frac{\log(R)}{\left\{ \log\left(1+\sqrt{R}\right) - \log(2) + \frac{\log(R)}{2} \right\}} \right] \le 2, \quad \textit{for all } R \ge 1, \ \varepsilon \in [0,1];$$

or: *(b)* $\varepsilon \to 1^-$, *for all* $\rho \ge 1$, $R \ge 1$;

If none of the conditions above are met, then $V^{SM}(MA) \le V^B$ *for* ε *sufficiently small.*

***Proof.** See Appendix.*

Remarks**.* In ***Figure *2*, we present the graph of simulations of the $\left\{V^{SM}(MA), V^B\right\}$ comparisons, in the space of the $\{\varepsilon, \rho, R\}$ parameters, using the same grid as that underlying Figure 1. On or above the drawn surface, the middle-aged generation is found to prefer a (constrained) transition to the stock market, relative to their stand-alone option of B, the *intra*generational bank. This is generally the case if their risk aversion is low, or if the proportion of early diers is high. However, for say $\varepsilon = 0.3$, $V^B \ge V^{SM}(MA)$ for all $\rho \ge 2.1$ for $R \ge 3.5$, which corresponds to an annual real rate of return of 3.18% over 40 years. The reason is that, for sufficient curvature in U(C), the "intertemporally inflexible" allocation of SM *plus* the selling constraint that the early diers among the initial (transitional) middle-aged are subjected to, lowers $V^{SM}(MA)$ sufficiently relative to V^{B}, if ε is low or moderate. For sufficiently high ε however ($\varepsilon \geq 0.6$ in our simulations), the effect of the tighter *intragenerational* resource constraint of the B allocation dominates, and we get $V^{SM}(MA) > V^{B}$ for all $\{\rho, R\}$, in the range simulated.

[INSERT FIGURE 2 ABOUT HERE]

Note that, in these simulations, for $\varepsilon = 0.3$, $R \geq 3.5$ and $\rho \in [\, 2.1 \, , \, 3.6 \,]$, $V^{SM}(NB) > V^{TIB}$ but at the same time $V^{SM}(MA) < V^{B}$. As we shall see below, this implies that the middle-aged would be credibly able to resist a self-interested proposal by the young to switch to SM from TIB[11]. These considerations suggest the theme and anticipate the results of the following section: given an intergenerational proposal game over choices among $\{B, TIB, SM\}$, agents could be *so* risk averse that *all* (middle aged and young) prefer TIB (which dominates B) to SM, or sufficiently risk-averse that $V^{SM}(MA) < V^{B}$ so SM is *blocked* by the middle-aged generation in favour of TIB. If however *neither* of these conditions holds, then an SM-proposal by the young will *prevail* and move the economy to what is, in the steady-state perspective, a worse equilibrium.

IV. INTERGENERATIONAL PROPOSAL GAMES AND EQUILIBRIA

IV.A Self-Sustaining Mechanisms

We now move on to define and analyze a class of non cooperative intergenerational proposal games, across the sequence of overlapping generations in our model. Each *stage game* of the proposal game, at times t = 1, 2, ..., ∞ , is modeled as a proposal *cum* counter-proposal game between the two generations, born at (t-1), the middle-aged, and at t, the new born, which at time t are in a position to make (real or financial) investment decisions. This represents an extension of the "transfer game" across generations analyzed by Esteban and Sakovics (1993), in which the authors allow only the young at any time t to make proposals for changing the previously agreed upon transfers to the old of the immediately prior generation. Our *two-sided* proposal / counter-proposal game is necessary, because two of the allowed proposals involve either the access to the initial endowment of the new born by the currently middle aged (TIB), or the utilization (acquisition at some price) of the inherited intermediate-maturity capital stock of the middle-aged by the young (the non-periodic, quantity constrained SM transition of part (b) of Proposition 5). Hence, because of this change relative to Esteban and Sakovics, we must specify not only the payoffs to agents taking into account *subsequent* stage games at times t+1, t+2, ..., but also their payoffs in the events of both *agreement* (regarding the allocational mechanism to be continued or switched to) as well as *disagreement* among the overlapping generations at any time t. We do so below assuming that:

(i) in the event of an *agreement* at time t, this is *binding* until time (t+1) at least, and

(ii) in the event of *disagreement* on proposals at time t:

- the currently middle-aged obtain their autarkic payoff given their endowment, and
- the new born at time t can always *revise* both their proposal/program at time t and their proposal (planned) at time (t+1).

We restrict the pure strategy set of each overlapping generation in the stage games at times t= 1, 2, ..., ∞ , to be the 3-tuple of allocation mechanisms (or Institutions, in the terminology of Esteban and Sakovics) {B, TIB, SM }, that were characterized in Sections II and III above. We do so for the following reasons. First, as we have suggested earlier, the TIB and SM intergenerational allocation mechanisms have the feature that, if at time t a current (agreed upon) mechanism is started and is continued through times t, t+1, t+2, ..., etc., then generations *subsequent* to the new born at time t are never worse off than the t-th generation. We postulate that reasonable proposal processes (e.g., in a legislature) satisfy this "minimal degree of fairness" property, even though the resulting mechanism need not guarantee, say, ultimate attainment of the Golden Rule optimum in the long run. The spirit of this restriction is identical to that underlying the transfer institutions or proposals described in Esteban and Sakovics [1993] or Boldrin and Rustichini [1995]. Second, while we have *not* proved that the three institutions / mechanisms which each generation is allowed to propose are the *only* ones that are (i) intergenerationally *stationary* for *future* generations on their continuation paths, and (ii) viable, they do represent the set of major significant alternatives in many aggregate (macroeconomic) settings, such as: generationally autarkic, pay as you go, and market-based *pension plan systems*. Once we step outside the class of these three mechanisms, we essentially open a Pandora's Box of proposals that satisfy the second criterion (ii) but not the first (i), that we deem desirable to impose[12].

