Finding the Influentials that Drive the Diffusion of New Technologies

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Finding the Influentials that Drive the Diffusion of New Technologies

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May 31, 2010

Abstract

In this article we consider the diffusion of similar technologies in a single market composed of many locations. We address the identification of the influential locations that drive the aggregate sales of these new technologies based on aggregate sales data and location specific online search data.

In this chapter we put forward a model where aggregate sales are a function of the online search of potential consumers at many locations. We argue that a location may be influential because of its power to drive aggregate sales and this power may be dynamic and evolving in time. Second, the influential locations may produce spillover effects over their neighbors and hence we may observe clusters of influence. We apply Bayesian Variable Selection (BVS) techniques and we use Multivariate Conditional Autoregressive Models (MCAR) to identify influentials locations and their clustering.

We apply our methodology to the video-game consoles market and to new search data of Google Insight. More precisely, we study the influential locations that drive the sales growth of the Nintendo Wii, the Sony PS3 and Microsoft Xbox 360. Specifically, we study the diffusion of these technologies at four different stages of their life-cycle. In this way, we can identify the group of influential locations and its composition in different sub-periods.

Our results indicate that the influential locations and their economic value (measured by search elasticities) vary over time. Moreover, we find significant geographical clusters of influential locations and the clusters composition varies during the life-cycle of the consoles. Finally, we find weak evidence that demographics explain the probability of a location to be influential. The main managerial implication of our results is the notion that the group of influential locations and their clustering varies during the life-cycle of a technology. Hence, managers should aim to identify the identity plus the locations and the dynamics of influentials.

keywords: diffusion, new products, variable selection, spatial modeling

1 Introduction

An important topic related to the diffusion of new technologies is the identification of influentials. Influentials play an important role as opinion leaders and trend setters and they critically affect the speed of adoption of new technologies (Van den Bulte and Joshi, 2007).

Recent attention is being given to the identification of the location and identities of these influentials. In the literature, influentials are defined as individuals or groups of individuals that influence the behavior of others in a significant way. Their influence has been studied at the individual level (Trusov et al., 2010), at the firm level (Albuquerque et al., 2007) and at the country level (van Everdingen et al., 2009). Influentials may have a specific location in a social network (Trusov et al. (2010), Christakis and Fowler (2009), Cho and Fowler (2007)) or a specific physical location (Choi et al. (2009), Goldenberg et al. (2009)). Their influence can be limited to a few others (Christakis and Fowler, 2009, page 28) but their impact may also exceed national boundaries (van Everdingen et al. (2009)).

In this article we study the diffusion of a number of similar and competing technologies and we address the identification of the influential locations that drive the aggregate sales of these new technologies. We put forward a model where sales are a function of the online search registered at many different locations. We will refer to this model as the sales-search model. We know that consumers search for technologies (or products) online and we posit that online search should be a good predictor of sales. However, people in many different locations search for products while only the consumers living in a subset of these locations may be the key groups driving the sales of new technologies. Moreover, the influential locations may not always be the same. And, the cross-influence among locations may be important and time-varying or fixed in time.

We present an approach that is new to the marketing literature and we study new search data obtained from Google Insight. Our novelty is that we use the sales-search model together with Bayesian Variable Selection techniques to select the locations that are most likely driving the aggregate sales of these three new technologies. We use this methodology because there are many possible important locations and a straightforward choice between them is not possible. In addition, we present a second model with Multivariate Conditional Autoregressive priors (known as MCAR priors) to study the cross-location influence, the significance of spatial clustering of influential locations and the competing relationships between technologies. We will refer to this model as the spatial model.

Our data consists of the aggregate weekly sales of the Nintendo Wii, the PlayStation 3 and the Microsoft Xbox 360 for the entire US market and online search data for each of these products. The online search data were obtained from Google Insight and these data consist of weekly indicators of online search for each of these technologies in each US state. The data cover a period from the launch time of each technology up to February 2010 (approximately four years) for both the sales and the online search data. This dataset is attractive because it allows us study three very successful technologies that receive worldwide interest. These three products were marketed simultaneously in all US states and this fact allows us to discard the explanation that a region may become influential because its products were available at an earlier introduction date relative to other regions.[1] Moreover, these technologies have unique names and they have kept these unique names for long periods of time and therefore we can obtain reliable online search data for all US states.[2] The sales data we observe can be easily classified in different periods of the products' life-cycle and we will identify the influential locations at these product life-cycle stages. We base these life-cycle stages on Rogers (2003) who suggests that innovations are characterized by five periods when different groups of people (innovators, early adopters, early majority, late majority and laggards) adopt an innovation. In this way we will be able to uncover the location of influential groups of adopters at

[1]For example, the launch time of the products studied by van Everdingen et al. (2009) differs across countries.

[2]Note that it is impossible to obtain state level sales data. We made inquiries at different market research firms, including NPD group, and to our knowledge there are no firms collecting these data.

different life-cycle phases of the products. Our results suggest that the influential regions driving aggregate sales differ across the life-cycle of a technology. Moreover, our approach uncovers geographical clustering of both influential and not influential regions. Influential regions seem to be close to each other but we find that their influence and the geographical clustering varies over time. In addition, we find only a weak association between demographic information and the probability that a region is influential. Finally, our results indicate that a 10% increase in local online search translates on average into a 1.5% percent increase in global sales but this number varies across regions and diffusion periods and its range goes from 0 up to 3%.

The plan of the paper is as follows. In Section 2 we discuss previous literature and its relationship to our work. In Section 3 we present our methodology. Later in Section 4 we give details about our data and some specific details regarding our model. In Section 5 we present our results and finally in Section 6 we conclude the paper. The statistical methodology that we use is presented in detail in Section A.1 and Section A.2.

2 Literature Review

The literature related to our work can be classified into micro-studies of adoption, like Choi et al. (2009), Goldenberg et al. (2009), Trusov et al. (2010), Garber et al. (2004) and Jank and Kannan (2005), and into macro-studies of technology diffusion, like van Everdingen et al. (2009), Albuquerque et al. (2007) and Putsis et al. (1997).

van Everdingen et al. (2009) examine the global spillover effects of product introductions and take-offs. They find that the product take-off in a country can help to predict the take-off of the same product in different countries. In addition, they report asymmetric patterns of influence and foreign susceptibility. The heterogeneity in the spill-over effects is significantly explained by economic and demographic characteristics. Moreover, van Everdingen et al. (2009) discuss briefly the time dimension of influence. Their results suggest that there are countries that have a large impact on others late in the diffusion process, while other countries may have a smaller influence but sooner. Albuquerque et al. (2007) study the global adoption of two ISO certification standards and their results indicate that cross-country influence is important and it improves the fit of their model. They find that the role of culture, geography and trade in the adoption process is different across the ISO standards. They use a multi-country diffusion model and therefore they assume that a firm's adoption is influenced by previous cumulative number of adoptions by other firms in different countries. Therefore, the global cumulative adoptions of ISO standards foster more adoptions. Albuquerque et al. (2007) also find that the influence of cumulative past adoptions is stronger among firms close to each other or between firms in neighboring countries. Finally, Putsis et al. (1997) study cross-country and inter-country diffusion patterns and they report important cross-country influence on diffusion. Their findings suggest that each country's influence varies from product to product.

The micro diffusion studies have documented the role and economic value of influential people in a social network (Trusov et al. (2010), Goldenberg et al. (2009)) and the formation of spatial clusters (Garber et al. (2004), Choi et al. (2009), Jank and Kannan (2005)). The study of Garber et al. (2004) deals with the spatial distribution of adoption. They discovered that the spatial pattern at early stages of the diffusion of a technology is an accurate predictor of new product success. They argue that spatial clustering is a sign of imitation and therefore if the spatial distribution of adoption shows clusters it is very likely that the diffusion process will continue and sales will eventually take off. They compare the spatial distribution of adoption against a uniform distribution of adoption and they find that successful products show an early spike of divergence between these two distributions (cross-entropy) while the cross-entropy of product failures remains relatively constant and low.

More recently, Choi et al. (2009) studied the temporal and spatial patterns of adoption in Pennsylvania and they discovered that the spatial clusters of adoption change over time and that the cross-region (cross zip code) influence decays over time. In the same way, Jank and Kannan (2005) report spatial clusters of customers with the same price sensitivity and preferences and they use spatial random effects to capture the geographical variation in preferences. The study of Hofstede et al. (2002) is focused in identifying spatial country and cross-country segments and they find evidence of contiguous and spatial clustering of consumer preferences. They argue that the spatial dependence in preferences should be useful to define distribution and marketing decisions across countries. Bradlow et al. (2005) provide an overview of spatial models and their relationship to marketing models. Finally, Trusov et al. (2010) and Goldenberg et al. (2009) suggest that influentials can have a significant economic value and they may foster the diffusion of new technologies.

In this paper we explore the time dimension and the spatial structure of influence at the level between micro and macro, that is at the regional level within a country. The objective of van Everdingen et al. (2009) and Albuquerque et al. (2007) is to identify the cross-country influence while our objective is to discover whether a region is influential and when it is influential. In contrast with previous research, in our study a region may be influential initially while later it may exert no influence at all or the other way around. That is, we consider the influence across the life-cycle of the products' diffusion while previous research has not focused particularly on this aspect. Moreover, the Bayesian Variable Selection technique that we use to detect influentials also distinguishes our study from previous work at a technical level. Finally, the visual inspection of our results suggests important geographical clusters of influential regions and we study whether these geographical clusters of influence are statistically relevant. For this latter purpose, we fit a spatial model with MCAR priors and perform tests to detect spatial clusters. It is the univariate version of this prior that has recently been applied in some marketing studies, an example is Duan and Mela (2009). The MCAR prior can incorporate both the spatial structure of the data as well as the relationship between technologies. To our knowledge, we are the first to use an MCAR prior on a marketing application while it must be mentioned that this prior is frequently used in bio-statistics and environmental studies.

3 Methodology

The approach we use consists of two main parts. First, in Section 3.1 we describe how we use Bayesian Variable Selection techniques to identify the regions and the sub-periods during which each region is likely to drive aggregate sales. The Bayesian Variable Selection technique will let us compute the posterior probability that a region is influential for any given sub-period. In Section 3.2 we specify a second model to study these posterior probabilities and our main objective in this section is to test whether there are important spatial clusters or demographic variables explaining these inclusion probabilities.

3.1 The Sales-Search Model

We observe the aggregate sales y_{it} of $i = 1, \ldots, M$ technologies at time $t = 1, \ldots, T$. We also observe the online search s_{ijt} for each of these i technologies at J different locations for $j = 1, \ldots, J$ and time periods $t = 1, \ldots, T$. In addition, s_{ijtn} will refer to the search observed at location j at a time t that is included in sub-period n, for $n = 1, \ldots, N$. We define sub-periods of diffusion because we are interested in studying the early, mid and late diffusion of the technologies.

The sales equation is specified as

$$y_{it} = \sum_j \sum_n \beta_{ijn} s_{ijtn} + \epsilon_{it} \text{ where } \epsilon_{it} \sim N(0, \sigma_i^2). \tag{1}$$

where both y_{it} and s_{ijtn} are in logs; sales are measured in hundred thousands and search is measured as an "interest indicator" and its range goes from 10 to 110. We give more details about the data in Section 4. We specify a technology i, sub-period n (for $n = 1, \ldots, N$) and region j specific coefficient β_{ijn} and the error term ϵ_{jt} is assumed to be normal with zero mean and variance σ_i^2.

This specification sums over all sub-periods n and locations j but estimating such a model may be impossible when the total number of regressors $J \times N$ is large relative to T. Note that in practice $J \times N$ can be even much larger than T. Moreover, it is very likely that many of the $\beta_{ijn} = 0$ because of the likely correlation among the s_{ijn} and the fact that some locations may simply do not drive sales. Hence, we need to select a subset location specific regressors that consists of the best set of all possible regressors. We will call the set of all possible regressors X and we will use X_γ to refer to the subset of best regressors. We will call q_γ to the total number of elements in X_γ and p to the total number of elements in X. That is, $X_\gamma \subset X$ and X is a set containing s_{ijn} for $j = 1, \ldots, J$ and $n = 1, \ldots, N$. The purpose is to select a model that sums only over this subset. Therefore we specify

$$y_{it} = \sum_j \sum_n \gamma_{ijn}\beta_{ijn}s_{ijn} + \epsilon_{it} \text{ where } \epsilon_{it} \sim N(0, \sigma^2) \tag{2}$$

as the sales equation where γ_{ijn} is a technology and region sub-period specific indicator that takes the value of 1 if s_{ijn} is in the subset X_γ and zero otherwise. Note that JN potential regressors result in 2^{JN} possible subsets and vectors γ_i where $\gamma_i = (\gamma_{i11}, \ldots, \gamma_{iJN})'$.

One could suggest for equation (2) that we could also sum over i on the right hand side and not only j and n. That is, the sales of a technology could be a function of the search for all technologies in the market. However, in our application there are over 2.57×10^{61} (that is $2^{51\times4}$ where 51 is the number of locations and 4 is the number of sub-periods) possible subsets of regressors and if we were to sum over i there would be more than 1.69×10^{184} (that is $2^{3\times52\times4}$) subsets of models. That is 6.61×10^{122} more subsets. Therefore, we study the relationship between technologies with a different model and we discuss this second model later in this section. A second issue is that sales are a function of search while at the same time search may be a function of sales. We are aware of this possible endogeneity of sales and search but as we are using local indicators for search and aggregate measures for sales we believe the endogeneity between them should be relatively weak. Finally, the right hand side could contain lags of the search indicators. However, the inclusion of lags forbids us to compare the inclusion reason across locations. For example, a location may be selected because it has an important lagged effect while another location because of its contemporaneous effect on sales. We restrict the model to a contemporaneous relationship between sales and search to be able to use the probability of a location regressor to be in X_γ at a later stage in the spatial model.

We use Bayesian Variable Selection (BVS) as presented in George and McCulloch (1997) and Chipman et al. (2001) to select the best subset of regressors. To use BVS we need proper priors, we specify $\pi(\beta_i|\sigma_i, \gamma_i)$ as in Equation (A-2) and $\pi(\sigma_i^2|\gamma_i)$ as in equation (A-4); these are the prior distributions of β_i coefficients and the variance σ_i^2 where $\beta_i = (\beta_{i11}, \ldots, \beta_{iJN})$ and we specify the prior distribution of the indicators $\pi(\gamma_i)$. We use equations (A-6) and (A-7) to define the prior on γ. BVS is an attractive technique because we can draw inferences on the probability of inclusion for each potential regressor in model (2). That is, we can draw inferences on the posterior distribution of the indicators given the data $\pi(\gamma_i|y_i)$ where $y_i = (y_{it}, \ldots, y_{iT})'$. We estimate model (2) for each of the technologies separately and details of our estimation approach are presented in the Appendix. In the Appendix we drop the sub-index i because we use the same prior specification for all technologies.

3.2 The Spatial Model

The indicator vector γ_i is composed of location and sub-period indicators and based on BVS we can compute for each element of the vector γ_i the probability that it equals one. That is, we can compute each region's posterior probability to be included at any sub-period and this posterior is available for each of the technologies. We will refer to the logit transformation[3] of this posterior probability as $\bar{p}_{ijn}$ where as before i refers to the technology, j to the location and n is the sub-period index.

[3]The function is log(p/(1-p)). A second transformation may be log(-log(p)). We tested both transformations and our results are similar.

Our objective is to test whether the variation in inclusion probabilities is explained by demographic variables and whether there are significant spatial effects in these inclusion probabilities. Hence, we propose a model where the posterior probabilities of inclusion depend on a set of covariates Z_n and their corresponding coefficients δ_n plus spatial effects Φ_n and some noise ε_n. We propose that

$$\bar{P}_n = Z_n\theta_n + \Phi_n + \varepsilon_n \tag{3}$$

where $\bar{P}_n = (\bar{p}'_{1n}, \ldots, \bar{p}'_{Mn})$, $\bar{p}'_{in} = (\bar{p}_{i1n}, \ldots, \bar{p}_{iJn})$. That is, $\bar{P}_n$ is a $J \times M$ matrix with the inclusion probabilities of each of the J locations for each technology in M columns. Z_n are covariates available for period n where Z_n is a $J \times K$ matrix where K is the number of covariates. We assume $\theta_n = \iota \otimes \delta_n$ is a $K \times M$ matrix with coefficients where ι is a row vector of ones of size M and δ_n is a $K \times 1$ vector of coefficients. $\Phi_n = (\phi'_{1n}, \ldots, \phi'_{Mn})$, $\phi'_{in} = (\phi_{i1n}, \ldots, \phi_{iJn})$ and $\varepsilon_n = (\varepsilon'_{1n}, \ldots, \varepsilon'_{Mn})$ with $\varepsilon'_{in} = (\varepsilon_{i1n}, \ldots, \varepsilon_{iJn})$. Both Φ_n and ε_n are $J \times M$ matrices.

The spatial effects Φ are a function of the relationships between technologies and the neighborhood structure of the market. The Φ matrix is composed of one spatial effect for each location and technology. Each spatial effect, in general terms, depends on the spatial effects of all technologies at neighboring locations. Hence, the spatial effects reflect spatial clustering but they do not detect the direction of influence between locations. This property of the spatial effects is specified in a prior distribution that depends on Λ, Ψ and ρ where Λ is a $M \times M$ matrix with the covariance structure between the technologies, Ψ is a $J \times J$ matrix that measures the neighborhood or the spatial structure of the market and ρ is a parameter that measures spatial auto-correlation. The element Ψ_{kl} is either a fixed distance between location k and l or an indicator that takes a value of 1 if the location k is a neighbor of l and zero otherwise. In the Appendix we provide details on how we draw inference about ρ, Λ, δ_n and the covariance matrix associated with ε_n. Note that Ψ is a fixed matrix with the neighborhood structure and hence we do not estimate it. We give more details about Ψ in the next section.

Next, we use this specification to explore if there are significant spatial effects Φ in the posterior probabilities of inclusion for each region during each sub-period n and if there is a relationship between the inclusion probabilities between technologies after controlling for the covariates in Z_n. Note that in the equation (3) we are pooling all technologies $i = 1, \dots, M$ together. The reason we pool technologies together is that their inclusion probabilities may be related to each other. For example, Texas could be the driver of growth for one technology but not for all technologies. That is, technologies may be competing against each other when the sign of the covariance terms in the Λ matrix are negative.

4 Data and Modeling Details

Weekly search indicators are available online from Google Insight for all US states and the weekly series of sales data for the video-game consoles were obtained from VGchartz.com. The data of VGchartz follows very closely the monthly figures of the NPD group. We use the latest (year 2000) demographic information of the US Census Bureau for all US states.

