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FINITE ELEMENT FORMULATION AND SOLUTION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

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FINITE ELEMENT FORMULATION AND SOLUTION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

May 1974

An Investigation Conducted by
STRUCTURAL ENGINEERING LABORATORY
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This report presents a general theory of contact-impact problems cast in a variational theorem suitable for implementation with the finite element method. The numerical scheme is described as is the structural analysis computer code in which it is contained.

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Introduction

In this report we consider the general problem of contact and impact between two bodies. The report is divided into three basic parts. These parts describe: (I) The general theory of contact-impact problems, (II) A numerical scheme for the analysis of contact-impact problems, and (III) The description of computer program FEAP 74 for the solution of contact-impact problems. In an appendix we include the program subroutines and general input description for FEAP 74.

In Sections 1 to 6, Part I, we deal with spatial aspects of the theory and in Section 7, Part I, we deal with temporal aspects. This splitting of the theory is motivated by the way we intend to numerically solve the equations, i.e., the finite element method spatially and a finite difference method temporally.

Part II considers a numerical implementation of the theory given in Part I. Section 9 deals with spatial notions of the numerical problem and Section 10 the temporal. The solution scheme for the resulting algebraic problem is discussed in Sections 11 and 12.

The computer program FEAP 74 was modified to incorporate the numerical contact-impact model. The program modifications and capabilities together with two numerical examples are contained in Part III.

Finally, in the appendix we give listings for the contact subroutines together with the data input instructions.

PART I

VARIATIONAL FORMULATION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

1. Preliminaries

Our conventions on indices are as follows:

Superscripts indicate to which body an entity pertains. Summation is to take place only when explicitly indicated.

Latin subscripts range over 1,2,3, while Greek subscripts range over 1,2. The summation convention is assumed to hold for both.

A body $\mathcal{B}$ is a nice connected region of $\mathbb{R}^3$ with a piecewise smooth boundary $\partial\mathcal{B}$. A contact* problem is a boundary value problem, or an initial-boundary value problem, in which two bodies, $\mathcal{B}^1$ and $\mathcal{B}^2$, interact according to the principles of mechanics. Thus the primary kinematic axiom of a contact problem is that configurations $\mathcal{b}^1$ and $\mathcal{b}^2$, of $\mathcal{B}^1$ and $\mathcal{B}^2$, respectively, do not penetrate each other, i.e.,

$$
\begin{aligned}
(\mathcal{b}^1)^\circ \cap \mathcal{b}^2 &= \emptyset, \\
\mathcal{b}^1 \cap (\mathcal{b}^2)^\circ &= \emptyset,
\end{aligned}
\tag{1}
$$

where $(\mathcal{b}^\alpha)^\circ$ denotes the interior of $\mathcal{b}^\alpha$, $\alpha = 1,2$.

On the other hand the unique conditon which characterizes contact problems is that material points on the boundaries of $\mathcal{B}^1$ and $\mathcal{B}^2$ may coalesce during the motion of the bodies. Thus we say $\mathcal{B}^1$ and $\mathcal{B}^2$ are in contact if $\partial\mathcal{b}^1 \cap \partial\mathcal{b}^2 \neq \emptyset$, and we define the contact surface $\mathcal{c}$ by

---

* It is usual for the term contact to have a static connotation while the term impact has a dynamic connotation. We shall use contact in the general sense to include static as well as dynamic phenomena.

$$\mathcal{C} = \partial b^1 \cap \partial b^2 \quad . \tag{2}$$

If $\mathcal{B}^1$ and $\mathcal{B}^2$ are never in contact then $\mathcal{C} = \emptyset$ for all configurations b^1 and b^2, and in this case an initial-boundary value problem for $\mathcal{B}^1$ and $\mathcal{B}^2$ reduces to one in which $\mathcal{B}^1$ and $\mathcal{B}^2$ may be treated separately. Thus a non-trivial contact problem is one in which $\mathcal{C} \neq \emptyset$ for at least one instant during the motion of $\mathcal{B}^1$ and $\mathcal{B}^2$. The picture (Fig. 1) illustrates these notions.

Equation (1) implies that $\mathcal{C}$ is a material surface with respect to both bodies, i.e., one which is not crossed by material particles. From this we may deduce the interface conditions on $\mathcal{C}$.

