Finite conjugate spherical aberration compensation in high numerical-aperture optical disc readout

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Spherical aberration arising from deviations of the thickness of an optical disc substrate from a nominal value can be compensated to a great extent by illuminating the scanning objective lens with a slightly convergent or divergent beam. The optimum conjugate change and the amount and type of residual aberration are calculated analytically for an objective lens that satisfies Abbe's sine condition. The aberration sensitivity is decreased by a factor of 25 for numerical aperture values of approximately 0.85, and the residual aberrations consist mainly of the first higher-order Zernike spherical aberration term A_{60}. The Wasserman–Wolf–Vaskas method is used to design biaspheric objective lenses that satisfy a ray condition that interpolates between the Abbe and the Herschel conditions. Requirements for coma by field use allow for only small deviations from the Abbe condition, making the analytical theory a good approximation for any objective lens used in practice. © 2005 Optical Society of America

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1. Introduction

In optical disc readout the beam is focused onto the data layer of the disc through a substrate layer of thickness d. The scanning objective lens is designed in such a way that the spherical aberration resulting from focusing through this layer is compensated for, so that the scanning spot at the data layer is nominally free from aberrations. A mismatch of the substrate thickness from the nominal value results in spherical aberration. The sensitivity for thickness mismatch-induced spherical aberration increases strongly with the numerical aperture (NA) of the objective lens. The wavelength is the natural measure for aberrations, so that a decrease of wavelength λ also results in greater aberration sensitivity. The increase of NA and decrease of λ used to increase the capacity of an optical disc (the size of the focal spot scales as λ/NA) thus deteriorates the tolerances of the system for substrate thickness mismatch.

For a Compact Disc (CD) ($\lambda = 0.785$ μm, NA of 0.50) sensitivity is 0.4 mλ/μm, for a Digital Versatile Disc (DVD) ($\lambda = 0.660$ μm, NA of 0.65) the sensitivity is 1.4 mλ/μm, and for the Blu-ray Disc (BD) ($\lambda = 0.405$ μm, NA of 0.85) the sensitivity is 10.1 mλ/μm.[1] (The aberration unit here is the milliwave; 1 mλ = $\lambda/1000$.) With a spacing of the two layers of a BD double-layer disc of 25 μm the amount of spherical aberration is ±126 mλ, if we assume that the objective lens is optimized for the focal position half-way between the two layers. This is more than the diffraction limit of $\lambda/8\sqrt{3}$ = 72 mλ and is hence unacceptably large. Clearly, means to compensate for spherical aberration must be introduced into the optical system. The compensating system must be switched into a first state when data are read from or written to the upper layer of a double-layer BD, and into a second state when data are read from or written to the lower layer of a double-layer BD.

The most straightforward way of compensating for spherical aberration is to use finite conjugate illumination of an objective lens. Spherical aberration depends on the conjugate of the lens.[2,3] It follows that this effect can be used to balance spherical aberration arising from other causes. In the practice of optical disc readout the lens is normally used at infinite conjugate, i.e., the lens is illuminated with a collimated beam. By changing to a slightly convergent or divergent beam the spherical aberration that is due to the thickness mismatch related to double-layer discs can be compensated for. Although strictly needed for the two discrete depths of a double-layer
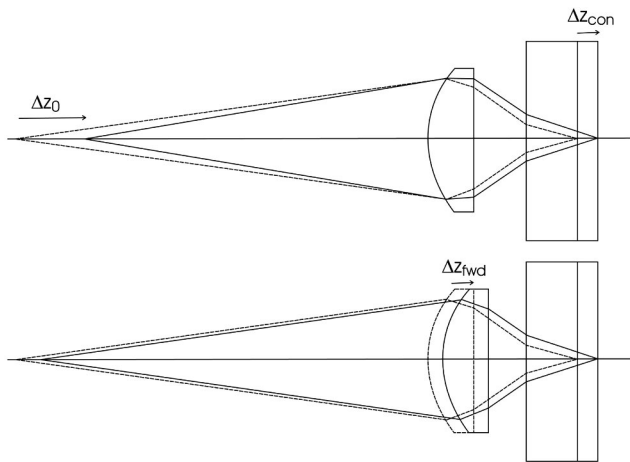
Fig. 1. Changing the axial focal position can be achieved by changing the conjugate of the objective lens (top) or by changing the free working distance (bottom).

