

# Fingerprints in the Ether: Using the Physical Layer for Wireless Authentication

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**Abstract**—The wireless medium contains domain-specific information that can be used to complement and enhance traditional security mechanisms. In this paper we propose ways to exploit the fact that, in a typically rich scattering environment, the radio channel response decorrelates quite rapidly in space. Specifically, we describe a physical-layer algorithm that combines channel probing ( $M$  complex frequency response samples over a bandwidth  $W$ ) with hypothesis testing to determine whether current and prior communication attempts are made by the same user (same channel response). In this way, legitimate users can be reliably authenticated and false users can be reliably detected. To evaluate the feasibility of our algorithm, we simulate spatially variable channel responses in real environments using the WiSE ray-tracing tool; and we analyze the ability of a receiver to discriminate between transmitters (users) based on their channel frequency responses in a given office environment. For several rooms in the extremities of the building we considered, we have confirmed the efficacy of our approach under static channel conditions. For example, measuring five frequency response samples over a bandwidth of 100 MHz and using a transmit power of 100 mW, valid users can be verified with 99% confidence while rejecting false users with greater than 95% confidence.

## I. INTRODUCTION

As wireless devices become increasingly pervasive and essential, they are becoming both a target for attack and the very weapon with which such an attack can be carried out. Traditional high-level computer and network security techniques can, and must, play an important role in combating such attacks, but the wireless environment presents both the means and the opportunity for new forms of intrusion. The devices that comprise a wireless network environment are low-cost commodity items that are easily available to potential intruders and also easily modifiable for such intrusion. In particular, wireless networks are open to intrusion from the outside without the need for a physical connection and, as a result, techniques which would provide a high level of security in a wired network have proven inadequate in a wireless network, as many motivated groups of students have readily demonstrated [1]–[3].

Although conventional cryptographic security mechanisms are essential to securing wireless networks, these techniques do not directly leverage the unique properties of the wireless domain to address security threats. The physical properties of the wireless medium are a powerful source of domain-specific information that can be used to complement and

enhance traditional security mechanisms. In this paper, we propose that a cross-layer approach can be used to augment the security of wireless networks. In particular, we believe that the nature of the wireless medium can be turned to the advantage of the network engineer when trying to secure wireless communications. The enabling factor in our approach is that, in the rich multipath environment typical of wireless scenarios, the response of the medium along any transmit-receive path is *frequency-selective* (or in the time domain, *dispersive*) in a way that is *location-specific*. This means:

- 1) The channel can be specified by a number of complex samples either in the frequency domain (a set of complex gains at a set of frequencies) or the time domain (a set of impulse response samples at a set of time delays).
- 2) Such sets of numbers decorrelate from one transmit-receive path to another if the paths are separated by the order of an RF wavelength or more.

Using the uniqueness of the channel between two locations, we believe it is possible to establish new forms of authentication that include information available at the physical layer. Rather than rely solely on higher-layer cryptographic mechanisms, wireless devices can authenticate themselves based upon their ability to produce an appropriate signal at the recipient.

While using the physical layer to enhance security might seem to be a radical paradigm shift for wireless systems, we note that this is not the first time that multipath and advanced physical layer methods have proven advantageous. Specifically, we are encouraged in our belief by two notable parallel paradigm shifts in wireless systems: (1) code division multiple access (CDMA) systems [4], where the use of Rake processing transforms multipath into a diversity-enhancing benefit; and (2) multiple-input multiple-output (MIMO) antenna techniques [5], which transform scatter-induced Rayleigh fading into a capacity-enhancing benefit.

In order to support the use of physical layer information for enhancing wireless security, it is necessary to understand the degree to which physical layer measurements can serve to discriminate between transmitters, and then to place this functionality in the context of a greater end-to-end security framework. In this paper, we tackle the first of these problems by providing an initial investigation into the ability of a receiver to distinguish between transmitters.