The stage games, at each time-point of the infinite horizon intergenerational proposal game, could themselves involve (at least) three alternative extensive forms:

A) The middle-aged moving first and proposing, and the young counter-proposing; or

B) The new born moving first and proposing, and the middle-aged counter-proposing; or

C) Simultaneous proposals by the two generations overlapping at any point in time.

Of these alternatives, cases (B) and (C) are relatively more in accordance with the spirit of the transfer-proposal game of Esteban and Sakovics (1993). Furthermore, the equilibrium outcome in case (A) is also somewhat obvious, given that $V^{TIB} > V^{B}$ (Proposition 4) and $V^{TIB} > V^{SM}(MA)$ (Proposition 6) *always*. It turns out, on the other hand, that (i) the equilibrium outcomes in cases (B) and (C), suitably defined, generically *coincide* with each other, and (ii) they depend on the *relative* magnitude of V^{TIB} versus $V^{SM}(NB)$, as well as on $V^{SM}(MA)$ versus V^{B}. We now proceed to describe the agents' strategies and payoffs more precisely, and also to define the notion of *Self-Sustaining Mechanisms* arising from these proposal games across current and future generations.

TABLE 2.

Payoffs in (First Stage) Proposal Game

MIDDLE-AGED / NEW BORN	B	TIB	SM
B	$\{V^B, V^B\}$	$\{V(c), V^B\}$	$\{V(c), V^B\}$
TIB	$\{V(c), V^B\}$	$\{V(c'), V^{TIB}\}$	$\{V(c), V^B\}$
SM	$\{V(c), V^B\}$	$\{V(c), V^B\}$	$\{V^{SM}(NB), V^{SM}(MA)\}$

In Table 2, we describe the pure strategy payoffs in the simultaneous moves case (C) of the proposal stage scheme, across generations 0 and 1, postponing the discussion of subsequent stage games until the subsequent result (Proposition 8). Notice that the *continuation payoff*, V(c) or V(c′), of the new born in the event of disagreement takes into account the optimal reswitching strategy of these agents. It is easy, using Table 2, to write the payoffs in the extensive form game trees of the stage games in cases (A) and (B) above. Mixtures of stage game pure strategies (uncorrelated) by players are allowed in all the cases.

We define the continuation payoff for the new born, at any stage game at time t, as:

$$V(c) = [V^{TIB}] \ , \ \text{if } \{TIB, TIB\} \text{ is in } [\, SSM\,(t+1) \mid H_{t+1} = (1, 0)\,] \qquad (22.a)$$

$$\text{and}: \ V(c) = [\, V^{SM}(MA) \mid H_{t+1} = (1, 0)\,], \ \text{otherwise}, \qquad (22.b)$$

where H_{t+1} is the vector of real capital stock (one-period old and maturing) which the current (as of time t) new born generation will be holding at time (t+1), and SSM (.) represents the notion of Self-Sustaining Mechanisms, to be defined below. The payoff $V(c')$ is analogously defined, with H_{t+1} now equaling the *inherited* capital stock vector along the TIB path, starting at $t = 0$.

We conclude this subsection with the following definition of equilibrium outcomes in the proposal game:

Definition. *A Self-Sustaining Mechanism (SSM) is a sequence of outcomes of the intergenerational stage games at times* $t = 1, 2, \ldots, \infty$, *such that the outcomes and equilibrium strategies (given* H_t*) at each time t,* $\{SSM(t)\}$, *satisfy the criteria that,* $\forall\ t \geq 1$:

(1) SSM(t) $\in NE(t)$, *Nash Equilibrium in stage t ;*

(2) $\{SSM(\tau)\}_{\tau=t}^{\infty} \in NE(\tau)$, *Subgame Perfect Equilibrium across stages (Selten, 1975);*

(3) $\{SSM(t)\} \in SPE(t)$ *in the stage game at time t ,in cases (A) and (B), or:*

$\{SSM(t)\} \in TPE(t)$, *the Trembling Hand Perfect Equilibrium (Selten, 1975) set at t , in case (C) (simultaneous-move) of the stage game.*

IV.B Equilibrium outcomes: Only the meek shall inherit the earth

We can now state and prove the result suggested in the previous section (Propositions 6 and 7). We do so assuming that V(c) = V(c′). While this assumption on V(c) is critical for an unique outcome in a subset of case (C), and it may appear to be unduly restrictive, in Proposition 8 below we shall prove that, under reasonable conditions, it is naturally satisfied[13].