disc, finite conjugate illumination can be used to compensate for any thickness value in a continuous range around the central value. In the field of microscopy this method of spherical aberration compensation is known as a change in the tube length (the distance between object and image).[4] I report on how effective the method is and which parameters determine the conjugate change required to bridge a given amount of thickness mismatch. The conclusions are reached with exact analytical means and compared with numerical ray-tracing methods. The question of the effectiveness of this method of spherical aberration compensation has been analyzed before within the context of confocal microscopy for the case of a small refractive-index mismatch of the medium in which the beam is focused with a nominal value.[5,6] This is in contrast with the case considered here in which the spherical aberration arises from a mismatch in thickness of the layer through which the beam is focused.

The content of this paper is as follows. Section 2 deals with the analytical theory of finite conjugate spherical aberration compensation for an aplanatic objective lens. The effects of deviations of aplanaticity are analyzed by numerical ray-tracing methods in Section 3.

2. Analytical Theory for an Aplanatic Objective Lens

Consider an objective lens that focuses a beam of light onto the data layer of an optical disc. There are two ways to shift the focal point in the disc in the axial direction, as illustrated in Fig. 1. The first way is to change the conjugate of the optical system, i.e., by axially shifting the source point, the image point will be shifted in the axial direction over a distance Δz_{con}. The second way is to bring the optical system as a whole a distance Δz_{fwd} closer to the disc, thus reducing the free working distance (fwd) of the objective lens. When the objective lens is used at infinite conjugate this is equivalent to bringing the objective lens closer to the disc, while keeping the other optical components at a fixed position. The diffraction focus,

i.e., the reference point for the aberration function giving rise to a minimum rms value of the aberration function, is in general a distance Δz_{ref} further in the disc. The total focal shift Δd then follows as

$$\Delta d = \Delta z_{con} + \Delta z_{fwd} + \Delta z_{ref}. \quad (1)$$

Clearly, given the total focal shift there are two degrees of freedom. These are fixed by the condition of minimum rms aberration. The form of the aberration function is determined by Abbe's sine condition.[2,3] According to this condition, if the optical system is free from (spherical) aberration when the object and image points are on the optical axis it is also free from (comatic) aberration when the object and image points are laterally displaced from the optical axis (to first order in the field angle). Alignment tolerances of an optical drive light path require that the design of the objective lens cannot deviate too much from the sine condition. Consequently, in many cases the objective lens may be approximated by an aplanat. The accuracy of this approximation is discussed in Section 3. The sine condition imposes a relation between the rays in object and image space, and this relation determines the aberration function resulting from the change in conjugate. In the folllowing an expression for the aberration function is derived and values are determined for Δz_{con} and Δz_{fwd} that give rise to a minimum rms value of the aberration function.

Consider an axisymmetric optical system and a ray passing through an object point and an image point, both on the optical axis, such that the ray makes an angle θ_0 with the optical axis in object space and an angle θ_1 with the optical axis in image space. Abbe's sine condition relates the ray angles θ_0 and θ_1 in object and image space by

$$n_0 \sin \theta_0 = M n_1 \sin \theta_1, \quad (2)$$

where n_0 and n_1 are the refractive indices in object and image space, and M is the (lateral) magnification from object to image space. A scaled pupil coordinate can be defined as

$$\rho = \frac{n_0 \sin \theta_0}{NA_0} = \frac{n_1 \sin \theta_1}{NA_1}, \quad (3)$$

where NA_0 and NA_1 are the NAs in object and image space. It follows that ρ has values between 0 and 1. For a small axial displacement of object and image points Δz_0 and Δz_1, respectively, the aberration then follows as

$$W_{con} = n_1 \Delta z_1 (\cos \theta_1 - 1) - n_0 \Delta z_0 (\cos \theta_0 - 1). \quad (4)$$

