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Research Article

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Finite-difference equations of quasistatic motion of the shallow concrete shells in nonlinear setting

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Abstract: The study solves a system of finite difference equations for flexible shallow concrete shells while taking into account the nonlinear deformations. All stiffness properties of the shell are taken as variables, *i.e.*, stiffness surface and through-thickness stiffness. Differential equations under consideration were evaluated in the form of algebraic equations with the finite element method. For a reinforced shell, a system of 98 equations on a 8×8 grid was established, which was next solved with the approximation method from the nonlinear plasticity theory. A test case involved computing a 1×1 shallow shell taking into account the nonlinear properties of concrete. With nonlinear equations for the concrete creep taken as constitutive, equations for the quasi-static shell motion under constant load were derived. The resultant equations were written in a differential form and the problem of solving these differential equations was then reduced to the solving of the Cauchy problem. The numerical solution to this problem allows describing the stress-strain state of the shell at each point of the shell grid within a specified time interval.

Keywords: concrete shell; differential equations; nonlinear deformations; finite difference; concrete creep; Cauchy problem

1 Introduction

Various classes of materials and structures working under creep conditions are widely used in modern engineering. Depending on the type of load and creep characteristics, these materials exhibit different behavior under various types of deformation (tension, compression, torsion) [1]. The use of these materials in technological aspects requires the development of appropriate deformation models (defining creep equations) and the study of creep behavior of structural elements [2]. The problems of evaluating shallow shells when taking into account their creep properties represent a separate class of problems in the theory of structures. There two major challenges associated with the evaluation process are the choice of physical equations to describe the creep properties of the shell material and the nonlinear character of these equations. The creep problems of layered cylinders and cylindrical shells were solved in the study [3]. However, under the chosen conditions it is impossible to obtain an exact analytical solution that satisfies all the boundary conditions.

In general, solutions to linearly elastic problems [4, 5] were obtained taking into account nonlinear factors, for example, nonlinear elasticity or plastic deformation, shells made of metal or composite materials [6]. The nonlinear task was formulated for composite elliptical cylindrical shells with an aperture to study the stress-strain state under the action of axial tensile forces [7] based on the refined theory of shallow shells of the Timoshenko type [8]. An analysis of the obtained solutions of the system of integral equations showed the influence of the mechanical and geometric parameters of the shell under the action of the axial tensile force on the distribution of stresses and strains near the circular aperture.

In [9], the example of a cylindrical shell under internal pressure was used to analyze the applicability of various shell models to determine the creep and damage of single-layer cylindrical shells. Shell solutions of different thicknesses based on the Kirchhoff – Love model are consistent with solving the spatial problem for an axisymmetrically loaded cylinder. A semi-analytical solution of nonlinear equations was adopted using the Galerkin method [10] for a nonlinear stability analysis of biconcave multilayer composite shallow shells. Such a solution has yielded nonlinear dependences of the load distribution of the four radii of curvature of curved shells, which are compared and

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verified using numerical solutions of finite elements. The use of nonlinear analysis in solving nonlinear equations is shown in [11–16] for various types of shells and loads.

A comparative analysis of the solutions of nonlinear and linear equations is also applicable to creep tasks [17]. In [18], a methodology was developed for solving a system of differential equations taking into account the nonlinear creep of three-layer plates and shallow shells with a light filler, using finite difference methods in combination with the Euler method. It was shown in the work that the use of the linear Maxwell-Thomson equation and the nonlinear Maxwell-Gurevich equation does not significantly affect the discrepancy between the results. Similar results were obtained in studies [19, 20].

To analyze the time dependences of the mechanical behavior of creep for various types of shells [21, 22], it is also necessary to solve nonlinear differential equations using a creep model based on the Norton law.

The works [23–25] use equations of quasistatic motion of a smooth shallow concrete shell as well as nonlinear equations of the theory of shallow shells [26], and equations of nonlinear concrete creep in differential form [27]. The solution of the problem was based on the Bubnov-Galerkin method. However, all the above methods of solution have a rather complicated structure, which limits their use for assessing the creep of structural elements in reinforced concrete structures. Finding a more elegant way of solving is an urgent task in this area. Therefore, the aim of this work is to solve the problem of quasistatic motion of a smooth shallow concrete shell by the nonlinear equations of the theory of shallow shells based on the numerical method of finite differences (grids).

