Finite-difference depth migration of exploration-scale 3-D seismic data

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Summary

Most migration methods are based on a variety of approximate versions of the wave equation, one notable exception being the finite-difference reverse-time depth migration algorithm. Since it requires enormous computer resources, its applications have been primarily restricted to 2-D synthetic data. Consequently, its potential for migrating real data and imaging complex 3-D structures by constructive interference of wave fronts has neither been recognized nor exploited.

Finite-difference depth migration being very similar to forward modeling, both these processes are subjected to the same conditions for avoiding grid dispersion and numerical instability. In the case of migrating real data, this necessitates interpolation both in space and time. One can, however, exploit the fact that in forward modeling, one tries to generate accurate reflection signals, whereas in migration the primary objective is to accomplish imaging from the pre-recorded signals and it may be attainable under less stringent conditions. Our investigations indicate that accurate imaging can be done without making any provision for grid dispersion in the lateral direction thereby obviating the use of interpolated traces. In the case of 3-D data, this cuts down the computational task by a factor of 50 or more with similar reductions in memory requirements. Further efficiency can be achieved by using a nonuniform grid in the vertical direction which adapts to the expansion and contraction of the downward propagating wavelet in response to variations in velocity and frequency content of the input data. Two applications on large exploration-scale post-stack 3-D field data are presented.

Introduction

In reverse-time depth migration, one sets up a model of the earth based on some estimated distribution of velocities in the subsurface. The model is excited along its surface boundary by using the unmigrated seismic traces at their respective physical locations but in the reverse order of time. In this sense, the entire procedure is very similar to forward modeling. Whitmore (1983) addressed the 2-D problem and based his treatment on the finite-difference method. A similar algorithm which works in the frequency domain was reported by Loewenthal and Mufti (1983). In both cases, the mechanism which yields the depth section is correct but a satisfactory explanation in support of their treatment is missing. Exactly the same algorithm but formulated as an initial- and boundary-value problem was reported by McMechan (1983) with very encouraging results. We shall address the 3-D problem and base our treatment on the finite-difference method.

A close and sequential examination of the various wave fronts generated during the reverse-time migration of synthetic seismic data computed for simple structures suggests that the migrated images are generated as a result of constructive interference of the seismic wave fronts. Since this method is based on the full wave equation, it does not suffer from any dip limitations. Consequently, it is capable of handling steeply dipping events including those involving turning waves with the same ease as the 15-degree features. These apparent advantages are accompanied by two problems described below.

(1) The downward propagating wave field gives rise to secondary reflections at each velocity interface. These spurious events must derive their energy from the input signals which become progressively weaker. Consequently any attempts to apply this algorithm to real data lead to meaningless results.

(2) In order to avoid grid dispersion which causes scattering of energy during computations, the grid spacing needed for the downward continuation of the wave field must, in most cases, be much finer than the trace interval of the data to be migrated. This necessitates the introduction of a large population of interpolated traces. In the case of 3-D data, the crossline separation is often significantly larger than the inline trace interval and the problem of interpolation becomes far more troublesome.

The first problem led to the development of approximate schemes (Baysal et al., 1986; Kosloff and Baysal, 1983) that would, if not eliminate, at least minimize the secondary reflections. More recently, Loewenthal et al. (1987) demonstrated that such reflections could be virtually eliminated by using sufficiently smoothed velocities for doing migration. The second problem is far more troublesome and requires a drastic reduction in the computational task without compromising the quality of results. This point will be stressed in the following treatment without going into the details of evaluating the wave field which have been adequately covered elsewhere (Mufti, 1990).

Practical and Computational Considerations

Consider a set of 3-D unmigrated data acquired or processed over a uniform rectangular grid. We shall choose the coordinate axes x and y such that they coincide with the inline and crossline directions respectively. For doing migration we set up a 3-D velocity model by introducing uniformly spaced grid lines $i = 0, 1, \ldots . I$ along the x-axis, $j = 0, 1, \ldots . J$ along the y-axis and $k = 0, 1, \ldots . K$ along the z-axis. Then the migration velocity at a point P with coordinates (x,y,z) can be expressed as $c_{i,j,k}$ where

$$x = i\,\Delta x,\ y = j\,\Delta y,\ z = k\,\Delta z$$

and Ax, Ay and Δz indicate grid intervals along the x, y and z directions respectively. We can also indicate the time t by the index n such that $t = n\,\Delta t\ (n = 0, 1, \cdots)$, where At is the time sampling interval.

In accordance with the experience gained with forward modeling, the maximum value of grid spacing which can be used without causing excessive dispersion of energy during the evaluation of the wave field is governed by the relation

$$h \le c_{min} / (w\, f_{max}) \qquad (1)$$

where

h = max (Ax, Ay, Az)
c_{min} = minimum value of the migration velocity
f_{max} = maximum frequency in the data to be migrated.

Moreover, for a given value of h, the system becomes numerically unstable unless the time sampling interval satisfies the condition

$$\Delta t \leq \mu\, h / c_{max} \qquad (2)$$

where c_{max} represents the maximum value of the migration velocity and μ is a constant. For 3-D problems, μ = 0.5 and the optimum value of the parameter w appearing in (1) is 3.5.