Let $\underset{\sim}{x}$ be a <u>persistent point</u> of $\mathcal{C}$ (one at which joining or releasing of the bodies is not instantaneously occurring) and $\underset{\sim}{v}$ be the velocity of $\underset{\sim}{x}$ ($\underset{\sim}{v} = \dot{\underset{\sim}{x}}$). Note that only the normal part of $\underset{\sim}{v}$ is independent of the parametrization of $\mathcal{C}$. Let $\underset{\sim}{v}^1$ and $\underset{\sim}{v}^2$ be the velocities of the material particles located at the points $\underset{\sim}{x}^1$ and $\underset{\sim}{x}^2$, contained in ∂b^1 and ∂b^2 , respectively, such that $\underset{\sim}{x} = \underset{\sim}{x}^1 = \underset{\sim}{x}^2$ at the present instant. Then since $\mathcal{C}$ is material and $\underset{\sim}{x}$ is persistant

$$\underset{\sim}{v} \cdot \underset{\sim}{n} = \underset{\sim}{v}^1 \cdot \underset{\sim}{n} = \underset{\sim}{v}^2 \cdot \underset{\sim}{n} \quad , \tag{3}$$

where $\underset{\sim}{n}$ is a unit normal vector to $\mathcal{C}$ at $\underset{\sim}{x}$. From this it follows that a necessary condition for momentum to be balanced at $\underset{\sim}{x}$ is that

$$(\underset{\sim}{t}^1 + \underset{\sim}{t}^2) \cdot \underset{\sim}{n} = \underset{\sim}{0} \quad , \tag{4}$$

where $\underset{\sim}{t}^\alpha$ is the Cauchy traction vector with respect to ∂b^α .

In addition we assume that no tensile tractions can occur on $\mathcal{C}$,

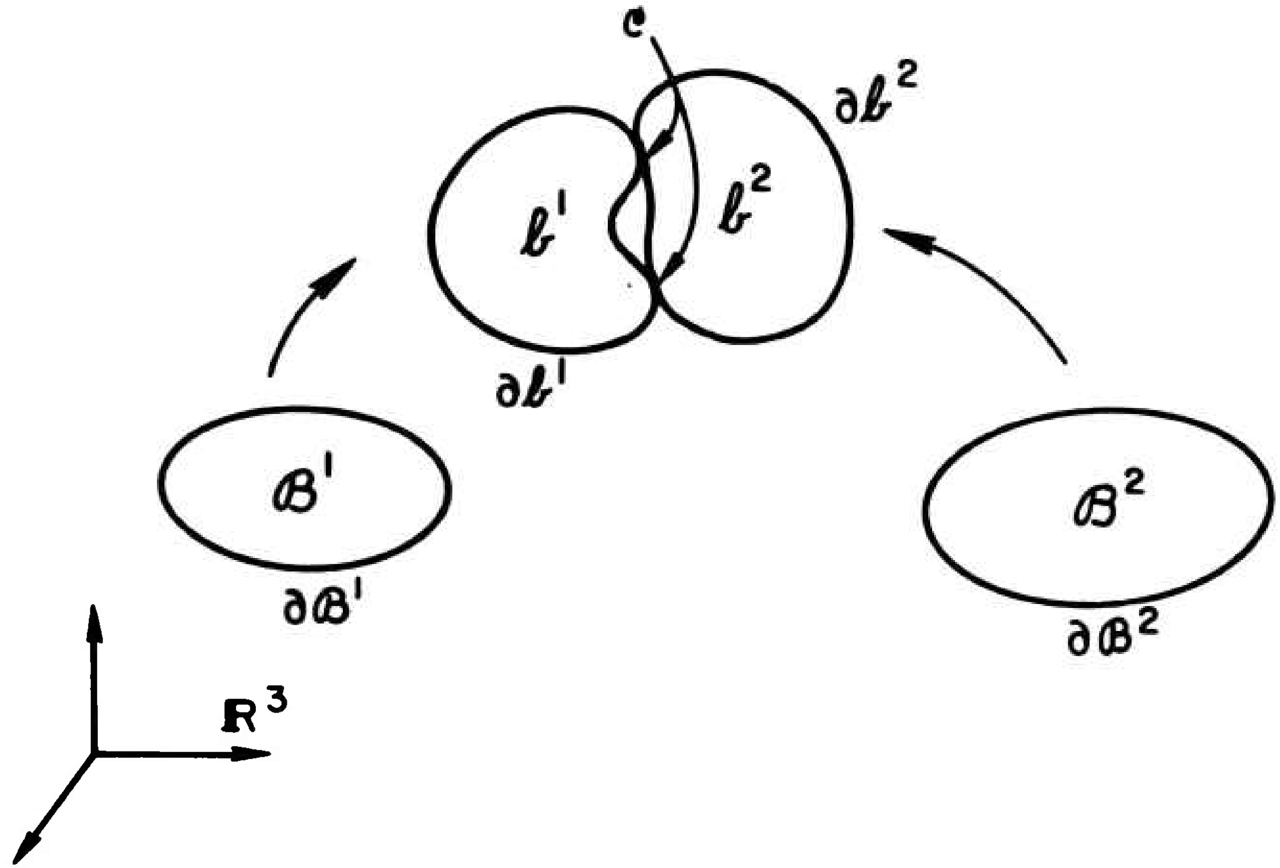


Figure 1

$$\underset{\sim}{t}^{\alpha} \cdot \underset{\sim}{n}^{\alpha} \leq 0 \; , \tag{5}$$

where $\underset{\sim}{n}^{\alpha}$ is the outward unit normal vector to $\partial\mathcal{B}^{\alpha}$. This condition excludes the possibility of the two bodies being glued together. Conditions (1-5) characterize our notion of a contact problem.