2 Research method

The authors of [28] presented the resolving equations of smooth shallow concrete shells, taking into account the physical and geometric nonlinearity as follows:

$$D_4 \cdot \frac{d^4F}{dx^4} + D_9 \cdot \frac{d^4F}{dy^4} + D_{14} \cdot \frac{d^4F}{dx^2dy^2} \tag{1}$$
$$+ \left(\frac{d^2D4}{dx^2} + \frac{d^2D8}{dy^2}\right) \cdot \frac{d^2F}{dx^2} + \left(\frac{d^2D5}{dx^2} + \frac{d^2D9}{dy^2}\right) \cdot \frac{d^2F}{dy^2}$$
$$+ \frac{dD4}{dx} \cdot \frac{d^3F}{dx^3} + \frac{dD9}{dy} \cdot \frac{d^3F}{dy^3} + \frac{dD812}{dy} \cdot \frac{d^3F}{dydx^2} + \frac{dD512}{dx}$$
$$\cdot \frac{d^3F}{dxdy^2} + D6 \cdot \frac{d^4W}{dx^4} + D11 \cdot \frac{d^4W}{dy^4} + D_{15} \cdot \frac{d^4W}{dx^2dy^2}$$
$$+ \left(\frac{d^2D6}{dx^2} + \frac{d^2D10}{dy^2}\right) \cdot \frac{d^2W}{dx^2} + \left(\frac{d^2D7}{dx^2} + \frac{d^2D11}{dy^2}\right)$$
$$\cdot \frac{d^2W}{dy^2} + 2 \cdot \left(\frac{d^2D13}{dxdy} \cdot \frac{d^2W}{dxdy} + \frac{dD6}{dx} \cdot \frac{d^3W}{dx^3} + \frac{dD11}{dy}\right.$$
$$\left.\cdot \frac{d^3W}{dy^3} + \frac{dD1013}{dy} \cdot \frac{d^3W}{dydx^2} + \frac{dD713}{dx} \cdot \frac{d^3W}{dxdy^2}\right)$$
$$+ \left(k_1 + \frac{d^2W}{dx^2}\right) \cdot \frac{d^2F}{dy^2} + \left(k_2 + \frac{d^2W}{dy^2}\right) \cdot \frac{d^2F}{dx^2} - 2 \cdot \frac{d^2W}{dxdy}$$
$$\cdot \frac{d^2F}{dxdy} + q(x, y) = 0; B_5 \cdot \frac{d^4F}{dx^4} + B_2 \cdot \frac{d^4F}{dy^4} + B_{14}$$
$$\cdot \frac{d^4F}{dx^2dy^2} + \left(\frac{d^2B5}{dx^2} + \frac{d^2B1}{dy^2}\right) \cdot \frac{d^2F}{dx^2} + \left(\frac{d^2B9}{dx^2} + \frac{d^2B2}{dy^2}\right)$$
$$\cdot \frac{d^2F}{dy^2} + \frac{d^2B8}{dxdy} \cdot \frac{d^2F}{dxdy} + 2 \cdot \left(\frac{dB5}{dx} \cdot \frac{d^3F}{dx^3} + \frac{dB2}{dy} \cdot \frac{d^3F}{dy^3}\right.$$
$$\left.+ \frac{dB98}{dy} \cdot \frac{d^3F}{dydx^2} + \frac{dB18}{dx} \cdot \frac{d^3F}{dxdy^2}\right) + B6 \cdot \frac{d^4W}{dx^4} + B_4$$
$$\cdot \frac{d^4W}{dy^4} + B_{15} \cdot \frac{d^4W}{dx^2dy^2} + \left(\frac{d^2B6}{dx^2} + \frac{d^2B3}{dy^2}\right) \cdot \frac{d^2W}{dx^2}$$
$$+ \left(\frac{d^2B7}{dx^2} + \frac{d^2B4}{dy^2}\right) \cdot \frac{d^2W}{dy^2} - \frac{d^2W}{dxdy} \cdot \frac{d^2B10}{dxdy} + 2$$
$$\cdot \left(\frac{dB6}{dx} \cdot \frac{d^3W}{dx^3} + \frac{dB3}{dy} \cdot \frac{d^3F}{dy^3} + \frac{dB310}{dy} \cdot \frac{d^3W}{dydx^2}\right.$$
$$\left.+ \frac{dB710}{dx} \cdot \frac{d^3W}{dxdy^2}\right) + \frac{d^2W}{dx^2} \cdot \frac{d^2W}{dy^2} - \left(\frac{d^2W}{dxdy}\right)^2 + k_1$$
$$\cdot \frac{d^2W}{dy^2} + k_2 \cdot \frac{d^2W}{dx^2} = 0$$

Where, D_i, B_i – stiffness properties, which have complex structure and are described in details [28]. To solve these equations (1), the authors use the finite difference technique [29], also called the grid method.

A rectangular shallow shell (Figure 1) with dimensions 2a×2b is considered in plan, on which a uniform vertical load of intensity **q** (kN/m^2) acts over the entire surface. The support conditions along the contour of the shell are movable hinged. The surface of the shell is divided into a grid with a cell of 8×8 (Figure 2). As can be seen from equations (1), not only the principal unknown functions F and W, but also the stiffness characteristics D_i, B_i are the variable parameters over the shell surface (x, y). The value and complexity of the proposed equations (1) lies in the fact that all parameters are subjected to finite-difference approximation: F, W, D_i, and B_i.

