Finding Better Securities while Holding Portfolios: *Is Stochastic Dominance the Answer?*

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Active portfolio managers are always looking for investments that will improve the performance of their portfolios, attempting to select assets that most greatly benefit their clients and to remove inferior asset positions. Indeed, with the constant flow of economic and financial news, asset prices change continuously, which compels managers to revise their portfolios. In turn, continuous trading affects securities prices. Faced with such a variable environment, analysts have difficulty generating optimal portfolios for their sponsors. Many trading schemes and models are designed to exploit this intertemporal disequilibrium in order to benefit investors. All of these models tackle the same problem—which assets should be included or removed from a portfolio in order to improve the investor's holdings? Some suggestions are offered in the various recommendation lists and newsletters published by trading analysts, most of which, unfortunately, tend to ignore the current positions of investors. Rather, these recommendations consider for the most part only the expected performance of the asset relative to the market, the industry, or the asset's proper merits, the most notable being its risk.

I address this problem by considering the class of all risk-averse investors and differentiating among them by the portfolios of risky assets they hold. More explicitly, I propose to use marginal conditional stochastic dominance (MCSD) rules, which were specifically designed for this purpose. MCSD was originally proposed by Shalit and Yitzhaki [1994] as a statistical device to find the dominant and dominated securities for *all* risk-averse investors, conditional that they hold a portfolio. If the dominant securities are added to the portfolio, investors benefit because of increasing expected utility of returns. Similarly, investors benefit if the dominated securities are removed from the portfolio.

Finding dominant and dominated assets is motivated by the fact that investors hold relatively small portfolios that are not optimal because financial markets are not generally in equilibrium. Hence, portfolio performance can be improved either by adding new assets or by changing the shares of dominating and dominated securities.

The MCSD model deals with risk-averse investors who maximize expected utility of returns. Therefore, the model is general enough to suit a wide range of clients and fund managers as applied by Shalit and Yitzhaki [2003] for popular advice portfolios. Nevertheless, it is also explicit enough to provide distinctive solutions for individuals holding different assets, because it is conditional on specific portfolios.

What can be gained by applying MCSD rules to existing portfolios? First, dominating and dominated asset positions can be traded to enhance portfolio performance. Second, if dominance cannot be found, investors and sponsors learn from the model whether their portfolios are sufficiently efficient and well managed. Third, fund managers can continuously change their holdings by investing in MCSD-dominant securities and dynamically improve their portfolios.

In this article, I will apply the MCSD methodology by assuming the investors hold portfolios that mimic the S&P 100 Index. Using two different approaches for MCSD, I compute the list of dominating and dominated stocks and compare the results to the standard mean-variance portfolio management.

THE MARGINAL CONDITIONAL STOCHASTIC DOMINANCE MODEL

Assume you are analyzing the portfolio of an investor who has the opportunity to invest in a new asset. Under what conditions would you recommend that the new asset be included in the portfolio? If the investor maximizes expected utility, you may recalculate the new allocation by including the new asset, changing the proportions of existing assets, and making the proper global adjustments. The task, however, is quite demanding, because you need to know the proper utility function and the joint probability distribution of returns.

Most financial analysts simplify this process by using mean-variance (MV) analysis for which the advantages and limitations are common knowledge. In brief, MV is easy to use and understand because the manager only needs to calculate the means and the variance-covariance matrix of asset returns. The main disadvantage is that the resulting efficient portfolios maximize expected utility only if returns are normally distributed or the utility function is quadratic. To see whether additional assets should be included in the portfolio when using MV, analysts calculate the covariance matrix with the new assets and minimize the variance of the portfolio subject to the required mean returns. Then, comparing the new and old portfolios, analysts decide whether to include the new asset.

Another approach to the problem is to use second-degree stochastic dominance (SSD) as the choice model. SSD, which complies with expected utility maximization without the need of a utility function, states the conditions under which all risk-averse expected utility maximizers prefer one portfolio over another portfolio that includes the new asset.[1] Usually, SSD compares the areas under the empirical cumulative probability functions of portfolio returns (see Hadar and Russell [1969], Hanoch and Levy [1969], and Rothschild and Stiglitz [1970]). A more convenient tool for SSD, developed by Shorrocks [1983], is to use generalized Lorenz curves (hereafter, Lorenz), or the expected cumulative returns on the portfolio. The Lorenz curves are easy to calculate because they involve ranking the portfolio returns in ascending order and then, for each observation, summing all the lower returns to that given observation. The Lorenz delineate the cumulative returns as a function of the empirical cumulative probability of the portfolio. For all risk-averse investors to prefer one portfolio of risky assets over another, its Lorenz must lie above the Lorenz of the alternative portfolio. Hence, by comparing the Lorenz curves of returns, analysts can establish the list of preferred assets to be included in the portfolio. Unfortunately, SSD cannot guarantee that an optimal portfolio is obtained.[2]

Whether MV or SSD is the model of choice, an entire optimization process has to be run on the portfolio. In the present study, instead of comparing the performance of entire portfolios, I propose to use MCSD, which compares the performance of individual assets conditional on holding a portfolio. Like SSD, MCSD is a technique that uses functions of cumulative probability distributions for determining whether to include new assets in the portfolio. But unlike SSD, MCSD considers the opportunities available to risk-averse investors conditional on their holding a portfolio. I will now proceed to explain how MCSD works.