We begin the paper in Section II by providing an overview of our proposed PHY-layer authentication service. We then examine the possibilities of achieving physical-layer authentication using a hypothesis testing framework in Section III. In

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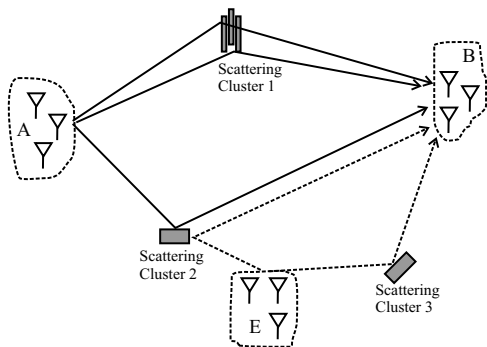


Fig. 1. The adversarial multipath environment involving multiple scattering surfaces. The transmission from Alice (A) to Bob (B) experiences different multipath effects than the transmission by the adversary, Eve (E).

order to validate our ideas, we have performed simulations using the WiSE propagation tool, and our results are in Section IV. Our objective is to understand the degree to which physical layer authentication is possible, and hence our initial performance studies reported in this paper are for a benign, static multipath environment. We wrap up the paper in Section V by providing concluding remarks and highlighting important areas for further investigation.

## II. PROBLEM OVERVIEW

Traditionally, authentication involves the verification of an entity's identity. In the context of physical layer authentication, however, we are not interested in identity, *per se*, but rather are interested in recognizing a particular transmitter device. The ability to distinguish between different transmitters would be particularly valuable in real wireless systems, as it would help prevent spoofing attacks, where one wireless device claims to be another wireless device. Currently, spoofing attacks are very easy to launch in many wireless networks. For example, in commodity networks, such as 802.11 networks, it is easy for a device to alter its MAC address by simply issuing an `ifconfig` command. This weakness is a serious threat, and there are numerous attacks, ranging from session hijacking [6] to attacks on access control lists [2], that are facilitated by the fact that an adversarial device may masquerade as another device.

Here, we seek to develop the notion of physical-layer authentication services by making use of the complexity associated with multipath propagation. Throughout the discussion, we shall borrow from the conventional terminology of the security community by introducing three different parties: Alice, Bob and Eve. For our purposes, these three entities may be thought of as wireless transmitters/receivers that are potentially located in spatially separated positions, as depicted in Figure 1. Our two "legal" protagonists are the usual Alice and Bob, and for the sake of discussion throughout this paper, Alice will serve as the transmitter that initiates communication, while Bob will serve as the intended receiver. Their nefarious adversary, Eve, will serve as an active opponent who injects undesirable communications into the medium in the hopes of impersonating Alice.

Our security objective, broadly speaking, is to provide authentication between Alice and Bob, despite the presence

of Eve. Authentication is traditionally associated with the assurance that a communication comes from a specific entity [7]. Returning to our communication scenario, this objective may be interpreted as follows. Since there is a potential adversary, Eve, who is within range of Alice and Bob, and who is capable of injecting her own signals into the environment to impersonate Alice, it is desirable for Bob to have the ability to differentiate between legitimate signals from Alice and illegitimate signals from Eve. He therefore needs some form of evidence that the signal he receives did, in fact, come from Alice.

In a multipath environment, the property of rapid spatial decorrelation can be used to authenticate a transmitter. To illustrate this, let us return to Figure 1 and consider a simple transmitter identification protocol in which Bob seeks to verify that Alice is the transmitter. Suppose that Alice probes the channel sufficiently frequently to assure temporal coherence between channel estimates and that, prior to Eve's arrival, Bob has estimated the Alice-Bob channel. Now, Eve wishes to convince Bob that she is Alice. Bob will require that each information-carrying transmission be accompanied by an authenticator signal. The channel and its effect on a transmitted signal between Alice and Bob is a result of the multipath environment. Bob may use the received version of the authenticator signal to estimate the channel response and compare this with a previous record for the Alice-Bob channel. If the two channel estimates are "close" to each other, then Bob will conclude that the source of the message is the same as the source of the previously sent message. If the channel estimates are not similar, then Bob should conclude that the source is likely not Alice.