Note that a different sign convention is used in Refs. 2 and 3. The axial displacements are related by

$$\Delta z_1 = M^2 \frac{n_1}{n_0} \Delta z_0, \quad (5)$$

so that

$$W_{\text{con}} = n_1 \Delta z_1 \left\{ \left(1 - \frac{\rho^2 \text{NA}_1^2}{n_1^2}\right)^{1/2} \right.$$
$$\left. - 1 + \frac{n_0^2}{n_1^2 M^2}\left[1 - \left(1 - \frac{\rho^2 \text{NA}_0^2}{n_0^2}\right)^{1/2}\right]\right\}$$
$$= \Delta z_1 \left[(n_1^2 - \rho^2 \text{NA}_1^2)^{1/2} - n_1 \right.$$
$$\left. + \frac{\rho^2 \text{NA}_1^2}{n_1 + (n_1^2 - \rho^2 \text{NA}_1^2 M^2 n_1^2 / n_0^2)^{1/2}} \right]. \quad (6)$$

In the particular case in which we are interested we have $\Delta z_1 = \Delta z_{\text{con}}$, and $M = 0$ (infinite conjugate). Using the abbreviations $n_1 = n$ and $\text{NA}_1 = \text{NA}$ it then follows that

$$W_{\text{con}} = \Delta z_{\text{con}} \left[(n^2 - \rho^2 \text{NA}^2)^{1/2} - n + \frac{p^2 \text{NA}^2}{2n} \right], \quad (7)$$

which is in agreement with the expression derived by Sheppard and Gu.[7] The decrease in free working distance introduces an aberration that is equivalent to the insertion of a layer of refractive index n with a thickness equal to the decrease in free working distance.[1]

$$W_{\text{fwd}} = \Delta z_{\text{fwd}} \left\{ (n^2 - \rho^2 \text{NA}^2)^{1/2} - n \right.$$
$$\left. - \left[(1 - \rho^2 \text{NA}^2)^{1/2} - 1 \right] \right\}. \quad (8)$$

The diffraction focus is found a distance Δz_{ref} deeper into the disc, which gives an aberration[1] of

$$W_{\text{ref}} = \Delta z_{\text{ref}} \left[(n^2 - \rho^2 \text{NA}^2)^{1/2} - n \right]. \quad (9)$$

Eliminating Δz_{ref} in favor of Δd the total aberration function follows as

$$W = W_{\text{con}} + W_{\text{fwd}} + W_{\text{ref}}$$
$$= \Delta d \left[(n^2 - \rho^2 \text{NA}^2)^{1/2} - n \right] + \Delta z_{\text{con}} \frac{\rho^2 \text{NA}^2}{2n}$$
$$- \Delta z_{\text{fwd}} \left[(1 - \rho^2 \text{NA}^2)^{1/2} - 1 \right]. \quad (10)$$

When the conjugate change $\Delta z_{\text{con}} = 0$ the aberration expression of Ref. 1 is recovered. For a given change in focal position Δd the changes in conjugate and free working distance are free parameters that can be varied to determine the optimum focal spot. The optimum corresponds to a minimum rms wavefront aberration. This minimization procedure can be done as follows. First, write

$$\frac{W}{\Delta d} = f - \sum_{i=1,\,2} \beta_i g_i, \quad (11)$$

with

$$f = (n^2 - \rho^2 \text{NA}^2)^{1/2}, \quad (12)$$

$$g_1 = -\frac{\rho^2 \text{NA}^2}{2n}, \quad (13)$$

$$g_2 = \frac{1}{n}(1 - \rho^2 \text{NA}^2)^{1/2}, \quad (14)$$

where the piston terms have been left out as they cancel from the expression for the rms wavefront aberration, and with

$$\beta_1 = \beta_{\text{con}} = \frac{\Delta z_{\text{con}}}{\Delta d}, \quad (15)$$

$$\beta_2 = \beta_{\text{fwd}} = \frac{n \Delta z_{\text{fwd}}}{\Delta d}. \quad (16)$$