It turns out that for the commonly encountered range of subsurface velocities and frequencies in the seismic data, the values of h are, in general, much too small as compared to the trace interval and crossline separation used for acquiring 3-D data in the field. Therefore, for migrating such data, it becomes necessary to introduce a huge population of traces and the resulting problem becomes much too large even for the most powerful computers available today. The severity of this problem can be best appreciated with the help of a concrete example. Let us consider a 3-D data set with the following parameters:

Number of seismic lines	= 256
Number of traces in each line	= 512
Trace spacing	= 25m
Crossline interval	= 75m
Trace length	= 4s
Time sampling rate	= 4ms
Maximum depth of interest	= 3000m

We shall assume that the data was acquired over an offshore prospect which includes a subsurface salt dome. Therefore, we must use a very fine grid spacing in order to accommodate for the low-velocity water layer. The presence of the dome will necessitate very fine time sampling. To be more specific, let us choose:

Velocity of water	= 1500 m/s
Velocity of salt	= 3100m/s
Maximum frequency in the data	= 65Hz.

Then, in accordance with conditions (1) and (2), a possible choice would be to use a uniform grid spacing of Ax = Ay = AZ = 6.25 m and At = 1 ms. The resulting dimensions of the migration model and the corresponding numerical task will be as follows:

Uniform grid spacing	= 6.25 m
Time-sampling rate	= 1ms
Grid size	= 3072 * 2048 * 512
Memory requirements	= 39 gigabytes
Number of time steps	=4000
Total number of grid-point operations	= 12,884 billions
Floating-point operations per grid point	= 27
Total number of floating-point operations	= 335 trillions

On a 64-node Connection Machine (CM-5), this problem will require about 93 hours of CPU time. Note that the velocity of salt chosen by us is rather low. If we had chosen a velocity of, say, 6200 m/s, the CPU time would have been more than a week. Since a problem of this size requires an out-of-core solution software, the actual computation time will be significantly larger. It is needless to say that the problem being considered is much smaller than the real-life problems to be presented later. We are obviously faced with a hopeless situation. We can, however, exploit the fact that in the case of forward modeling, the primary interest is to generate accurate reflection signals, whereas in migration, the main objective is to accomplish imaging from the pre-recorded signals, and it may be attainable under less stringent conditions. We must also keep in mind that the real data is never free from noise. These considerations led to the idea of doing migration without using any interpolated traces. After a series of tests both on synthetic and real data, we were able to obtain good results along these lines. In practical terms, it is a very significant step. The problem under consideration now requires only 268 billion grid-point operations and it can be finished in only about 1.9 hours. This amounts to improvement in performance by a factor of 50 with similar gains in the reduction of memory requirements.

To our surprise, the choice of Az is rigidly controlled by condition (1) but this does not stop us from using a variable vertical grid which would minimize the computational effort. This idea has attracted the attention of several investigators (e. g., Mufti, 1985; Jastram and Behle, 1992) and is motivated by the fact that the velocity of the earth materials, in general, increases with depth. Equally compelling is the argument that the frequency content of the seismic pulse decreases with depth. Therefore, one should be able to use larger values of Δz at the deeper parts of the model. Going back to our example considered earlier, condition (1) which will depend on the velocity of water will determine the largest permissible value of Δz in order to avoid grid dispersion in the results. In the absence of water, we could obviously afford a larger value of Az. Similarly, condition (2) would permit larger values of At if there were no salt dome in the model. By using the variable grid one can cut down the task of computations significantly. This "adaptive" procedure which is frequently used for solving fluid flow problems (e. g., Pita and Sundaresan, 1993) does not give rise to grid dispersion in the presence of low-velocity zones in the subsurface such as a gas reservoir. It is worth mentioning here that the improvement in computational performance by using the adaptive grid is structure dependent but in most cases it will more than compensate for the extra work introduced by the variable grid. In one of our real data applications, we were able to cut down the CPU time as well as memory requirements by about 40 % as compared to using a constant value of Δz. Note that the quantity f_{max} appearing in (1) can be treated as a monotonically decreasing function of depth. This provides for further additional expansion of the vertical grid which adapts to variations in the frequency content of the input data.