In the proposed problem, the central difference operators [29] are used from the 1st to the 4th orders, including those of the mixed type:

$$\frac{dS}{dx}; \frac{dS}{dy}; \frac{d^2S}{dx^2}; \frac{d^2S}{dy^2}; \frac{d^3S}{dx^3}; \frac{d^3S}{dy^3}; \frac{d^4S}{dx^4}; \frac{d^4S}{dy^4}; \tag{2}$$
$$\frac{d^2S}{dxdy}; \frac{d^4S}{dx^2dy^2}; \frac{d^3S}{dx^2dy}; \frac{d^3S}{dxy^2}$$

For the center point of the shell surface - O (i, j) (Figure 2), the derivatives of any function S (x, y) are written by the

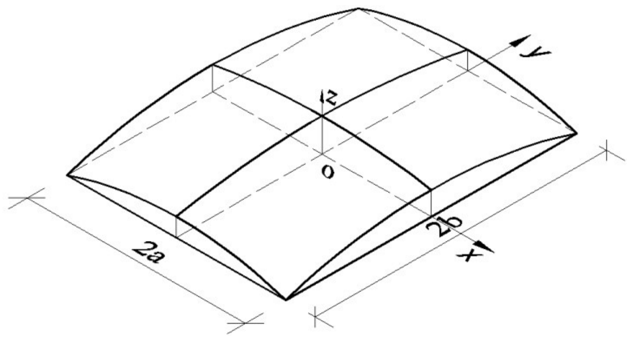

Figure 1: A rectangular shallow shell with dimensions 2a×2b , on which a uniform vertical load of intensity q (kN/m2) acts over the entire surface

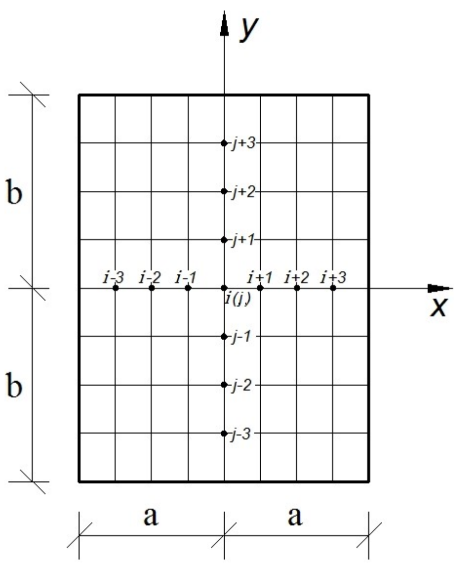

Figure 2: The surface of the shell is divided into a grid with a cell of 8×8

following difference operators:

$$\frac{dS}{dx} = \frac{[S_{(i+1),j} - S_{(i-1),j}]}{\Delta X} \quad (3)$$

$$\frac{dS}{dy} = \frac{[S_{i,(j+1)} - S_{i(j-1)}]}{\Delta y} \quad (4)$$

$$\frac{d^2S}{dx^2} = \frac{[S_{(i+1),j} - 2 \cdot S_{i,j} + S_{(i-1),j}]}{(\Delta X)^2} \quad (5)$$

$$\frac{d^2S}{dy^2} = \frac{[S_{i,(j+1)} - 2 \cdot S_{i,j} + S_{i,(j-1)}]}{(\Delta y)^2} \quad (6)$$

$$\frac{d^2S}{dxdy} = \frac{[S_{(i+1),(j+1)} - S_{(i+1),(j-1)} - S_{(i-1),(j+1)} + S_{(i-1),(j-1)}]}{(\Delta y \Delta x)} \quad (7)$$

$$\frac{d^4S}{dx^4} = \frac{[S_{(i+2),j} - 4 \cdot S_{(i+1),j} + 6 \cdot S_{i,j} - 4 \cdot S_{(i-1),j} + S_{(i-2),j}]}{(\Delta x)^4} \quad (8)$$

$$\frac{d^4S}{dy^4} = \frac{[S_{i,(j+2)} - 4 \cdot S_{i,(j+1)} + 6 \cdot S_{i,j} - 4 \cdot S_{i,(j-1)} + S_{i,(j-2)}]}{(\Delta y)^4} \quad (9)$$

$$\begin{aligned} \frac{d^4S}{dx^2dy^2} &= S_{(i+1),(j+1)} - 2 \cdot S_{(i+1),j} + S_{(i+1),(j-1)} \quad (10) \\ &- 2 \cdot [S_{i,(j+1)} - 2 \cdot S_{i,j} + S_{i,(j-1)}] + S_{(i-1),(j+1)} \\ &- 2 \cdot S_{(i-1),j} + S_{(i-1),(j-1)} / [(\Delta X)^2 (\Delta y)^2] \frac{d^3S}{dx^3} \\ &= [S_{(i+2),j} - 2 \cdot S_{(i+1),j} + 2 \cdot S_{(i-1),j} - S_{(i-2),j}] / (\Delta X)^3 \end{aligned}$$