There are several important issues related to such a procedure that should be addressed before it can be a viable authentication mechanism. First is the specification of the authenticator signal that is used to probe the channel. There are many standardized techniques to probe the channel, ranging from pulse-style probing to multi-tonal probing [8], and we may use these techniques to estimate the channel response. Regardless of what probing method is employed, the channel response can be characterized in the frequency domain, and throughout this paper we will represent our channels in that domain.

Next, at the heart of our idea, we use the fact in a richly scattered multipath environment (typical of indoor wireless environments) it is difficult for an adversary to create or precisely model a waveform that is transmitted and received by entities that are more than a wavelength away from the adversary. The difficulty of an adversary to predict the environment is supported by the well-known Jakes uniform scattering model [9], which states that the received signal rapidly decorrelates over a distance of roughly half a wavelength, and that spatial separation of one to two wavelengths is sufficient for assuming independent fading paths. The implication of such a scattering model in a transmitter identification application remains to be tested, and one of the objectives behind this study is to examine the utility of a typical indoor multipath environment for discriminating between Alice-Bob and Eve-Bob channels. It should also be noted that the multipath channel will change

with time due to both terminal mobility and changes in the environment. As mentioned earlier, in practice it will be necessary to guarantee the continuity of the authentication procedure by probing the channel at time intervals less than the channel's coherence time. However, even before issues of temporal variability can be brought into the picture, it is necessary to first examine the ability to distinguish between transmitters in a static multipath environment. This paper examines the ability to authenticate transmitters in such an environment, and serves to illustrate the potential for new forms of physical layer security.

### III. ANALYSIS

In this section, we provide a formulation of physical layer authentication as a hypothesis testing problem.

#### A. System Model

We assume that Bob first measures and stores the frequency response of the channel connecting Alice with him. Though the true channel response is  $H_{AB}(f)$ , Bob stores a noisy version,  $\hat{H}_{AB}(f)$ , due to his receiver noise. After a while, he has to decide whether a transmitting terminal is still Alice, his decision being based on a noisy measured version,  $\hat{H}_t(f)$ , of that terminal's channel response to Bob (the true response being  $H_t(f)$ ). By sampling  $\hat{H}_{AB}(f)$  and  $\hat{H}_t(f)$ ,  $f \in (f_o - W/2, f_o + W/2]$ , Bob obtains two vectors  $\underline{\hat{H}}_{AB}$  and  $\underline{\hat{H}}_t$ ,

$$\underline{\hat{H}}_{AB} = \underline{H}_{AB} e^{j\phi_1} + \underline{N}_1 \quad (1)$$

$$\underline{\hat{H}}_t = \underline{H}_t e^{j\phi_2} + \underline{N}_2 \quad (2)$$

where the elements of vector  $\underline{A} = [A_1, \dots, A_M]^T$  are samples from  $A(f)$ . More specifically,  $A_m = A(f_o - W/2 + m\Delta f)$ ,  $m = 1, \dots, M$ , where  $\Delta f = W/M$ ;  $M$  is the sample size;  $W$  is the measurement bandwidth;  $f_o$  is the center frequency of the measurement; and all elements of  $\underline{N}_1$  and  $\underline{N}_2$  are i.i.d complex Gaussian noise samples  $CN(0, \sigma^2)$ . Considering the fact that the phase of Bob's receiver local oscillator (LO) can change between one measurement and another, we introduce  $\phi_1$  and  $\phi_2 \in [0, 2\pi)$  to represent measurement errors in the phase of the channel frequency response.