Note that thus far we have said nothing about the tangential parts of $\underset{\sim}{v}^{\alpha}$ and $\underset{\sim}{t}^{\alpha}$. These remaining conditions are determined by the frictional nature of the contact. We shall study two simple cases.

Case I: If we assume that points, once in contact, move with $\mathcal{C}$ until released, we have that

$$\underset{\sim}{v}^{1} = \underset{\sim}{v}^{2} \; , \tag{6}$$

and therefore

$$\underset{\sim}{t}^{1} + \underset{\sim}{t}^{2} = \underset{\sim}{0} \; . \tag{7}$$

For this model we say that a no-slip, or perfect friction, condition is achieved on $\mathcal{C}$. Thus condition (5) and equations (6) and (7) are the interface conditions for this case.

Case II: We may create the interface conditions for a frictionless, sliding contact by asserting that the tangential part of each $\underset{\sim}{t}^{\alpha}$ is identically zero,

$$\underset{\sim}{t}^{\alpha} - (\underset{\sim}{t}^{\alpha} \cdot \underset{\sim}{n}^{\alpha})\underset{\sim}{n}^{\alpha} = \underset{\sim}{0} \; . \tag{8}$$

Eq. (8), along with (3-5), are the interface conditions for this case.

2. Variational Theorems

We will formulate a variational theorem for the contact problem of finite elastodynamics. We point out, however, that our treatment is entirely general and could be used in conjunction with any field theory, as the only unique feature of the formulation involves the handling of interface conditions. At the same time finite elastodynamics, though lending itself to a clean and simple variational statement, is a case of wide practical interest.

We shall first obtain a variational theorem for the usual initial-boundary value problem of finite elastodynamics by a trivial generalization of some work done by S. Nemat-Nasser [1].

For notational simplicity let $\mathcal{A}$ denote $\partial\mathcal{B}$, and let $d\mathcal{A}$ and $d\mathcal{B}$ denote area and volume forms for $\mathcal{B}$ and $\mathcal{A}$, respectively. Let $\mathcal{A}_T \subset \mathcal{A}$ be that part of $\mathcal{A}$ where surface tractions are prescribed, and denote by $\bar{\underset{\sim}{T}}$ the Piola - Kirchhoff traction vector representing these prescribed tractions. Call ρ_0 the density of $\mathcal{B}$ in the initial configuration, $\underset{\sim}{F}$ the extrinsic body force vector and let $\underset{\sim}{x} = \underset{\sim}{x}_t(\underset{\sim}{X})$ represent the position at time t of the material particle located at $\underset{\sim}{X}$ in the initial configuration. For convenience we take $\mathcal{B}$ to be the initial configuration. We denote by $\partial\underset{\sim}{x}/\partial\underset{\sim}{X}$ the deformation gradients and by $\Phi(\partial\underset{\sim}{x}/\partial\underset{\sim}{X})$ the strain energy density. Then if $\underset{\sim}{x}$ satisfies the kinematic boundary conditions

$$\underset{\sim}{x} = \bar{\underset{\sim}{x}} \tag{9}$$

on $\mathcal{A}_x \subset \mathcal{A}$, where

$$\mathcal{A}_x \cup \mathcal{A}_T = \mathcal{A} ,$$
$$\mathcal{A}_x \cap \mathcal{A}_T = \emptyset ,$$

the functional Π defined by

$$\Pi(\underset{\sim}{x}) = \int_0^t \Big\{ \int_{\mathcal{B}} \big(\Phi(\partial\underset{\sim}{x}/\partial\underset{\sim}{X}) - \rho_0\, \dot{\underset{\sim}{x}}\cdot\dot{\underset{\sim}{x}}/2 - \rho_0 \underset{\sim}{F}\cdot\underset{\sim}{x} \big) d\mathcal{B} - \int_{\mathcal{A}_T} \underset{\sim}{x}\cdot\bar{\underset{\sim}{T}}\, d\mathcal{A} \Big\} dt \,, \tag{10}$$

is stationary, i.e., its first variation vanishes

$$0 = \delta\Pi(\underset{\sim}{x},\delta\underset{\sim}{x}) = \int_0^t \Big\{ \int_{\mathcal{B}} \big(\rho_0 (\ddot{\underset{\sim}{x}} - \underset{\sim}{F}) - \mathrm{DIV}\, \underset{\sim}{P} \big)\cdot\delta\underset{\sim}{x}\, d\mathcal{B} + \int_{\mathcal{A}_T} (\underset{\sim}{T} - \bar{\underset{\sim}{T}})\cdot\delta\underset{\sim}{x}\, d\mathcal{A} \Big\} dt \,, \tag{11}$$