$$\frac{d^3S}{dy^3} = \frac{[S_{i,(j+2)} - 2 \cdot S_{i,(j+1)} + 2 \cdot S_{i,(j-1)} - S_{i,(j-2)}]}{(\Delta y)^3} \quad (11)$$

$$\frac{d^3S}{dx^2dy} = \frac{[S_{(i+1),(j-1)} - 2 \cdot S_{i,(j-1)} + 2 \cdot S_{i,(j+2)} - S_{(i-1),(j-1)}]}{[(\Delta x)^2 \Delta y]} \quad (12)$$

$$\frac{d^3S}{dxdy^2} = \frac{[S_{(i-1),(j+1)} - 2 \cdot S_{(i-1),j} + 2 \cdot S_{(i+2),j} - S_{(i+1),(j+1)}]}{[(\Delta y)^2 \Delta x]} \quad (13)$$

where: $\Delta x = a/4$; $\Delta y = b/4$.

We obtain a system of 2 nonlinear algebraic equations applying the difference operators (2-13) to equations (1) for the central point O (i, j):

$$\begin{aligned} & c_1 \cdot W_{(i-2)} + c_2 \cdot W_{(i-1)} + c_3 \cdot W_i + c_4 \cdot W_{(i+1)} \quad (14) \\ & + c_5 \cdot W_{(i+2)} + c_6 \cdot W_{(i-1)} \cdot F_{(i-1)} + c_7 \cdot W_i F_{(i-1)} \\ & + c_8 \cdot W_{(i+1)} F_{(i-1)} + c_9 W_{(i-1)} \cdot F_i + c_{10} \cdot W_i F_i \\ & + c_{11} \cdot W_{(i+1)} F_i + c_{12} \cdot W_{(i-1)} \cdot F_{(i+1)} + c_{13} \cdot W_i F_{(i+1)} \\ & + c_{14} \cdot W_{(i+1)} F_{(i+1)} = -q(x, y) \end{aligned}$$

$$\begin{aligned} & e_1 \cdot F_{(i-2)} + e_2 \cdot F_{(i-1)} + e_3 \cdot F_i + e_4 \cdot F_{(i+1)} \quad (15) \\ & + e_5 \cdot F_{(i+2)} + e_6 \cdot W_{(i-1)} \cdot W_{(i-1)} + e_7 \cdot W_i \cdot W_{(i-1)} \\ & + e_8 \cdot W_{(i+1)} W_{(i-1)} + e_9 \cdot W_{(i-1)} \cdot W_i + e_{10} \cdot W_i \cdot W_i \\ & + e_{11} \cdot W_{(i+1)} W_i + e_{12} \cdot W_{(i-1)} \cdot W_{(i+1)} + e_{13} \cdot W_i \cdot W_{(i+1)} \\ & + e_{14} \cdot W_{(i+1)} W_{(i+1)} = 0 \end{aligned}$$

Here, the coefficients **c, e** contains the stiffness and geometrical parameters of the shell.

Equations similar to system (14, 15) are written for all 49 grid points (Figure 2). Thus, a system of 98 algebraic equations for the functions **F** and **W** is obtained. The values of these functions at the contour and boundary points are determined from the support conditions for the shell edges:

$$W = 0; \ \frac{d^2F}{dx^2} = 0; \ \frac{d^2F}{dy^2} = 0; \ \frac{d^2W}{dx^2} = 0; \ \frac{d^2W}{dy^2} = 0 \quad (16)$$

The values of the stiffness parameters D_i, B_i at the contour and boundary points of the grid are set to zero.

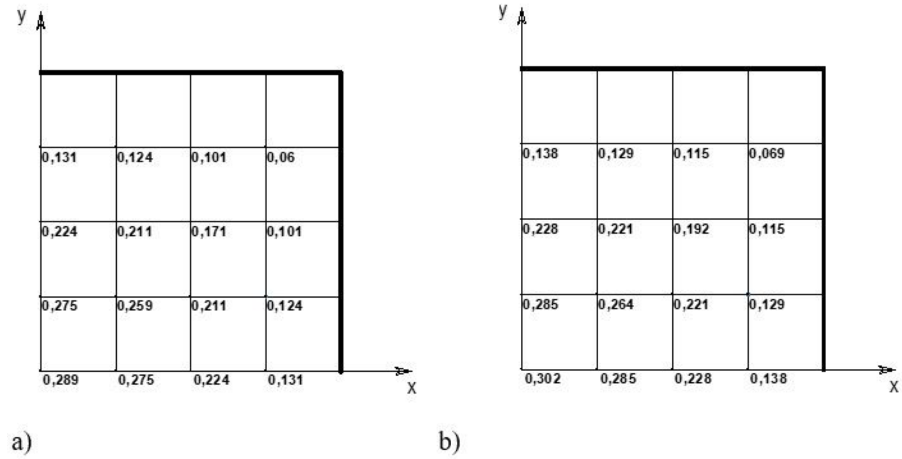

Figure 3: Shell deflections (cm): a) according to a linear scheme; b) according to a nonlinear scheme

Like any physically nonlinear theory of plasticity problem, this problem has no direct solution. To solve it, the method of variable elasticity parameters is used [30], with recalculation at each step of the approximation of the stiffness characteristics D_i, B_i and a solution to the system of 98 algebraic equations.