The rms wavefront aberration can now be written as

$$\frac{W_{\text{rms}}^2}{(\Delta d)^2} = \frac{\langle W^2 \rangle - \langle W \rangle^2}{(\Delta d)^2}$$
$$= f_{\text{rms}}^2 - 2 \sum_{i=1,\,2} v_i \beta_i + \sum_{i,j=1,2} R_{ij} \beta_i \beta_j, \quad (17)$$

where

$$f_{\text{rms}}^2 = \langle f^2 \rangle - \langle f \rangle^2, \quad (18)$$

$$v_i = \langle f g_i \rangle - \langle f \rangle \langle g_i \rangle, \quad (19)$$

$$R_{ij} = \langle g_i g_j \rangle - \langle g_i \rangle \langle g_j \rangle, \quad (20)$$

the angle brackets indicate the pupil average defined by

$$\langle A \rangle = \frac{1}{\pi} \int_P d^2 \rho A(\rho), \quad (21)$$

and the integration is over the unit circle. Explicit analytical expressions for the relevant pupil averages are given in Appendix A. Minimization with respect to β_i leads to

$$\beta_i = \sum_{j=1,\,2} R_{ij}^{-1} v_j, \quad (22)$$

$$\frac{W_{\text{rms}}^2}{(\Delta d)^2} = f_{\text{rms}}^2 - \sum_{i,j=1,\,2} R_{ij}^{-1} v_i v_j, \quad (23)$$

where R^{-1} is the inverse of the 2×2 matrix R. This completes the analytical theory. The dependence of the residual aberration sensitivity $W_{\text{rms}}/\Delta d$ and the parameters β_{con} and β_{fwd} on NA are discussed in the following.
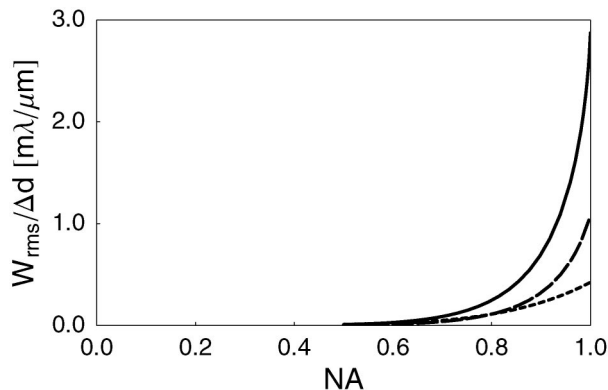
Fig. 2. Residual aberration sensitivity as a function of NA (solid curve) and approximation by the single Zernike term A_{60} (long-dashed curve) and by the Seidel term proportional to NA^6 (short-dashed curve).
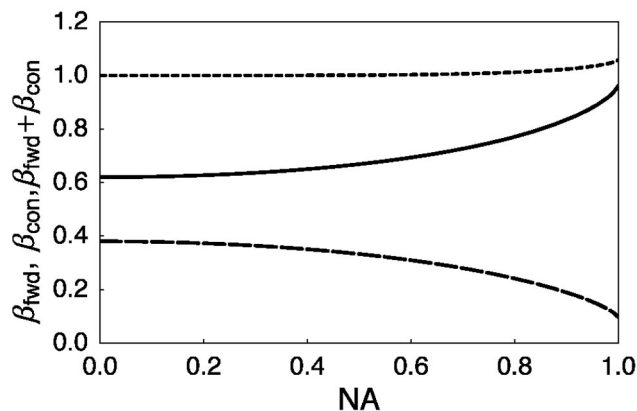

Fig. 3. Parameter β_{con} that describes the relative change of conjugate (solid curve), the parameter β_{fwd} that describes the relative change in free working distance (long-dashed curve), and the sum of the two parameters $\beta_{con} + \beta_{fwd}$ (short-dashed curve) as a function of NA.

Figure 2 shows the residual aberration sensitivity as a function of NA. For BD conditions (NA of 0.85 NA, $\lambda = 405$ nm, and $n = 1.6$) the sensitivity is 0.41 mλ/μm compared with the 10.07 mλ/μm for the case without conjugate change,[1] which amounts to a reduction by a factor of 25. For a BD dual-layer disc with a nominal spacer layer thickness of 25 μm this results in only a ± 5.1 mλ rms aberration, assuming that the objective lens is optimized for the focal position halfway between the two layers. Clearly, conjugate change is an effective way to compensate for spherical aberration.