In Figure 1 we present a printed screen with the exact keywords that we used to retrieve the search data from Google Insights for Search (`http://www.google.com/insights/search/`). In Table 1 we provide the R code to automatically retrieve the data from `http://www.vgchartz.com/`.

To estimate the parameters of equation (2) we used MCMC and the chain ran for 210 thousand iterations and we discarded the first 10 thousand. The equation that we used includes a spline term that captures the seasonal fluctuation of y_i and its overall level. We fit a smoothing spline of y_i as a function of time and we use 10 degrees of freedom as the smoothing parameter; we refer to Hastie et al. (2001, page 127-137) for mode details on fitting smoothing splines. Sloot et al. (2006) also use spline terms to capture seasonal fluctuations. The spline term is always included on the right-hand side of the model and we do not use BVS on this term. Finally, note that we used the logs of y_i and the s_{ijn} and that y_{it} are the sales of the technology i at the end of week t and s_{ijt} is the online search index for the technology i at state j during the week t.

Next, we use MCMC to estimate the parameters of equation (3) and the chain ran for 2000 iterations and we discarded the first 1000. We used much less draws than before because convergence for a linear model is quite fast. We run the estimation for each sub-period separately and therefore we estimated the parameters of equation (3) for each period.

We divide the sales data of each consoles in four periods of equal length. These periods roughly correspond with the first four stages of adoption proposed by Rogers (2003). It is likely that in practice the length of each period varies per product or industries. For example, we know that the time to take-off is different across countries while within a country the take offs tend to occur at a systematic time difference relative to other countries (van Everdingen et al., 2009; Golder and Tellis, 1997; Tellis et al., 2003). Additionally, we choose periods of equal length to be able to compare the influential locations across products for exactly the same period of time. In this way we can naturally make cross-product comparisons.

We estimate equation (2) and equation (3) separately because we prefer not to impose any spatial structure on the prior probability of including regressors in the prior for the indicator variables, that is $\pi(\gamma)$. We estimate equation (3) for each life-cycle stage. The disadvantage of treating equations (2) and (3) separately is that the uncertainty of the first model is not taken into account in the second model. A technical reason to keep the estimation of these equations separately is that the posterior probabilities of inclusion are computed using the full MCMC chain and therefore we know them only at the end of the estimation. However, the most important reason to keep the estimation in two steps is not to impose a priori a spatial structure in the inclusion probabilities. In this way, we leave the task of testing for spatial clustering as a second step and we may be able to provide stronger evidence of any spatial structure.

We checked for convergence of the MCMC chains visually. We give more details about the estimation approach and about the MCAR models in the Appendix.

5 Results

In this section we first discuss the results for the sales-search model in equation (2) and then for the spatial model in equation (3).

5.1 Sales-Search Model Results

In Figure 2 we report the posterior distribution of the number of regressors included in the model, that is q_γ. The average number of regressors included in the model is around 17 with a minimum near 5 and a maximum of 35 regressors. If the regressors were uniformly distributed among diffusion periods this would mean an average of 4 regressors per diffusion period.[4]

In Figure 3, 4 and 5 we graphically report the posterior means of the inclusion probabilities for all US states and for the Nintendo Wii, the Sony PS3 and the Xbox 360, respectively. All these probabilities are also reported in Table 2, 3 and 4. In Figures 3, 4 and 5 the lighter (green) colors represent high posterior probabilities while the darker (red) colors represent low inclusion probabilities. We include a map of the USA including state names in Figure 17 to facilitate the reading of these figures.

In Figure 3 we can observe that the states with the higher inclusion probabilities during the first diffusion period of the Nintendo Wii are Washington, Texas, Alabama, Wyoming, Kansas and New Hampshire. So, this means that these states are more likely to drive the sales of the Wii at an early stage of the Wii's life-cycle. It is noticeable too that the Western states are more likely to be included in the first diffusion period

[4]Note that we chose $v_1 = 7$ and $a = 50$ and $b = 100$ (the parameters of the distribution of the prior inclusion probability w, see equations (A-6) and (A-7)) and this set-up results in a relatively small number of selected regressors q_γ.

while the North-Eastern states have very low probability of inclusion. However, during the third diffusion period the Western states are not likely to be included in the model while it is more likely to include states in the center and North-East of the US. In the last diffusion period we find that very few states have high probabilities and these are Montana, North Dakota and New Hampshire. That is, there are many locations driving the growth of the Wii at early life-cycle stages and relatively few engines of growth at the end.

The geographical pattern for the Sony PS3 is slightly similar to the pattern of the Nintendo Wii. However, we find that during the first diffusion period there are many more states (relative to the Wii) with high probability of inclusion. Again, all states in the West (California, Nevada, Oregon and Washington) have higher inclusion probabilities but for the PS3 many states in the East and North-East also have high probabilities during this first period. In fact, there are very few states with low probability of inclusion during the first period and these are North and South Dakota and Minnesota together with Kentucky and West Virginia. The opposite happens during the last diffusion period where many states have low probability of being included in the model. The probability of the West Coast states is high at the beginning and their influence seems to diminish in subsequent periods. The maps seem to be revealing a *boom bust* pattern. That is, many states may be influential during the first diffusion period but of this first set of countries very few remain influential in the last diffusion period and other states take the influential position.

The geographical pattern for the Microsoft Xbox 360 is very different from the other two consoles. The states with higher probabilities at each diffusion period are fewer than for the other two consoles and the influential states seem to be far from each other. However, for all regions, with the exception of Washington and Oregon, the states that seem more likely to be included in the model are in the North and North-East of the US.

An immediate question about these results is whether there is evidence of geographical clusters. At first glance, influential regions seem to be neighbors of other influential regions while not influential regions seem to be clustered together too. However, we may have some bias when judging probability distributions (Kahneman et al., 1982, page 32) and therefore we need some formal way to measure spatial association. Two statistics that can measure spatial association in aereal data are the Moran's I and the Geary's C (Banerjee et al., 2004, page 71).

We computed both the Moran's I and Geary's C for all sub-periods and technologies and we compared these two statistics, computed with the estimated inclusion probabilities, against the distribution of these two statistics when we assume that the probability of inclusion is uniformly distributed. Garber et al. (2004) also compare the observed spatial distribution of adoption against the uniform distribution. High spatial association is indicated by high Moran's I or by low Geary's C statistics. In Figure 6 we report the statistics computed with the real inclusion probabilities (in vertical dashed lines) and the distribution of both statistics (in the histograms) assuming the inclusion probabilities follow a uniform distribution.[5] As we can observe in Figure 6, when the inclusion probabilities are uniformly distributed the chances are very low to obtain the statistics in the extremes where the Moran's I and Geary's C based on the estimated inclusion probabilities appear. In the next section we discuss the results regarding the spatial model (equation (3)) where we further investigate the significance of the spatial clustering.

In the left panel of Figure 7, 8 and 9 we report the histogram of the posterior mean of the β coefficients for all sub-periods of the Nintendo Wii, the PS3 and Xbox 360, respectively. We report the distribution of the $\beta|\gamma = 1$ coefficients. That is, we report their distribution given that their corresponding regressor was included in the selected subset of regressors X_γ and we refer to these coefficients simply as β. In the right hand panel of the same figures we report the distribution of the posterior mean of the β coefficients divided by their posterior standard deviation. As we can see, the size of the β coefficients seems to be centered around 0.15 for the Nintendo Wii and the Xbox

[5]We assume that the inclusion probability of each state is independent and identically distributed from other states and they follow a uniform with range [0,1]. We draw the probability for every state from the uniform and then we compute the Moran's I and Geary's C for L number of draws to obtain the probability distribution of these two statistics.

360 and around 0.12 for the PS3. This means that on average a local (state) increase of 10% in search translates into a 1.5% or 1.2% increase in the global (nation) sales. The significance of the β coefficients varies from 1 up to 2 and there are approximately 25 regressors with a ratio (posterior mean over posterior standard deviation) higher than 1.5 and this number is quite satisfactory for a model with an average number of 17 regressors included.

In Figures 3, 4 and 5 we noticed that the probability of inclusion of different regions varies depending on the time period. In Figures 10, 11 and 12 we draw a scatter plot between the posterior mean of the search elasticity (the β coefficients) for each state and their probability of inclusion for the Nintendo Wii, the PS3 and Xbox 360, respectively. The vertical and horizontal lines correspond with the average inclusion probability and the average search elasticity, respectively. What we see in all three figures is that the place where states appear varies not only relative to their inclusion probabilities but also relative to the search elasticities. For example, in Figure 10 we see that the states with above average search elasticity and above average inclusion probability (upper right quadrant) during the first period are Kansas, New Hampshire, Delaware, New Mexico, Nebraska, Arizona, New Jersey and California. However, the upper right quadrant states that appear in the following periods are different. For example, during the forth period the upper-quadrant states are North Dakota, Montana, Maine and New Hampshire. The Figures 11 and 12 for the PS3 and Xbox 360 confirm the same pattern, different groups of states appear at each quadrant of the scatter plots at each sub-period. These results point that some states may be important earlier in the diffusion of a technology while other states become important during later states of the diffusion. Note that this result is not explained by different introduction dates as the three consoles were launched simultaneously in all US states.

The sales-search model takes into account the relationship between aggregate sales and the online search at many different locations. This provides with interesting inclusion probabilities and we can rank the states according to their power to drive the aggregate sales. If we were to ignore all these details and we run a simple regression between aggregate sales and aggregate online search we obtain the results reported in Table 13. The overall sensitivity of sales to aggregate search (an indicator of search for all US) is larger than the sensitivity of sales to state-specific search. The estimates range from 0.17 up to 0.46, see the coefficient of search in this table. These last results seem intuitive but we miss the detailed region-specific analysis and a possible spatial story behind the results of the sales search model.

5.2 Results of the Spatial Model

In Table 5 we report the posterior mean and the posterior standard deviation of the δ coefficients of the spatial model (3). In the Table we report the δ coefficients for a set of seven variables. We tested other demographic variables measuring the ethnic origin and age distribution but we did not find them as significant and they were highly correlated with the set of seven variables that we kept in the model.

As we can observe, our results indicate that there is not a very strong association between demographic variables and the inclusion probabilities at each state. The reason why the posterior standard deviations might be large is because we have only 48 states in the probability model and therefore we have very few observations to estimate the coefficients. A second reason may be that we observe a relatively small variation in our dependent variable. Nonetheless, we find some interesting features in the δ_n coefficients.

The variables that seem to be relevant are the percentage of the population in college dorms and the percentage of the population that is married (percentage of households with married couples). Both of these variables are somewhat significant during the first and second diffusion periods. The effect of travel time to work is not significant but it is most of the time negative, as we would expect given than longer commuting time reduces leisure time to play video games or to search for consoles. Population density and income per capita seem to be slightly more important in the last diffusion stage while in the first stages of diffusion they are not. A last important feature to notice is that in many cases the size and sign of the δ_n coefficients may vary according to the diffusion stage of the products. For example, it may be that students and married couples tend to buy more video-game consoles at an early stage, as a high proportion of these groups increases the chance of a state being influential, while these groups may not buy at the end of the diffusion when we see that other parameters like population density and income per capita are slightly more important.

We estimate the spatial random effects Φ_n along side with the δ_n coefficients and we report their posterior mean and their posterior mean divided by their posterior standard deviation in Tables 6, 7, 8 and 9 for the first, second, third and fourth diffusion periods, respectively. In contrast with the δ_n coefficients, several of the spatial effects are significant. For example, in Table 6 we see that the spatial effect of Texas is significant both for the Nintendo Wii and the PS3 while it is not for the Xbox 360. This means that Texas is more likely to be driving the sales of the Wii and PS3 relative to the Xbox 360 during the first diffusion period. In the same table we notice that Ohio, South Dakota and Washington are positive and significant for the Xbox 360. The spatial effect of Washington is significant for all three technologies. Tables 7, 8 and 9 show similar many significant spatial effects during the rest of the diffusion periods.

In Figures 13, 14, 15 and 16 we report the distribution of the spatial effects for the Nintendo Wii and the first, second, third and fourth diffusion periods, respectively. In Figure 13 we can observe that for the first diffusion period the states with higher posterior spatial effects are Alabama, Delaware, Kentucky, Texas, Washington and Wyoming. The states with the lowest spatial effects are Georgia, Massachusetts, Missouri and Rhode Island. Texas and Wyoming continue to have a high spatial effect in the next diffusion period, see Figure 14 but the other states that had a high spatial effect in the first period no longer continue to be high in the second. In general, the spatial effect for each state varies according to the diffusion time of the technologies. For example, according to our results Texas is very influential for the Nintendo Wii at an early stage of its life-cycle while this state is not influential at the end of the life-cycle of the Wii.

We are finding significant spatial random effects for several states and all diffusion periods. However, a natural concern is whether the δ_n coefficients may have a different level of significance if we were to exclude the spatial effects from equation (3). In Table 10 we report the same δ_n coefficients estimated with ordinary least squares and their level of significance is relatively the same as before. Again, the population in college dorms and the percentage of households with married couples seem to be the more important variables. That is, the spatial effects explain geographical variation without affecting the inference we draw from the posteriors of the δ_n coefficients.

In Table 11 we present the posterior distribution of the correlations derived from the matrix Λ. The matrix Λ is a 3×3 covariance matrix and it measures the covariance between the spatial effects of different technologies. In the first diffusion period, for example, we find that the correlation of the spatial effects of the Xbox 360 are negatively correlated with the spatial effects of the PS3. The posterior mean of the correlation is -0.257 and the association is significant (zero is almost excluded from the 95% highest posterior density region). This negative correlation implies that if a state is likely to drive the sales of the Xbox 360 then it is not likely to drive the sales of the PS3. The association between the spatial effects of the Wii and those of the Xbox and PS3 are not different from zero (in these cases 0 is almost in the middle of the highest posterior density region) during the first diffusion period. We find some other significant associations during the third and fourth periods while in the second period we find no association between the spatial effects of the different technologies. The variation in correlation structure shows that at an early stage there is some degree of competition only between the PS3 and Xbox 360 (because of the negative correlation in their spatial effects) while at later stages technologies seem to nurture each other (because we find significant positive correlations in their spatial effects).

Finally, in Table 12 we report the highest posterior density region for the ρ coefficients. We find roughly the same spatial decay (or spatial correlation) during all diffusion periods. The posterior mean of the ρ_n for all n is around 0.82. This number should be between

0 and 1 and numbers close to 1 indicate high spatial correlation between a state and its neighbors. The estimate of the ρ coefficient together with the Φ_n spatial effects are evidence of significant clusters of spillover effects between states. We do not know the direction of influence between the states but the model parameters capture significant spatial dependence among neighboring states.

6 Conclusions

We applied Bayesian variable selection methods to identify the influential locations for the diffusion of new technologies. We define influential locations as those that are more likely to drive the aggregate sales of the technologies. For our particular data on game consoles, we find that the influential locations change over time and that there is geographical clustering that is significantly captured by the spatial random effects in the probability model and by different measures of spatial association.

Moreover, we find variation in the groups of influential locations over time and the size of their associated search elasticity varies over time too. The search elasticity for the technologies at influential locations is on average 0.15. That is, an increase of 10% in local (state) search translates into a 1.5% increase in country level sales. Finally, we find some evidence of time variation in the association between spatial affects. Our results suggest that the geographical clustering is not driven by demographic heterogeneity and we find some evidence that suggests that the demographic effects vary over time.

In summary, our results suggest that influential locations may change over time together with the relationship between technologies and the relevance of demographics. The main managerial implications of this research is the notion that the group of influential locations is not fixed and therefore when a manager is looking to identify influentials, she or he should expect influentials to play a role at different locations and at different times. If managers were to ignore the spatial heterogeneity they will miss the valuable insights of how to allocate their marketing efforts based on the important locations for their products. The relevant question is not only who is influential but where and when and for how long a consumer or a group of consumers is influential.

7 Tables and Figures

```
library(RCurl)
library(XML)
wii_sales<-rep(0,416)
week.numbers<-seq((39838)-2184,40358,by=7)
for(i in 1:416)
{
part1<-"http://vgchartz.com/hwtable.php?cons[]=Wii&reg[]=America&start="
part2<-"&end="
week<-week.numbers[i]
url.dir<-paste(part1,week,part2,week,sep="")
url.text <- getURL(url.dir)
doc <- htmlParse(url.text,useInternalNodes=TRUE, error=function(...){})
x = xpathSApply(doc, "//table//td|//table//th", xmlValue)
wii_sales[i]<-as.numeric(gsub(",", ".", x[12]))
}
write.csv(wii_sales,file="wii_data.csv")
```

Note that the keyword `Wii` should be changed to `PS3` or `X360` to retrieve the data for each of these consoles.