#### B. Hypothesis Testing

Bob uses a simple hypothesis test [10] to decide if the transmitting terminal is Alice or a would-be intruder, e.g., Eve. The null hypothesis,  $\mathcal{H}_0$ , is that the terminal is not an intruder, i.e. the claimant is Alice; and Bob accepts this hypothesis if the test statistic he computes,  $L$ , is below some threshold,  $k$ . Otherwise, he accepts the alternative hypothesis,  $\mathcal{H}_1$ , that the claimant terminal is someone else.

$$\mathcal{H}_0 : \underline{H}_t = \underline{H}_{AB} \quad (3)$$

$$\mathcal{H}_1 : \underline{H}_t \neq \underline{H}_{AB} \quad (4)$$

The test statistic is chosen to be

$$L = \min_{\phi} \frac{1}{\sigma^2} \sum_{m=1}^M | \hat{H}_{tm} - \hat{H}_{ABm} e^{j\phi} |^2. \quad (5)$$

The minimization over the phase  $\phi$  is necessary to account for measurement errors in the phase of the frequency response,  $\phi_1$  and  $\phi_2$ . Without this adjustment by Bob, the transmitting terminal can be rejected even if it is in fact Alice. It is easy to show that the minimizing value of  $\phi$  is

$$\phi^* = \text{Arg} \left( \sum_{m=1}^M \hat{H}_{tm} \hat{H}_{ABm}^* \right). \quad (6)$$

For the sake of analytical tractability, we will use for  $\phi^*$  the value corresponding to a noiseless channel ( $\hat{H}_{AB}(f) = H_{AB}(f)$  and  $\hat{H}_t(t) = H_t(f)$ ); for the high-SNR conditions where the system must operate, this is a very reasonable approximation.

Subject to this approximation, it is easy to show that, when the transmitting terminal is Alice, the test statistic  $L$  is a chi-square random variable with  $2M$  degrees of freedom [11], i.e.,

$$L = \frac{1}{\sigma^2} \left( \sum_{m=1}^M n_{rm}^2 + \sum_{m=1}^M n_{im}^2 \right) \sim \chi_{2M,0}^2, \quad (7)$$

where  $n_{rm}$  and  $n_{im}$  are i.i.d Gaussian variables  $N(0, \sigma^2)$ . When the transmitting terminal is Eve, however,  $L$  becomes a non-central chi-square variable with a non-centrality parameter  $\mu_L$ , i.e.,

$$L = \frac{1}{\sigma^2} \left( \sum_{m=1}^M (\Delta h_{rm}^* + n_{rm})^2 + \sum_{m=1}^M (\Delta h_{im}^* + n_{im})^2 \right) \sim \chi_{2M, \mu_L}^2, \quad (8)$$

where  $\Delta h_{rm}^*$  and  $\Delta h_{im}^*$  are the real and imaginary part of  $(H_{EBm} - H_{ABm} e^{j\phi^*})$ , respectively, with  $\phi^*$  given by (6), and  $\mu_L = \frac{1}{\sigma^2} \sum_{m=1}^M |H_{EBm} - H_{ABm} e^{j\phi^*}|^2$ .

We define the rejection region for  $H_0$  as  $L > k$ , where  $k$  is the threshold. Thus, the "false alarm rate" (or Type I error) is

$$\alpha = P_{H_0}(L > k) = 1 - F_{\chi_{2M}^2}(k), \quad (9)$$

and the "miss rate" (or Type II error) is

$$\beta = P_{H_1}(L < k) = F_{\chi_{2M, \mu_L}^2}(k), \quad (10)$$

where  $F_X(\cdot)$  is the CDF of the random variable  $X$ . For a specified  $\alpha$ , the threshold of the test is  $k = F_{\chi_{2M}^2}^{-1}(1 - \alpha)$ , and the miss rate is  $\beta = F_{\chi_{2M, \mu_L}^2}(F_{\chi_{2M}^2}^{-1}(1 - \alpha))$ , which decreases with  $\mu_L$ . More specifically, with  $\alpha$  fixed,  $\beta$  rises with  $\sigma^2$  (because  $k$  does) and falls with  $\sum_{m=1}^M |H_{EBm} - H_{ABm} e^{j\phi^*}|^2$ .