A risk-averse expected utility–maximizing investor holds an existing portfolio P and considers adding a new security, X, to his assets. The question that arises is how an analyst would recommend including asset X in the portfolio by changing the portfolio share of existing security Y. The analyst would approve of the move if *all* risk-averse investors benefit from the change. The answer to the question is provided by the MCSD conditions in terms of cumulative conditional expected returns relative to the portfolio. The approach is promising because it addresses the relative performance of risky assets given a portfolio that could very well be specific to the investor. For a given portfolio, MSCD finds the sets of dominating and dominated assets by comparing their absolute concentration curves (ACC), an asset's cumulative conditional expected return weighted by the probability distribution of the portfolio.

To understand the notion of an ACC, consider the conditional expected return of security X for a given portfolio return r_P. For analysts who normally calculate the beta (systematic risk) of a security, the ACC should be a familiar concept since the conditional expected return is measured by the regression line of the expected return of X given the portfolio return r_P, or $E(r_X | P = r_P) = \alpha + \beta r_P$. The ACC of security X is the cumulative sum of the conditional expected returns of X for a given portfolio return r_P, each one multiplied by the probability that the portfolio yields r_P.

By its construction, the ACC provides much more information than beta. Indeed, beta measures systematic risk as an average of the sensitivity of asset return to portfolio return and, therefore, does not capture the complete relation between asset return, asset risk, and portfolio risk. By contrast, the ACC complies with the risk and return preferences of *all* risk-averse investors over the entire range of portfolio returns. To see this, let us first consider a portfolio whose returns have been ranked from the lowest to the highest. The ranking concurs with the preferences of *all* risk-averse investors because it is compatible with decreasing marginal utility. Indeed, the lowest portfolio returns yield the highest marginal utilities and the highest returns yield the lowest marginal utilities.

Next, let us consider a portfolio mix that has a return of, at most, r_P. Let $F(p)$ be the probability that the portfolio will earn *at most* r_P. To calculate the ACC of asset X for $F(p)$, add all the returns on X, each one having been multiplied by the probabilities $F(p)$, obtained from the worst return on portfolio P to the point where it reaches a return of, at most, r_P. Repeat this process for all returns on the portfolio. In a sense, the ACC of asset X measures the cumulative returns ranked and weighted by the probabilities of the portfolio. I can now state the following MCSD rule:

> Conditional to holding portfolio P, asset X is preferred to asset Y by all risk-averse investors, if the ACC of X lies above the ACC of Y. The converse is also true. If the ACCs intersect, no preference between assets can be determined for all risk-averse investors.

In a sample of N discrete returns, the ACC of asset X is obtained by ranking all the asset returns with respect to the portfolio in ascending order, from the worst to the best return. Then, the rank divided by the number of observations N is a consistent estimate of the cumulative probability distribution of the portfolio's return. As asset X is ranked with respect to the portfolio's increasing returns, sum, for every observation, the returns on X from the first data point to the current one. This operation is repeated for every observation in the ranked sample. Next, divide the results by the total number of observations N to obtain the ACC of asset X, which is the function that relates cumulative asset returns to the rank of portfolio P.

The MCSD rule allows us to determine the preference between all pairs of securities held in and out of the portfolio. If the ACC of asset X lies above the ACC of asset Y over the entire range of portfolio returns, asset X dominates asset Y, and all risk-averse investors will benefit by increasing the share of asset X and decreasing the share of asset Y in their portfolios. Increasing holdings of dominating securities and decreasing holdings of dominated securities will yield a higher return to the entire portfolio. In this way, better securities can be found while holding a portfolio of stocks.

AN EXAMPLE

To understand the mechanics of MCSD, consider a portfolio of four stocks whose daily returns for a specific week are given in Exhibit 1, Panel A. The portfolio P is composed of 25% AA, 25% BB, 25% CC, and 25% DD. If we would like to compute the ACC for each stock, given the performance of portfolio P when the entire sample consists only of the returns for that week, first, sort all the daily returns according to the increasing returns on the portfolio as done in Panel B. Then for each stock, calculate the cumulative returns starting from zero, as shown in Panel C. Finally, to obtain the ACCs in Panel D, divide each cumulative return by the number of days (five), since 1/days is the estimated probability of occurrence of portfolio returns. The last row in Panel D presents the mean return of each stock for that week. The right-most column in Panel D shows the cumulative probability of portfolio returns.

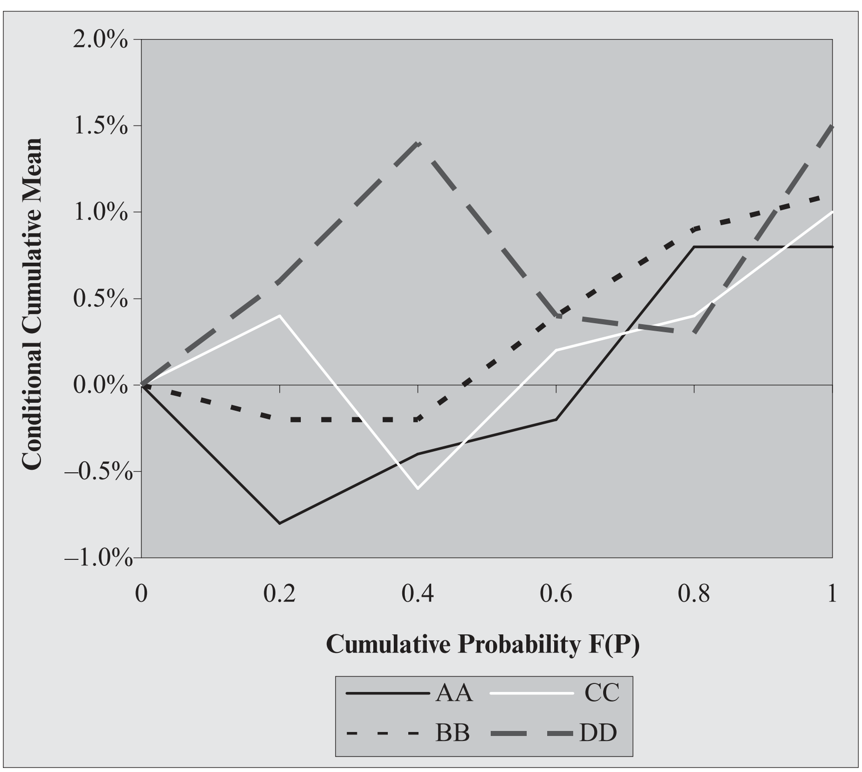
The ACCs for the four stocks are plotted in Exhibit 2 as a function of the cumulative probability of portfolio returns. The main issue in MCSD is to determine which ACCs intersect and which ACCs lie entirely above others. As Exhibit 2 shows, the ACC of stock BB lies entirely above the ACC of AA. All others intersect. Hence, following the MCSD rule, if the composition of the portfolio is modified by increasing the share of the dominating stock BB and reducing the share of AA, all risk-averse investors will benefit from the move.