The exact residual aberration sensitivity can be expanded in powers of NA. This corresponds to an expansion of the aberration function in terms of Seidel polynomials. The lowest-order terms are

$$\frac{W_{\mathrm{rms}}}{\Delta d} = \frac{(n^2 - 1)\mathrm{NA}^6}{320\sqrt{7}n^5} + \frac{(n^2 - 1)(2n^2 + 5)\mathrm{NA}^8}{1280\sqrt{7}n^7}.$$
(24)

The dominant term originates from the higher-order Seidel spherical aberration term proportional to ρ^6. Figure 2 shows the lowest-order term of the series as a function of NA. Clearly, the NA^6 term seriously underestimates the exact result. In fact, for a NA of 0.85 it is 61% too low. A better approximation is in terms of the Zernike polynomials. It appears that the dominant contribution is the lowest-order Zernike spherical aberration term $20\rho^6 - 30\rho^4 + 12\rho^2 - 1$ with coefficient A_{60}. Figure 2 shows the A_{60} coefficient as a function of NA. Apparently, approximating the exact aberration function by use of this single Zernike term is quite accurate, even for high-NA values. For example, for a NA of 0.85 the error is only 4%.

Figure 3 shows the relative change in conjugate and free working distance as a function of NA. It was determined that $\beta_{con} + \beta_{fwd}$ is close to unity throughout the entire range of NA values. For example, for BD conditions it was found that $\beta_{con} = 0.80$ and $\beta_{fwd} = 0.22$, giving $\beta_{con} + \beta_{fwd} = 1.02$.

The relative change in conjugate and free working distance can also be expanded in powers of NA. The lowest-order terms are

$$\beta_{con} = \frac{n^2 - 1}{n^2} + \frac{3(n^2 - 1)\mathrm{NA}^2}{4n^4}$$
$$+ \frac{3(n^4 - 51n^2 + 50)\mathrm{NA}^4}{280n^6},$$
(25)

$$\beta_{fwd} = \frac{1}{n^2} - \frac{3(n^2 - 1)\mathrm{NA}^2}{4n^4}$$
$$+ \frac{3(3n^4 + 22n^2 - 25)\mathrm{NA}^4}{140n^6},$$
(26)

$$\beta_{con} + \beta_{fwd} = 1 + \frac{3(n^2 - 1)\mathrm{NA}^4}{40n^6}.$$
(27)

It follows that the approximation $\beta_{con} + \beta_{fwd} = 1$ is exact for small NA values.

3. Effects of Deviating from the Sine Condition

The change in conjugate is made in practice by an axial translation of a lens in front of the objective lens, for example, the collimator lens that converges the beam emitted by the laser diode into a parallel beam. The entrance NA_0 of this lens is much smaller than the exit NA of the objective lens. Additional spherical aberration contributions that arise from the change of conjugate for this lens can therefore be neglected. It then follows that the required translation of this lens is given by

$$\Delta z_0 = \frac{\Delta z_{con}}{nM^2} = \beta_{con} \frac{\Delta d}{n} \frac{\mathrm{NA}^2}{\mathrm{NA}_0^2}.$$
(28)

In practice it is advantageous to keep this axial

stroke Δz_0 as small as possible. This can be achieved by deviating slightly from Abbe's sine condition. This would possibly allow a decrease in the parameter β_{con}. This is investigated here by numerical means, in particular with automated lens design methods and ray tracing.