Table 1: R Code to Retrieve Data from VGChartz.com

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	**0.111**	0.083	0.080	**0.117**
Alaska	0.074	0.069	0.091	**0.115**
Arizona	0.092	**0.110**	0.070	0.075
Arkansas	0.091	0.093	**0.103**	0.089
California	0.093	0.093	0.081	**0.116**
Colorado	0.076	0.069	0.098	0.079
Connecticut	0.074	0.057	**0.103**	0.071
Delaware	**0.105**	0.058	0.079	**0.103**
District of Columbia	0.076	**0.108**	0.077	0.083
Florida	0.096	0.061	0.090	0.077
Georgia	0.056	0.079	0.089	0.076
Hawaii	0.072	0.096	0.077	0.099
Idaho	0.086	0.082	0.075	**0.112**
Illinois	0.073	0.092	0.080	0.066
Indiana	0.079	0.059	0.065	0.082
Iowa	0.077	0.077	0.083	**0.125**
Kansas	**0.108**	0.085	0.088	0.083
Kentucky	0.075	0.091	0.093	0.099
Louisiana	**0.102**	**0.122**	0.081	0.065
Maine	0.079	0.080	0.090	**0.137**
Maryland	0.059	0.088	0.057	0.079
Massachusetts	0.084	**0.119**	0.096	0.074
Michigan	0.070	0.079	0.086	0.086
Minnesota	0.078	0.098	0.088	0.074
Mississippi	0.058	0.092	**0.105**	0.060
Missouri	0.086	0.075	0.088	0.093
Montana	0.095	0.084	0.099	**0.173**
Nebraska	0.092	0.073	0.093	0.090
Nevada	0.096	0.096	0.068	0.094
New Hampshire	**0.105**	0.097	0.076	**0.154**
New Jersey	0.095	**0.127**	**0.103**	0.073
New Mexico	0.099	0.096	**0.113**	**0.105**
New York	0.078	0.068	0.080	0.054
North Carolina	0.096	0.071	0.083	0.066
North Dakota	0.081	0.086	0.082	**0.190**
Ohio	0.078	0.090	**0.102**	0.089
Oklahoma	0.082	0.098	0.081	0.078
Oregon	0.098	**0.144**	0.063	0.055
Pennsylvania	0.064	0.081	0.065	0.062
Rhode Island	0.086	0.074	0.082	**0.101**
South Carolina	0.090	0.075	0.083	0.097
South Dakota	0.098	0.079	0.070	0.098
Tennessee	0.092	0.073	**0.119**	0.068
Texas	**0.129**	0.075	0.086	0.094
Utah	0.097	0.097	0.089	0.097
Vermont	0.076	0.073	**0.136**	0.091
Virginia	**0.100**	0.070	0.079	0.086
Washington	**0.126**	0.073	0.065	0.095
West Virginia	0.073	0.062	**0.108**	**0.119**
Wisconsin	0.072	0.076	**0.131**	0.060
Wyoming	**0.107**	**0.115**	**0.107**	**0.119**
Note: In bold probabilities larger than 0.10				

Table 2: State Inclusion Probabilities for Each Diffusion Period for the Nintendo Wii

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	0.088	0.073	0.094	0.086
Alaska	0.081	0.094	0.084	**0.185**
Arizona	0.081	0.063	0.091	0.057
Arkansas	**0.101**	0.090	0.098	0.093
California	0.096	0.096	0.088	0.080
Colorado	**0.106**	0.092	0.092	0.099
Connecticut	**0.104**	**0.102**	0.093	0.076
Delaware	0.086	0.078	**0.118**	0.090
District of Columbia	0.088	0.099	0.098	0.075
Florida	**0.100**	0.079	0.091	0.083
Georgia	0.095	0.097	**0.103**	0.067
Hawaii	0.098	0.080	0.094	0.088
Idaho	0.092	0.085	0.080	0.076
Illinois	0.091	**0.107**	0.082	0.088
Indiana	0.085	0.081	**0.104**	0.085
Iowa	0.087	**0.102**	0.087	0.093
Kansas	0.079	0.094	0.083	0.080
Kentucky	0.070	0.098	0.084	0.085
Louisiana	0.087	0.093	0.091	0.076
Maine	0.086	0.071	0.073	**0.115**
Maryland	0.095	**0.119**	0.085	0.093
Massachusetts	0.095	0.093	0.082	0.071
Michigan	0.089	**0.109**	0.081	0.086
Minnesota	0.073	0.068	0.081	0.086
Mississippi	0.086	0.085	0.087	0.078
Missouri	0.084	0.093	0.087	0.084
Montana	0.091	0.089	0.089	**0.103**
Nebraska	0.092	**0.109**	0.089	0.093
Nevada	0.096	0.087	0.090	0.072
New Hampshire	0.091	0.087	0.090	**0.140**
New Jersey	0.090	0.094	0.072	0.071
New Mexico	0.083	0.097	0.069	**0.105**
New York	0.096	0.089	0.093	0.064
North Carolina	**0.103**	0.083	0.082	0.071
North Dakota	0.070	0.076	0.084	0.094
Ohio	0.088	0.097	**0.105**	0.073
Oklahoma	0.080	0.091	0.091	0.084
Oregon	**0.104**	0.077	**0.101**	**0.102**
Pennsylvania	0.089	0.091	0.079	0.074
Rhode Island	0.081	0.087	0.082	**0.130**
South Carolina	0.090	0.092	0.076	0.090
South Dakota	0.066	0.068	0.079	0.094
Tennessee	0.089	0.087	0.095	0.091
Texas	**0.108**	0.093	**0.113**	0.065
Utah	**0.101**	0.072	**0.109**	0.097
Vermont	0.090	0.086	**0.100**	**0.141**
Virginia	0.096	0.083	0.062	0.073
Washington	**0.101**	0.081	0.095	0.069
West Virginia	0.074	0.083	**0.106**	0.097
Wisconsin	0.089	0.087	0.092	0.095
Wyoming	0.092	0.086	0.094	0.090

Note: In bold probabilities larger than 0.10

Table 3: State Inclusion Probabilities for Each Diffusion Period for the Sony PS3

	Posterior Inclusion Probabilities			
	1st Period	2nd Period	3rd Period	4th Period
Alabama	0.085	0.091	0.090	0.079
Alaska	**0.104**	**0.188**	0.080	**0.199**
Arizona	0.077	0.074	0.080	0.054
Arkansas	0.098	0.082	0.087	0.075
California	0.099	0.082	0.075	0.074
Colorado	0.078	0.088	0.087	0.084
Connecticut	0.078	0.081	0.091	**0.101**
Delaware	**0.116**	**0.136**	0.075	**0.204**
District of Columbia	0.091	0.096	0.097	0.071
Florida	**0.102**	0.066	**0.100**	0.065
Georgia	0.084	0.073	**0.115**	0.092
Hawaii	0.089	**0.115**	0.055	0.076
Idaho	0.087	**0.109**	0.076	**0.137**
Illinois	0.086	0.075	**0.105**	**0.100**
Indiana	0.086	0.087	0.074	0.064
Iowa	0.097	**0.126**	0.083	**0.100**
Kansas	**0.114**	0.082	0.087	0.081
Kentucky	**0.103**	0.078	**0.101**	**0.109**
Louisiana	0.067	0.074	0.082	0.058
Maine	0.097	**0.113**	0.097	0.095
Maryland	0.087	0.066	0.085	0.087
Massachusetts	0.095	**0.100**	0.085	0.079
Michigan	0.092	0.076	0.096	0.082
Minnesota	0.097	0.092	0.073	0.095
Mississippi	0.096	0.062	**0.131**	0.080
Missouri	0.079	0.087	0.098	0.073
Montana	0.071	0.059	0.087	0.096
Nebraska	0.084	0.067	0.071	0.095
Nevada	0.093	0.074	0.071	0.084
New Hampshire	0.089	0.098	0.089	**0.119**
New Jersey	0.085	**0.110**	0.095	0.071
New Mexico	0.091	**0.112**	0.071	**0.100**
New York	0.083	**0.106**	**0.101**	0.093
North Carolina	0.091	**0.103**	0.090	0.066
North Dakota	**0.129**	0.082	**0.113**	**0.113**
Ohio	0.099	0.094	0.083	0.079
Oklahoma	0.086	0.085	0.084	0.094
Oregon	**0.116**	0.081	0.087	0.081
Pennsylvania	0.096	0.087	0.093	0.085
Rhode Island	**0.102**	**0.113**	0.092	**0.127**
South Carolina	0.090	0.081	0.082	0.073
South Dakota	**0.132**	0.097	**0.118**	**0.102**
Tennessee	0.084	0.096	**0.134**	0.089
Texas	0.093	0.073	0.078	0.063
Utah	0.085	0.088	0.077	0.059
Vermont	0.082	**0.110**	**0.152**	0.082
Virginia	**0.103**	0.074	0.078	0.064
Washington	**0.111**	**0.101**	**0.100**	0.079
West Virginia	0.085	0.065	**0.140**	**0.114**
Wisconsin	0.090	0.087	0.093	0.086
Wyoming	0.070	**0.125**	**0.105**	0.084

Note: In bold probabilities larger than 0.10

Table 4: State Inclusion Probabilities for Each Diffusion Period for the Microsoft Xbox 360

	MCAR First Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3684	0.0163	-145.2131
Male Female Ratio	0.0103	0.0326	0.3152
Population Density	0.0042	0.0270	0.1569
Population in College Dorms	0.0337	0.0200	1.6820
Married Couple	0.0236	0.0171	1.3834
Travel Time to Work	-0.0015	0.0194	-0.0751
Income per Capita	0.0106	0.0157	0.6723

	MCAR Second Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4029	0.0177	-135.4291
Male Female Ratio	0.0345	0.0361	0.9561
Population Density	0.0357	0.0285	1.2506
Population in College Dorms	0.0304	0.0232	1.3138
Married Couple	-0.0208	0.0202	-1.0332
Travel Time to Work	-0.0183	0.0221	-0.8307
Income per Capita	-0.0185	0.0185	-1.0007

	MCAR Third Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3715	0.0214	-110.8737
Male Female Ratio	-0.0192	0.0409	-0.4694
Population Density	-0.0294	0.0343	-0.8562
Population in College Dorms	-0.0245	0.0251	-0.9758
Married Couple	0.0231	0.0251	0.9208
Travel Time to Work	0.0163	0.0258	0.6329
Income per Capita	-0.0103	0.0199	-0.5181

	MCAR Fourth Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3987	0.0283	-84.6213
Male Female Ratio	-0.0305	0.0589	-0.5171
Population Density	0.0464	0.0474	0.9795
Population in College Dorms	-0.0218	0.0339	-0.6436
Married Couple	-0.0013	0.0324	-0.0402
Travel Time to Work	-0.0157	0.0352	-0.4457
Income per Capita	0.0191	0.0267	0.7141

Note: The first column reports the posterior mean of the coefficient. The second column reports the posterior standard deviation and the third column reports the ratio of the posterior mean over the posterior standard deviation, called here t-value.

Table 5: Posterior of MCAR δ coefficients

	MCAR First Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.260	3.990	0.025	0.380	-0.011	-0.196
Arizona	0.053	0.780	-0.066	-1.203	-0.131	-2.336
Arkansas	0.041	0.770	0.141	2.329	0.110	2.093
California	-0.038	-0.481	-0.001	-0.035	0.023	0.356
Colorado	-0.164	-2.412	0.154	2.322	-0.141	-2.268
Connecticut	-0.213	-2.762	0.135	1.903	-0.155	-2.167
Delaware	0.201	3.464	-0.005	-0.103	0.302	5.033
Florida	0.081	1.306	0.132	2.304	0.150	2.406
Georgia	-0.462	-6.174	0.063	1.322	-0.055	-1.172
Idaho	-0.243	-3.420	0.053	0.952	-0.034	-0.525
Illinois	-0.070	-1.349	-0.007	-0.261	-0.053	-1.149
Indiana	-0.232	-4.463	-0.014	-0.237	-0.066	-1.417
Iowa	-0.138	-2.370	-0.058	-1.276	-0.051	-0.861
Kansas	-0.162	-3.003	-0.037	-0.728	0.074	1.578
Kentucky	0.205	3.492	-0.101	-2.010	0.255	4.228
Louisiana	-0.097	-1.705	-0.174	-2.826	0.218	3.517
Maine	0.183	2.815	0.010	0.151	-0.254	-4.285
Maryland	-0.099	-1.467	-0.020	-0.460	0.105	1.764
Massachusetts	-0.443	-5.272	0.027	0.278	-0.064	-0.882
Michigan	-0.078	-1.512	0.041	0.791	0.047	0.952
Minnesota	-0.264	-4.806	-0.037	-0.761	0.009	0.107
Mississippi	-0.068	-0.913	-0.137	-2.282	0.152	2.150
Missouri	-0.409	-5.690	-0.020	-0.524	0.080	1.520
Montana	-0.013	-0.253	-0.037	-0.700	-0.095	-1.832
Nebraska	0.069	1.258	0.026	0.548	-0.217	-3.822
Nevada	0.056	0.629	0.055	0.717	-0.030	-0.324
New Hampshire	0.060	0.952	0.060	1.082	0.027	0.455
New Jersey	0.134	1.442	-0.011	-0.131	-0.034	-0.400
New Mexico	0.129	2.338	0.066	1.228	0.012	0.143
New York	0.052	0.622	-0.116	-1.556	-0.029	-0.410
North Carolina	-0.160	-3.347	0.046	0.966	-0.099	-1.794
North Dakota	0.089	1.459	0.153	2.331	0.037	0.568
Ohio	-0.126	-2.172	-0.266	-3.950	0.342	5.245
Oklahoma	-0.125	-2.507	0.003	0.129	0.118	2.365
Oregon	-0.065	-1.420	-0.081	-1.685	-0.020	-0.365
Pennsylvania	0.027	0.433	0.075	1.114	0.187	2.729
Rhode Island	-0.289	-3.190	0.059	0.587	0.133	1.393
South Carolina	0.035	0.472	0.044	0.780	0.043	0.771
South Dakota	0.108	2.093	-0.276	-4.334	0.409	6.168
Tennessee	0.034	0.688	0.006	0.111	-0.054	-1.247
Texas	0.314	4.926	0.128	2.476	-0.020	-0.262
Utah	0.009	0.147	0.047	0.716	-0.124	-1.475
Vermont	-0.139	-2.786	0.037	0.498	-0.057	-1.164
Virginia	0.082	1.547	0.038	0.831	0.109	2.372
Washington	0.353	6.191	0.121	2.318	0.224	4.035
West Virginia	-0.160	-2.826	-0.142	-2.508	-0.011	-0.243
Wisconsin	-0.242	-4.815	-0.024	-0.626	-0.013	-0.288
Wyoming	0.180	2.908	0.033	0.611	-0.239	-3.781

Note: The numbers correspond to the Φ parameters of the MCAR model for the first diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 6: Posterior of MCAR Spatial Effects

	MCAR Second Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.005	0.082	-0.106	-1.394	0.096	1.139
Arizona	0.231	2.843	-0.278	-2.827	-0.148	-1.867
Arkansas	0.096	1.393	0.061	0.972	-0.024	-0.365
California	-0.004	-0.039	0.019	0.185	-0.134	-1.459
Colorado	-0.211	-2.324	0.053	0.683	0.005	0.091
Connecticut	-0.392	-4.157	0.172	1.713	-0.061	-0.599
Delaware	-0.365	-4.093	-0.080	-0.950	0.482	5.568
Florida	-0.350	-4.939	-0.090	-1.367	-0.284	-3.924
Georgia	-0.072	-1.085	0.118	1.746	-0.153	-2.014
Idaho	0.113	1.363	-0.057	-0.792	0.288	3.083
Illinois	-0.063	-0.859	-0.020	-0.334	0.217	2.850
Indiana	0.046	0.592	0.180	2.409	-0.152	-2.212
Iowa	-0.349	-4.061	-0.047	-0.740	0.017	0.274
Kansas	-0.100	-1.419	0.159	2.145	0.382	4.606
Kentucky	0.005	0.129	0.095	1.494	-0.030	-0.495
Louisiana	0.072	0.934	0.127	1.718	-0.089	-1.073
Maine	0.406	5.404	0.123	1.628	-0.104	-1.115
Maryland	-0.035	-0.398	-0.140	-1.670	0.305	3.512
Massachusetts	-0.054	-0.581	0.232	2.550	-0.321	-3.198
Michigan	0.306	4.063	0.053	0.796	0.129	1.833
Minnesota	-0.084	-1.101	0.223	2.738	-0.111	-1.645
Mississippi	0.139	1.745	-0.203	-2.384	0.085	1.070
Missouri	0.082	1.156	0.018	0.254	-0.278	-3.091
Montana	-0.163	-2.117	0.044	0.563	-0.017	-0.213
Nebraska	-0.020	-0.301	0.034	0.612	-0.351	-3.717
Nevada	-0.212	-1.893	0.161	1.571	-0.299	-2.607
New Hampshire	0.176	2.273	0.074	0.955	-0.086	-1.124
New Jersey	0.091	0.830	-0.013	-0.146	0.108	1.013
New Mexico	0.372	4.661	0.069	1.076	0.234	2.975
New York	0.029	0.272	0.032	0.391	0.182	1.950
North Carolina	-0.259	-3.558	0.010	0.128	0.177	2.434
North Dakota	-0.237	-2.899	-0.075	-1.059	0.134	1.736
Ohio	-0.036	-0.403	-0.137	-1.909	-0.087	-1.222
Oklahoma	0.052	0.714	0.102	1.671	0.087	1.135
Oregon	0.128	1.746	0.050	0.816	-0.012	-0.202
Pennsylvania	0.442	4.291	-0.143	-1.726	-0.105	-1.268
Rhode Island	-0.139	-1.234	-0.022	-0.172	-0.079	-0.691
South Carolina	-0.133	-1.663	0.080	1.223	-0.055	-0.710
South Dakota	-0.102	-1.486	-0.216	-2.726	0.096	1.267
Tennessee	-0.137	-1.954	0.027	0.502	0.108	1.570
Texas	-0.171	-2.121	0.043	0.518	-0.207	-2.642
Utah	0.160	1.554	-0.118	-1.298	0.078	0.766
Vermont	-0.137	-1.648	0.025	0.432	0.269	3.278
Virginia	-0.181	-2.476	-0.013	-0.137	-0.125	-1.923
Washington	-0.156	-1.911	-0.051	-0.760	0.164	1.966
West Virginia	-0.251	-2.873	0.018	0.230	-0.211	-2.454
Wisconsin	-0.143	-1.858	0.000	-0.016	-0.011	-0.187
Wyoming	0.254	2.904	-0.031	-0.453	0.333	3.692

Note: The numbers correspond to the Φ parameters of the MCAR model for the second diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 7: Posterior of MCAR Spatial Effects