To support this claim, I use the preferences suggested by Levy and Spector [1996]. First, let us consider the logarithmic utility function $U_1(W) = \ln(1 + P)$ and then compute the expected utility for the original portfolio,

EXHIBIT 1
An Example: How to Construct ACCs

Panel A: Daily Stock Returns						Panel B: Returns Sorted w.r.t. P					
Days	**AA**	**BB**	**CC**	**DD**	**P**	**AA**	**BB**	**CC**	**DD**	**P**	
Mon	5.0%	2.5%	1.0%	–0.5%	2.0%	–4.0%	–1.0%	2.0%	3.0%	0.0%	
Tue	0.0%	1.0%	3.0%	6.0%	2.5%	2.0%	0.0%	–5.0%	4.0%	0.3%	
Wed	–4.0%	–1.0%	2.0%	3.0%	0.0%	1.0%	3.0%	4.0%	–5.0%	0.8%	
Thu	2.0%	0.0%	–5.0%	4.0%	0.3%	5.0%	2.5%	1.0%	–0.5%	2.0%	
Fri	1.0%	3.0%	4.0%	–5.0%	0.8%	0.0%	1.0%	3.0%	6.0%	2.5%	
Panel C: Accumulated Sorted Returns						**Panel D: Absolute Concentration Curves**					
	AA	**BB**	**CC**	**DD**	**P**	**f(P)**	**AA**	**BB**	**CC**	**DD**	**F(P)**
	0.0%	0.0%	0.0%	0.0%	0.0%		0.0%	0.0%	0.0%	0.0%	0
	–4.0%	–1.0%	2.0%	3.0%	0.0%	0.20	–0.8%	–0.2%	0.4%	0.6%	0.2
	–2.0%	–1.0%	–3.0%	7.0%	0.3%	0.20	–0.4%	–0.2%	–0.6%	1.4%	0.4
	–1.0%	2.0%	1.0%	2.0%	1.0%	0.20	–0.2%	0.4%	0.2%	0.4%	0.6
	4.0%	4.5%	2.0%	1.5%	3.0%	0.20	0.8%	0.9%	0.4%	0.3%	0.8
	4.0%	5.5%	5.0%	7.5%	5.5%	0.20	0.8%	1.1%	1.0%	1.5%	1

EXHIBIT 2
An Example: The ACCs for Four Stocks



which amounts to $EU_1(W) = 1.08928\%$. If the share of BB is increased to 30% and reduce the share of AA to 20%, leaving the other stocks at the same level, expected utility increases to 1.10437. Expected utility also increases for the modified portfolio when the family of myopic preference functions $U(W) = (1 + P)^{1-\alpha}/(1 - \alpha)$ is used for various $\alpha \neq 1$. Applying an alternative concave utility function, such as the piecewise linear function,

$$U_2(W) = \begin{cases} 1+P & \text{if } W \leq 1 \\ 1+0.3P & \text{if } W > 1 \end{cases}$$

expected utility increases as a result of the new portfolio from 1.0033 to 1.003345. In addition, for the improved portfolio, the mean return increases from 1.1% to 1.115% and the standard deviation falls from 1.098% to 1.069%, which indicates that the new portfolio is also MV efficient.

To verify the converse of MCSD when ACCs intersect, it is necessary to ascertain that dominance cannot be found for all risk-averse investors. To do this, consider two stocks whose ACCs intersect—BB and CC—and modify the share of BB to 30% and the share of CC to 20%. Given $U_1(W) = \ln(1 + P)$, expected utility increases to 1.09432. However, when using the $U_2(W)$ function, expected utility decreases from 1.0033 to 1.0031. This example shows that dominance cannot be ascertained when ACCs intersect; for some concave utilities, the move is beneficial, and for others, it is detrimental.

AN APPLICATION

To apply the MCSD methodology to stock market data, consider a portfolio that mimics the performance of the S&P 100 Index. For three different holding periods lasting one year each (1987 with 252 trading days, 2000 with 251 trading days, and 2006 with 250 trading days), I analyze the relative performance of the stocks that make up the S&P 100 Index and establish a list of securities that can improve an investor's portfolio. For each of the periods, daily returns are computed and ranked from worst to best relative to the daily performance of the S&P 100 Index return. Then, for each asset, the cumulative returns are calculated and the ACCs are obtained by dividing these cumulative returns by the number of observations for each year.