### IV. SIMULATION AND NUMERICAL RESULTS

#### A. Simulating the Transfer Functions

In order to test the proposed scheme, it is necessary to model not only "typical" channel responses, but the spatial variability of these responses. Only in that way can we discern the success in detecting would-be intruders like Eve. To that end, we make use of the WiSE tool, a ray-tracing software package developed by Bell Laboratories [12]. One input to WiSE is the 3-dimensional plan of a specific building, including walls, floors, ceilings and their material properties. With this

information, WiSE can predict the rays at any receiver from any transmitter, including their amplitudes, phases and delays. From this, it is straightforward to construct the transmit-receive frequency response over any specified interval.

We have done this for one particular office building, for which a top view of the first floor is shown in Fig. 2. This floor of this building is 120 meters long, 14 meters wide and 4 meters high. For our numerical experiment, we placed Bob in the hallway (the filled-in circle) at a height of 2 m. For the positions of Alice and Eve, we considered four rooms at the extremities of the building (shown shaded). For each room, we assumed Alice and Eve both transmitted from a height of 2 m, each of them being anywhere on a uniform horizontal grid of points with 0.2-meter separations. With  $N_s$  grid points in a room, there were  $N_s(N_s - 1)/2$  possible pairs of Alice-Eve positions. For Rooms 1, 2, 3 and 4, the numbers of grid points were  $N_s = 150, 713, 315$  and  $348$ , respectively. For each Alice-Eve pair, (1) WiSE was used to generate the Alice-Bob and Eve-Bob channel responses ( $H_{AB}(f)$  and  $H_{EB}(f)$ ); and (2) the hypothesis test described above was used to compute  $\beta$  for a specified  $\alpha$ . The set of all  $\beta$ -values in a room were used to compute a room-specific mean,  $\bar{\beta}$ , for each of several selected combinations of bandwidth ( $W$ ), number of tones ( $M$ ) and transmit power ( $P_T$ ).

### B. Transmit Power and Receiver Noise

Assume that, in conjunction with WiSE, we obtain the various transfer functions as dimensionless ratios (e.g., received  $E$ -field/transmitted  $E$ -field). Then the proper treatment of the noise variance,  $\sigma^2$ , in the hypothesis test is to define it as the receiver noise power per tone,  $P_N$ , divided by the transmit power per tone,  $P_T/M$ , where  $P_T$  is the total transmit power. Noting that  $P_N = \kappa T N_F b$ , where  $\kappa T$  is the thermal noise density in mW/Hz,  $N_F$  is the receiver noise figure, and  $b$  is the measurement noise bandwidth per tone in Hz, we can write

$$\sigma^2 = \frac{\kappa T N_F b}{P_T/M} = \frac{M}{\Gamma} \quad (11)$$

where  $P_T$  is in mW,  $\Gamma = P_T/P_N$  and we will henceforth refer to  $\Gamma$  by its decibel value.

### C. Simulation Results

In the simulations, we set  $\alpha = 0.01$ ,  $f_0 = 5$  GHz,  $N_F = 10$ , and  $\Gamma = 90, 100, 110, 120$  dB, which may be viewed as combinations of  $b = 2.5$  MHz and  $P_T = 0.1, 1, 10, 100$  mW, respectively. As noted earlier, we place Alice and Eve on dense grids in each of four rooms at the corners of a particular building, with Bob in the hallway, Fig. 2.

We obtain a miss rate  $\beta$  for each Alice-Eve pair, and then calculate the mean value  $\bar{\beta}$  for each room with  $M = 1 \sim 10$  and  $W = 0.05 \sim 0.5$  GHz. The results verify the utility of our algorithm and show that, if  $P_T = 100$  mW, most values of  $\bar{\beta}$  are below 0.05, even at the farthest corners of the building.