The proposed algorithm for solving the bending of a shallow concrete shell is as follows.

1. In the linear formulation, a system of algebraic equations of the 98th order is solved and the values of the functions F and W are determined for each grid point. Using the stiffness coefficients of the *elastic* shell, the values of deformations, stresses and their intensities σ_i, ϵ_i at each given point in thickness and surface of the shell are calculated.
2. According to the known diagram of instantaneous deformation of concrete $\sigma_i \sim \epsilon_i$, the values of the obtained variables of σ_i, the secant modulus E_c are corrected and the stiffness coefficients D_i, B_i are calculated from these new values of deformation parameters.
3. Thus obtaining a new field of stiffness, the system of 98 equations is again solved and, due to obtained values of F and W, the process moves to the next step. The iterative process continues until a given point is achieved (by deflection, load, or other parameters).

Figure 3 shows the calculation results of a shallow concrete shell with the following data: a = 100 cm; b = 100 cm; k1 = k2 = $0.00055\frac{1}{\text{cm}}$, h = 1.33cm, q = $0.13\ \frac{\text{kN}}{\text{cm}^2}$.

To describe the quasistatic motion of a shallow concrete shell under load, let us differentiate the well-known resolving equations of flexible shallow shells [30] in the mixed mode according to the time parameter - t:

$$\frac{d^2\dot{M}_1}{dx^2}+\frac{d^2\dot{M}_2}{dy^2}+2\frac{d^2\dot{M}}{dxdy}+\dot{N}_1\left(k_1+\frac{d^2W}{dx^2}\right) \\ +\dot{N}_2\left(k_2+\frac{d^2W}{dy^2}\right)+N_1\cdot\frac{d^2\dot{W}}{dx^2}+N2\cdot\frac{d^2\dot{W}}{dy^2}+2\frac{d^2W}{dxdy}\cdot\dot{T} \\ +2\cdot T\cdot\frac{d^2\dot{W}}{dxdy}=0 \tag{17}$$

$$\frac{d^2\dot{\epsilon}_1}{dx^2}+\frac{d^2\dot{\epsilon}_2}{dy^2}-\frac{d^2\gamma}{dxdy}-2\cdot\frac{d^2W}{dxdy}\cdot\frac{d^2\dot{W}}{dxdy} \\ +\left(k_2+\frac{d^2W}{dy^2}\right)\cdot\frac{d^2\dot{W}}{dx^2}+\left(k_1+\frac{d^2W}{dx^2}\right)\cdot\frac{d^2\dot{W}}{dy^2}=0 \tag{18}$$

The equations of the nonlinear creep of aging theory have been written for a biaxial stress state and are taken as physical ones:

$$\dot{\sigma}_{11}=\frac{E\,(t)\cdot\ [\dot{\epsilon}_{11}+\nu\cdot\dot{\epsilon}_{22}-P_1\,(t)-\nu\cdot P_2\,(t)]}{1-\nu^2} \tag{19}$$

$$\dot{\sigma}_{22}=\frac{E\,(t)\cdot\ [\dot{\epsilon}_{22}+\nu\cdot\dot{\epsilon}_{11}-P_2\,(t)-\nu\cdot P_1\,(t)]}{1-\nu^2} \tag{20}$$

$$\dot{\tau}_{12}=\frac{E(t)\cdot\ [\dot{\gamma}_{12}(t)-P_3(t)]}{2\cdot(1+\nu)} \tag{21}$$

Here,

$$P_1\,(t)=\frac{1}{E\,(0)}\,[\sigma_{11}-\nu\cdot\sigma_{22}]\cdot f\,(\sigma_i)\cdot\dot{\phi}\,(t) \tag{22}$$

$$P_2\,(t)=\frac{1}{E\,(0)}\,[\sigma_{22}-\nu\cdot\sigma_{11}]\cdot f\,(\sigma_i)\cdot\dot{\phi}\,(t) \tag{23}$$

$$P_3(t)=\frac{2\cdot(1+\nu)\cdot\tau_{12}\cdot f(\sigma_i)\cdot\dot{\phi}(t)}{E(0)} \tag{24}$$

$$\phi(t) = \phi_{\infty} \cdot (1 - B_1 \cdot e^{-\gamma_1 \cdot t} - B_2 \cdot e^{-\gamma_2 \cdot t}) \tag{25}$$

where: B_1, B_2, γ_1, γ_2 are the parameters chosen from the experimental creep curves of concrete.