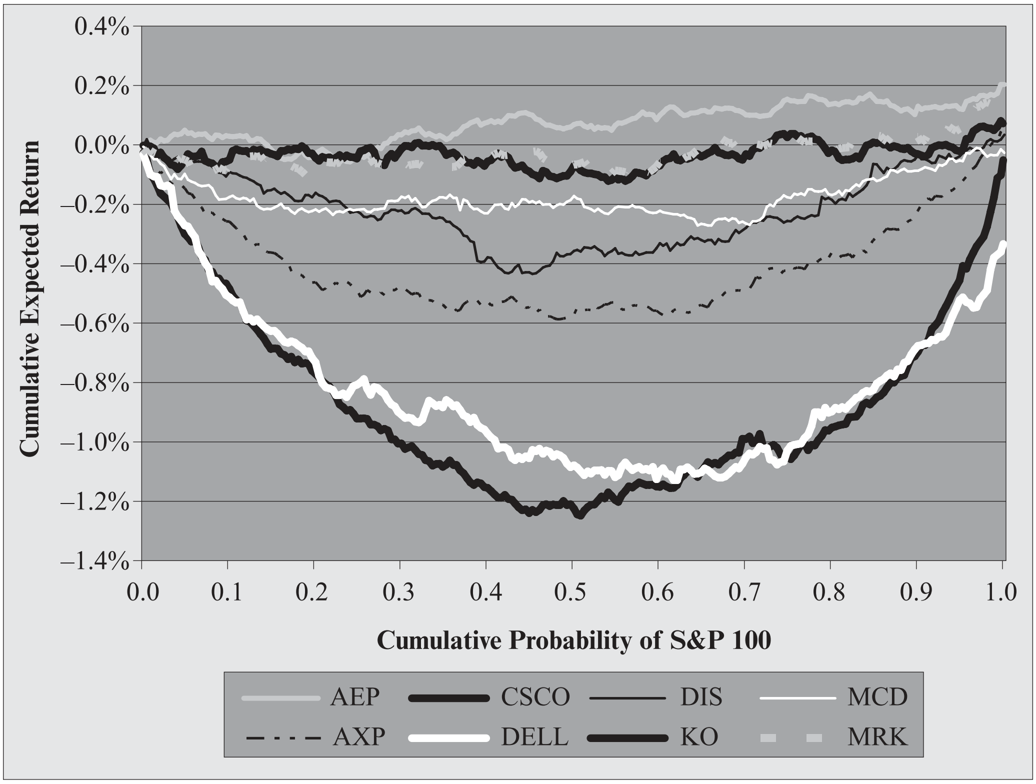
Exhibit 3 reports the ACC*s* for a select subset of eight stocks for the year 2000. The ACCs start at the origin (0,0) of the axes and end at their respective means, shown on the vertical axis at the right. The ACCs of Cisco (CSCO) and Dell (DELL) lie entirely at the bottom of the chart. Neither stock is preferred because their lines cut each other, and both are dominated by all six remaining stocks in the chart. The ACC of American Electric Power (AEP) lies above the ACCs of American Express (AXP), Cisco, Dell, Walt Disney (DIS), McDonald's (MCD), and Merck (MRK) for the entire range of the cumulative probability function and does not dominate Coca-Cola (KO) because their ACCs intersect. Similarly, McDonald's dominates American Express, and that Coca-Cola dominates Disney. Exhibit 3 shows that a necessary condition for one security to dominate another is that its mean return must be greater than the mean return of the stock it dominates. Indeed, the values of the ACCs on the right-hand vertical axis are the calculated mean returns for each stock, since they are the cumulative returns for the entire sample divided by the number of observations.

A list of better securities can be assessed by this type of pairwise analysis. Dominance is found when the ACC of one stock lies entirely above other stocks, but preference cannot be substantiated when their ACCs intersect. For each of the three years in the analysis and for every stock in the S&P 100 Index, I establish the list of dominated stocks subject to holding the S&P 100 portfolio. Then, I determine the set of dominating securities that are not dominated by any other stocks included the S&P 100 Index in that year.[3]

For the year 1987, the dominating securities are Alcoa (AA), American Electric Power (AEP), Amgen (AMGN), Baxter International (BAX), Black & Decker (BDK), Burlington Northern Sante Fe (BNI), Anheuser-Busch Inbev (BUD), Caterpillar (CAT), Cigna (CI), Campbell Soup (CPB), Dow Chemical (DOW), Excelon (EXC), Ford (F), Home Depot (HD), Harrah's Entertainment (HET), H.J. Heinz (HNZ), Hewlett Packard (HPQ), Intel (INTC), Medtronic (MDT), Merck, Microsoft (MSFT), National Semiconductor (NSM), Pepsi (PEP), Raytheon (RTN), Sprint (S), Texas Instruments (TXN), and Williams Companies (WMB).

For the year 2000, the securities that dominated stocks in the S&P 100 Index, but are not dominated by any other stock in the index are American Electric Power (AEP), Avon Products (AVP), Baker Hughes (BHI), Cigna, Disney, El Paso Corporation (EP), Entergy (ETR), Excelon, General Dynamics (GD), Harrah's Entertainment, H.J. Heinz, Coca Cola, Lehman Brothers (LEH), Altria Group (MO), Pepsi, Raytheon, and The Southern Company (SO).