subject to the constraint on variations $\delta\underset{\sim}{x}_t = \delta\underset{\sim}{x}_0 = \underset{\sim}{0}$, (12)

if and only if the equations of motion and traction boundary conditions are satisfied

$$\rho_0(\ddot{\underset{\sim}{x}} - \underset{\sim}{F}) = \mathrm{DIV}\,\underset{\sim}{P} \,, \qquad \text{in } \mathcal{B} \,, \tag{13}$$

$$\underset{\sim}{T} = \bar{\underset{\sim}{T}} \,, \qquad \text{on } \mathcal{A}_T \,, \tag{14}$$

where $\underset{\sim}{P} = \partial\Phi/\partial(\partial\underset{\sim}{x}/\partial\underset{\sim}{X})$ is the first Piola - Kirchhoff stress tensor, $\underset{\sim}{T} = \underset{\sim}{N}\cdot\underset{\sim}{P}$ is the Piola - Kirchhoff traction vector, and $\underset{\sim}{N}$ is the outward unit normal vector to $\mathcal{A}$. The solution to the initial-boundary value problem must also satisfy the given initial conditions

$$\left.\begin{array}{l} \underset{\sim}{x} = \underset{\sim}{x}_0 \\ \underset{\sim}{v} = \underset{\sim}{v}_0 \end{array}\right\} \quad \text{in } \mathcal{B} \,. \tag{15}$$

To interpret this variational theorem for two (non-interacting) bodies set

$$\mathcal{B} = \mathcal{B}^1 \cup \mathcal{B}^2 \,,$$

$$\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2 \,, \quad \text{etc.} \,,$$

and write

$$\Pi(\underset{\sim}{x}) = \Pi^1(\underset{\sim}{x}^1) + \Pi^2(\underset{\sim}{x}^2) .$$

The next step is to add to Π terms manifesting the interface conditions on $\mathcal{C}$ and to stipulate the constraints under which the vanishing of the first variation of the appended functional corresponds to a solution of the contact problem. To do this we must consider further the kinematics and geometry of $\mathcal{C}$.

Define two piecewise smooth, invertible maps $\underset{\sim}{\chi}^1, \underset{\sim}{\chi}^2$ by the condition

$$(\underset{\sim}{\chi}^\alpha)^{-1} : \mathcal{C} \longrightarrow \mathcal{C}^\alpha \subset \mathcal{Q}^\alpha , \tag{16}$$

where each $\underset{\sim}{\chi}^\alpha$ identifies points on the boundary of the initial configuration $\mathcal{B}^\alpha$ which map into the contact surface $\mathcal{C}$ at each instant of time. If $\underset{\sim}{x} \in \mathcal{C}$, then $\underset{\sim}{X}^1 = (\underset{\sim}{\chi}^1)^{-1}(\underset{\sim}{x})$ and $\underset{\sim}{X}^2 = (\underset{\sim}{\chi}^2)^{-1}(\underset{\sim}{x})$ are the positions of particles in $\mathcal{Q}^1$ and $\mathcal{Q}^2$, respectively, which have coalesced at $\underset{\sim}{x} \in \mathcal{C}$. It is clear what the $\underset{\sim}{\chi}^\alpha$'s really are, viz., if $\underset{\sim}{x}^\alpha = \underset{\sim}{x}^\alpha_t(\underset{\sim}{X}^\alpha)$, for all $\underset{\sim}{X}^\alpha \in \mathcal{B}^\alpha$, represents the motion of body $\mathcal{B}^\alpha$ from the original configuration $\mathcal{B}^\alpha$ to the present one $\mathcal{B}^\alpha$, then $\underset{\sim}{\chi}^\alpha$ is the restriction of $\underset{\sim}{x}^\alpha$ to $\mathcal{C}^\alpha$,

$$\underset{\sim}{\chi}^\alpha(\underset{\sim}{X}^\alpha) = \underset{\sim}{x}^\alpha(\underset{\sim}{X}^\alpha) , \tag{17}$$

for each $\underset{\sim}{X}^\alpha \in \mathcal{C}^\alpha$, $\alpha = 1,2$. For the time being we consider the $\underset{\sim}{\chi}^\alpha$'s as maps defined independently of the $\underset{\sim}{x}^\alpha$'s and consider (17) a constraint on possible motions.