Theorem. *Self-Sustaining Mechanisms in the <u>first stage</u> Intergenerational Proposal Games are characterized as follows:*

a. *In case (A), middle-aged move first,* $SSM(t) = \{\text{TIB}, \text{TIB}\}$.

b. *In case (B), new born move first, then either of the following equilibria may result:*

b.1. $SSM(t) = \{\text{TIB}, \text{TIB}\}$

if either: *(b.1.1)* $V^{\text{SM}}(NB) < V^{\text{TIB}}$;

or: *(b.1.2)* $V^{\text{SM}}(NB) \geq V^{\text{TIB}}$ *and* $V^{\text{SM}}(MA) < V^{\text{B}}$;

b.2. $SSM(t) = \{\text{SM}, \text{SM}\}$

if: $V^{\text{SM}}(NB) > V^{\text{TIB}}$ *and* $V^{\text{SM}}(MA) \geq V^{\text{B}}$;

b.3. *Both* $\{\text{SM}, \text{SM}\}$ *and* $\{\text{TIB}, \text{TIB}\} \in \text{SSM(t)}$

if: $V^{\text{SM}}(NB) = V^{\text{TIB}}$ *and* $V^{\text{SM}}(MA) \geq V^{\text{B}}$.

c. *In case (C), the following equilibria may result:*

c.1. $SSM(t) = \{\text{TIB}, \text{TIB}\}$

if either: (c.1.1) $V^{\text{SM}}(NB) < V^{\text{TIB}}$;

or: (c.1.2) $V^{\text{SM}}(MA) < V^{\text{B}}$.

c.2. $SSM(t) = \{\text{SM}, \text{SM}\}$

if: $V^{\text{SM}}(NB) > V^{\text{TIB}}$ *and* $V^{\text{SM}}(MA) \geq V^{\text{B}}$.

c.3. *Both* $\{\text{SM}, \text{SM}\}$ *and* $\{\text{TIB}, \text{TIB}\} \in \text{SSM}(t)$

if: $V^{\text{SM}}(NB) = V^{\text{TIB}}$ *and* $V^{\text{SM}}(MA) \geq V^{\text{B}}$.

Proof. Part (a) follows from Propositions (4) and (6).

In part (c.2), the fact that $\{\text{SM}, \text{SM}\}$ is uniquely in SSM(t) follows from using the Trembling Hand Perfect equilibrium concept and from the maintained hypothesis that $V(c) = V(c')$; see below the discussion of Proposition 8.

The remainder of parts (b) and (c) are obvious, on examination of Table 2 and equations (22).

Remarks. In *economic terms*, the above Theorem, together with Propositions 4 , 5 and 6 above, implies the following:

- The very first generation will never start a stock market in long lived capital (SM), nor an intragenerationally autarkic bank (B).
- The slowest turnpike transition path (TIB) to steady state (Golden Rule) intergenerational banking would survive (continue) at time t if *either* it is a dominant strategy for both new born and middle aged at time t *or*, at least, the middle aged at time t can *credibly resist* a switch to the (steady state, non-periodic) stock market (SM) at time t, by *threatening* to revert to their intragenerationally autarkic banking allocation (B).

In particular, the most desirable steady state outcome (the Golden Rule supported by intergenerational banking, IB) will be reached when people are *sufficiently risk averse* and are *not too likely to require early utilization* of their savings (or liquidity), so that Only the Meek Shall Inherit the Golden Rule steady state!

For the TIB path to be maintained, we must have that it is *also* an equilibrium outcome of the intergenerational proposal games at all dates along the accumulation path, until the switch to the Golden Rule steady state can actually take place. We conclude this section by showing that if $\{SM, SM\}$ is not a Nash equilibrium outcome in the initial stage game, it will continue not to be so for all intergenerational stage games along the accumulation path in a wide class of cases. We do this in the following Proposition 8, which serves to justify the assumption that V(c) = V(c′), which was used in the proof of the Theorem above. Then, given V(c) = V(c′), the proof of the Theorem also applies in *all* stage games). Furthermore, the *same* equilibrium outcome would arise in all current and subsequent stages.

The thrust of our argument is the following. Assume TIB to be the unique Nash equilibrium outcome in the first stage intergenerational proposal game. Then in cases (b.1.1) and (c.1.1) of the preceding Theorem it will clearly continue to be the chosen equilibrium over time, since the terms of choice between TIB and competing mechanisms will be invariant across stage games, even as capital accumulation proceeds along the transition path. In the other cases, where the new born might prefer a move to SM, but the middle-aged can credibly resist such a proposal *in the first stage* of the game, by credibly threatening to revert to the intragenerationally autarkic allocation B, then the terms of this choice will change as capital accumulation proceeds along TIB. We can still prove, for the class of *homothetic* (instantaneous) *utility functions*, that this credible threat to revert to B will persist along the accumulation path. We can thus state that:

Proposition 8. *If TIB is a (Nash, and SPE or TPE) equilibrium allocation in the first stage of the intergenerational proposal game, it will continue to be an equilibrium allocation at all subsequent stage games, for $t \geq 2$, for all homothetic* $U(C) = \dfrac{C^{1-\rho}}{1-\rho}$, $\rho > 0$.

Proof. *See Appendix.*

Remark. Given Proposition 8 and our main Theorem, and definitions (22.a, b), V(c′) = V(c) holds true for continuation payoffs if: $\left[V^B \mid H_1 = (1,0)\right] > \left[V^{SM}(MA) \mid H_1 = (1,0)\right]$. Alternatively, suppose $\left[V^B \mid H_1 = (1,0)\right] < \left[V^{SM}(MA) \mid H_1 = (1,0)\right]$, but still {SM, SM } is not the unique Trembling-Hand Perfect SSM at time t=1 in case C (simultaneous proposals) because, by hypothesis, V(c′) > V(c). But then {TIB, TIB} must also be in SSM (t+1) because V(c′) > V(c) will continue to hold also at time t = 2, in light of the proof of Proposition 8. But then it contradicts the definition of V(c) in (22.a, b) to assume that V(c′) ≠ V(c) at t=1.