Let us introduce into consideration the function of the velocity of forces F (t, x, y):

$$\dot{N}_1(t) = \frac{d^2\dot{F}}{dx^2} \tag{26}$$

$$\dot{N}_2(t) = \frac{d^2\dot{F}}{dy^2} \tag{27}$$

$$\dot{T}(t) = -\frac{d^2\dot{F}}{dxdy} \tag{28}$$

Using classical techniques and transformations [30], let us express formula of moments in equations (17, 18) through the function of displacement velocity - $\dot{W}$, and force speeds (22-24) – through $\dot{F}$. After complex transformations, we arrive at a system of 2 equations for the speeds of the functions - $\dot{W}_H\dot{F}$:

$$\begin{aligned} & d_1 \cdot \dot{W}_{(i-2)} + d_2 \cdot \dot{W}_{(i-1)} + d_3 \cdot \dot{W}_i + d_4 \cdot_{\dot{W}(i+1)} \\ & + d_5 \cdot \dot{W}_{(i+2)} + d_6 \cdot \dot{W}_{(i-1)} \cdot F_{(i-1)} + d_6 \cdot W_{(i-1)} \cdot \dot{F}_{(i-1)} \\ & + d_7 \cdot \dot{W}_i F_{(i-1)} + d_7 \cdot W_{i\dot{F}(i-1)} + d_8 \cdot_{\dot{W}(i+1)} F_{(i-1)} \\ & + d_8 \cdot W_{(i+1)\dot{F}(i-1)} + d_{9\dot{W}(i-1)} \cdot F_i + d_9 W_{(i-1)} \cdot \dot{F}_i \\ & + d_{10} \cdot \dot{W}_i F_i + d_{10} \cdot W_{i\dot{F}i} + d_{11} \cdot_{\dot{W}(i+1)} F_i + d_{11} \cdot W_{(i+1)}\dot{F}i \\ & + d_{12} \cdot \dot{W}_{(i-1)} \cdot F_{(i+1)} + d_{12} \cdot W_{(i-1)} \cdot \dot{F}_{(i+1)} + d_{13} \cdot \dot{W}_i F_{(i+1)} \\ & + d_{13} \cdot W_{i\dot{F}(i+1)} + d_{14\dot{W}(i+1)} F_{(i+1)} + d_{14} \cdot W_{(i+1)\dot{F}(i+1)} \\ & = G1(x, y, t, W, F, \ldots.) \end{aligned} \tag{29}$$

$$\begin{aligned} & g_1 \cdot \dot{F}_{(i-2)} + g_2 \cdot \dot{F}_{(i-1)} + g_3 \cdot \dot{F}_i + g_4 \cdot_{\dot{F}(i+1)} \\ & + g_5 \cdot \dot{F}_{(i+2)} + 2 \cdot g_6 \cdot W_{(i-1)} \cdot \dot{W}_{(i-1)} + g_7 \cdot \dot{W}_i \cdot W_{(i-1)} \\ & + g_7 \cdot W_i \cdot \dot{W}_{(i-1)} + g_8 \cdot_{\dot{W}(i+1)} W_{(i-1)} + g_8 \cdot W_{(i+1)\dot{W}(i-1)} \\ & + g_9 \cdot \dot{W}_{(i-1)} \cdot W_i + g_9 \cdot W_{(i-1)} \cdot \dot{W}_i + 2 \cdot g_{10} \cdot W_i \cdot \dot{W}_i \\ & + g_{11} \cdot W_{(i+1)\dot{W}i} + g_{11} \cdot_{\dot{W}(i+1)} W_i + g_{12} \cdot \dot{W}_{(i-1)} \cdot W_{(i+1)} \\ & + g_{12} \cdot W_{(i-1)} \cdot \dot{W}_{(i+1)} + g_{13} \cdot \dot{W}i \cdot W_{(i+1)} + g_{13} \cdot W_i \dot{W}_{(i+1)} \\ & + 2 \cdot g_{14} \cdot W_{(i+1)\dot{W}(i+1)} = G2(x, y, t, W, F, \ldots.) \end{aligned} \tag{30}$$

Here, the coefficients *d*, *g* has a structure similar to the coefficients *c*, *e* - in equations (14, 15).

3 Results

Solving the obtained system - (25) as a system of algebraic equations for the derivatives $\dot{W}_i$ and $\dot{F}_I$, we arrive at a system of first-order differential equations of the following form:

$$\left.\begin{aligned} \dot{Y}_1 &= L1[w_i(x, y), f_i(x, y), d_k, g_k]; \\ \dot{Y} &= L2[w_i(x, y), f_i(x, y), d_k, g_k]; \\ \dot{Y} = L_n &= Ln[w_i(x, y), f_i(x, y), d_{k,} g_k]; \end{aligned}\right\} \tag{31}$$

Thus, the problem of determining the parameters of the quasistatic motion of a shallow concrete shell is come down to solving the Cauchy problem - (19-21), which can be implemented by the Runge – Kutta numerical method [30].