	MCAR Third Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	-0.155	-2.445	0.003	0.000	-0.045	-0.739
Arizona	-0.247	-4.132	0.003	0.112	-0.122	-2.098
Arkansas	0.085	1.788	0.038	0.831	-0.075	-1.489
California	-0.016	-0.192	0.060	0.748	-0.093	-1.187
Colorado	0.111	1.571	0.056	0.795	0.002	0.000
Connecticut	0.216	2.797	0.107	1.416	0.093	1.206
Delaware	-0.120	-2.013	0.292	4.726	-0.169	-2.349
Florida	0.033	0.730	0.040	0.932	0.141	3.158
Georgia	-0.003	-0.161	0.134	2.776	0.256	5.227
Idaho	-0.209	-2.808	-0.011	-0.166	-0.548	-5.979
Illinois	-0.143	-3.097	-0.077	-1.455	-0.135	-2.773
Indiana	-0.104	-2.571	-0.069	-1.580	0.176	3.609
Iowa	-0.343	-5.740	0.131	2.517	-0.213	-3.606
Kansas	-0.089	-1.768	-0.039	-0.748	-0.090	-1.730
Kentucky	-0.055	-1.379	-0.114	-2.935	-0.068	-1.564
Louisiana	0.013	0.159	-0.083	-1.145	0.102	1.350
Maine	-0.147	-2.509	-0.026	-0.453	-0.136	-2.225
Maryland	0.029	0.381	-0.183	-2.747	0.101	1.594
Massachusetts	-0.336	-3.908	0.069	0.842	0.068	0.830
Michigan	0.107	2.373	-0.049	-1.242	-0.023	-0.579
Minnesota	-0.028	-0.632	-0.086	-1.840	0.081	1.874
Mississippi	-0.049	-0.737	-0.129	-1.942	-0.244	-3.254
Missouri	0.149	2.612	-0.041	-0.930	0.369	5.740
Montana	-0.035	-0.683	-0.039	-0.789	0.084	1.485
Nebraska	0.081	1.574	-0.028	-0.577	-0.045	-0.986
Nevada	0.074	0.727	0.034	0.368	-0.194	-1.727
New Hampshire	-0.316	-5.361	-0.027	-0.516	-0.273	-4.767
New Jersey	-0.079	-0.704	0.090	0.850	0.074	0.646
New Mexico	0.118	2.261	-0.233	-3.983	0.036	0.626
New York	0.340	3.580	-0.147	-1.705	-0.120	-1.397
North Carolina	-0.092	-2.102	0.059	1.387	0.142	3.221
North Dakota	-0.079	-1.263	-0.082	-1.350	0.007	0.153
Ohio	-0.046	-0.761	-0.010	-0.245	0.280	4.138
Oklahoma	0.106	2.612	0.127	3.062	-0.109	-2.295
Oregon	-0.098	-2.227	0.018	0.431	-0.061	-1.229
Pennsylvania	-0.286	-3.265	0.176	2.233	0.033	0.389
Rhode Island	-0.222	-1.949	-0.023	-0.192	0.144	1.246
South Carolina	-0.089	-1.493	-0.176	-2.974	-0.105	-2.070
South Dakota	-0.252	-4.378	-0.132	-2.261	0.261	4.083
Tennessee	0.276	5.944	0.047	1.130	0.396	6.843
Texas	-0.020	-0.361	0.250	4.013	-0.115	-1.882
Utah	-0.094	-0.951	0.098	0.941	-0.242	-2.204
Vermont	0.411	7.663	0.088	1.529	0.530	9.155
Virginia	-0.113	-2.047	-0.343	-6.009	-0.124	-2.607
Washington	-0.321	-5.615	0.069	1.359	0.119	2.074
West Virginia	0.117	1.873	0.100	1.595	0.386	5.353
Wisconsin	0.404	7.422	0.040	0.977	0.056	1.251
Wyoming	0.180	2.606	0.042	0.677	0.154	2.353

Note: The numbers correspond to the Φ parameters of the MCAR model for the third diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 8: Posterior of MCAR Spatial Effects

	MCAR Fourth Diffusion Period					
	Wii	t-value	PS3	t-value	Xbox360	t-value
Alabama	0.311	3.294	-0.008	-0.171	-0.092	-1.108
Arizona	-0.092	-1.076	-0.367	-4.305	-0.412	-4.807
Arkansas	0.044	0.613	0.078	1.217	-0.123	-1.864
California	0.377	3.187	-0.005	-0.083	-0.080	-0.759
Colorado	-0.040	-0.449	0.178	1.738	0.022	0.201
Connecticut	-0.386	-3.484	-0.306	-2.844	-0.017	-0.144
Delaware	0.079	0.999	-0.045	-0.631	0.806	9.027
Florida	-0.136	-2.313	-0.065	-1.059	-0.313	-5.411
Georgia	-0.103	-1.827	-0.228	-3.586	0.088	1.323
Idaho	0.197	1.842	0.072	0.719	-0.077	-0.745
Illinois	0.277	4.295	-0.109	-1.978	0.485	7.363
Indiana	-0.258	-4.541	0.022	0.413	0.158	3.015
Iowa	-0.063	-0.839	-0.021	-0.271	-0.293	-4.108
Kansas	0.395	6.189	0.079	1.320	0.156	2.558
Kentucky	-0.027	-0.567	-0.062	-1.127	-0.049	-0.780
Louisiana	0.155	1.626	-0.006	-0.162	0.254	2.682
Maine	-0.307	-3.621	-0.149	-1.864	-0.412	-4.844
Maryland	0.372	3.837	0.191	2.154	0.002	-0.065
Massachusetts	-0.233	-2.048	-0.065	-0.597	-0.140	-1.277
Michigan	-0.155	-2.323	-0.191	-3.465	-0.089	-1.414
Minnesota	0.006	0.061	0.007	0.110	-0.038	-0.671
Mississippi	-0.137	-1.491	0.007	-0.006	0.113	1.163
Missouri	-0.374	-4.820	-0.103	-1.523	-0.076	-1.138
Montana	0.118	1.436	0.006	0.001	-0.132	-1.815
Nebraska	0.729	8.505	0.187	2.936	0.118	1.865
Nevada	0.132	0.819	0.160	1.043	0.180	1.101
New Hampshire	0.066	0.846	-0.196	-2.587	-0.054	-0.703
New Jersey	0.394	2.581	0.285	1.962	0.124	0.840
New Mexico	-0.151	-2.031	-0.168	-2.577	-0.181	-2.489
New York	0.219	1.758	0.203	1.710	0.162	1.303
North Carolina	-0.462	-7.091	-0.277	-4.951	0.081	1.415
North Dakota	-0.247	-3.025	-0.166	-1.989	-0.251	-3.191
Ohio	0.805	8.115	0.075	0.965	0.260	3.472
Oklahoma	0.050	1.012	-0.146	-2.722	-0.071	-1.272
Oregon	-0.073	-1.372	-0.004	-0.084	0.109	1.615
Pennsylvania	-0.443	-3.824	0.165	1.630	-0.056	-0.544
Rhode Island	-0.566	-3.806	-0.380	-2.586	-0.243	-1.660
South Carolina	0.121	1.680	0.038	0.472	-0.175	-2.585
South Dakota	0.149	2.292	0.100	1.566	0.191	2.710
Tennessee	-0.235	-3.317	0.053	0.929	0.029	0.457
Texas	0.163	1.913	-0.198	-2.502	-0.239	-3.037
Utah	0.178	1.235	0.169	1.186	-0.324	-2.171
Vermont	0.052	0.736	0.489	6.011	-0.060	-0.965
Virginia	-0.011	-0.171	-0.166	-2.960	-0.304	-5.075
Washington	0.133	1.688	-0.188	-2.728	-0.060	-0.839
West Virginia	0.355	3.908	0.144	1.711	0.302	3.604
Wisconsin	-0.351	-5.133	0.099	1.862	0.008	0.121
Wyoming	0.375	3.677	0.088	0.941	0.018	0.213

Note: The numbers correspond to the Φ parameters of the MCAR model for the fourth diffusion period. We report the posterior mean of the spatial effects for the Wii, PS3 and X360 and the ratio of the posterior mean over the posterior standard deviation.

Table 9: Posterior of MCAR Spatial Effects

	OLS First Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3730	0.0131	-180.7930
Male Female Ratio	0.0185	0.0195	0.9480
Population Density	0.0039	0.0228	0.1720
Population in College Dorms	0.0263	0.0164	1.6040
Married Couple	0.0198	0.0161	1.2250
Travel Time to Work	0.0131	0.0194	0.6740
Income per Capita	-0.0028	0.0217	-0.1300

	OLS Second Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4053	0.0159	-151.0310
Male Female Ratio	0.0218	0.0237	0.9230
Population Density	0.0239	0.0277	0.8620
Population in College Dorms	0.0257	0.0199	1.2910
Married Couple	-0.0092	0.0196	-0.4710
Travel Time to Work	-0.0136	0.0235	-0.5780
Income per Capita	-0.0194	0.0263	-0.7390

	OLS Third Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.3730	0.0131	-180.7930
Male Female Ratio	0.0185	0.0195	0.9480
Population Density	0.0039	0.0228	0.1720
Population in College Dorms	0.0263	0.0164	1.6040
Married Couple	0.0198	0.0161	1.2250
Travel Time to Work	0.0131	0.0194	0.6740
Income per Capita	-0.0028	0.0217	-0.1300

	OLS Fourth Diffusion Period		
	Coefficient	St. Dev.	t-value
Intercept	-2.4008	0.0203	-118.0540
Male Female Ratio	-0.0197	0.0302	-0.6520
Population Density	0.0355	0.0354	1.0030
Population in College Dorms	-0.0237	0.0254	-0.9310
Married Couple	0.0210	0.0250	0.8400
Travel Time to Work	0.0189	0.0300	0.6290
Income per Capita	0.0182	0.0336	0.5410

Note: These are parameter estimates of the model in equation (3) obtained by OLS and with no spatial effects.

Table 10: OLS δ coefficients

	MCAR First Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.075	-0.185	0.337
Λ_{13} (Wii-Xbox)	0.115	-0.139	0.352
Λ_{23} (PS3-Xbox)	-0.257	-0.488	0.036

	MCAR Second Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	-0.082	-0.344	0.210
Λ_{13} (Wii-Xbox)	0.096	-0.156	0.354
Λ_{23} (PS3-Xbox)	-0.103	-0.377	0.179

	MCAR Third Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.061	-0.204	0.302
Λ_{13} (Wii-Xbox)	0.401	0.145	0.600
Λ_{23} (PS3-Xbox)	0.117	-0.137	0.368

	MCAR Fourth Period		
	Mean	5%	95%
Λ_{12} (Wii-PS3)	0.409	0.156	0.601
Λ_{13} (Wii-Xbox)	0.349	0.090	0.552
Λ_{23} (PS3-Xbox)	0.311	0.058	0.534

Note: We present the posterior mean and the posterior 95% highest density region of the correlation matrix obtained from the Λ matrix. The Λ matrix measures the covariance between the spatial effects of the three products.

Table 11: Posterior of MCAR Λ correlations

	HPDR		
	95%	50%	5%
MCAR 1st period ρ	0.975	0.805	0.150
MCAR 2nd period ρ	0.975	0.825	0.150
MCAR 3rd period ρ	0.975	0.825	0.150
MCAR 4th period ρ	0.975	0.815	0.200
Note:			

Table 12: Highest Posterior Density Region (HPDR) for the ρ coefficient.

	Aggregate Sales Model for the Wii		
Variable	Estimate	Std. Error	t-value
spline	0.663	0.088	7.554
Search Wii	0.468	0.121	3.862

	Aggregate Sales Model for the PS3		
Variable	Estimate	Std. Error	t-value
spline	0.862	0.108	7.974
Search PS3	0.171	0.133	1.287

	Aggregate Sales Model for the X360		
Variable	Estimate	Std. Error	t-value
spline	0.722	0.122	5.916
Search X360	0.375	0.163	2.304

Note: The dependent variable is aggregate sales for each of the consoles (in logs). The right hand side includes a spline term and the logs of the search index for the console. The R^2 is higher than 0.95 for all three regressions.

Table 13: OLS Regressions between Aggregate Sales Data and Aggregate Online Search Data

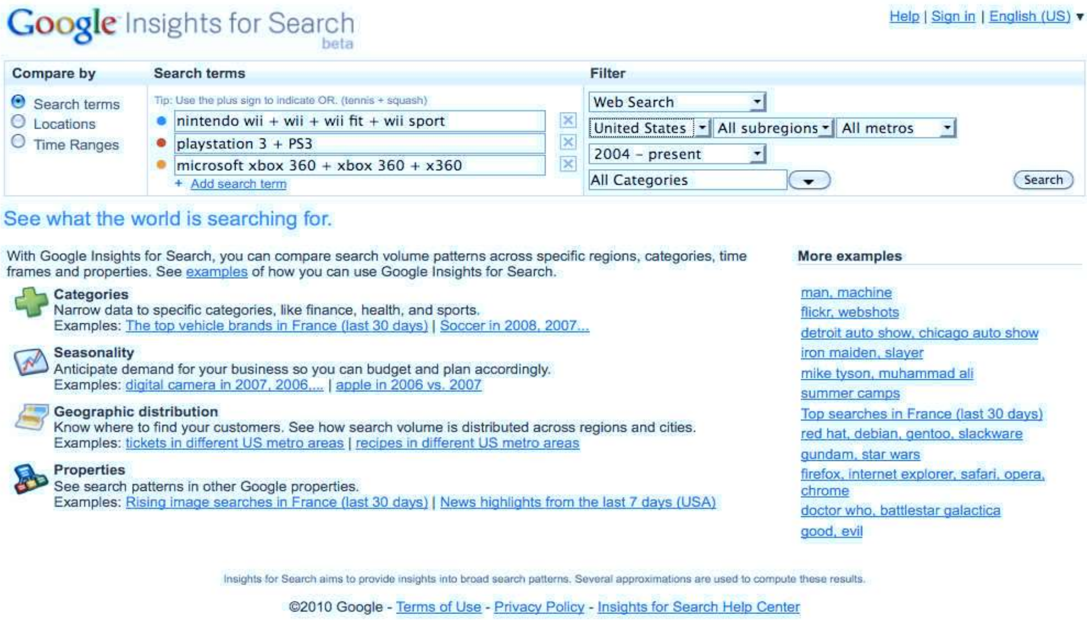

Figure 1: Google Insights for Search

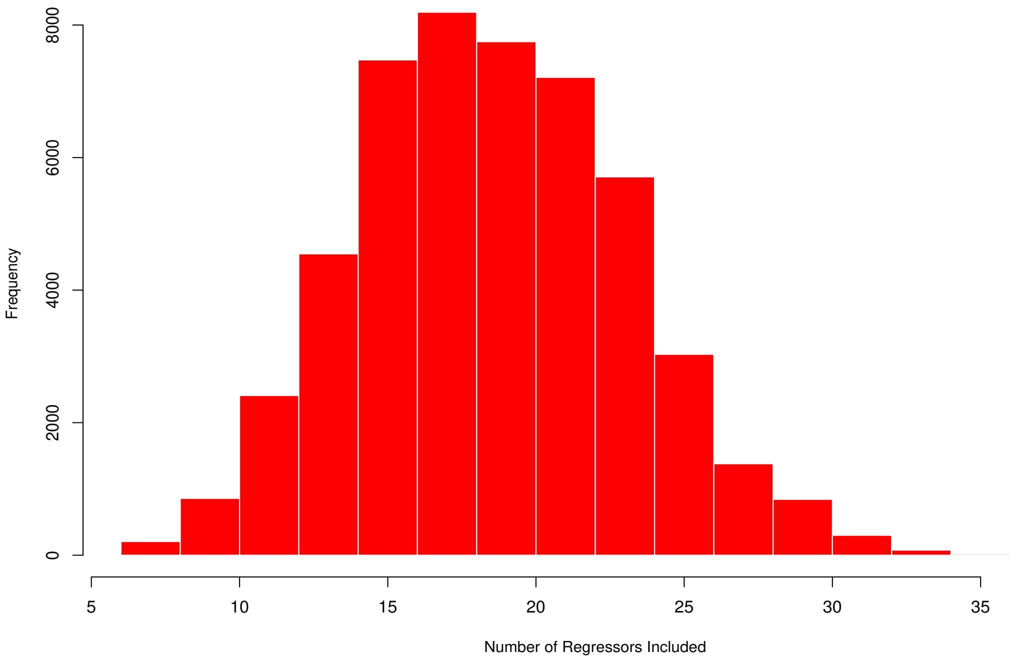

Figure 2: Model Size: Posterior Distribution of the Number of Regressors Included in the Model for the Nintendo Wii

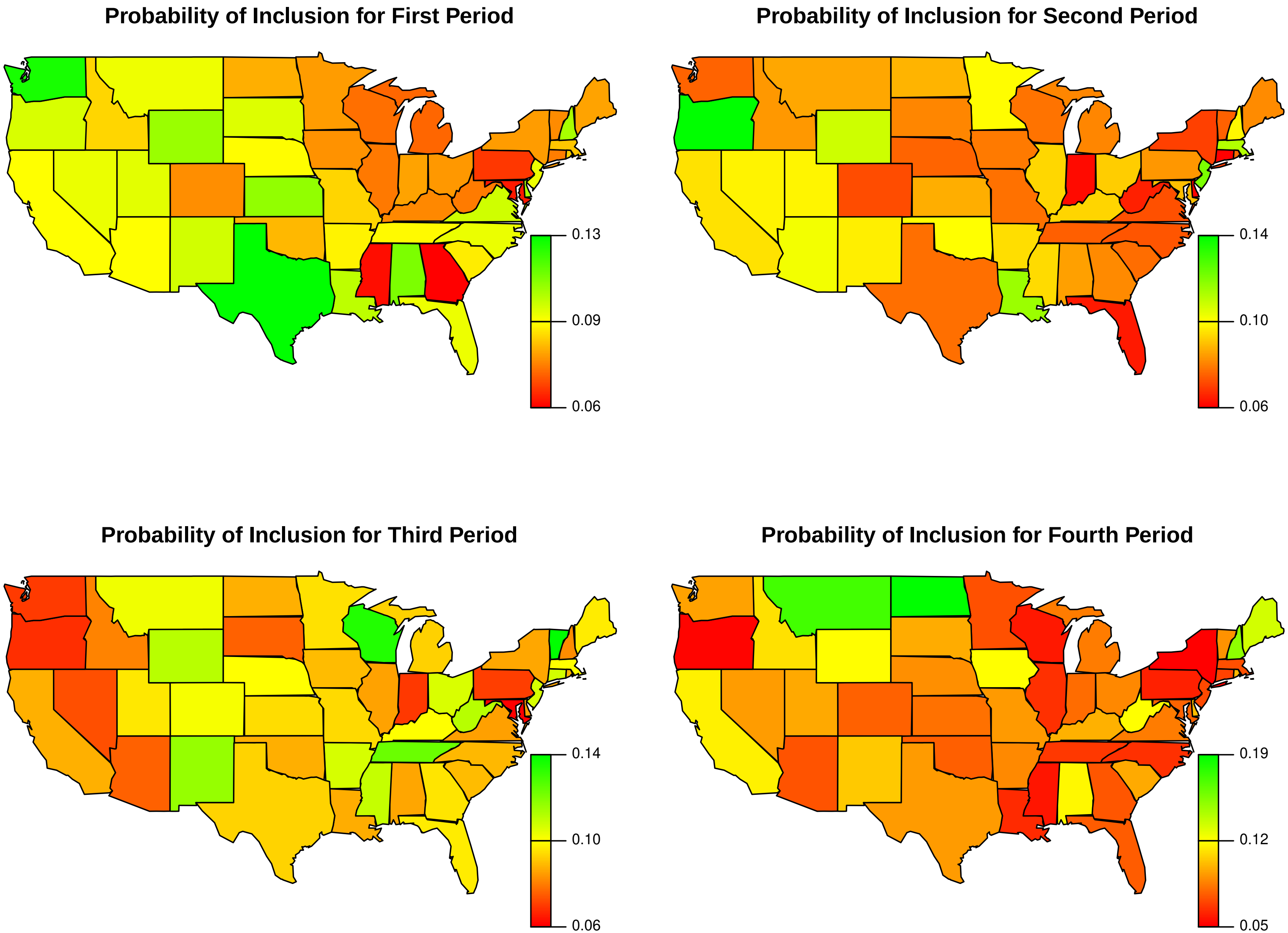

Figure 3: State Inclusion Probabilities for Each Diffusion Period of the Nintendo Wii