Figures 3-6 show our computed results for Rooms 1-4, respectively. They show that, in terms of minimizing  $\beta$ , increasing transmit power can be most beneficial, while increasing the bandwidth and number of tones has less impact. In all cases,

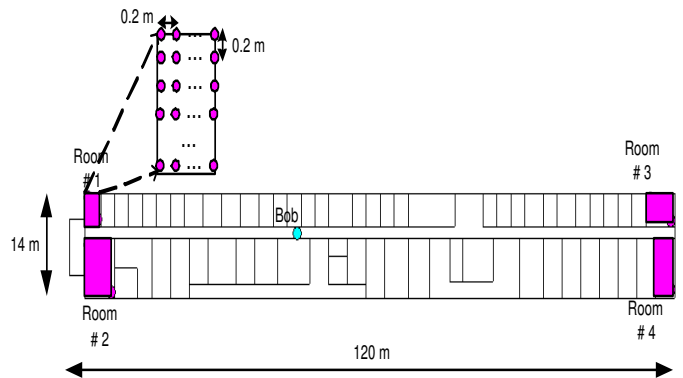


Fig. 2. System topology assumed in the simulations. Bob is located at [45.6, 6.2, 3.0] m in a 120 m  $\times$  14 m  $\times$  4 m office building. Alice and Eve are located on dense grids at a height of 2 m. The sizes of the grids are  $N_s = 150, 713, 315$ , and  $348$ , respectively, for Room 1, 2, 3 and 4.

there is little benefit (or even a deficit) in increasing  $M$  beyond  $\sim 5$ ; and in most cases, there is little benefit in increasing  $W$  beyond  $\sim 100 - 200$  MHz. This finding, however, applies to the case where there are no temporal variations in the levels or shapes of the transfer functions, a topic we discuss in the last section.

Finally, the figures show the effects of distance (path length), which influences the per-tone signal-to-noise ratios at Bob's receiver. Rooms 3 and 4, which are farther from Bob than Rooms 1 and 2, have clearly poorer performance in rejecting Eve. Since the four rooms are at the building extremities, we can assume that this set of results lower-bounds the capabilities of our PHY-layer authentication algorithm.

## V. CONCLUSION & FUTURE WORK

We have described and studied a physical layer technique for enhancing authentication in a wireless in-building environment. The technique uses channel frequency response measurements and hypothesis testing to discriminate between a legitimate user (Alice) and a would-be intruder (Eve). The study used the ray-tracing tool WiSE to generate realistic spatially varied responses, and results were obtained for several most-distant (i.e., worst-case) rooms of one particular building. They confirm the efficacy of the algorithm for realistic values of the measurement bandwidth (e.g.,  $W \sim 100$  MHz), number of response samples (e.g.,  $M \leq 5$ ) and transmit power (e.g.,  $P_T \sim 100$  mW). Computed results not shown here (but suggested by the left side of Fig. 3a) indicate good performance down to  $W = 20$  MHz, so that the method can be used within bandwidths typical of existing WLANs.

Moving forward, further investigation is needed to test other buildings and to look at multiple Bob locations within the same building, thereby establishing required power levels for a wider class of cases. Another important topic is the temporal variations of the measured channel responses, e.g., variations due to movements within the building, slow time changes in the transmit power and/or receiver noise level, etc. Our preliminary investigations in [13] have confirmed the efficacy of our approach in time-variant channels and showed that the

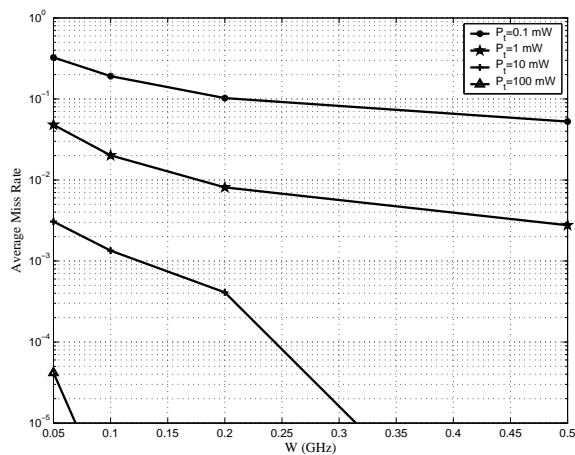
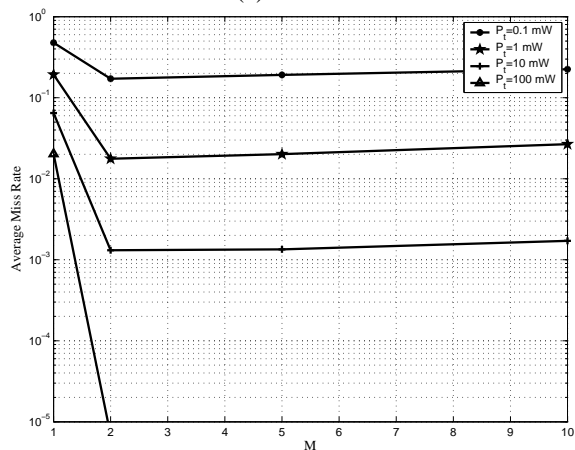
(a)  $M = 5$ (b)  $W = 0.1$  GHz