V. CONCLUDING REMARKS

In this paper we have attempted to extend and synthesize the extant and emerging theoretical frameworks pertaining to (a) static liquidity sharing (consumption smoothing) mechanisms, (b) intergenerational (OLG) tradeoffs and modifications to these, in and out of steady states, and (c) non-cooperative intergenerational proposal games related to Sustainable Mechanisms or Institutions in these settings. Our results show that, if currently young generations are constrained to be "minimally altruistic" towards their descendants (i.e., not to do relative harm to them) in their "feasible" (allowed) proposals, then transition to the long-run optimal Golden Rule of Phelps [1961] outcome may be obtained as the unique non-cooperative equilibrium outcome among generations, but only if the agents involved have preferences embodying a significant degree of relative risk aversion, or curvature in the (additive) utility functions for consumption at different points of their stochastic lifespans, and also relatively low likelihood of liquidity or early withdrawal needs. By embodying such a reasonably restricted notion of intergenerational "autonomy", we have improved on the essentially planning-theoretic modeling of, for example, Allen and Gale [1995], and also enriched (in context) the methodology of Esteban and Sakovics [1993], obtaining quite different results vis-à-vis the universality of long-run efficiency attainment.

Our model may also be usefully compared to that of Boldrin and Rustichini [1995]: with no long-lived capital and, hence, no transfer of property rights to capital *across* generations taking place, they find that whether or not PAYG Social Security (intergenerational transfer) systems are voted into existence (and maintained) depends on the return to capital and the dynamics of stochastic population growth (a high rate of growth will favor the adoption of a PAYG system)[14]. Keeping in mind that both our transition path to intergenerational banking (TIB) and also the intergenerational banking steady state (IB) share an essential similarity with PAYG systems, our results point to the fact that whether or not a PAYG outcome will be chosen, in an environment with *long lived capital stock and stochastic lifetimes*, may also depend on the extent of agents' aversion to risk and random needs for early withdrawal or liquidity. A synthesis of these two types of modeling, with roles for both endogenous trade or exchange of property rights to capital and for aggregate shocks to state variables pertaining to (i) agents' (preference) characteristics, (ii) demographic changes, and (iii) return to capital, should help provide important and policy-relevant answers to intriguing economic questions regarding the nature and efficiency of equilibrium financial intermediation *and* social security systems.

ENDNOTES

Section I

1 The background assumption is that there are many small uninsurable risks, to endowments (such as accidents), health, or family size, for which it is extremely costly to develop separate insurance markets, in part due to private observability of outcomes, which could be monitored by insurers only at a prohibitive cost. The lack of such markets gives rise to uncertain indirect utility functions for agents, over their intertemporal withdrawal patterns from their invested savings.

2 In Bhattacharya and Gale [1987] it is shown that the opening of an *interim* stock market in the Diamond - Dybvig [1983] model leads to *over* investment, relative to the ex ante optimal level, in the long-term technology, when agents have relative risk aversion greater than unity and they make investment choices individually. See Bhattacharya and Padilla [1996] for further discussion of these contrasting results, which arise from the dominance of different incomplete markets effects, within and across generations.

3 The obvious reason is that each subcoalition of current and future generations has an incentive to deviate from the Golden Rule allocation, by denying the anticipated positive old-age transfer to the immediately prior generation, when each generation's endowment pattern is tilted towards its youth.

4 Hence, the reliance on *intra*-generational mechanisms in an OLG context, as in Bencivenga and Smith [1991] and others, appears to be not very well justified.

5 In addition, we also show the existence of a steady-state periodic stock market equilibrium.

6 Hence, our results differ from those of Esteban and Sakovics [1993] in their simpler "transfer games" context (with no endogenous real investments), in which - as the cost of making new proposals goes to zero - they obtain nearly efficient steady-state outcomes as unique equilibria.

Section II

7 The assumption that $Q = 1$ is used by Diamond and Dybvig [1983] to prove the existence of Panic Bank Runs, of early withdrawals for storage at $(t+1)$ by agents who do not wish to consume from savings until time $(t+2)$. However, simple measures, such as suspension of convertibility, can eliminate such Runs, so we ignore this problem. This assumption considerably simplifies the analysis of *transitions* to optimal Steady-State allocations.

8 This intragenerational optimal allocation (B) is the unique equilibrium outcome of such a contract if banks can suspend early withdrawal rights once a proportion ε of their depositors has been withdrawn (that is, a fraction   of their investments has been liquidated). All agents would then deposit all their initial endowments in such banks, and not invest in real investments on their own. They would also withdraw early if and only if they are early diers.

Section III

9 One may think of a 40 year period as made up of ages 40 to 80, so that a (disabled) individual may take early retirement at the age of 60, or continue working until 80 and then enjoy an increased level of consumption, with both dying soon after retirement.

10 In thinking of V^B as the outside option of the currently middle-aged, when the next generation proposes a transition to SM, we are implicitly assuming that *intragenerational solidarity* among the middle-aged is present (or, more plausibly, that times of death are unresolved for them) when the next generation proposes.