EXHIBIT 3
ACCs When Holding the S&P 100 Portfolio (Year 2000)



The same analysis for the year 2006 indicates that the dominating securities are AES Corporation (AES), Allegheny Technologies (ATI), Avon Products, Baxter International, Baker Hughes, Colgate Palmolive (CL), Comcast (CMCSA), Campbell Soup, Disney, El Paso Corporation, Entergy, Excelon, General Dynamics, General Motors (GM), Halliburton (HAL), H.J. Heinz, Hewlett Packard, Lucent Tech Cap Tr 1 (LUTHP.PK), McDonald's, Medtronic, Merck, Office Max (OMX), Oracle (ORCL), Radio Shack (RSH), Raytheon, Schlumberger (SLB), AT&T (T), and Unysis (UIS).

Following the MCSD rule that I have used, the portfolio can be improved by increasing the shares of dominating securities and, in parallel, reducing the shares of dominated securities.

THE TWO-PARAMETER APPROACH TO MCSD

One shortcoming of the ACC method is that it does not provide a complete preference ranking of assets conditional on holding a portfolio. The results are limited to "asset *X* dominates asset *Y*" or "asset *Y* dominates asset *X*" or neither dominates. Furthermore, the process of computing ACCs and searching for dominance is considered complex by some analysts. Finally, as ACCs are sample dependent, they might be subject to statistical inference. Indeed, to what extent a computed ACC intersects another ACC with a certain degree of statistical significance is a topic that is still being researched.

A simpler parametric approach based on computed statistics would be preferable, especially when a complete ordering of investment alternatives is required. In this case, the analyst can use the following two necessary conditions for MCSD established by Shalit and Yitzhaki [1994]:

If asset *X* is preferred to asset *Y* conditional on holding the portfolio *P*, then:

1. the mean return of *X* must be equal to or greater than the mean return of *Y*, and
2. the risk-adjusted mean return of *X* must be equal to or greater than the risk-adjusted mean return on asset *Y*.

The risk-adjusted mean return used here is defined by the mean of asset X less the mean-Gini systematic risk of X multiplied by the Gini of the portfolio P.[4] Mathematically, the second necessary condition is written as

$$(\mu_X - \beta_X^\Gamma \Gamma_P) \geq (\mu_Y - \beta_Y^\Gamma \Gamma_P)$$

where μ_X is the mean return of X, and β_X^Γ is the mean-Gini beta of asset X, which is calculated as

$$\beta_X^\Gamma = \frac{2\,\text{cov}[X, F(P)]}{2\,\text{cov}[P, F(P)]}$$

where cov(.) is the covariance function, $F(P)$ is the portfolio cumulative probability, and G_P is the Gini of portfolio P defined as $2\text{cov}[P, F(P)]$. One way of estimating the cumulative probability is to use the portfolio performance rankings. In that case, the portfolio's Gini is twice the covariance between the portfolio's return and its rank divided by the number of data points. The second necessary condition provides a relation as powerful as the standard CAPM that relates the security's expected return to its systematic risk. Indeed, if asset X dominates asset Y, conditional on holding portfolio P, the difference in their expected returns must exceed the difference in their systematic risks. This is expressed in Gini terms as

$$\mu_X - \mu_Y \geq (\beta_X - \beta_Y)\Gamma_P$$

Dominance is obtained by first ranking the securities in descending order of their mean return, μ. According to the first necessary condition, the securities at the top of the list dominate the securities that are ranked below it. The analyst can then check whether the ranking remains the same for the risk-adjusted mean return, $\mu_X - \beta_X\Gamma_P$. If the ranking is not maintained, the security does not dominate the securities ranked lower in the list.

For the year 2006, the portfolio comprises of 98 stocks whose statistics are shown in Exhibit 4.[5] To find MCSD dominance using the necessary conditions, rank the stocks in decreasing order of mean returns and review their risk-adjusted mean returns to determine if the ranking is preserved. Dominance does not exist if the ranking with respect to the mean return does not coincide with the ranking according to the risk-adjusted mean return.

More can be learned about the relative dominance by mapping out the stocks in the mean return/risk-adjusted mean return space. Because of the large number of stocks in the portfolio, the display might appear somewhat confusing, and thus it may be difficult to identify the dominating/dominated stocks from the chart. A way to alleviate this problem is to first number the securities in order of each security's mean return (from 1 to 98) and then to repeat the numbering process using each security's risk-adjusted mean (from 1 to 98). Using the two numbers as coordinates, I can chart the securities in the ranking mean return/ranking risk-adjusted mean return space, as illustrated in Exhibit 5. The stocks in the upper-right corner dominate the stocks below them and to their left. For example, Intel (INTC) in the lower-left corner is dominated by most stocks, but not by National Semiconductor (NSM). Hence, according to MCSD, investors holding a portfolio that emulates the S&P 100 Index will benefit if they decrease their holdings of Intel and increase their holdings of most others stocks, excluding National Semiconductor. Of course, these recommendations are based on estimated statistical parameters and not on the true expected return and true systematic risks. Thus, the dominating/dominated relationships are preserved and the chart is more readable. Furthermore, each security's dominating stocks are located in the upper-right quadrant defined by the virtual lines crossing the stock.

COMPARING MCSD TO STANDARD PRACTICES

I have presented two approaches for using marginal conditional stochastic dominance, both of which enable an investor to find the preferred stocks while holding a risky portfolio. The next question is whether these approaches are more advantageous to practitioners who use standard procedures such as the Markowitz mean-variance (MV) criterion or the Sharpe-Lintner-Mossin CAPM. To answer this question, I make the following comparisons.