Fig. 3. Results for Room 1. Alice and Eve are placed within Room 1, while Bob is located in the center of the building, as depicted in Fig. 2. For each combination of Alice and Eve locations, the corresponding channel responses to Bob were used to estimate the miss rate. The average miss rate for Room 1,  $\bar{\beta}$ , is reported as: (a) a function of bandwidth ( $W$ ) for fixed number of tones ( $M$ ); and (b) as a function of  $M$  for fixed  $W$ .

temporal variations even improve the performance in some cases. Finally, as part of our ongoing efforts, we are working to integrate physical layer authentication into a holistic cross-layer framework for wireless security that will augment traditional “higher-layer” network security mechanisms with physical layer methods.

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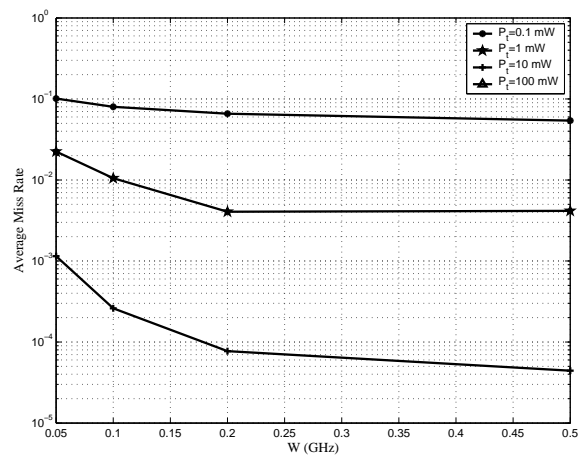
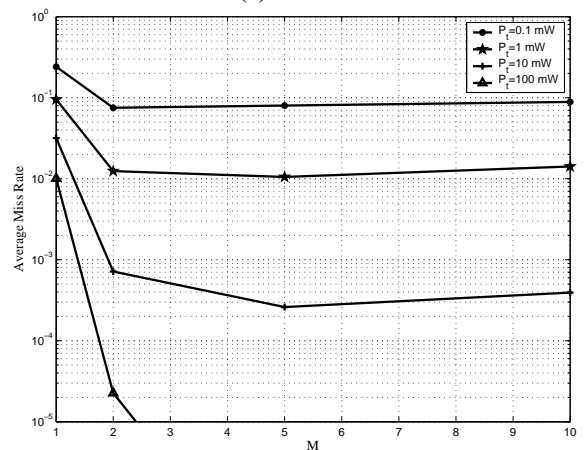
(a)  $M = 5$ (b)  $W = 0.1$  GHz

Fig. 4. The average miss rate,  $\bar{\beta}$ , for Room 2, is reported as: (a) a function of bandwidth ( $W$ ) for fixed number of tones ( $M$ ); and (b) as a function of  $M$  for fixed  $W$ .