11 In contemplating these transitions, we are not allowing the middle-aged generation to do *both* the constrained trading with the young, and also further within-generation risk-sharing as in (B); otherwise, $V^{SM}(MA)$ would always exceed V^B. Our rationale is that we think of the SM-mechanism as being more individually decentralised than either B or TIB, which require intragenerational coordination at least. If, on the other hand, the middle-aged could renegotiate (B) among themselves, the economy would shift to SM (following a proposal from the new born) whenever .

Section IV.

12 For example, generation 1 could offer to buy ε units of real, one-period old investments from generation 0 at time t = 1, *at unit price*, and invest the remainder of its endowment (1-ε) in real investments, thus guaranteeing *its* early and late diers consumption levels of $\{C_{1,1} = R$, $C_{1,2} = R\}$. Generation 0 would not have a better outside option (as this would involve physical liquidation) and generation 2 would be strictly worse off than generation 1.

13 In other words, we are analyzing cases in which the new born at t do not "go along with" the middle aged, on {TIB, TIB}, *just* to acquire $H_{t+1} > 1$ to do better in a move to SM, i.e. for $[V^{SM}(MA) \mid H_{t+1}]$, at the *next* stage (t+1) game.

Section V

14 In common with our model and Esteban and Sacovics [1993], Boldrin and Rustichini [1995] also assume that a proposed social security transfer to the old embodies in it the same proportional transfers to the current young when they would be old (unless this is then modified by a new proposal by the then young agents).

APPENDIX

Proof of Proposition 5

Parts (a) and (b) are self-evident from Proposition 3 and the Remarks following. To show the nonexistence of other transition paths, notice first that the no arbitrage condition, equation (6.d), implies that $p_{t+i} = p_{t+i+2}$ for all $i \geq 0$ (or, $i \geq 1$ without prior generations participating). Thus, iterating in equation (9) implies that:

$$1 - I_{t+k+2} = p_{t+k+2}\varepsilon - R\left[\varepsilon - \left(1 - I_{t+k}\right)\right]$$

or $$R\left(1 - I_{t+k}\right) - \left(1 - I_{t+k+2}\right) = \varepsilon\left(R - p_{t+2}\right) \tag{A.1}$$

This implies, for example, that if $I_{t+k+2} \gtreqless I_{t+k}$, then $I_{t+k+4} \gtreqless I_{t+k+2}$, etc. Equation (A.1) has the solutions of parts (a) and (b) above of the proposition, but any other I_{t+k} leads to I_{t+k+2n}, $n \geq 1$, to ultimately explode to the boundaries of *either* $I = 0$ *or* $I = 1$.

However, from the lower boundary $I_t = 0$ the forward-looking equilibrium equation (9) cannot be satisfied, with $\{I_\tau\} \in [0,1]$ and $\{p_\tau \geq 0\}$ for all τ thereafter, since $0 < \varepsilon < 1$. From the upper boundary $I_t = 1$ the only consistent continuation is the periodic solution of Proposition 3(b), which is consistent with perfect foresight only if it held for $\tau < t$. **QED.**

Proof of Proposition 6, Part (b)

The constrained (for the initial (t-1)-th generation) transition to the intergenerational stock market steady-state (Part (b) of Proposition 5), characterized by the stock market price $p_\tau = \sqrt{R}$, $\forall \tau \geq t$, gives the generations from the t-th onwards the same consumption profiles as defined in eq.(12), that is:

$$C_{\tau,1}^{SM(NB)} \equiv C_{\tau,1}^{SM} = \sqrt{R}; \quad C_{\tau,2}^{SM(NB)} \equiv C_{\tau,2}^{SM} = R; \quad \forall \tau \geq t \tag{A.2.a, b}$$

for early and late diers respectively. Comparing (A.2.a) to condition (21), and noting that:

$$\frac{2R}{(1+R)} < \sqrt{R}, \qquad \forall R \geq 0$$

by completion of squares, we reach the conclusion that *if:*

$$C_2^{TIB} \equiv \sqrt{R} C_1^{TIB} \tag{A.3}$$

then $V^{TIB} < V^{SM}(NB)$. Now, solving the problem of maximizing eq. (1) subject to (14) (as defined in Proposition 6), and for the class of utility functions:

$$U_i(C_i) = \frac{(C_i)^\gamma}{\gamma} \tag{A.4}$$

with relative risk aversion coefficient $(1-\gamma) \equiv \rho$, we see that the first-order conditions imply that:

$$C_2^{TIB} = \left[\frac{(1+R)}{2}\right]^{1/\rho} C_1^{TIB} \tag{A.5}$$

Hence, condition (A.3) is satisfied for all ρ in some open neighborhood of:

$$\rho = 2\frac{\log((1+R)/2)}{\log(R)} \tag{A.6}$$

To see that $V^{TIB} < V^{SM}(NB)$ is also true for ρ less than the right-hand side of (A.6), we note that, rearranging (14):

$$\varepsilon C_1^{TIB} + (1-\varepsilon)C_2^{TIB} < R - \frac{(R-1)\varepsilon C_1^{TIB}}{2} \tag{A.7}$$

whereas, using using equations (A.2.a, b) above, the steady-state stock market allocation satisfies:

$$\varepsilon C_1^{SM} + (1-\varepsilon)C_2^{SM} = \varepsilon\sqrt{R} + (1-\varepsilon)R = R - \varepsilon\left(R - \sqrt{R}\right) \tag{A.8}$$