An alternative to Abbe's sine condition is the Herschel condition.[2,3] An imaging system that satisfies the latter condition does not suffer from spherical aberration when the conjugate of the imaging system is changed (to first order in the shift of the object and image points). In the present terms this means that $\beta_{con} = 1$ and $\beta_{fwd} = 0$, i.e., the conjugate change is sufficient for a focal shift with minimum induced aberrations, the minimum being zero in this case. A lens design condition that interpolates between Abbe and Herschel is[8,9]

$$n_0 \sin\left(\frac{\theta_0}{q}\right) = Mn_1 \sin\left(\frac{\theta_1}{q}\right), \qquad (29)$$

where q is a real parameter. For $q = 1$ the Abbe condition is retrieved, whereas for $q = 2$ the Herschel condition is retrieved. Values of $1 < q < 2$ therefore interpolate between the two conditions. For values of $q < 1$ we are on the other side of Abbe, so to speak. The trend in β_{con} as a function of q derived from the two analytically available points ($\beta_{con} = 1$ for $q = 2$ and $\beta_{con} = 0.80$ for $q = 1$) suggests that the wanted small values of β_{con} can be found in the regime $q < 1$.

The q condition can be used to fully specify two aspheric surfaces in an imaging system by use of an algorithm that is based on the methods of Wasserman and Wolf,[10] Vaskas,[11] and Braat and Greve[12] for the special case of the sine condition. This approach to lens design has been used in Ref. 13 for the design of lenses with a so-called flat intensity profile. The objective lens was taken to be a biasphere operating under BD conditions, i.e., the wavelength was taken to be $\lambda = 0.405$ µm, the NA was 0.85, and the focus is at a depth of 87.5 µm in a layer of polycarbonate ($n = 1.62231$). The glass refractive index is $n_{lens} = 1.71055$, lens thickness $b = 2.75$ mm, the free working distance is 0.75 mm, and the stop of diameter 4.0 mm was taken to be at the vertex of the first refractive surface of the lens. For a given q value the surface sag of the two aspheres was determined by use of an automated design tool that solves the Wasserman–Wolf differential equations. The lens design thus obtained was futher analyzed with the Zemax ray-tracing software package[14] by considering a cover layer that is 1 µm thinner or thicker than the nominal 87.5 µm. This is sufficiently small to be in agreement with the linearized treatment in axial displacements of the analytical theory in Section 2. The distance between the objective lens and the disc and the location of the object point is then adjusted for a minimum rms aberration. This allows for a numerical evaluation of parameters β_{con} and β_{fwd}. Figure 4 shows the resulting values as a function of the q
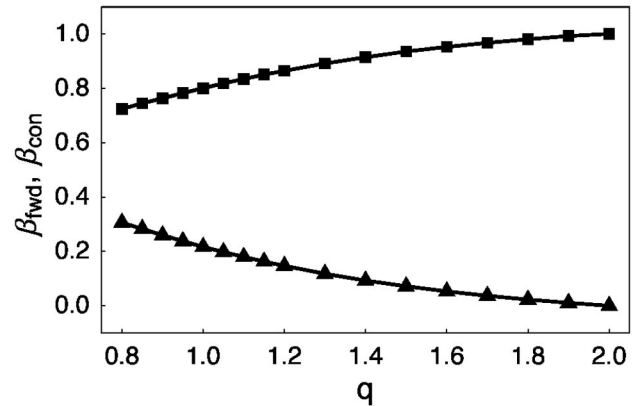

Fig. 4. Numerically determined parameters β_{con} (squares) and β_{fwd} (triangles) for a lens with 0.85 NA as a function of the q parameter that interpolates between the Abbe and the Herschel conditions.

parameter. The agreement with the exact results for $q = 1$ and $q = 2$ is quite good, with an estimated error of less than 0.3%.