We are interested in to what extent the relation

$$\underset{\sim}{x} = \underset{\sim}{\chi}'^{1}(\underset{\sim}{X}^{1}) = \underset{\sim}{\chi}'^{2}(\underset{\sim}{X}^{2}) \quad , \tag{18}$$

is smooth in time and analogously under what circumstances the variations of the $\underset{\sim}{\chi}'^{\alpha}$'s are equal. In general the $\underset{\sim}{\chi}'^{\alpha}$'s will not even be continuous in time since contact surfaces can be instantaneously created or destroyed. If we eliminate such exceptional instants and consider only persistent points, the bodies still may slide with respect to each other, as depicted in Fig. 2. Thus tangential velocities are seen to be unequal in general. However, when $\underset{\sim}{x}$ is persistent, the impenetrability condition (1) forces the normal velocity components to be equal, and concomitantly the normal components of variations of the $\underset{\sim}{\chi}'^{\alpha}$'s are also equal

$$\delta\underset{\sim}{\chi}'^{1}\cdot\underset{\sim}{n} = \delta\underset{\sim}{\chi}'^{2}\cdot\underset{\sim}{n} \tag{19}$$

For sliding contact (Case II), Eq. (19) characterizes the constraint on variations of the $\underset{\sim}{\chi}'^{\alpha}$'s eqivalent to the velocity constraint (3).

For no-slip contact (Case I),

$$\delta\underset{\sim}{\chi}'^{1} = \delta\underset{\sim}{\chi}'^{2} \quad , \tag{20}$$

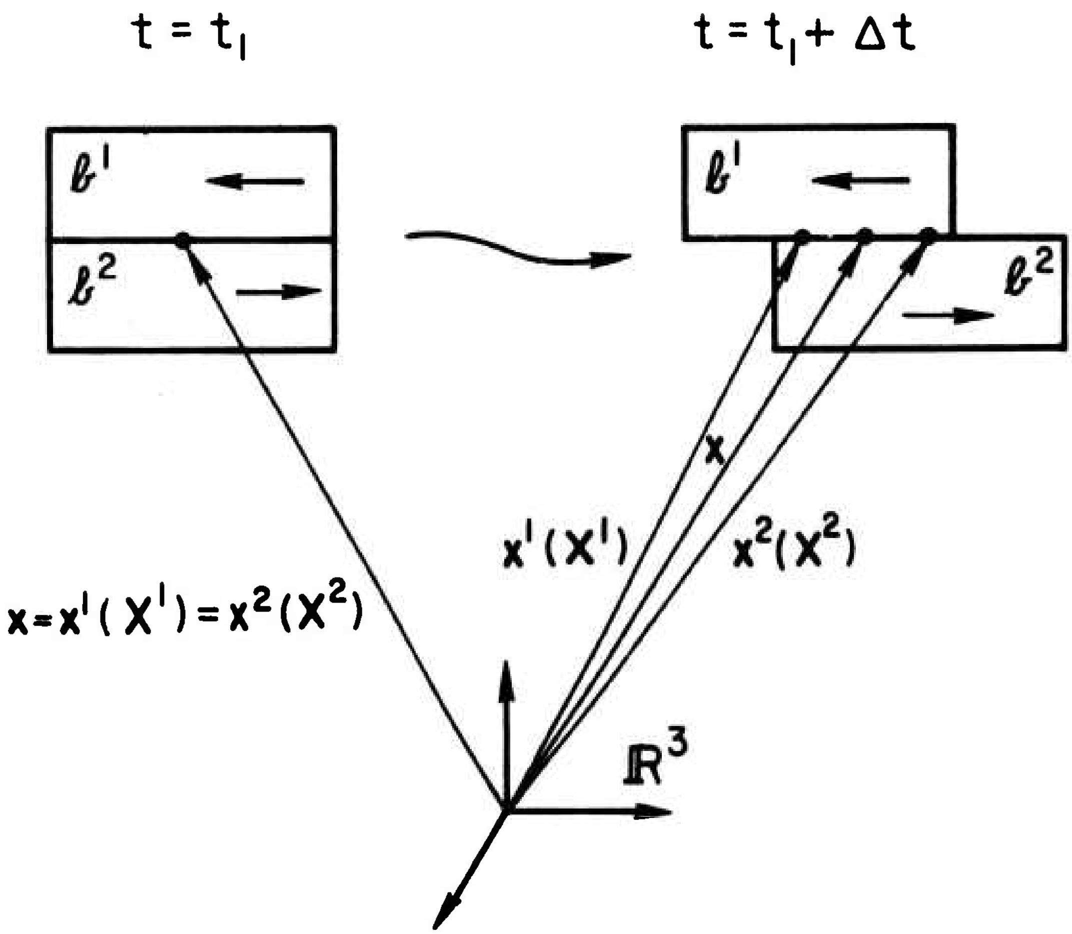
is easily seen to be the condition on variations equivalent to Eq. (6). We shall see that Eqs. (19) and (20) lead to the proper interface conditions in the variational theorems.