Using equations (A.7), (A.8), and (A.5), we see that the $\{C_1^{SM}, C_2^{SM}\}$ "lottery" (across early and late diers) stochastically dominates the $\{C_1^{TIB}, C_2^{TIB}\}$ lottery, if ρ is less than the right hand side of (A.6) *and* it is the case that:

$$\left(R - \sqrt{R}\right) \le \frac{(R-1)}{2} C_1^{TIB} \tag{A.9}$$

Next, we know that at ρ *equalling* the RHS of (A.6), $C_2^{TIB} = \sqrt{R} C_1^{TIB}$. Hence, using (A.7) we have that:

$$C_1^{TIB} = \frac{2R}{2\sqrt{R} + \varepsilon\left(1 + R - 2\sqrt{R}\right)} \ge \frac{2R}{1+R}, \qquad \forall\, \varepsilon \in [\,0\,,1\,) \tag{A.10}$$

The last inequality in (A.10) holds for all $\rho \ge 1$ also, since for ρ = 1 (log utility) equation (A.5) implies that $C_2^{TIB} = \frac{C_1^{TIB}(1+R)}{2}$, which using (14) implies (21). Hence, using (21) in (A.9), we obtain: $\left(R - \sqrt{R}\right) \le \frac{(R-1)}{2}\frac{2R}{(1+R)}$

or: $R^2 - R^{3/2} + R - \sqrt{R} \le R^2 - R$

or: $R^{3/2} - R \geq R - \sqrt{R}$ (A.11)

which is true for all $R \geq 1$. **QED.**

Proof of Proposition 7

(a) We know that the transitional (middle-aged) generation, faced with a switch to SM, obtains the consumption tuple:

$$C_1^{SM(MA)} = \frac{2\sqrt{R}}{(1+\sqrt{R})} > 1 \quad \text{(A.12.a)}$$

$$C_2^{SM(MA)} = R \quad \text{(A.12.b)}$$

for its early and late diers respectively. Thus:

$$\varepsilon C_1^{SM(MA)} + (1-\varepsilon)\frac{C_2^{SM(MA)}}{R} = R - \varepsilon\left\{R - \frac{2\sqrt{R}}{1+\sqrt{R}}\right\} \quad \text{(A.13)}$$

In contrast, by eliminating $\bar{L}$ between equations (3.a, b) and assuming the constraint is satisfied with equality, we see that the *intragenerational* optimum allocation $\{C_1^B, C_2^B\}$ satisfies:

$$\varepsilon C_1^B + (1-\varepsilon)\frac{C_2^B}{R} = R - (R-1)\varepsilon C_1^B \quad \text{(A.14)}$$

Hence, with $C_1^B \geq 1$, average consumption is higher in SM(MA) if

$$R - \frac{2\sqrt{R}}{1-\sqrt{R}} < R - 1,$$

or if $\dfrac{2\sqrt{R}}{1+\sqrt{R}} > 1$

which is always true. Furthermore, if $C_1^B < C_1^{SM}(MA)$, then SM(MA) dominates B by FOSD. For the class of utility functions satisfying (A.4) we have that:

$$C_1^B = \frac{1}{\varepsilon + (1-\varepsilon)R^{\frac{1-\rho}{\rho}}}.$$

Note that for $\rho \geq 1$, C_1^B is maximized as $\varepsilon \to 0$ at $C_1^B = R^{\frac{\rho-1}{\rho}}$. Hence $C_1^B < C_1^{SM}(MA)$ if

$$C_1^B = R^{\frac{(\rho-1)}{\rho}} < \frac{2\sqrt{R}}{\left(1+\sqrt{R}\right)}$$

or $\log(R) - \frac{\log(R)}{\rho} < \log(2) + \frac{\log(R)}{2} - \log\left(1+\sqrt{R}\right)$

or $\frac{\log(R)}{\rho} > \log\left(1+\sqrt{R}\right) - \log(2) + \frac{\log(R)}{2}$

or $\rho < \dfrac{\log(R)}{\log\left(1+\sqrt{R}\right) - \log(2) + \frac{\log(R)}{2}}$.

Finally note that

$$\frac{\log(R)}{\log\left(1+\sqrt{R}\right) - \log(2) + \frac{\log(R)}{2}} > 1$$

or $\frac{\log(R)}{2} > \log\left(1+\sqrt{R}\right) - \log(2)$

or $\frac{\log(R)}{2} + \log(2) > \log\left(1+\sqrt{R}\right)$

or $\log\left(\sqrt{R}\right) + \log(2) > \log\left(1+\sqrt{R}\right)$

or (antilog) $2\sqrt{R} > \left(1+\sqrt{R}\right)$

(b) Following a procedure similar to the one used in (a), part (b) may immediately be verified by noting that as $\varepsilon \to 1$, then $C_1^B \to 1 < C_1^{SM(MA)}$ and $C_2^B \to R = C_2^{SM(MA)}$. **QED.**

Proof of Proposition 8

Notice first that, by definition, $V^{SM}(NB)$ and V^{TIB} are both independent from the stock of capital accumulated along the TIB path. Hence, if conditions (b.1.1) and (c.1.1) hold at t=1, they will continue to hold at $t \geq 2$, and $SSM(t) = \{TIB, TIB\}$ for all $t \geq 1$. For the remaining cases, we will now show that if $V_0^{SM}(MA) < V_0^B$, then also $V_{t-1}^{SM}(MA) < V_{t-1}^B$ for $t \geq 2$ for all homothetic utility functions $U(C) = \dfrac{(C)^{1-\rho}}{1-\rho}$, with $\rho > 0$. To prove this, consider the inherited endowment level at time t, net of the contractual obligations to the generation born at time (t-1), given by

$$Z_t = RI_{t-2} + I_{t-1} - (1-\varepsilon)C_2^{TIB}.$$

Note that Z_t represents the maximum liquidable resources of the (t-1)th middle-aged generation at time t. We have the following:

Claim 1. $\quad Z_t \geq Z_1 = 1 \quad \forall t \geq 2.$

Proof of Claim 1

From equations (15.a,b) we obtain:

$$Z_t = 1 + \delta\frac{R^j - 1}{R-1} + R\left[1 + \delta\frac{R^{J-1} - 1}{R-1}\right] - R^j\varepsilon C_1^{TIB} - (1-\varepsilon)C_2^{TIB}, t = 2j+1; j \geq 1; \quad (A.15)$$

and:

$$Z_t = (1+R)\left[1 + \delta\frac{R^J - 1}{R-1}\right] - R^j\varepsilon C_1^{TIB} - (1-\varepsilon)C_2^{TIB}, \quad t = 2j+2; \ j \geq 1; \qquad (A.16)$$

from which we also note that: $Z_{2j+2} > Z_{2j+1}$. Hence, to prove Claim 1 it suffices to restrict ourselves to the case in which Z_t is valued in odd periods. This is satisfied if, in (A.15):

$$\delta\frac{R^j - 1}{R-1} + R\left[1 + \delta\frac{R^{J-1} - 1}{R-1}\right] > R^j\varepsilon C_1^{TIB} + (1-\varepsilon)C_2^{TIB},$$

that is if:

$$R^j\left(\frac{2\delta}{R-1} - \varepsilon C_1^{TIB}\right) + R - \delta\frac{R+1}{R-1} > (1-\varepsilon)C_2^{TIB}.$$

Adding and subtracting εC_1^{TIB} and using the definition (15.c) of δ we obtain:

$$R^j\left(\frac{2\delta}{R-1}-\varepsilon C_1^{TIB}\right)>\delta\frac{R+1}{R-1}-\delta-\varepsilon C_1^{TIB}=\frac{2\delta}{R-1}-\varepsilon C_1^{TIB} \tag{A.17}$$

which, given that $R > 1$, is true for all $j \geq 1$. This completes the proof of Claim 1.

Now let $\{C^B_{t-1,1}, C^B_{t-1,2}\}$ be the consumption allocation available to the (t-1)th generation from forming an Intragenerational Bank which uses the inherited stock of capital, after providing for the survivors of the previous generation (stand-alone option). Let $\{C^{SM(MA)}_{t-1,1}, C^{SM(MA)}_{t-1,2}\}$ be the alternative quantity constrained Stock Market transition, in which early diers are allowed to trade $1/(1+\sqrt{R})$ with the new born of the following generation. Equations (20.a, b) define the consumption levels available in the latter solution, if the (t-1)th generation has ineherited at time t one unit of one period old investment *per-capita* (See Proposition 6 for details). The corresponding consumption levels available in case of a constrained transition to the Stock Market at some point along TIB (i.e., with the inherited capital stock of the middle aged possibly greater than one) are defined by:

$$C^{SM(MA)}_{t-1,1}=Z_t+\frac{\sqrt{R}-1}{1+\sqrt{R}} \tag{A.18.a}$$

$$C^{SM(MA)}_{t-1,2}=\begin{cases} R\,Z_t & \text{if: } R\,I_{t-2}-(1-\varepsilon)C_2^{TIB}\leq 0 \\ Z_t+(R-1)I_{t-1} & \text{if: } R\,I_{t-2}-(1-\varepsilon)C_2^{TIB}>0. \end{cases} \tag{A.18.b}$$

The consumption available to the early diers of generation (t-1) is equal to the value of the inherited net endowment Z_t, plus the net gains from quantity constrained trade with the next generation, giving (A.18.a). Similarly, the consumption available to the late diers of generation (t-1) is obtained as follows. If $RI_{t-2}-(1-\varepsilon)C_2^{TIB}\leq 0$, this generation must liquidate part of capital invested in the previous period to fulfill their obligations to the previous generation. This would sustain the next period a consumption level equal to $R\left[I_{t-1}+R_{t-2}-(1-\varepsilon)C_2^{TIB}\right]=RZ_t$. If instead $RI_{t-2}-(1-\varepsilon)C_2^{TIB}\geq 0$, this generation may invest the residual and liquidate it the next period, leading to a consumption level equal to $RI_{t-1}+RI_{t-2}-(1-\varepsilon)C_2^{TIB}=Z_t+(R-1)I_{t-1}$. These considerations lead to (A.18.b).

Now define the following multiples:

$$B_{1t}\equiv\frac{C^B_{t-1,1}}{C^B_{0,1}};\, B_{2t}\equiv\frac{C^B_{t-1,2}}{C^B_{0,2}};\, M_{1t}\equiv\frac{C^{SM(MA)}_{t-1,1}}{C^{SM(MA)}_{0,1}};\, M_{2t}\equiv\frac{C^{SM(MA)}_{t-1,2}}{C^{SM(MA)}_{0,2}}. \tag{A.19}$$

We will now show that if an allocation $\{M_{1t}, M_{2t}\}$ is feasible in a quantity constrained transition, then the allocation $\{B_{1t} = M_{1t}, B_{2t} \geq M_{2t}\}$ will also be feasible under the *intragenerational* mechanism (B) for the (t-1)th generation.