A deviation from Abbe's sine condition results in a nonzero sensitivity of coma for field use. Figure 5 shows the coma sensitivity as a function of the q parameter. As expected, the sensitivity crosses zero for $q = 1$, i.e., when the lens satisfies the Abbe condition. For a sufficiently small NA, this sensitivity is proportional to $1 - q^{-2}$. The numerical data fit quite well with this function, even though the NA is as high as 0.85. We found a coma sensitivity of 491 mλ/deg $\times (1 - q^{-2})$. The upper bound for this coma sensitivity is typically approximately 50 mλ/deg. It then follows that the lens design must satisfy $1.06 > q > 0.95$, i.e., only small deviations from the Abbe condition are allowed. As a consequence, the parameter for relative conjugate change cannot deviate much from the Abbe value $\beta_{con} = 0.80$. By use of the numerically calculated values it was found that for the range of q values $1.06 > q > 0.95$ the conjugate parameter is in the range $0.82 > \beta_{con} > 0.78$, i.e., the
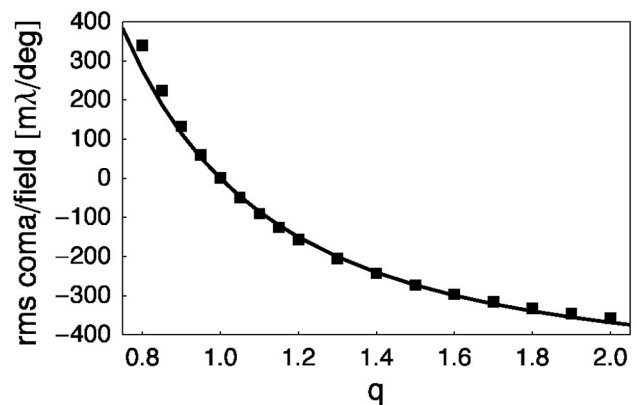

Fig. 5. Sensitivity of coma for field use of the lens as a function of the q parameter that interpolates between the Abbe and the Herschel conditions (squares) and a fit proportional to $1 - q^{-2}$ (solid curve).

decrease in required conjugate change compared with the Abbe case is of the order of a few percent.

In conclusion, the analytical theory of finite conjugate spherical aberration compensation presented in Section 2 is a good aproximation for any objective lens used in practice. It allows for a simple, straightforward evaluation of the conjugate change required to minimize the rms value of the aberration function and, of this minimum, residual aberration sensitivity. A possible decrease of the required conjugate change by non-Abbe objective lens designs is relatively small. Finally, note that ways to break the sine condition other than the q condition can be explored, but it cannot be expected that such an approach will result in an outcome substantially different from this one.

Appendix A

All the pupil averages were evaluated by use of Mathematica[15] and yield

$$\langle f \rangle = \frac{2}{3\mathrm{NA}^2} \left[n^3 - (n^2 - \mathrm{NA}^2)^{3/2} \right], \tag{A1}$$

$$\langle f^2 \rangle = n^2 - \frac{1}{2}\mathrm{NA}^2, \tag{A2}$$

$$\langle g_1 \rangle = -\frac{\mathrm{NA}^2}{4n}, \tag{A3}$$

$$\langle g_2 \rangle = \frac{2}{3n\mathrm{NA}^2} \left[1 - (1 - \mathrm{NA}^2)^{3/2} \right], \tag{A4}$$

$$\langle fg_1 \rangle = -\frac{2n^5 + (3\mathrm{NA}^4 - n^2\,\mathrm{NA}^2 - 2n^4)(n^2 - \mathrm{NA}^2)^{1/2}}{15n\mathrm{NA}^2}, \tag{A5}$$

$$\langle fg_2 \rangle = \frac{1}{4n\mathrm{NA}^2} \left\{ n^3 + n - (n^2 + 1 - 2\mathrm{NA}^2) \right.$$
$$\times (n^2 - \mathrm{NA}^2)^{1/2}(1 - \mathrm{NA}^2)^{1/2} + (n^2 - 1)^2$$
$$\left. \times \log\left[\frac{(n^2 - \mathrm{NA}^2)^{1/2} + (1 - \mathrm{NA}^2)^{1/2}}{n + 1} \right] \right\}, \tag{A6}$$

$$\langle g_1{}^2 \rangle = \frac{\mathrm{NA}^4}{12n^2}, \tag{A7}$$

$$\langle g_2{}^2 \rangle = \frac{2 - \mathrm{NA}^2}{2n^2}, \tag{A8}$$

$$\langle g_1 g_2 \rangle = -\frac{2 + (3\mathrm{NA}^4 - \mathrm{NA}^2 - 2)(1 - \mathrm{NA}^2)^{1/2}}{15n^2\mathrm{NA}^2}. \tag{A9}$$

Some of these integrals can also be found in Ref. 1.

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