Introduce vector valued Lagrange multipliers $\underset{\sim}{\tau}^{\alpha}$, and add

$$\mathcal{K} = -\sum_{\alpha=1}^{2}\int_{0}^{t}\int_{\mathcal{C}^{\alpha}} \underset{\sim}{\tau}^{\alpha}\cdot(\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}'^{\alpha})\, d\mathcal{C}^{\alpha} dt \quad , \tag{21}$$

to the functional Π (Eq. 10). Note that when $\mathcal{C} \neq \emptyset$,

$$\mathcal{A}^{\alpha} = \mathcal{A}^{\alpha}_{x} \cup \mathcal{A}^{\alpha}_{T} \cup \mathcal{C}^{\alpha} \quad ,$$



$t = t_1$
$t = t_1 + \Delta t$
$\mathcal{B}^1$
$\mathcal{B}^2$
$\mathcal{B}^1$
$\mathcal{B}^2$
x
$x^1(X^1)$
$x^2(X^2)$
$x = x^1(X^1) = x^2(X^2)$
$\mathbb{R}^3$

Figure 2

and assume for consistency's sake that

$$\mathcal{C}^\alpha \subset \mathcal{A}^\alpha_T ,$$

$$\underset{\sim}{T} = \underset{\sim}{0} \qquad \text{on} \quad \mathcal{C}^\alpha . \tag{22}$$

This condition will preclude the ambiguous circumstance of non-zero tractions being specified on the contact area. Upon taking variations of $\mathcal{J} = \Pi + \mathcal{K}$ we get Eqs. (13), (14) and,

$$0 = - \sum_{\alpha=1}^{2} \int_0^t \int_{\mathcal{C}^\alpha} \{ \delta \underset{\sim}{\tau}^\alpha \cdot (\underset{\sim}{x}^\alpha - \underset{\sim}{z}^\alpha) + \tag{23}$$

$$+ \delta \underset{\sim}{x}^\alpha \cdot (\underset{\sim}{\tau}^\alpha - \underset{\sim}{T}^\alpha) - \delta \underset{\sim}{z}^\alpha \cdot \underset{\sim}{\tau}^\alpha \} d\mathcal{C}^\alpha dt \quad +$$

$$+ \text{ transversality condition.}$$

The transversality condition is the classical terminology for variations associated with the domain $\mathcal{C}^\alpha$.

The first summand of (23) gives us (17) which insures that the $\underset{\sim}{x}^\alpha$'s map into $\mathcal{C}$ properly. The second summand identifies $\underset{\sim}{\tau}^\alpha$ as the Piola - Kirchhoff traction vector $\underset{\sim}{T}^\alpha$ on $\mathcal{C}^\alpha$. Let us investigate the third summand.

Consider first Case I and define

$$\delta \underset{\sim}{z} = \delta \underset{\sim}{z}^\alpha \quad , \quad \alpha = 1,2 , \tag{24}$$

which makes sense because of Eq. (20). This condition is equivalent to insisting

$$\dot{\underset{\sim}{z}} = \dot{\underset{\sim}{z}}^\alpha \quad , \quad \alpha = 1,2 ,$$

thus the first summand of (23) also implies (6) holds whenever we have a persistent point. Let j^α denote the Jacobian determinant associated with $\underset{\sim}{\chi}^\alpha$,

$$d\mathcal{E} = j^\alpha d\mathcal{C}^\alpha. \tag{25}$$

Notice then that since $\underset{\sim}{\tau}^\alpha$ is the Piola - Kirchhoff traction vector, $(1/j^\alpha)\underset{\sim}{\tau}^\alpha$ is the corresponding Cauchy traction vector. With these we have for the third summand,

$$0 = \sum_{\alpha=1}^{2} \int_{\mathcal{C}^\alpha} \delta\underset{\sim}{\chi}^\alpha \cdot \underset{\sim}{\tau}^\alpha d\mathcal{C}^\alpha = \int_{\mathcal{E}} \delta\underset{\sim}{\chi} \cdot ((1/j^1)\underset{\sim}{\tau}^1 + (1/j^2)\underset{\sim}{\tau}^2)d\mathcal{E} \quad , \tag{26}$$

which in words means the Cauchy traction vectors are in equilibrium. Thus the momentum balance, Eq. (7), is satisfied on $\mathcal{E}$.