Claim 2. *If a multiple M_1 is feasible for SM(MA), than a multiple $B_1 \geq M_1$ is also feasible for the stand-alone option, B.*

Proof of Claim 2. Substitute from (A.18.a) and (20.a) into (A.19) to obtain:

$$M_1 = \frac{Z_t + \frac{\sqrt{R}-1}{\sqrt{R}+1}}{1+\frac{\sqrt{R}-1}{\sqrt{R}+1}} < Z_t \qquad \text{(A.20)}$$

Also, note that: $\varepsilon C^B_{0,1} \leq Z_1 = 1$, and that: $\varepsilon C^B_{t-1,1} \leq Z_1, \forall t \geq 1$. Then, $C^B_{t-1,1} \geq M_1 C^B_{1,1}$ is feasible. This completes the proof of Claim 2.

Claim 3. *If $\{M_1, M_2\}$ is feasible for SM(MA), then with $B_1 = M_1$ there will be a multiple $B_2 \geq M_2$ that is feasible for B.*

Proof of Claim 3. Consumption levels in an Intragenerational Bank will be constrained by:

$$\varepsilon C^B_{t-1,1} \leq Z_t, \quad \forall t \geq 1 \qquad \text{(A.21.a)}$$

$$(1-\varepsilon) C^B_{t-1,2} = \begin{cases} R\left[Z_t - \varepsilon C^B_{t-1,1}\right] & \text{if: } R\,I_{t-2} - (1-\varepsilon)C_2^{TIB} - \varepsilon C^B_{t-1,1} \leq 0 \\ Z_t + (R-1)I_{t-1} - \varepsilon C^B_{t-1,1} & \text{if: } R\,I_{t-2} - (1-\varepsilon)C_2^{TIB} - \varepsilon C^B_{t-1,1} > 0 \end{cases} \qquad \text{(A.21.b)}$$

These are obtained by noting that the maximum per-capita consumption that early diers may obtain in an intragenerational bank is given by the per-capita value of the current net endowment, giving (A.21.a). The maximum per-capita consumption of late diers is obtained in a way similar to (A.18.b), after accounting for the consumption of the early diers of the same generation $C^B_{t-1,1}$, giving (A.21.b).

Let us evaluate separately the two cases in (A.21.b). For the first one, we consider two subcases. *(i.a)* If $R\,I_{t-2} - (1-\varepsilon)C_2^{TIB} \leq 0$, note from the definition of M_2 in equation (A.19) and from (A.18.b) that $M_2 = R\,Z_t / R = Z_t$. Hence $Z_t > M_1$ from equation (A.20) and referring to the homotheticity assumption implies that:

$$B_2 = \frac{R\left(Z_t - M_1 \varepsilon C_{0,1}^{B}\right)}{R\left(1 - \varepsilon C_{0,1}^{B}\right)} > Z_t \equiv M_2 \qquad \text{(A.22)}$$

is feasible.

(i.b) If: $R I_{t-2} - (1-\varepsilon)C_2^{TIB} > 0$, but: $R I_{t-2} - (1-\varepsilon)C_2^{TIB} - M_1 \varepsilon C_{0,1}^{B} < 0$, then Claim 3 is true if:

$$B_2 = \frac{R\left[Z_t - M_1 \varepsilon C_{0,1}^{B}\right]}{R\left[1 - \varepsilon C_{0,1}^{B}\right]} > \frac{Z_t + (R-1)I_{t-1}}{R} = M_2 \qquad \text{(A.23)}$$

From (A.22) we have that (A.23) will hold if: $R Z_t > Z_t + (R-1) I_{t-1}$, or if:

$$R[R I_{t-2} + I_{t-1} - (1-\varepsilon) C_2^{TIB}] > R I_{t-2} + R I_{t-1} - (1-\varepsilon) C_2^{TIB},$$

which is the case for $R I_{t-2} - (1-\varepsilon) C_2^{TIB} > 0$.

(ii) If: $R I_{t-2} - (1-\varepsilon) C_2^{TIB} - \varepsilon\, C_{t-1,1}^{TIB} > 0$, we have that: $M_2 = \frac{Z_t + (R-1) I_{t-1}}{R}$, and:

$$\begin{aligned} B_2 &= \frac{Z_t + (R-1) I_{t-1} - M_1 \varepsilon C_{0,1}^{B}}{R\left(1 - \varepsilon C_{0,1}^{B}\right)} = \\ &= \frac{(R-1) I_{t-1}}{R\left(1 - \varepsilon C_{0,1}^{B}\right)} + \frac{Z_t - M_1 \varepsilon C_{0,1}^{B}}{R\left(1 - \varepsilon C_{0,1}^{B}\right)} \geq \frac{Z_t + (R-1) I_{t-1}}{R} = M_2 \end{aligned} \qquad \text{(A.24)}$$

This completes the proof of Claim 3.

We have thus shown that there is a feasible allocation $\{B_1, B_2\}$ such that $B_1 = M_1$ and $B_2 \geq M_2$. Hence for all homothetic utility functions $U(C_i)$, i = 1, 2, if: $\left[V^B \mid H_1 = (1,0)\right] \geq \left[V^{SM}(MA) \mid H_1 = (1,0)\right]$, then: $\left[V^B \mid H_t\right] \geq \left[V^{SM}(MA) \mid H_t\right]$, for all $t \geq 2$ and all $\{H_t\}$ along a continuation of the path TIB. **QED.**

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