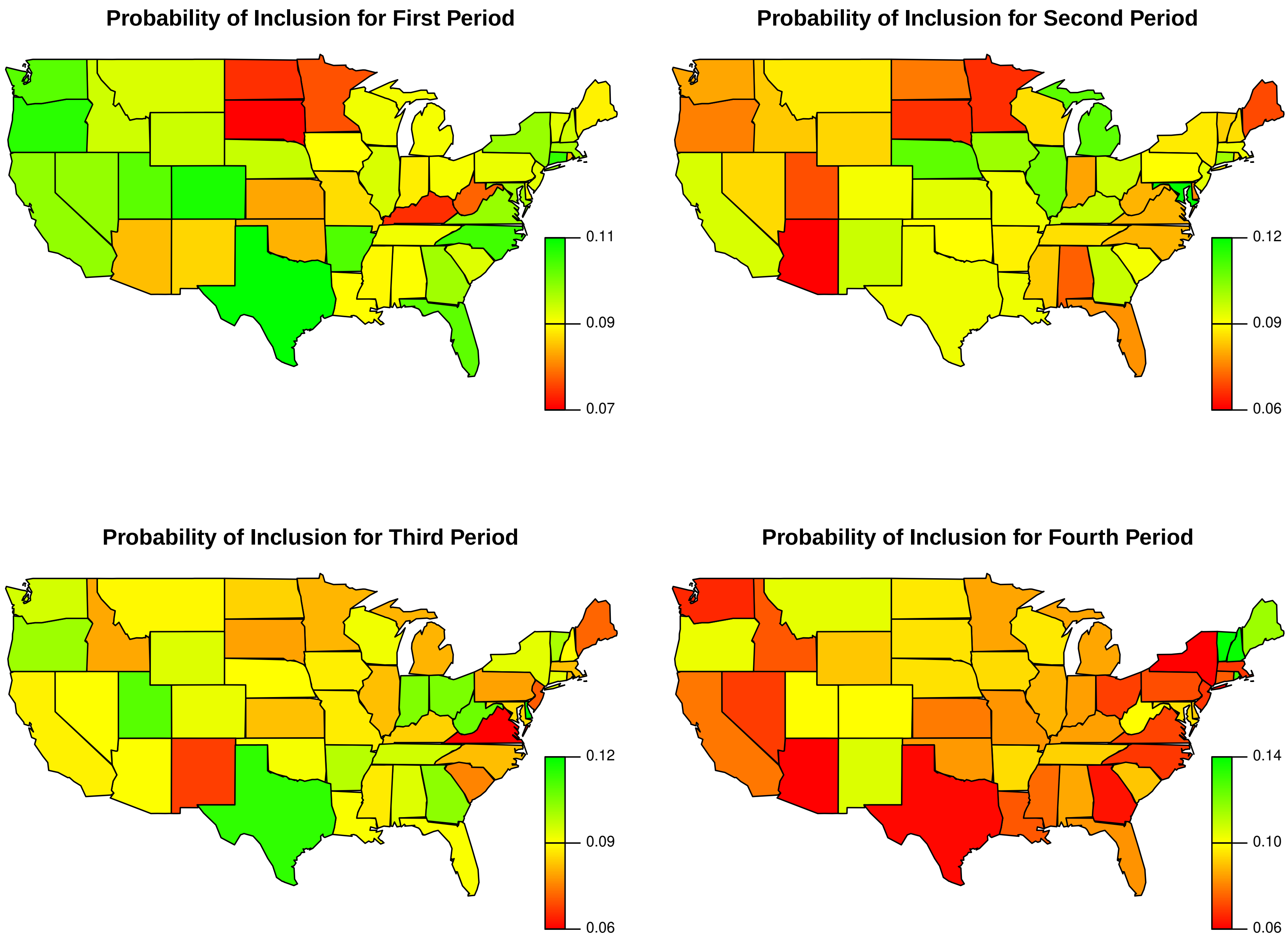

Figure 4: State Inclusion Probabilities for Each Diffusion Period of the Sony PS3

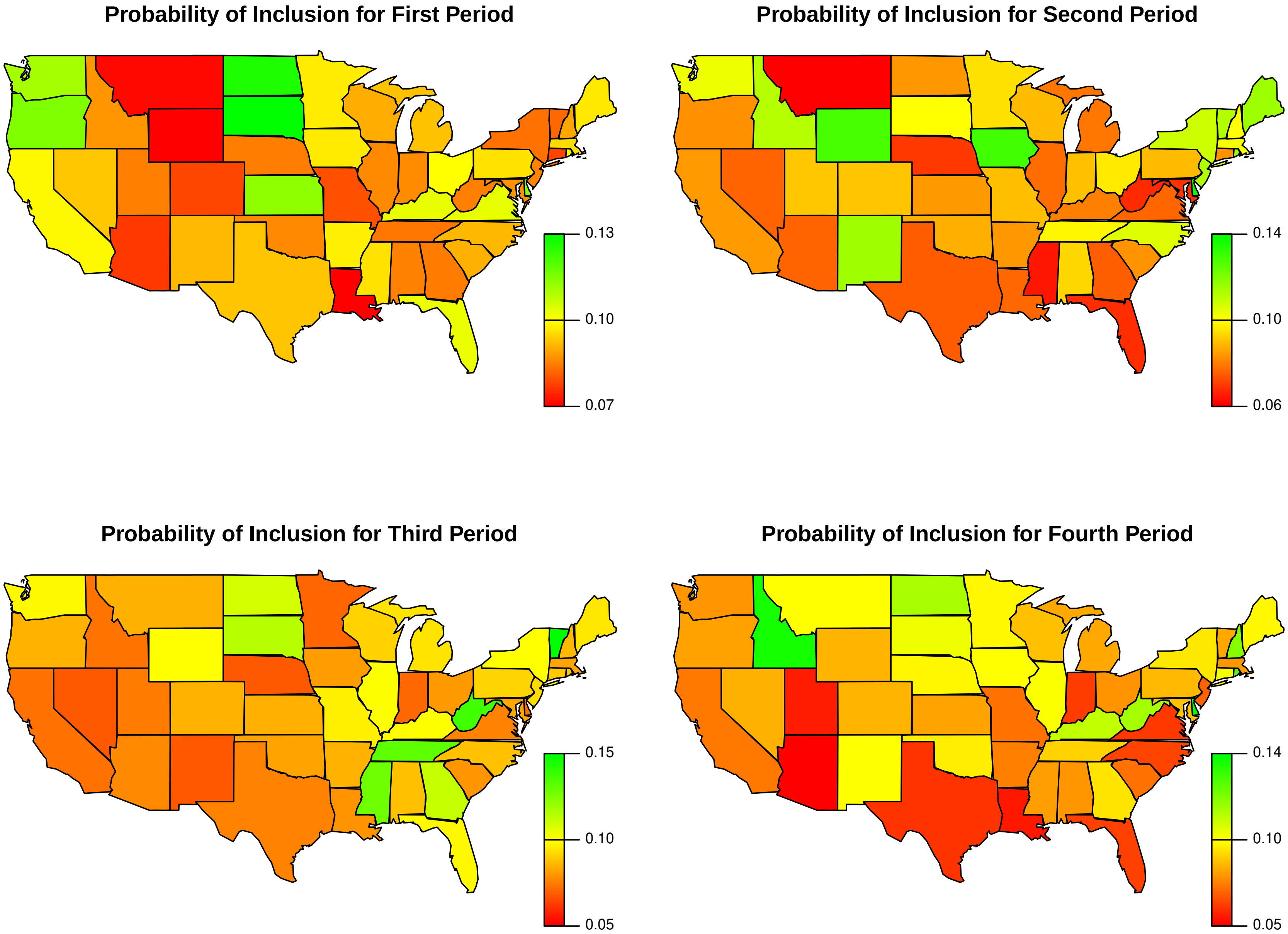

Figure 5: State Inclusion Probabilities for Each Diffusion Period of the Microsoft Xbox 360

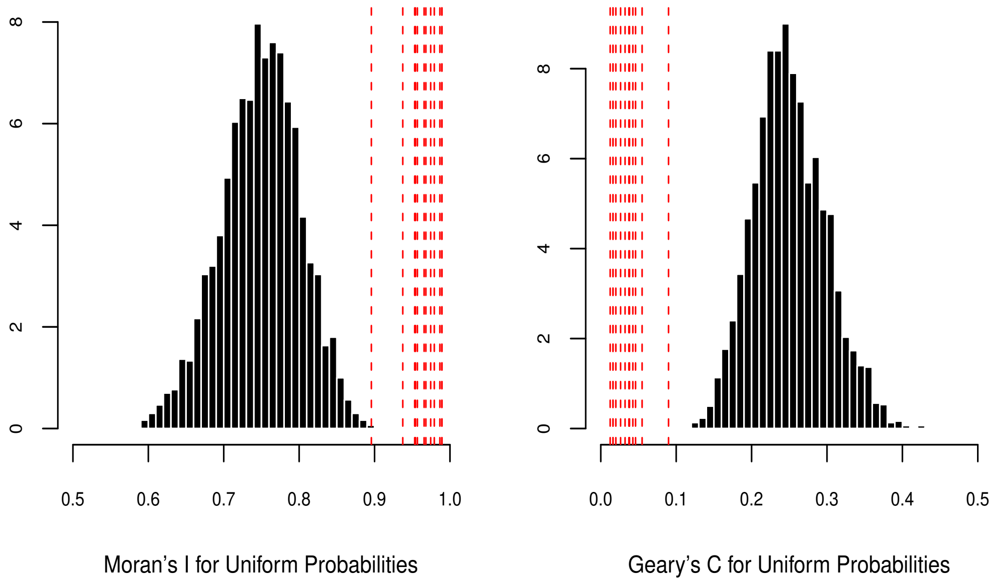

Figure 6: Moran's I and Geary's C for Uniform Probabilities (Histogram) and Moran's I and Geary's C for all Diffusion Periods and Technologies (Vertical Lines)

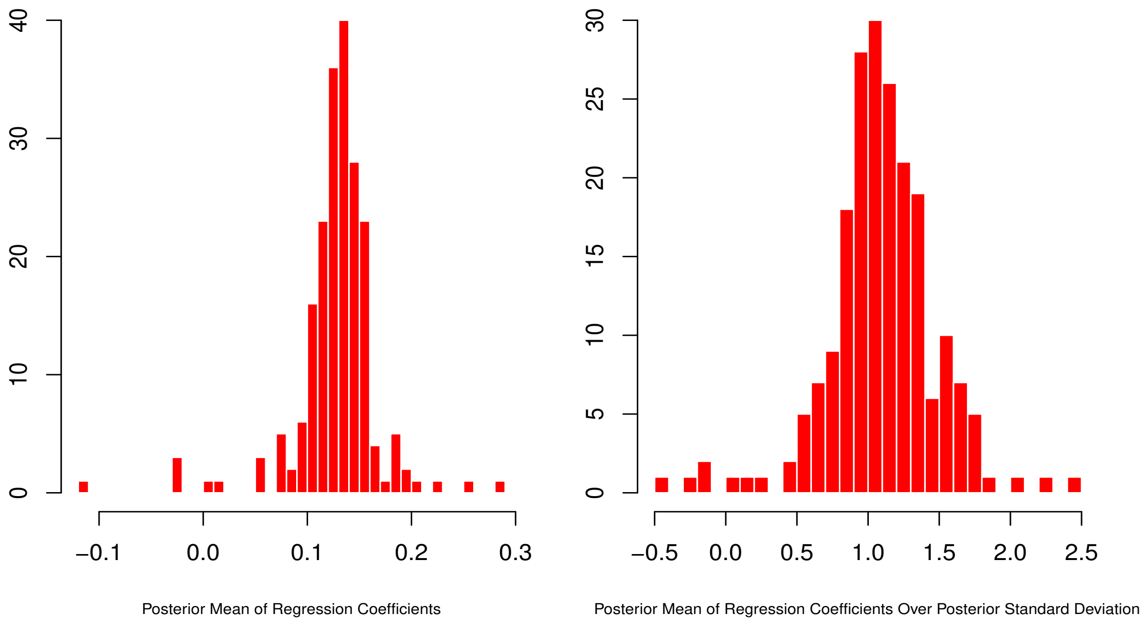

Figure 7: Nintendo Wii Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

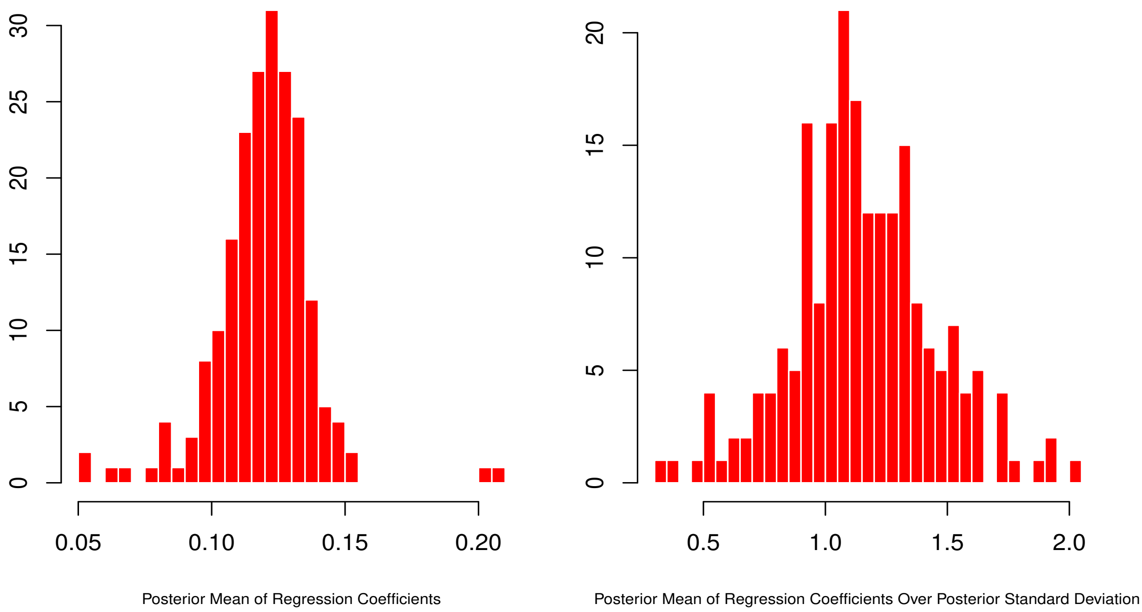

Figure 8: Sony PS3 Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

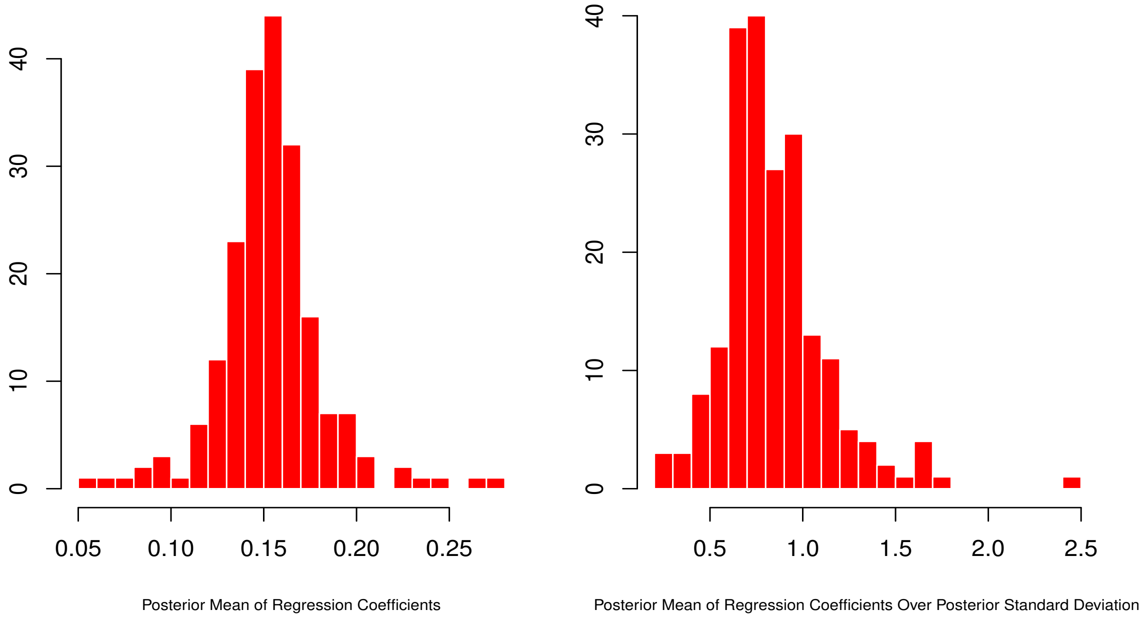

Figure 9: Microsoft Xbox Model: Histogram of the Posterior Mean of the Regression Coefficient for all US States and All Time Periods (Left Panel) and Posterior Mean Over Posterior Standard Deviation (Right Panel)