In Case II we only have that (19) holds, so define

$$\delta\chi(n) = \delta\underset{\sim}{\chi}^\alpha \cdot \underset{\sim}{n} \quad , \quad \alpha = 1,2 . \tag{27}$$

This requirement also insures that,

$$\dot{\underset{\sim}{\chi}}^1 \cdot \underset{\sim}{n} = \dot{\underset{\sim}{\chi}}^2 \cdot \underset{\sim}{n} \quad ,$$

thus the first summand of (23) implies (3). For this case the third summand takes the form,

$$\begin{aligned} 0 = \sum_{\alpha=1}^{2} \int_{\mathcal{C}^\alpha} \delta\underset{\sim}{\chi}^\alpha \cdot \underset{\sim}{\tau}^\alpha d\mathcal{C}^\alpha &= \int_{\mathcal{E}} \delta\chi(n)((1/j^1)\underset{\sim}{\tau}^1 \cdot \underset{\sim}{n} + (1/j^2)\underset{\sim}{\tau}^2 \cdot \underset{\sim}{n})d\mathcal{E} \\ &+ \sum_{\alpha=1}^{2} \int_{\mathcal{C}^\alpha} (\delta\underset{\sim}{\chi}^\alpha - \delta\chi(n)\underset{\sim}{n}) \cdot \underset{\sim}{\tau}^\alpha d\mathcal{C}^\alpha . \end{aligned} \tag{28}$$

The integral over $\mathcal{E}$ gives us Eq. (4). The significance of the second integral hinges on the observation that $(\delta\underset{\sim}{\chi}^\alpha - \delta\chi(n)\underset{\sim}{n})$ is a tangent vector to $\mathcal{E}$ for each α . Thus the tangential part of each $\underset{\sim}{\tau}^\alpha$ is identically zero, which is equivalent to the shear free condition, Eq. (8), which we require for Case II.

A standard calculation enables us to write the transversality condition as,

$$0 = \sum_{\alpha=1}^{2} \int_{\partial \mathcal{C}^\alpha} (\delta \boldsymbol{\chi}^\alpha \cdot \mathbf{T}^\alpha)(\boldsymbol{\tau}^\alpha \cdot (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha))\, d(\partial \mathcal{C}^\alpha) \,, \tag{29}$$

where the transversal $\mathbf{T}^\alpha$ is a unit vector field tangent to $\mathcal{C}^\alpha$, and perpendicular and pointing outward with respect to $\partial \mathcal{C}^\alpha$, Fig. 3. Thus (29) implies that

$$\boldsymbol{\tau}^\alpha \cdot (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha) = 0 \quad \text{on } \partial \mathcal{C}^\alpha, \ \alpha = 1,2 \tag{30}$$

Assuming continuity of the integrands of (21) on the closure of $\mathcal{C}^\alpha$, condition (30) is already implied by the first summand of (23). This assumption precludes $\boldsymbol{\tau}^\alpha$ taking the form of a δ-distribution on $\partial \mathcal{C}^\alpha$.

Although this assumption is warranted here it may not be true when one employs certain approximate theories in mechanics. For instance consider the case where a Bernoulli-Euler beam is uniformly loaded and sits on a rigid parabolic surface (Fig. 4). At the contact points a, a', concentrated reactions must exist to balance shear forces. This example is actually from a completely different class of contact problems in that contact is made along a part of the interior rather than the boundary. Such problems as the contact of plates and shells also fall into this class. We could summarize such situations by the description -- m-dimensional contact of m-dimensional bodies, e.g., for the beam m=1, and for plates and shells m=2. The case under investigation in this paper (m=3) is an example of the (m-1)-dimensional contact of m-dimensional bodies.

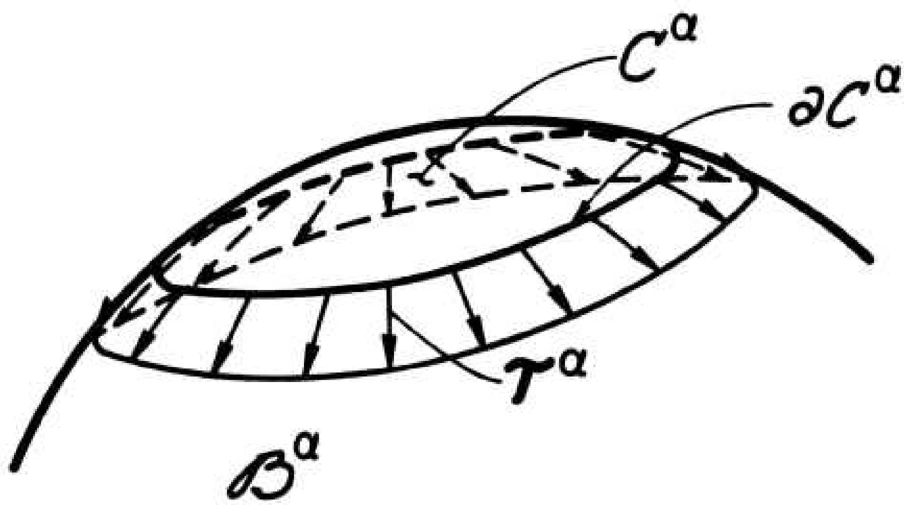


Figure 3

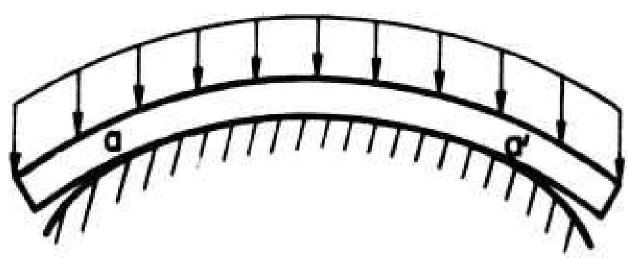


Figure 4

It is good to keep in mind cases such as that illustrated in Fig. 4 when considering specific boundary value problems.