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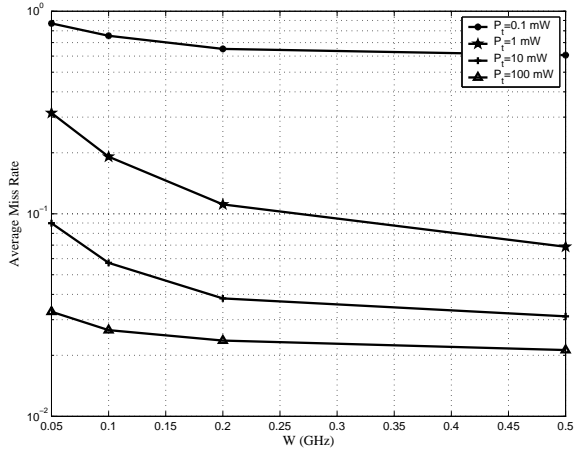
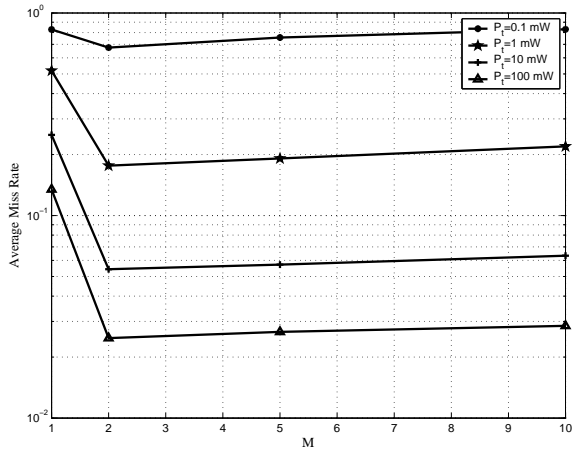
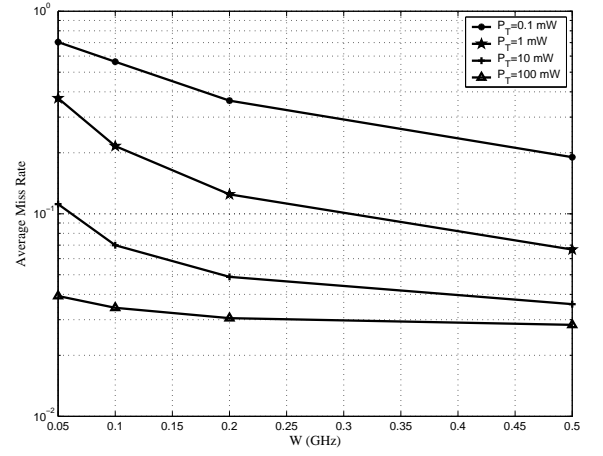
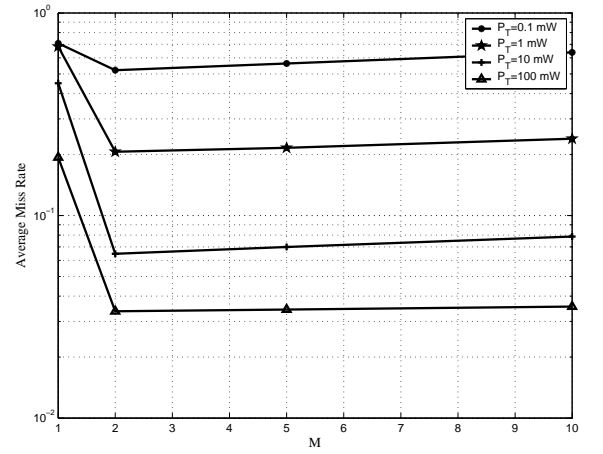
(a)  $M = 5$ (b)  $W = 0.1$  GHz(a)  $M = 5$ (b)  $W = 0.1$  GHz

Fig. 5. The average miss rate,  $\bar{\beta}$ , for Room 3, is reported as: (a) a function of bandwidth ( $W$ ) for fixed number of tones ( $M$ ); and (b) as a function of  $M$  for fixed  $W$ .

Fig. 6. The average miss rate,  $\bar{\beta}$ , for Room 4, is reported as: (a) a function of bandwidth ( $W$ ) for fixed number of tones ( $M$ ); and (b) as a function of  $M$  for fixed  $W$ .