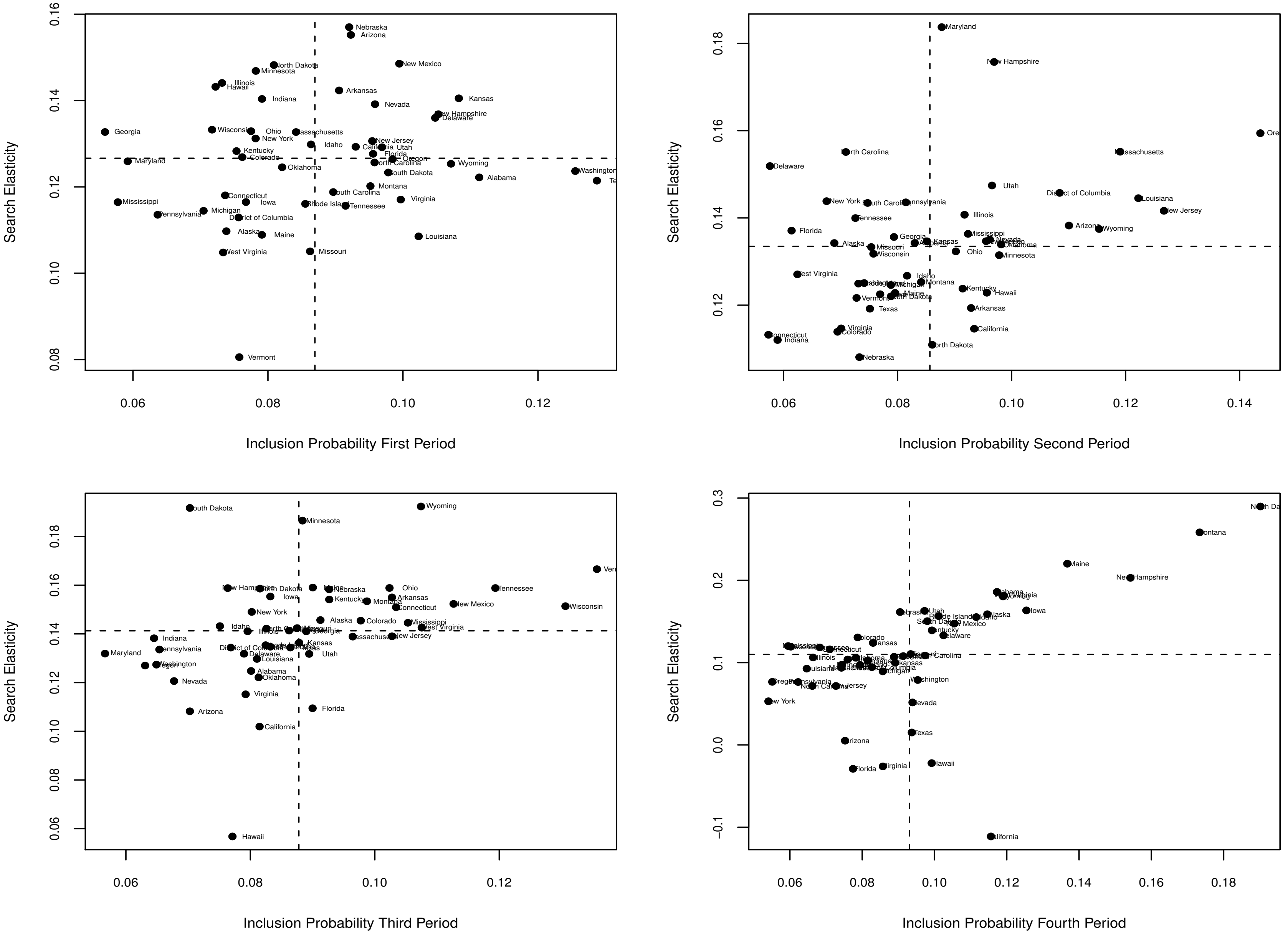

Figure 10: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Nintendo Wii)

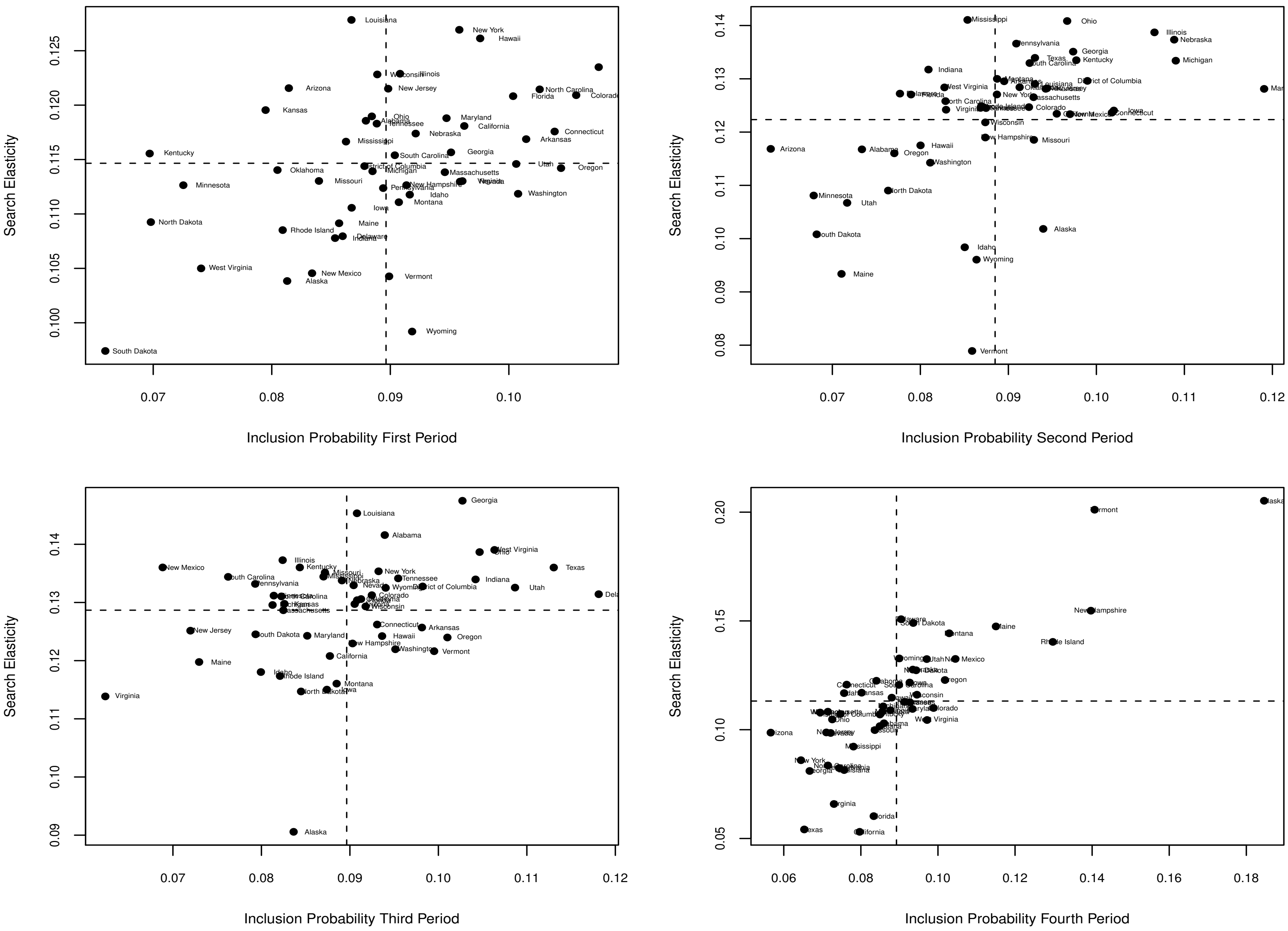

Figure 11: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Sony PS3)

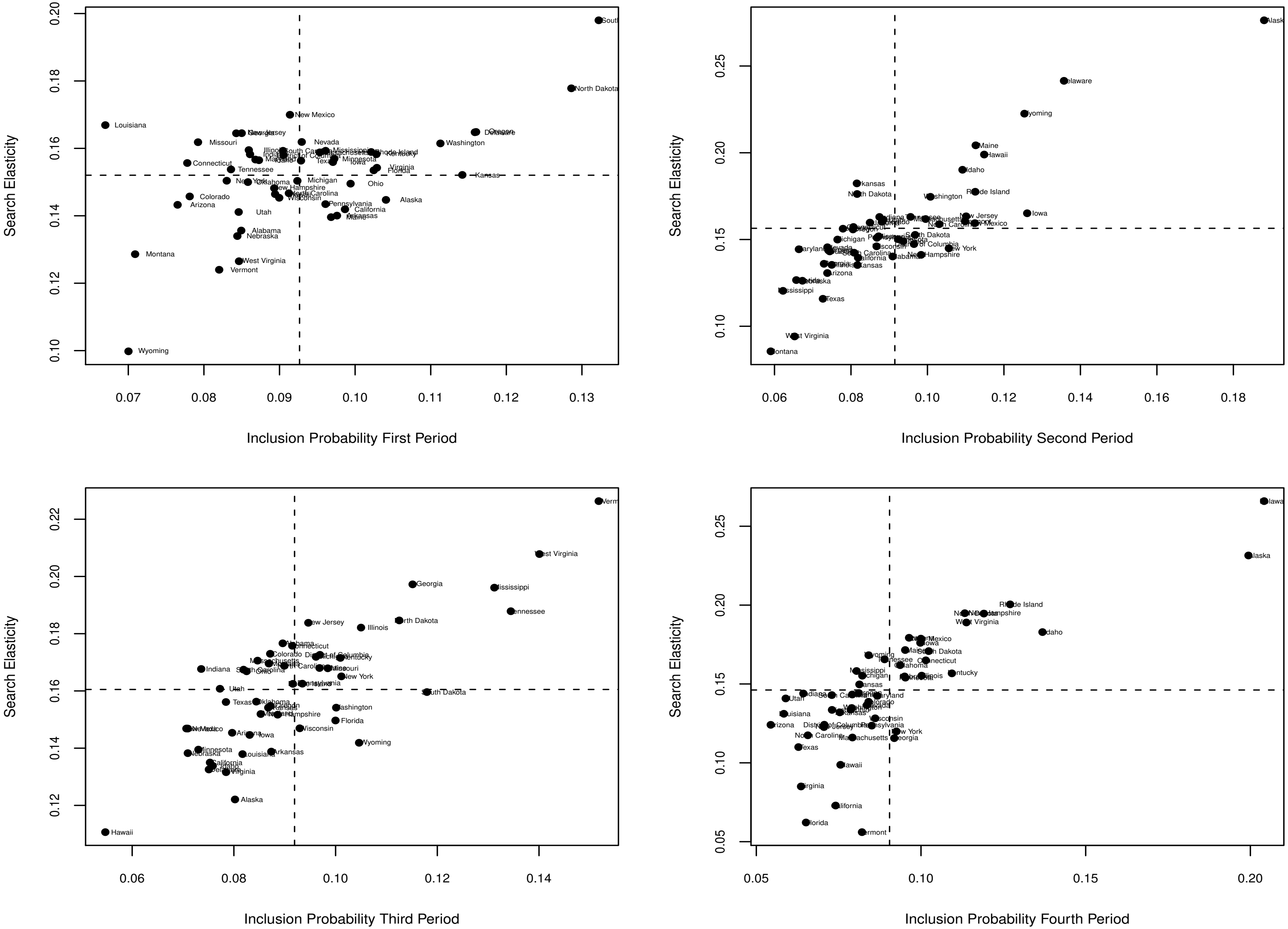

Figure 12: Scatter Plots between Inclusion Probabilities for Each Diffusion Period and Search Elasticity (Xbox 360)

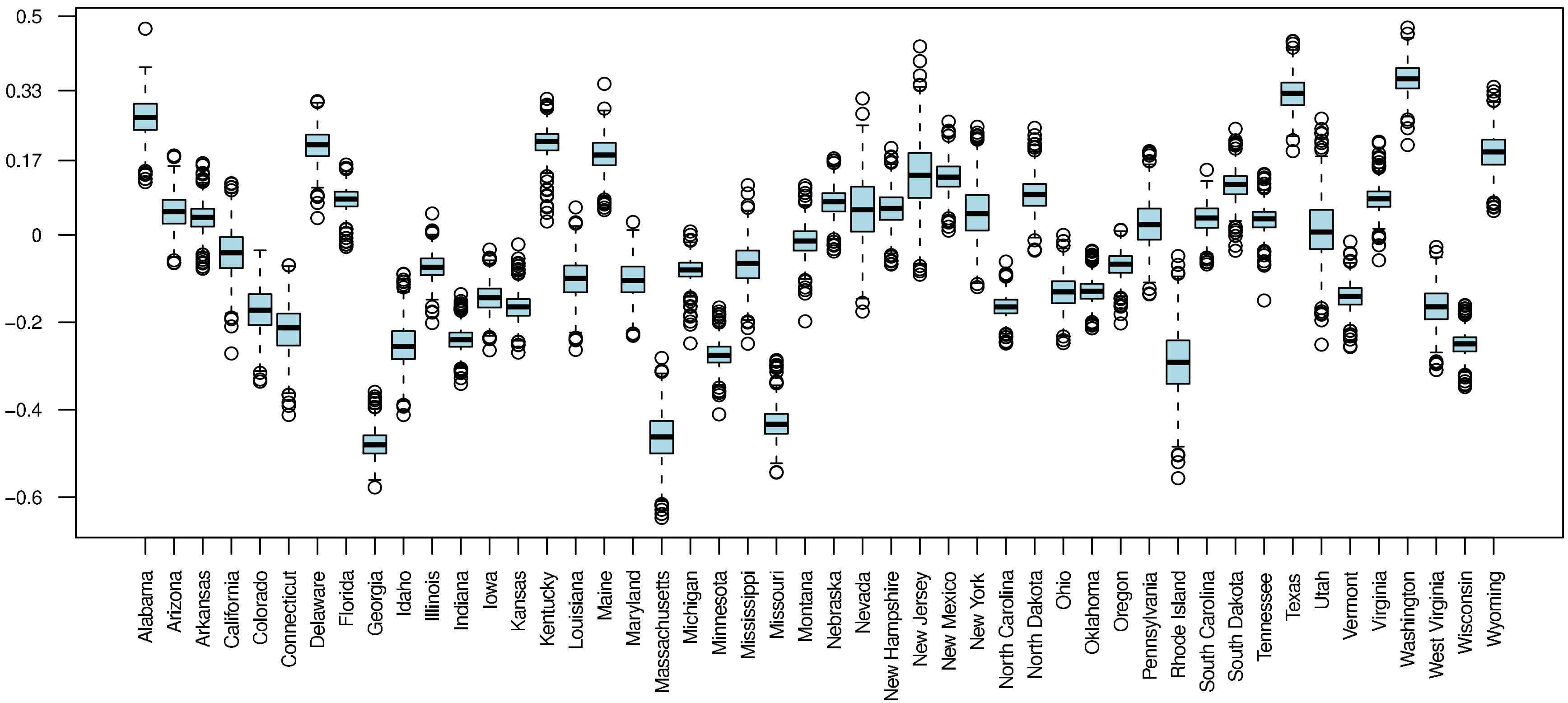

Figure 13: Distribution of the Spatial Effects of the Nintendo Wii during First Diffusion Period

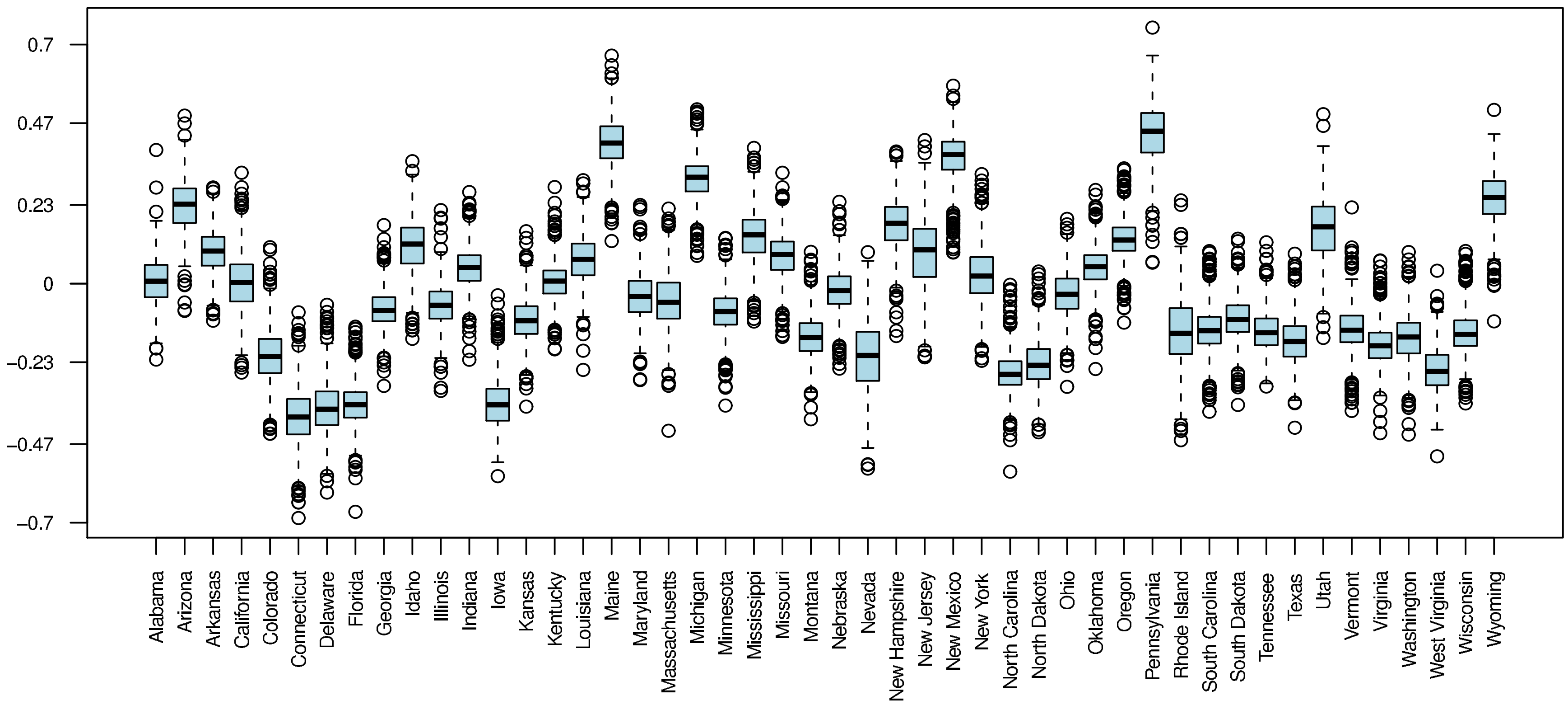

Figure 14: Distribution of the Spatial Effects of the Nintendo Wii during Second Diffusion Period