Using the MV criterion, dominance is obtained for all stocks that have a higher mean return and a lower standard deviation. The usual method is to draw the stocks in the mean–standard deviation space. The securities located in the upper-left corner dominate the securities located in the lower-right corner. As the chart for the year 2006 is not entirely clear, I use the ranking technique described earlier. First, I sort the stocks in ascending order from 1 to 98

EXHIBIT 4
Statistics for 98 Stocks, 2006

Stocks	Mean	St. Dev.	Risk-Adjusted Mean	Beta	Stocks	Mean	St. Dev.	Risk-Adjusted Mean	Beta
AA	0.024%	1.679%	–0.356%	1.310	HPQ	0.161%	1.642%	–0.282%	1.287
ABT	0.099%	1.049%	–0.159%	0.758	IBM	0.077%	0.896%	–0.241%	0.873
AEP	0.073%	0.892%	–0.153%	0.642	INTC	–0.071%	1.673%	–0.638%	1.484
AES	0.135%	1.516%	–0.071%	0.585	IP	0.027%	1.299%	–0.385%	1.280
AIG	0.019%	0.837%	–0.278%	0.850	JNJ	0.039%	0.718%	–0.124%	0.428
ALL	0.083%	0.899%	–0.158%	0.712	JPM	0.092%	1.081%	–0.346%	1.295
AMGN	–0.058%	1.170%	–0.310%	0.727	KO	0.080%	0.676%	–0.156%	0.626
ATI	0.419%	3.064%	–0.465%	2.877	LEH	0.093%	1.783%	–0.589%	2.094
AVP	0.082%	1.430%	–0.122%	0.610	LTD	0.125%	1.536%	–0.242%	1.193
AXP	0.066%	0.985%	–0.290%	1.046	LUTHP.PK	0.057%	1.237%	0.051%	0.014
BA	0.109%	1.439%	–0.294%	1.203	MCD	0.128%	1.124%	–0.182%	0.877
BAC	0.070%	0.796%	–0.214%	0.828	MDT	–0.018%	1.416%	–0.191%	0.410
BAX	0.088%	1.165%	–0.117%	0.605	MER	0.136%	1.241%	–0.311%	1.394
BDK	–0.018%	1.575%	–0.395%	1.274	MMM	0.010%	1.134%	–0.268%	0.828
BHI	0.090%	2.230%	–0.388%	1.501	MO	0.077%	1.046%	–0.144%	0.543
BMY	0.079%	1.460%	–0.289%	1.013	MRK	0.138%	1.189%	–0.158%	0.851
BNI	0.041%	1.846%	–0.536%	1.772	MSFT	0.057%	1.298%	–0.260%	0.824
BUD	0.062%	0.918%	–0.152%	0.632	MS	0.148%	1.260%	–0.349%	1.546
C	0.069%	0.903%	–0.249%	0.917	NSC	0.085%	2.044%	–0.431%	1.494
CAT	0.046%	1.776%	–0.433%	1.538	NSM	–0.042%	2.212%	–0.689%	1.961
CCU	0.060%	1.108%	–0.173%	0.668	OMX	0.304%	2.074%	–0.120%	1.257
CI	0.084%	2.011%	–0.241%	1.074	ORCL	0.135%	1.552%	–0.222%	1.020
CL	0.082%	0.934%	–0.094%	0.440	PEP	0.028%	0.706%	–0.107%	0.367
CMCSA	0.199%	1.216%	–0.132%	0.981	PFE	0.058%	1.315%	–0.253%	0.853
CPB	0.116%	0.816%	–0.029%	0.416	PG	0.048%	0.843%	–0.174%	0.601
CSC	0.031%	1.684%	–0.381%	1.057	ROK	0.028%	1.676%	–0.370%	1.320
CSCO	0.195%	1.788%	–0.287%	1.349	RSH	–0.063%	2.342%	–0.300%	0.780
CVX	0.108%	1.273%	–0.159%	0.848	RTN	0.124%	1.029%	–0.128%	0.747
DD	0.068%	1.026%	–0.254%	0.947	S	–0.026%	1.659%	–0.281%	0.723
DELL	–0.061%	1.903%	–0.539%	1.366	SLB	0.109%	2.258%	–0.346%	1.500
DIS	0.147%	1.211%	–0.153%	0.855	SLE	–0.021%	1.205%	–0.357%	0.905
DOW	–0.015%	1.256%	–0.370%	1.091	SO	0.041%	0.776%	–0.159%	0.591
EK	0.055%	1.946%	–0.427%	1.446	T	0.173%	1.064%	–0.114%	0.825
EMC	–0.003%	1.516%	–0.379%	1.062	TGT	0.028%	1.282%	–0.318%	0.974
EP	0.108%	1.891%	–0.233%	1.109	TWX	0.093%	0.888%	–0.140%	0.630
ETR	0.123%	0.840%	–0.054%	0.498	TXN	–0.041%	1.712%	–0.522%	1.451
EXC	0.067%	1.056%	–0.140%	0.610	TYC	0.023%	1.224%	–0.273%	0.853
F	0.023%	2.326%	–0.338%	1.041	UIS	0.140%	1.992%	–0.159%	0.813
FDX	0.032%	1.434%	–0.462%	1.497	UPS	0.012%	1.217%	–0.286%	0.893
GD	0.115%	1.191%	–0.150%	0.761	USB	0.090%	0.643%	–0.104%	0.572
GE	0.035%	0.787%	–0.251%	0.769	UTX	0.053%	1.139%	–0.266%	1.032
GM	0.243%	2.629%	–0.156%	1.130	VIA.B	0.004%	1.413%	–0.335%	1.936
GS	0.189%	1.479%	–0.369%	1.713	VZ	0.121%	1.021%	–0.222%	0.970

EXHIBIT 4 (*Continued*)