Application Examples

The migration procedure outlined above was successfully applied to several sets of 3-D data acquired in different parts of the world. The example chosen for this abstract includes the data acquired over a salt dome with very steep flanks and located in a medium characterized by variations of velocity in all spatial dimensions. The data was provided on a rectangular 512 * 512 grid with a time length of 7 s. The following parameters were used for migration:

Number of lines in the input data	= 512
Number of traces in each line	= 512
Depth of investigation	= 6.25 km
Line spacing	=25m
Trace interval	= 25 m
Maximum frequency in the input data	=60Hz
Time step for computing the wave field	= 1 ms

For setting up the velocity model, we chose Ax and Ay equal to the trace interval and line spacing respectively but used a variable grid in the vertical direction in response to variations in velocity which ranged from 1500 m/s to 5000 m/s.. The computations took a total of 5.5 hours of CPU time on a 64-node Connection Machine (CM-5). Figure 1 shows the depth-migrated section for crossline 240. It shows the various features including the flanks of the salt dome very clearly due to preservation of higher frequencies. Equally striking is the absence of grid dispersion. The outline of the dome on the left

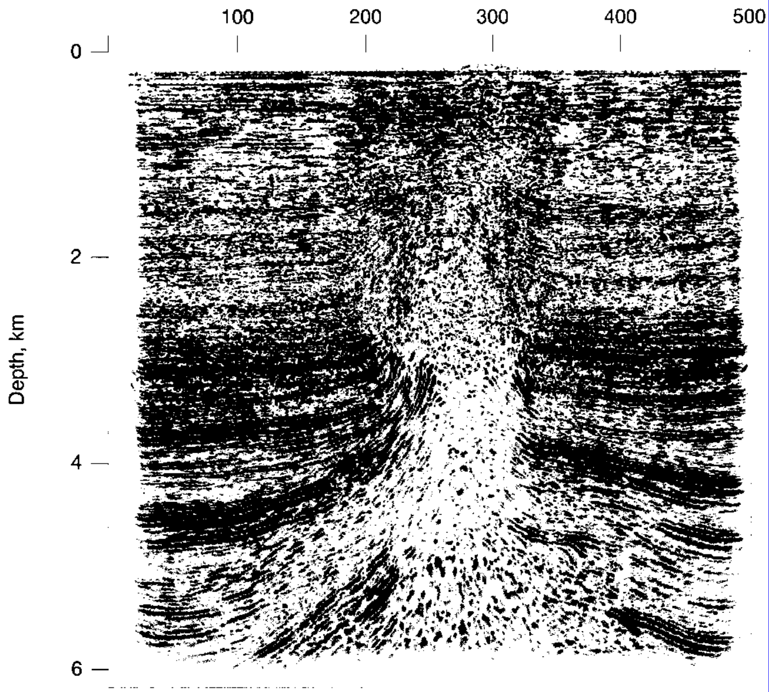

Figure 1. Depth-migrated section for crossline 240.

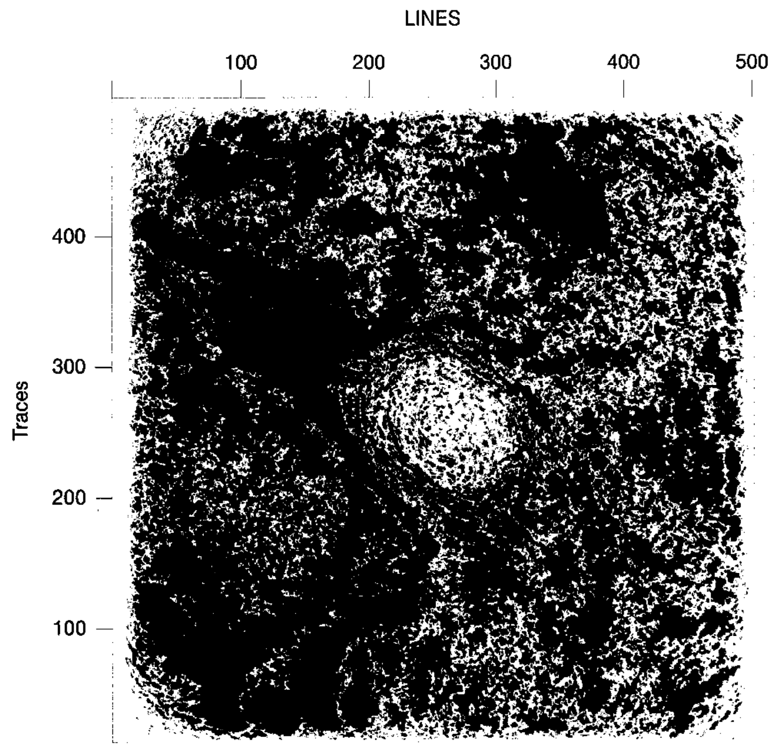

Figure 2. Seismic depth slice at 3000 meters.

side indicates the presence of a very prominent apparent overhang at a depth of about 3000 m. It demonstrates the capability of the algorithm to accommodate features with dips exceeding 90 degrees. Figure 2 shows the seismic depth slice of the subsurface at a depth of 3000 m. The most prominent feature here is the elliptic cross-section of the dome.

Conclusions

The method of reverse-time migration is known to yield excellent results, but it is computationally very expensive. As such, its applications have been mostly restricted to low-frequency 2-D synthetic data. Consequently, its potential as a powerful, flexible tool for solving complex 3-D real-data imaging problems could never be explored. The procedure presented by us is based on very coarse sampling of the wave field in all spatial dimensions which cuts down the cost of computations by about two orders of magnitude. The results obtained along these lines retain the higher frequency content of the input data with no signs of grid dispersion. The examples presented by us also demonstrate the capability of this algorithm to migrate steeply dipping interfaces.

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