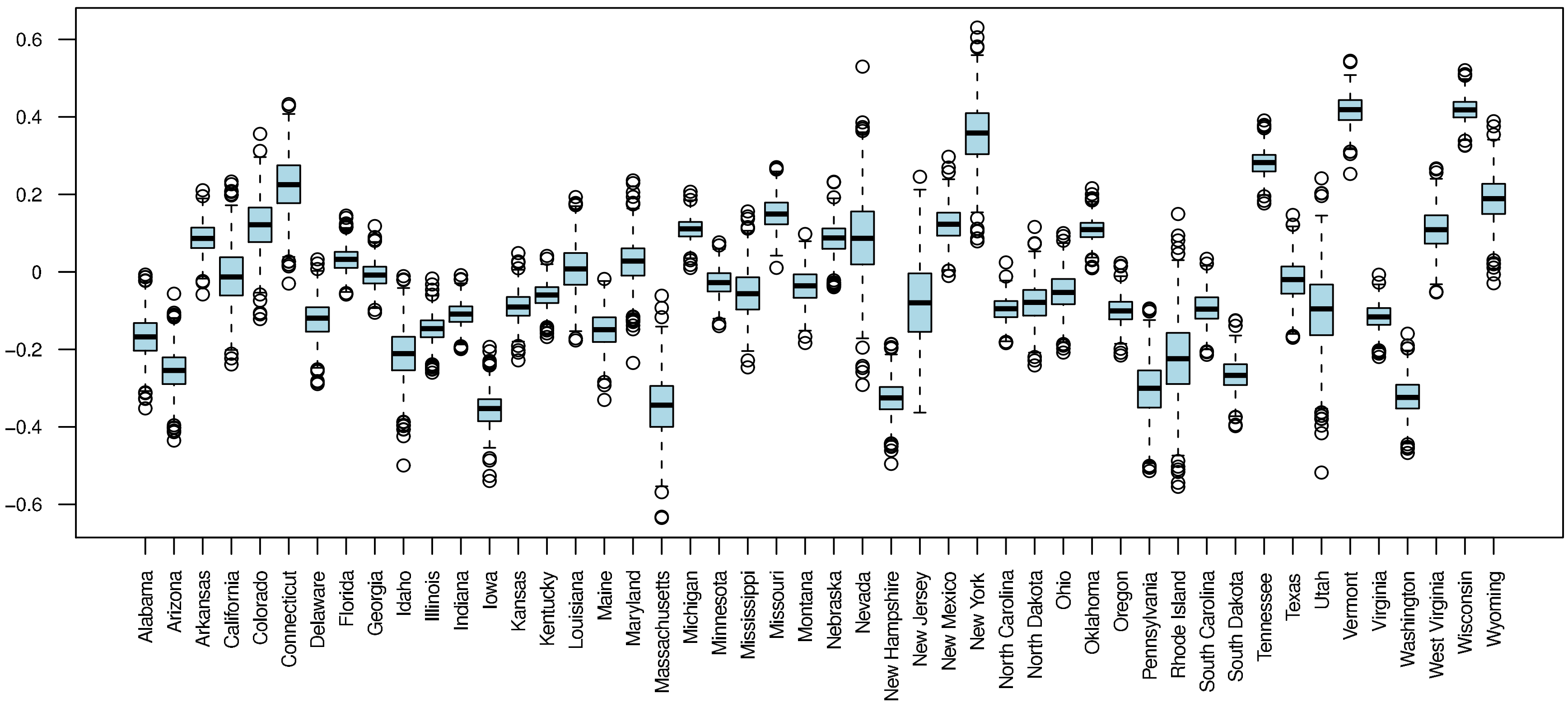

Figure 15: Distribution of the Spatial Effects of the Nintendo Wii during Third Diffusion Period

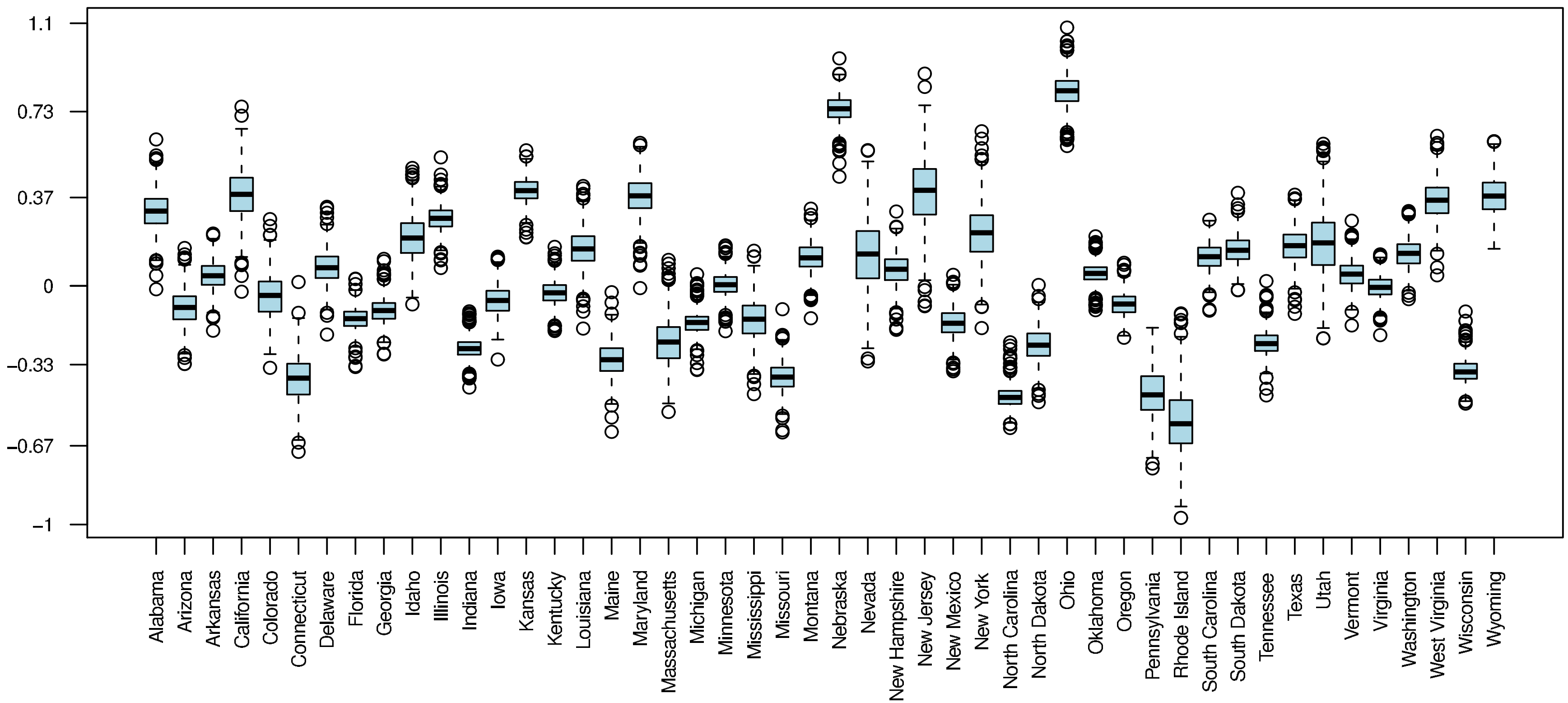

Figure 16: Distribution of the Spatial Effects of the Nintendo Wii during Fourth Diffusion Period

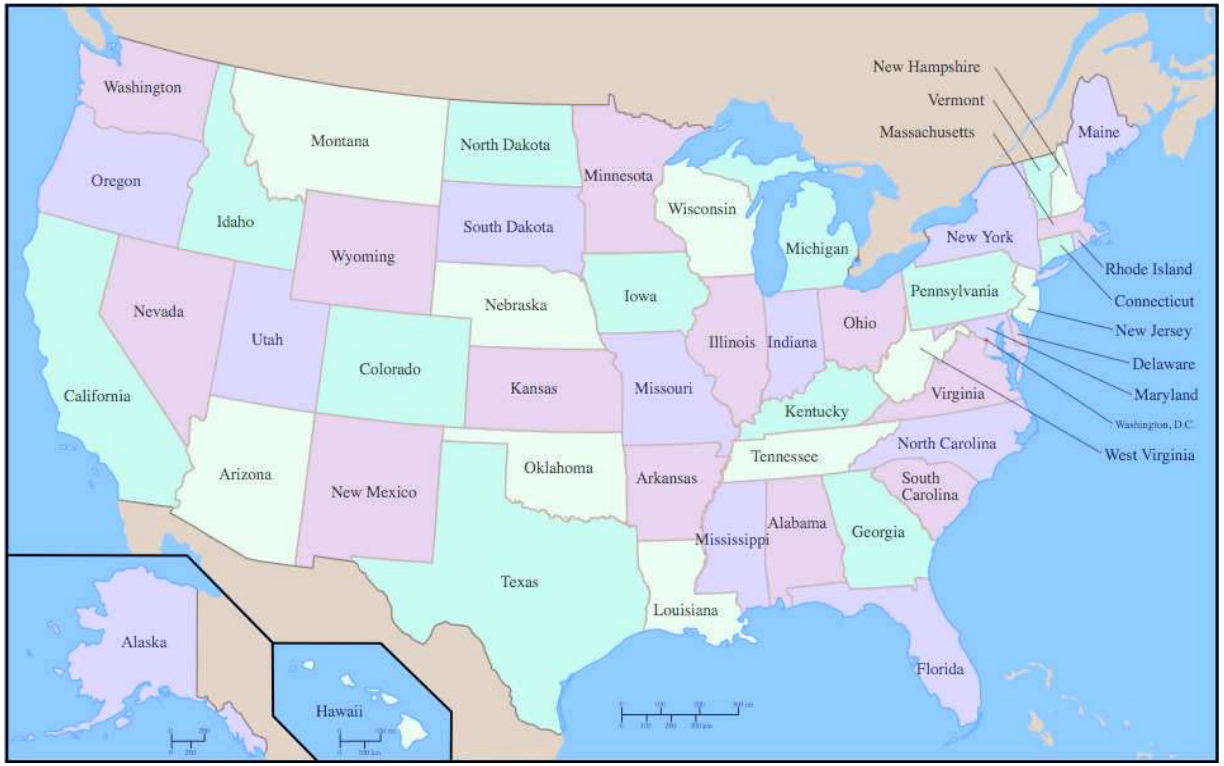

Figure 17: US State Map (Source: Wikipedia)

A Methodology

In this appendix we discuss the BVS method and the MCAR model estimation we use to study the probabilities of inclusion of the different regions and locations.

A.1 Bayesian Variable Selection

In what follows we follow closely the presentation of George and McCulloch (1997) section 4. In Section 4 they discuss the specification of conjugate priors for β and σ. We chose to use conjugate priors because it facilitates the integration of β and σ out of the posterior distribution of the indicators γ and hence the computation of the posterior of γ becomes simple and fast.

The likelihood is specified as

$$f(Y|\beta,\sigma) = \phi(Y; X_\gamma \beta_\gamma, \sigma^2 I) \tag{A-1}$$

where $Y = y_i = (y_{i1}, \ldots, y_{iT})$, X_γ is a subset of potential regressors for which $\gamma = 1$, I is an identity and $\phi(y; x, \Sigma)$ is the Normal distribution density with mean x and variance Σ evaluated at y. The prior for β is

$$\pi(\beta|\sigma,\gamma) = \phi(\beta; 0, \sigma^2 D_\gamma R D_\gamma), \tag{A-2}$$

where D_γ is a diagonal matrix with elements

$$D_\gamma^{kk} = \begin{cases} \upsilon_0 & \text{when} \quad \gamma_k = 0 \\ \upsilon_1 & \text{when} \quad \gamma_k = 1, \end{cases} \tag{A-3}$$

and R is a correlation matrix. $R \propto I$ or $R \propto (X_\gamma' X_\gamma)^{-1}$ are attractive choices when $\upsilon_0 = 0$. The scalars υ_0 and υ_1 are chosen according to the objectives of the modeler. The choice of υ_0 and υ_1 affect the number of regressors included in the subset X_γ and the threshold after which an element of β is distinguished from zero. See George and

McCulloch (1997, page 346-347) for more details.

George and McCulloch (1997) discuss how different choices of υ_0 and υ_1 affect the selection of variables and the size of the β coefficients that are included in the model. The suggestion is to set υ_0 small and υ_1 large such that when the posterior supports that $\gamma_k = 0$ then the prior specification is narrow enough to keep β_k close to zero. A popular choice in the literature is to set $\upsilon_0 = 0$ and to specify $\pi(\beta|\gamma) = \pi(\beta_\gamma|\gamma) \times \pi(\beta_{\bar{\gamma}}|\gamma)$ where $\pi(\beta_\gamma|\gamma) = \phi(\beta_\gamma; 0, \sigma^2\Sigma_\gamma)$ and $\pi(\beta_{\bar{\gamma}}|\gamma) = 1$ being β_γ and $\beta_{\bar{\gamma}}$ the coefficients included and excluded in the model, respectively. The attractiveness of this last specification is that we can select β_k depending on how significantly they are different from zero rather than selecting them depending on their relative size when $\upsilon_0 \neq 0$.

The prior for σ^2 is

$$\pi(\sigma^2) = IG(\nu/2, \nu\lambda/2) \tag{A-4}$$

where ν are the degrees of freedom and λ is the scale of the inverse gamma (IG) distribution. What is left to specify is the prior for the indicators γ. They are usually specified as

$$\pi(\gamma) = \prod_k w_k^{\gamma_k}(1 - w_k)^{1-\gamma_k}, \tag{A-5}$$

where w_k is the probability of including the k regressor in the model. A popular choice in the literature is to use $w_k = w$ and therefore

$$\pi(\gamma) = w^{q_\gamma}(1 - w)^{p-q_\gamma}, \tag{A-6}$$

where q_γ is the number of regressors included out of a total set of size p. This last prior can be combined with a conjugate prior on w and set $w \sim Beta(a, b)$ and the prior becomes

$$\pi(\gamma) = \frac{B(a + q_\gamma, b + p - q_\gamma)}{B(a, b)}, \tag{A-7}$$

where $B(x, y)$ is the beta function with x and y parameters. See Chipman et al. (2001) for other choices of $\pi(\gamma)$. Careful selection should be given to the scalars υ_1 and w (or a

and b) as they directly affect model size. Large υ_1 and small w concentrate the prior on parsimonious models with large coefficients while large w and small υ_1 concentrate the prior on saturated models with small coefficients (Clyde and George, 2004, page 86).

The joint density $\pi(Y, \beta, \sigma^2|\gamma) = \pi(Y|\beta, \sigma^2, \gamma)\pi(\beta|\sigma, \gamma)\pi(\sigma^2|\gamma)$ has a closed form expression when $\upsilon_0 = 0$ and after integrating over β and σ^2 and that is

$$\pi(Y|\gamma) \propto |X'_\gamma X_\gamma + \Sigma_\gamma^{-1}|^{-1/2}|\Sigma_\gamma|^{-1/2}(\nu\lambda + S_\gamma^2)^{-(T+\nu)/2}, \tag{A-8}$$

where

$$S_\gamma^2 = Y'Y - Y'X_\gamma(X'_\gamma X_\gamma + \Sigma_\gamma^{-1})X'_\gamma Y, \tag{A-9}$$

and $\Sigma_\gamma = D_\gamma R D_\gamma$. The posterior of the indicators is straightforward to compute as $\pi(\gamma|Y) \propto \pi(Y|\gamma)\pi(\gamma)$ and the Metropolis-Gibbs sampler is straightforward and it proceeds by sampling $\pi(\gamma|Y)$, $\pi(\beta_\gamma|Y, \sigma^2, \gamma)$ and $\pi(\sigma^2|Y, \beta, \gamma)$ sequentially.

We use $a = 50$ and $b = 100$ for the prior on w (in equation (A-7)). The prior of σ^2 has $\nu = 1000$ and $\lambda = 0.30$. We follow the recommendation of George and McCulloch (1997, page 341) who suggest to set λ such that the posterior of σ^2 assigns substantial probability to an interval close to the sample variance of Y and the variance of the residual of a saturated model. The prior on β in equation (A-2) and (A-3) has $\upsilon_0 = 0$ and $\upsilon_1 = 7$ and we use $R = (X'_\gamma X_\gamma)^{-1}$.

A.2 A short review of aereal data models

Aereal data usually refers to cross sectional or panel data collected across different regions or areas with well defined boundaries. Therefore aereal data consists of aggregate or summary measures at different locations. The CAR and SAR models are among the most popular models applied to aereal data but there are many other popular approaches like *kriging* or spatial interpolation. In this review we focus on the CAR model and its multivariate extensions.

CAR stands for *Conditional Autoregressors* and SAR stands for *Simultaneous Au-*

toregressors and hence CAR models are usually referred as Conditionally Autoregressive models and the SAR as Simultaneous Autoregressive models.

The CAR and SAR models are discussed in several sources. A basic reference is Cressie (1992). Cressie covers topics that range from model specification, classical and Bayesian estimation to the theoretical foundations of the CAR and SAR models. Many other topics in spatial analysis are discussed in Cressie (1992). Banerjee et al. (2004) focus on Bayesian analysis and estimation of spatial models. Held and Rue (2002) review many of the computational methods and sampling techniques usually applied to the Bayesian analysis of CAR models and to more general spatial models referred to as *Gaussian Markov Random Fields*.

Wall (2004) compares the CAR and SAR models and offers some insights about the different correlation between locations implied by these two models. The CAR and SAR models might be equivalent under certain conditions, for example see Assunçao (2003) or Banerjee et al. (2004, page 86). We intend to apply spatial priors to the distribution of model parameters. Therefore, in what follows we focus on the CAR model as it is better suited than the SAR both as a hierarchical prior specification on a model's parameters and for Bayesian modeling (Banerjee et al., 2004, page 86).

The main assumption of the CAR model is that a measurement at a location has a conditional distribution with a mean that is proportional to a weighted sum of the measurement at neighboring locations. Both the joint distribution and the conditional distribution of the spatial effects given all other spatial parameters can be derived in closed form and they are presented in Banerjee et al. (2004, page 79) and in the references therein. However, there are alternative specifications to the joint distribution of the spatial effects and a common approach is to use the pairwise difference specification (Besag et al., 1991). Haran et al. (2003) present how to use block updating when some of the coefficients in a linear regression follow the pairwise difference prior.

The CAR is suited for univariate aereal data and Mardia (1988) presents an extension to the multivariate case, usually referred to as *multivariate CAR* or simply as MCAR. It is common to have more than one measurement at each location and the MCAR allows to model both the correlation among measurements of neighboring sites and the correlation among the different measures across sites. Gelfand and Vounatsou (2003) and Carlin and Banerjee (2003) apply Bayesian analysis to the MCAR of Mardia (1988) and present applications with two and up to five dimensional data. On the other hand, Gamerman et al. (2003) present a multivariate version of the pairwise difference specification (used as a prior) and its sampling schemes.

Other extensions of the CAR model incorporate dynamics into its spatial coefficients. Waller et al. (1997), Nobre et al. (2005) and Gelfand et al. (2005) propose models that use a random walk specification for the mean or for the variance of the spatial effects. Gelfand et al. (2005) provide a review of spatio-temporal models.