The initial conditions of the problem (at t = 0) are determined by solving the system (14, 15) by the above method. At each step of integration over time — t_k, we will obtain new values of W (t) and F (t) at each grid point.

To test this technique, the test problem of long-term deformation of the shell, discussed earlier (Figure 3), was solved.

The parameters of creep and nonlinearity have the following meanings:

$$\varphi_{\infty} = 0,52;\ B_1 = 1;\ B_2 = 0;\ \gamma_1 = 0,04$$

The following Figures show the results of the calculation of the deformation of a given concrete shell.

4 Discussion

The obtained results indicate the effectiveness of the selected finite difference method (grids). From the above figures, it is seen that the breakdown of the shell into the grid allows one to determine the dynamics of concrete shell's deformation over time. In addition, analyzing the obtained results, the authors conclude that the points with the highest values of deflections deform faster with time (Figure 6). The above methodology for solving the problem of quasistatic motion of a smooth and sloping shell is unique, applied to a concrete shell.

Similar approaches to the development of methods for solving creep problems were carried out in various works, studying shallow shells of arbitrary shape for metal alloys. The creep of isotropic shallow shells and plates of arbitrary shape from AK4-1T aluminum alloy [31] at various loads was studied. The study showed that the problem was solved when the nonlinear initial-boundary value problem was formulated using the R-function, Ritz, and Runge – Kutta – Merson methods. According to the results of studying the influence of the direction of the external load, it was found that the values of the deformation parameters depended on the sign of the external transverse load. For example, under the influence of external pressure, tensile stresses decrease, and under the influence of

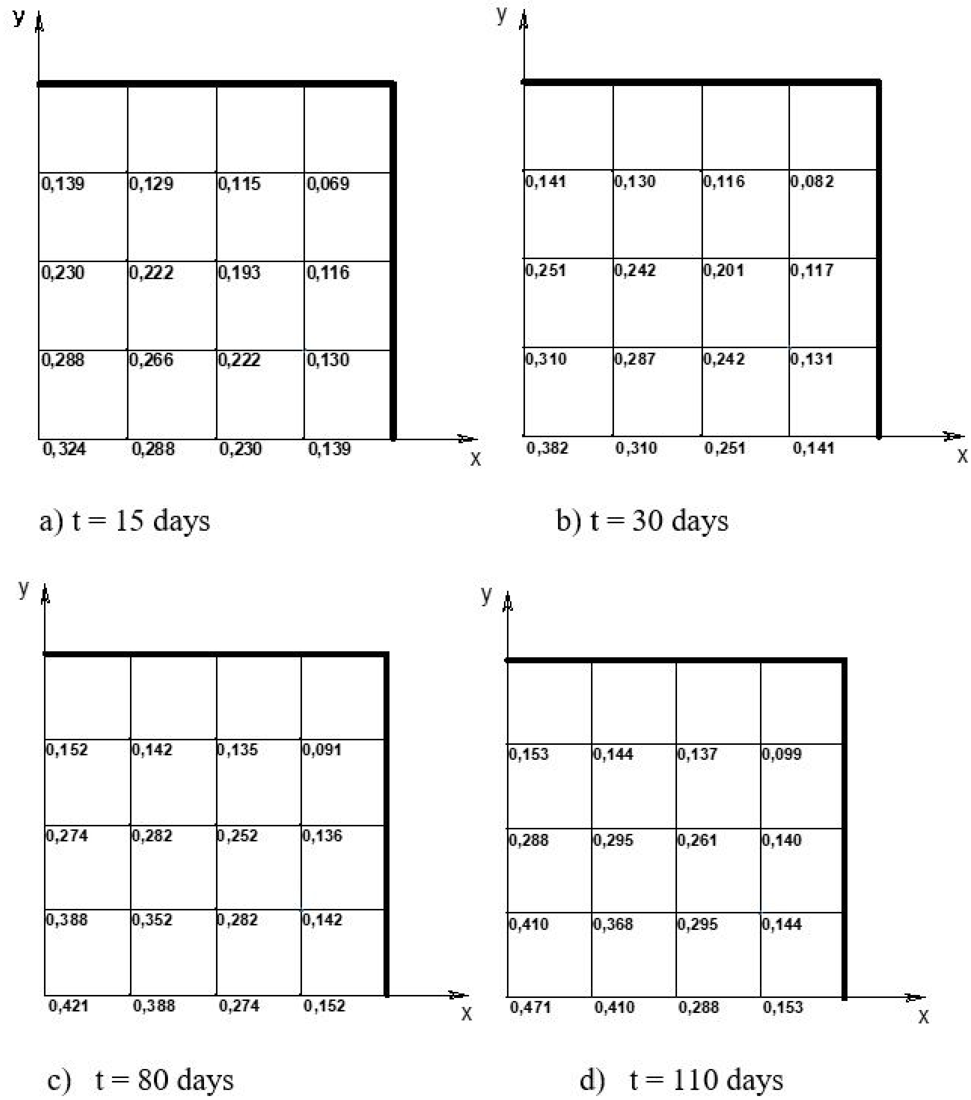

Figure 4: The growth of deflection (cm) in sections of the shell in the calculated time intervals