Stocks	Mean	St. Dev.	Risk-Adjusted Mean	Beta	Stocks	Mean	St. Dev.	Risk-Adjusted Mean	Beta
HAL	0.007%	2.196%	–0.486%	1.558	WFC	0.059%	0.810%	–0.186%	0.716
HD	0.005%	1.276%	–0.362%	1.105	WMB	0.056%	1.681%	–0.251%	1.055
HET	0.074%	1.548%	–0.213%	0.851	WMT	0.011%	1.069%	–0.307%	0.904
HIG	0.038%	1.174%	–0.334%	1.082	WY	0.046%	1.369%	–0.321%	1.220
HNZ	0.132%	0.931%	–0.010%	0.458	XOM	0.123%	1.186%	–0.191%	0.953
HON	0.091%	1.189%	–0.310%	1.207	XRX	0.059%	1.273%	–0.277%	1.004

EXHIBIT 5

Ranking MCSD Stocks, 2006

according to their mean returns and use the ranking number as the mean-return coordinate. Then, I sort the stocks according to their standard deviations and use the ranking number as the standard-deviation coordinate. For the year 2006, the 98 securities, whose statistics are presented in Exhibit 4, are charted in the MV ranking space shown in Exhibit 6. The efficient (nondominated) stocks are U.S. Bancorp (USB), Campbell Soup (CPB), Entergy (ETR), H.J. Heinz (HNZ), AT&T (T), Comcast (CMSA), Office Max (OMX), and Allegheny Technologies (ATI); all are located at the left and above all other securities. These stocks cannot be dominated by any other efficient stock in the list.

EXHIBIT 6
Ranking MV Stocks, 2006

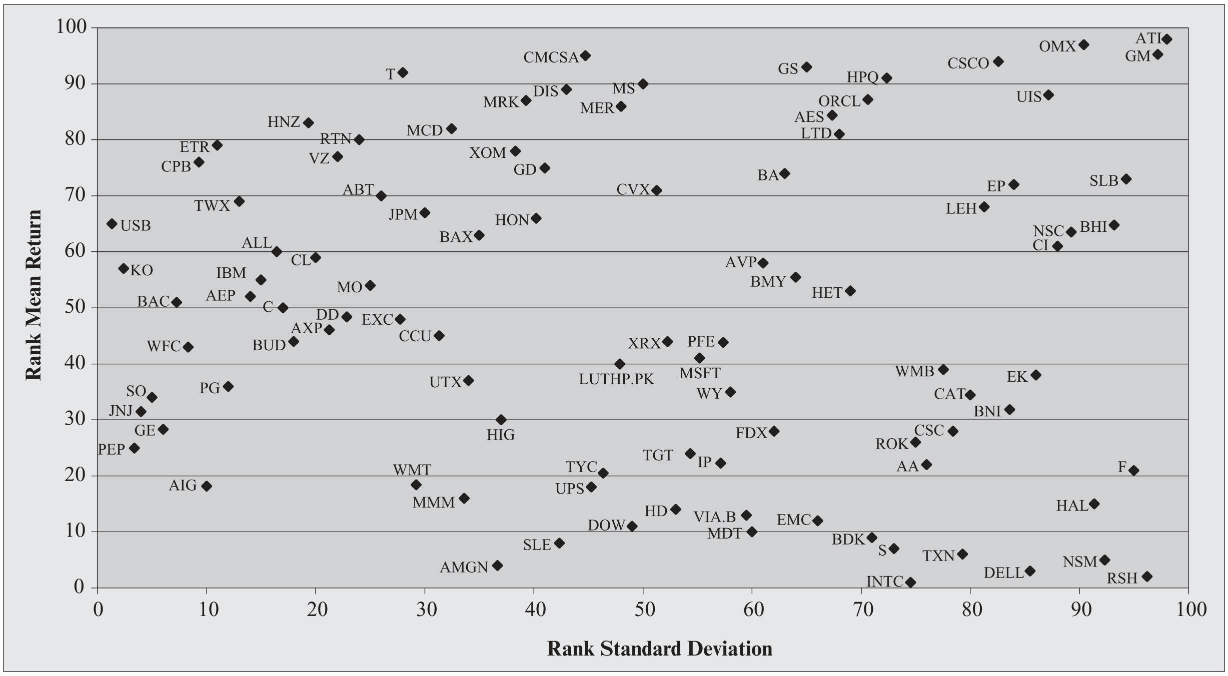
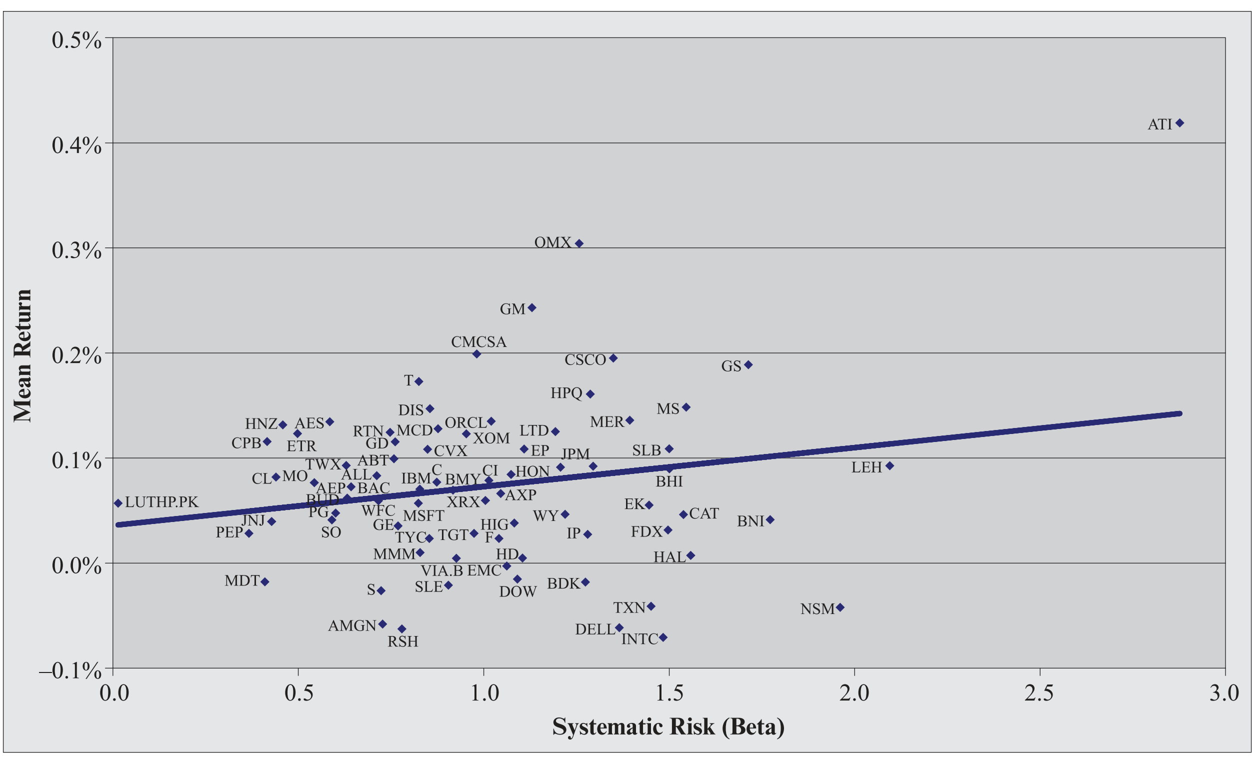