A.2.1 Linear Model with CAR Prior

Next we work out the specification and sampling for the model

$$y_i = x_i\beta + \phi_i + \epsilon_i, \tag{A-10}$$

where y_i is measured at i locations for $i = 1, \ldots, p$, x_i is a set of k covariates at i and β is a coefficient column vector $k \times 1$ while ϵ_i and ϕ_i are random effects meant to capture overall variability and spatial heterogeneity, respectively. We define $y^{'} = (y_1, \ldots, y_p)$, $\phi^{'} = (\phi_1, \ldots, \phi_p)$ and $X = (x_1, \ldots, x_k)$. The distribution of ϵ_i is

$$\epsilon \sim N(0, \Sigma), \tag{A-11}$$

where $\epsilon^{'} = (\epsilon_1, \ldots, \epsilon_p)$, $\Sigma = \sigma^2 I$ and σ^2 is the variance of ϵ. $N(\mu, \Sigma)$ refers to a normal distribution with mean μ and covariance matrix Σ. We define $\lambda_\epsilon = 1/\sigma^2$. The prior distribution of the spatial effects ϕ_i follows

$$\phi_i | \phi_{j \sim i} \sim N\big(\sum_{j \sim i} c_{ij} \phi_j, \tau_i^2\big). \tag{A-12}$$

This form states that the distribution of ϕ_i given its j neighbors, denoted as $j \sim i$, has a normal distribution with a mean that is a weighted sum (using weights c_{ij}) of the neighboring values and variance τ_i^2. Besag (1974) shows that the joint distribution of the spatial effects in (A-12) can we written in the form

$$\phi \sim N(0, \Omega), \tag{A-13}$$

where $\phi = (\phi_1, \ldots, \phi_p)$ and Ω is a $p \times p$ symmetric and positive semi-definite or positive definite matrix. In the literature it is common to define the elements of Ω^{-1} directly instead of specifying Ω. For example, Banerjee et al. (2004, page 79) assume that $\tau_i^2 = \tau^2 / w_{i+}$ and that $c_{ij} = w_{ij}/w_{i+}$ where w_{ij} takes the value of 1 if $j \sim i$ and zero otherwise and where w_{i+} is the total number of neighbors of i. Given these assumptions $\Omega^{-1} = T^{-1}(I - C)$ and given that T is a diagonal matrix with elements $T_{ii} = \tau^2 / w_{i+}$ and $C_{ij} = c_{ij}$ then Ω^{-1} can be written as

$$\Omega^{-1} = \frac{1}{\tau^2}(I_{w_{i+}} - W), \tag{A-14}$$

where $I_{w_{i+}}$ is a diagonal matrix with elements w_{i+} and $W_{ij} = w_{ij}$. This last specification for Ω results in an improper distribution given that the rows of $(I_{w_{i+}} - W)$ sum to zero. A solution to this issue is to specify Ω as

$$\Omega^{-1} = \frac{1}{\tau^2}(I_{w_{i+}} - \rho W), \tag{A-15}$$

where ρ takes a value (between 0 and 1) that makes Ω^{-1} positive definite and consequently the distribution of ϕ becomes proper. For a discussion on the *impropriety* of the CAR distribution and the role of the ρ parameter see Banerjee et al. (2004, page 163), Eberly and Carlin (2000), Sahu and Gelfand (1999) or Best et al. (1999). This latter form implies that

$$\phi_i|\phi_{j\sim i} \sim N\big(\rho \sum_{j\sim i} c_{ij}\phi_j, \tau_i^2\big). \tag{A-16}$$

The distribution of ϕ is usually referred as $CAR(\tau^2)$ when the conditional distributions of the spatial effects are defined as in equation (A-12) and it is referred as $CAR(\rho, \tau^2)$ when its conditional distribution follows (A-16). In what follows we use $\Omega^{-1} = \lambda_\phi Q$ with $Q = I_{w_{i+}} - \rho W$ and $\lambda_\phi = 1/\tau^2$. To carry out Bayesian inference and to complete the model specification we need to define the priors for β, λ_y, λ_ϕ and ρ. We specify them as

$$\begin{aligned} p(\beta) &\propto 1 \\ p(\lambda_y) &\propto {\lambda_y}^{a_y} e^{-b_y \lambda_y} \\ p(\lambda_\phi) &\propto \lambda_\phi^{a_\phi} e^{-b_\phi \lambda_\phi} \\ p(\rho) &\propto \text{discretized prior} \end{aligned} \tag{A-17}$$

We use $p(\cdot)$ generically to denote a probability density. That is, the prior for β is non-informative, the priors for λ_y and λ_ϕ have the form of a Gamma distribution. Finally, for ρ we give probability mass to a discrete set of values with a high proportion of them near 1. Gelfand and Vounatsou (2003) suggest the use of discretized priors for ρ. The model specification is now complete and next we describe the sampling steps to estimate equation (A-10).

A.2.2 Sampling Steps for the CAR

To sample the parameters of the model in equation (A-10) we can apply the Gibbs sampler and MCMC. To derive the posterior of β we can write the likelihood of equation (A-10) as

$$L(y|\beta, \lambda_y) \propto |M|^{-1/2} e^{-\frac{1}{2}(y - X\beta)' M^{-1} (y - X\beta)}, \tag{A-18}$$

where $M = (\frac{1}{\lambda_\phi}Q^{-1} + \frac{1}{\lambda_\epsilon}I)$. The posterior of β is then

$$p(\beta|y, \lambda_y, \lambda_\phi) \propto |M|^{-1/2} e^{-\frac{1}{2}(\beta-b)'(X'M^{-1}X)^{-1}(\beta-b)}, \tag{A-19}$$

with $b = (X'M^{-1}X)^{-1}X'M^{-1}y$. Therefore β can be sampled from $N(b, (X'M^{-1}X)^{-1})$.

Next we derive the posterior distribution of the spatial effects ϕ. To do so we write the density of y conditional on β. That is

$$L(y|\beta, \phi, \lambda_y) \propto \lambda_y^{p/2} e^{-\frac{\lambda_y}{2}(\tilde{y}-\phi)'(\tilde{y}-\phi)}, \tag{A-20}$$

with $\tilde{y} = y - X\beta$. Therefore, the posterior of ϕ is

$$p(\phi|\tilde{y}, \lambda_y, \lambda_\phi) \propto \lambda_y^{p/2} e^{-\frac{1}{2}((\phi-a)'R^{-1}(\phi-a))}, \tag{A-21}$$

where $a = (\lambda_y I + \lambda_\phi Q)^{-1}\lambda_y \tilde{y}$ and $R^{-1} = (\lambda_y I + \lambda_\phi Q)$. That is ϕ can be sampled form $N(a, R)$.

The posterior of λ_y and λ_ϕ are

$$\begin{aligned} p(\lambda_\phi|\tilde{y}, \phi, \lambda_y) &\propto \lambda_\phi^{p/2+a_\phi} e^{-\lambda_\phi(\frac{1}{2}\phi' Q\phi + b_\phi)} \\ p(\lambda_y|\tilde{y}, \phi, \lambda_\phi) &\propto \lambda_y^{p/2+a_y} e^{-\lambda_y(\frac{1}{2}(\tilde{y}-\phi)'(\tilde{y}-\phi)+b_y)}. \end{aligned} \tag{A-22}$$

That is $\lambda_\phi \sim \Gamma(p/2 + a_y, b_y + 1/2\phi' Q\phi)$ and $\lambda_y \sim \Gamma(p/2 + a_\phi, b_\phi + 1/2(\tilde{y} - \phi)'(\tilde{y} - \phi))$.

Finally we need to sample the ρ in the Q matrix. We know that

$$p(\rho|\phi, y, \lambda_y, \lambda_\phi) \propto |Q|^{1/2} e^{-\frac{1}{2}\phi' Q\phi} p(\rho). \tag{A-23}$$

A common method to sample ρ is to assume that $p(\rho)$ is a uniform distribution with range $(0, 1)$ and to sample it with the Metropolis-Hastings algorithm. A second popular choice is to discretize ρ in a set of values and to draw them proportional to their posterior probability. We use the following set $0.01, 0.10, 0.20, 0.30, \ldots, 0.70, 0.71, 0.72, \ldots, 0.99$.

In summary we use the next steps in the Gibbs sampler

1. $\beta \sim N((X'M^{-1}X)^{-1}X'M^{-1}y, (X'M^{-1}X)^{-1})$

2. $\phi \sim N((\lambda_y I + \lambda_\phi Q)^{-1}\lambda_y \tilde{y}, (\lambda_y I + \lambda_\phi Q))$

3. $\lambda_y \sim \Gamma(p/2 + a_y, b_y + 1/2\phi' Q\phi)$

4. $\lambda_\phi \sim \Gamma(p/2 + a_\phi, b_\phi + 1/2(\tilde{y} - \phi)'(\tilde{y} - \phi))$

5. $\rho \sim p(\rho|\phi, y, \lambda_y, \lambda_\phi)$

where $x \sim \Gamma(a, b)$ means that x follows a Gamma distribution with the form $cx^a e^{-bx}$ where c is a constant. At the end of the sampling step 2 we center the ϕ vector around its own mean following Eberly and Carlin (2000) and Best et al. (1999). The re-centering is equivalent to sampling with the restriction $\sum \phi_i = 0$. Rue and Held (2005) show a general form to sample with linear restrictions and that is equivalent to centering around a mean.

A.2.3 Multivariate Linear Model with MCAR Prior

Next we expand the linear model of Section A.2.1 to a multivariate setting. The exposition follows Carlin and Banerjee (2003) and Gelfand and Vounatsou (2003).

In this setting we observe J different measurements at each location. That is we use the notation y_{ji} to refer to the j^{th} measurement at location i. We use the notation y_j for $(y_{j1}, \ldots, y_{jp})'$ and Y is a $p \times J$ matrix with columns $(y_1, \ldots, y_J)$. The same notation is used for the spatial effects ϕ_{ij} and the error terms ϵ_{ij}. That is $\phi_j = (\phi_{j1}, \ldots, \phi_{jp})'$, $\Phi = (\phi_1, \ldots, \phi_J)$ and finally $\epsilon_j = (\epsilon_{j1}, \ldots, \epsilon_{jp})'$, $E = (\epsilon_1, \ldots, \epsilon_J)$. We observe a common group of N covariates X where $X = (x_1, \ldots, x_N)$ and $x_i = (x_{i1}, \ldots, x_{ip})'$. Hence we can write

$$\underset{(p\times J)}{Y} = \underset{(p\times N)}{X} \cdot \underset{(N\times J)}{B} + \underset{p\times J}{\Phi} + \underset{(p\times J)}{E} \tag{A-24}$$

To carry out Bayesian inference we define the following priors

$$
\begin{aligned}
&p(B) \propto 1 \\
&p(\Sigma) \propto |\Sigma|^{-\frac{v}{2}} e^{-\frac{1}{2} tr \Sigma^{-1} V_\Sigma} \\
&p(\Phi|\Lambda, \Psi) \propto |\Psi|^{-J/2} |\Lambda|^{-p/2} e^{-\frac{1}{2} tr(\Psi \Phi \Lambda \Phi')} \\
&p(\Lambda) \propto |\Lambda|^{-\frac{v_0}{2}} e^{-\frac{1}{2} tr \Lambda V_\Lambda}
\end{aligned}
\tag{A-25}
$$

Above Σ is a $J \times J$ covariance matrix of E and $\text{vec}(E) \sim N(0, \Sigma \otimes I)$; Λ is $J \times J$ and it is the inverse of the covariance matrix between the columns of Φ while Ψ is $p \times p$ and it is the inverse covariance matrix between the rows of Φ. That is, $\text{vec}(\Phi) \sim N(0, \Lambda^{-1} \otimes \Psi^{-1})$.

The form of Ψ might be identical to the form of the Q matrix in the CAR prior. That is $\Psi = (I_{w_{i+}} - \rho W)$ where W and $I_{w_{i+}}$ are defined as before. A second choice for Ψ might be $\Psi = (I_{w_{i+}} - W)$. Carlin and Banerjee (2003) and Gelfand and Vounatsou (2003) use the first form while Gamerman et al. (2003) use the second. A third choice is to define a general form for $\Lambda \otimes \Psi$ as Gelfand and Vounatsou (2003) propose. Gelfand and Vounatsou (2003) propose a form of Q that allows an item (J items) specific ρ parameters. They first define $Q_j = (I_{w_{i+}} - \rho_j W)$ and its Choleski factorization $Q_j = P_j' P_j$. Then they define

$$
\Lambda \otimes \Psi = \mathbf{P}' (\Lambda \otimes I_{p \times p}) \mathbf{P}, \tag{A-26}
$$

where $\mathbf{P}$ is a diagonal matrix with P_j blocks. This last form may allow for a more flexible correlation structure of the Φ parameters. In the application we assume $\rho_j = \rho$ for all j.

A.2.4 Sampling the Multivariate Linear Model with MCAR Prior

If we condition on Φ and define $\bar{Y} = Y - \Phi$ we obtain the traditional multivariate regression model

$$
\bar{Y} = X \cdot B + E. \tag{A-27}
$$

Given this last expression we can write the density of the model as

$$p(\bar{Y}|X, B, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}tr(\bar{Y}-XB)'(\bar{Y}-XB)\Sigma^{-1}}. \tag{A-28}$$

The joint posterior of B and Σ can be written as

$$\begin{aligned} p(B, \Sigma|X, Y) &= p(Y|X, B, \Sigma)p(B)p(\Sigma) \\ &\propto |\Sigma|^{-\frac{p+v}{2}} e^{-\frac{1}{2}tr\Sigma^{-1}G}, \end{aligned} \tag{A-29}$$

where $G = (\bar{Y} - XB)'(\bar{Y} - XB) + V_{\Sigma}$. Furthermore, we can write $G = S + V + (B - \tilde{B})'(X'X)(B - \tilde{B})$ where $S = (\bar{Y} - X\tilde{B})'(\bar{Y} - X\tilde{B})$ and $\tilde{B} = (X'X)^{-1}X'\bar{Y}$. This last form of G allows us to easily integrate out either B or Σ in the last equation and to obtain the posteriors of B and Σ respectively. Therefore

$$\begin{aligned} p(B|X, Y, \Sigma) &\propto |\Sigma|^{-\frac{p+v}{2}} e^{\Sigma^{-1}(B-\tilde{B})'(X'X)(B-\tilde{B})} \\ p(\Sigma|X, Y) &\propto |\Sigma|^{-\frac{p+v}{2}} e^{-\frac{1}{2}tr\Sigma^{-1}(V_{\Sigma}+S)}, \end{aligned} \tag{A-30}$$

and we can sample B and Σ using these last forms for a matric-variate normal for B and a Inverse Wishart for Σ.

If we condition equation (A-24) on B and we take $\tilde{Y} = Y - XB$ then we have a multivariate regression model

$$\underset{(p\times J)}{\tilde{Y}} = \underset{(p\times J)}{\Phi} + \underset{(p\times J)}{E}, \tag{A-31}$$

and given equation (A-31) we can write the density of $\tilde{Y}$ as

$$p(\tilde{Y}|\Phi, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}tr(\tilde{Y}-\Phi)'(\tilde{Y}-\Phi)\Sigma^{-1}}. \tag{A-32}$$

If we use $\phi = \text{vec}(\Phi)$, $y = \text{vec}(\tilde{Y})$ then equation (A-32) can be expressed as

$$p(y|\phi, \Sigma) \propto |\Sigma|^{-p/2} e^{-\frac{1}{2}(y-\phi)'(\Sigma^{-1}\otimes I_{p\times p})(y-\phi)}. \tag{A-33}$$

In the same way the prior for Φ can be expressed in vectorized form as

$$p(\phi) \propto |\Psi|^{-J/2}|\Lambda|^{-p/2}e^{-\frac{1}{2}\phi'(\Psi\otimes\Lambda)\phi}. \tag{A-34}$$

We use the vectorized forms to derive the posterior of ϕ. That is $p(\phi|y,\Sigma,\Psi) \propto p(y|\phi,\Sigma)\times p(\phi)$ and therefore

$$p(\phi|y,\Sigma,\Psi) \propto |\Lambda|^{-\frac{(2p+v_0)}{2}}|\Sigma|^{-p/2}e^{-\frac{1}{2}((\phi-a)'M^{-1}(\phi-a)+S_\phi)} \tag{A-35}$$

where $S_\phi = y'Hy + a'M^{-1}a$, $M^{-1} = (H+F)$, $H = \Sigma^{-1}\times I$, $F = \Psi\otimes\Lambda$ and $a = MHy$.

The posterior of Λ can be derived from the third and fourth line of equation (A-25) as follows

$$p(\Lambda|\Phi,Y,\Sigma,\Psi) \propto |\Lambda|^{-\frac{(p+v_0)}{2}}e^{-\frac{1}{2}tr\Lambda(V_\Lambda+\Phi'\Psi\Phi)}. \tag{A-36}$$

If the form of Ψ contains a ρ or ρ_j parameters Gelfand and Vounatsou (2003) suggest to sample them from a discretized prior. The posterior of the ρ parameters is

$$p(\rho|\Phi,Y,\Sigma,\Lambda) \propto |\Psi|^{-J/2}e^{-\frac{1}{2}tr(\Psi\Phi'\Lambda\Phi)}. \tag{A-37}$$

In summary we use the following Gibbs steps

1. $\beta|X,\bar{Y},\Phi,\Lambda,\Psi \sim N(\mathrm{vec}((X'X)^{-1}X'\bar{Y}),\Sigma\otimes(X'X)^{-1})$
2. $\phi|B,X,Y,\Lambda,\Psi \sim N((\Sigma^{-1}\otimes I+\Psi\otimes\Lambda)^{-1}(\Sigma^{-1}\otimes I)y,(\Sigma^{-1}\otimes I+\Psi\otimes\Lambda))$
3. $\Sigma|Y,B,\Phi,\Lambda,\Psi \sim IW((p+v)/2,V_\Sigma+S)$
4. $\Lambda|\Psi,B,X,Y,\Sigma \sim IW((p+v_0)/2,V_\Lambda+\Phi'\Psi\Phi)$
5. $\rho|\Phi,\Lambda,B,X,Y,\Sigma \sim p(\rho|\Phi,Y,\Sigma,\Lambda)$

In the paper we set $V_\Sigma = I_3$ and $V_\Lambda = I_3$ and $v_0 = 5$ while $v = 3$ and $p = 48$. We use 48 states because we leave out Hawaii and Alaska. The matrix Ψ is defined based on the neighborhood structure of the US states where the element Ψ_{kj} takes the value of one when the state k is neighbor of the state j and zero otherwise. We further assume that $\rho_j = \rho$ and we sample this parameter based on the discretized prior described above. Finally, we assume that $\Sigma = \sigma^2 I$ and the β coefficients are equal across technologies.

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