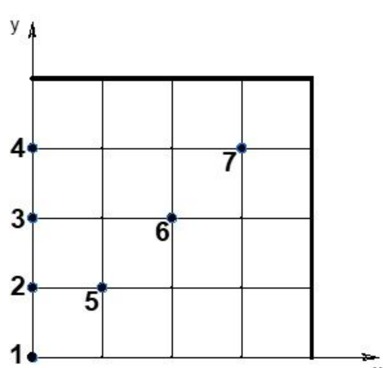

Figure 5: Calculated cross section of the shell when calculating the long-term load

internal pressure, compressive stresses increase first and then decrease. The use of Runge – Kutta – Merson method for solving two-layer shells with different ratios of layer thicknesses [32, 33], axisymmetrically loaded hollow cylinders, for predicting the creep time during failure showed its high accuracy. Thus, its application to the tasks of shallow shells is very effective.

In [34], a similar method was used — radial point interpolation, in which the damage algorithm with inverse mapping allows one to obtain the necessary internal variable fields and the displacement field. The developed algorithm was tested on concrete blocks under the influence of uniaxial and biaxial pressure and on three-point bending. In [34], a similar method was used — radial point interpolation, in which the damage algorithm with inverse mapping allows one to obtain the necessary internal variable fields

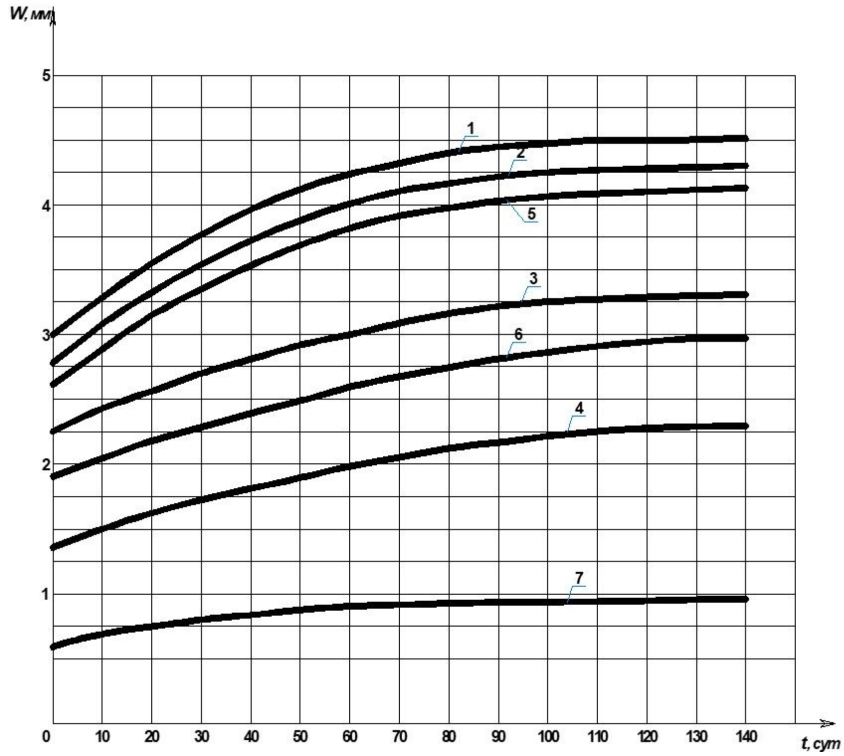

Figure 6: The growth of deflection (cm) in the calculated cross sections of the shell in time

and the displacement field. The developed algorithm was tested on concrete blocks under the influence of uniaxial and biaxial pressure and on three-point bending.

5 Conclusion

Thus, according to the experimental results, a comparative analysis of such studies in the discussion, the following conclusion is made. The proposed numerical method of finite differences for solving the problem of quasistatic motion of a smooth shallow concrete shell, using the theory of nonlinear equations, is effective and allows one to get a clear and connected picture of the change in the parameters of the stress-strain state over time. The developed calculation algorithm allows solving long-term deformation problems of shallow shells in more complex and relevant formats: determination of long-term critical loads, prediction of buckling time, which allows evaluating the behavior of concrete structures throughout the entire life cycle. In addition, this method takes into account the predicted calculations when the load changes, which is relevant when assessing the mechanical capabilities of concrete structures.

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