EXHIBIT 7
Mean Return of Beta Stocks, 2006



MV dominance and MCSD dominance can be compared by analyzing Exhibits 5 and 6. In general, the dominant securities obtained by using MV also appear to dominate with respect to MCSD. The MCSD stocks are more discriminatory, however, in the sense that the set of dominant stocks is much smaller. Furthermore, MCSD sets a clear pairwise comparison among stocks. For example, under MCSD, Office Max, which is located in the upper-right corner, dominates most of the other stocks. However, under MV, because Office Max has a larger standard deviation, its dominance can be determined versus most other stocks only in the presence of specific preferences.

A word of caution is called for. I am using only the necessary conditions for the two criteria. MV, as is well known, is compatible with expected utility only if returns are normally distributed or if the utility function is quadratic. Levy and Markowitz [1979] and Markowitz [1991] showed that MV provides a good approximation to expected utility maximization. Hence, MV would be suitable for most risk-averse investors. MCSD, in contrast, is appropriate for *all* risk-averse investors. Thus, it is imperative for the analyst to find MCSD-dominant stocks when their dominance relationship is ambiguous under MV.

A more severe problem encountered with the MV criterion is that, in contrast to MSCD, it entirely ignores portfolio performance. Under the MV criterion, stocks with large standard deviations usually do not dominate other stocks. Under MCSD, however, what matters is the risk-adjusted return with risk expressed relative to the portfolio.

Now, let us compare MCSD dominance with the results obtained by applying the standard CAPM approach. Practitioners consult the so-called security market line (SML) that delineates all securities by the mean return relative to the security's systematic risk given by the mean-variance beta. As a result, practitioners will generally buy the securities above the SML and short those below the SML, because favored stocks have a higher mean return for a given level of systematic risk. Some 75 securities in our sample are charted in Exhibit 7 with an estimated SML. Following the CAPM, the securities above the line dominate the ones below it. For example, Office Max (OMX), General Motors (GM), and Comcast (CMSA), which have higher mean returns relative to their betas, are also MCSD-dominating stocks, according to Exhibit 5. The difference, however, is that using MCSD allows analysts to discriminate among the dominating stocks and find the better securities, a feat not achieved by SML analysis.

CONCLUSION

In this article, I have presented a different approach for looking at relative preferences for stocks, given that an investor already holds a portfolio. With the same data and information used in MV modeling, an analyst can obtain a more powerful evaluation of the stocks in a given portfolio by solving for MCSD. In a sense, ranking the stocks according to MCSD removes some of the ambiguity that exists with MV, especially when dominant stocks exhibit a larger variance. The main reason for this result is rooted in the way historical data is analyzed. Indeed, by ranking stock return performance relative to portfolio performance, additional information is produced so that a more discriminating dominance relationship is created. Thus, analysts can establish a list of dominant and dominated securities and change the securities' respective proportions in the portfolio. Because different investors hold different portfolios, the selection process is, in most cases, specifically different, and the recommended better securities are tailored to individual portfolios.

ENDNOTES

I am grateful to Yair Markovits for collecting and analyzing the data. I thank Shlomo Yitzhaki and an anonymous referee for their useful comments.

[1]Levy [1992, 2006] provides an excellent survey of stochastic dominance.

[2]SSD requires that the expected return on the dominating portfolio must at least equal the expected return on the dominated portfolio; otherwise, an investor would prefer the latter. This implies that when short-selling is allowed, no stochastically dominating portfolio can be found if assets have different expected returns.

[3]The lists for each year are available upon request.

[4]Mean-Gini analysis in finance originated with Shalit and Yitzhaki [1984].

[5]For the year 2006, only 98 stocks of the S&P 100 Index were analyzed because of missing trading days for two firms.

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