A further point worth mentioning here is that the transversality condition will in general be an independent one in a numerical algorithm. For example, if the fields in the integrand of (21) are approximated by a family of trial functions, Eq. (23) only implies that some weighted integrals over the $\mathcal{C}^{\alpha}$'s vanish. The condition (29) requires that weighted integrals over the $\partial\mathcal{C}^{\alpha}$'s also vanish.

We now summarize our results in the following theorems:

Theorem I: Let (1), (2), (5), (9), (12), (15), and (20) hold. Then $\underset{\sim}{x}$ is a solution to the no-slip contact problem (Case I), that is, (6), (7), (13), (14) and (17) also hold if, and only if, $\delta\mathcal{J}=0$ for arbitrary variations of $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{\chi}^{\alpha}$ and $\underset{\sim}{\tau}^{\alpha}$, $\alpha=1,2$.

Theorem II: Let (1), (2), (5), (9), (12), (15), and (19) hold. Then $\underset{\sim}{x}$ is a solution to the sliding contact problem (Case II), that is, (3), (4), (8), (13), (14) and (17) also hold if, and only if, $\delta\mathcal{J}=0$ for arbitrary variations of $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{\chi}^{\alpha}$ and $\underset{\sim}{\tau}^{\alpha}$, $\alpha=1,2$.

3. Consideration of Theorems I and II as Computational Tools

Theorems I and II may be employed to generate numerical algorithms for the solution of contact problems. The basic idea is to represent $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{r}^{\alpha}$ and $\underset{\sim}{\mathcal{C}}^{\alpha}$ as the product of known functions on $\mathbb{R}^3$ with unknown parameters depending on time. Then Theorems I and II provide us with a method for generating an approximate system of equations (e.g., by the classical Ritz-Galerkin technique) in terms of these unknown parameters, which then can be solved incrementally and/or iteratively, subject to the side conditions of the theorems. The constraints (1) and (5) will both take the form of inequalities in actual computations, thus the ideas of optimization theory will probably be useful in the actual construction of a numerical algorithm.

The finite element method is a powerful technique for obtaining a system of approximate equations, and it is of interest to find out how amenable are Theorems I and II to a finite element formulation. Unfortunately the term $\mathcal{X}$ would result in a terrible mess if the integrand was represented by typical finite element functions. This is because the boundaries of the $\mathcal{C}^{\alpha}$'s are unknown and thus a parametric integration would bury the defining parameters of the $\mathcal{C}^{\alpha}$'s in the arguments of Heaviside functions representing the supports of the elements. Note that a classical Ritz-Galerkin approximation would not be subject to this pitfall, since the associated trial functions could be chosen to be real analytic and thus easily integrated parametrically to a relatively simple form. However, such a formulation is restricted to a geometrically simpler class of problems. Thus it is desirable to seek a generalization that will lend itself cleanly to a finite element formulation.

4. <u>Variational Theorems Without Transversality Conditions</u>

Let $\tilde{\mathcal{E}}^{\alpha}$ be a fixed part of $\mathcal{Q}_{\tau}^{\alpha}$ such that

$$\tilde{\mathcal{E}}^{\alpha} \supset \mathcal{E}^{\alpha} \quad , \tag{31}$$

and

$$\underset{\sim}{T} = \underset{\sim}{0} \quad \text{on} \quad \tilde{\mathcal{E}}^{\alpha} \sim \mathcal{E}^{\alpha} . \tag{32}$$

Define a scalar valued function η^{α} on $\tilde{\mathcal{E}}^{\alpha}$ such that

$$\eta^{\alpha}(\underset{\sim}{X}^{\alpha}) = 0 \qquad \text{if} \quad \underset{\sim}{X}^{\alpha} \in \tilde{\mathcal{E}}^{\alpha} \sim \mathcal{E}^{\alpha} . \tag{33}$$

Let $\tilde{e} \supset e$, and define the maps $\underset{\sim}{\chi}^{\alpha}$ by the condition

$$(\underset{\sim}{\chi}^{\alpha})^{-1} : \tilde{e} \longrightarrow \tilde{\mathcal{E}}^{\alpha} ,$$

where, as before, $\underset{\sim}{\chi}^{\alpha}$ represents $\underset{\sim}{x}^{\alpha}$ on $\mathcal{E}^{\alpha}$; but on $\tilde{\mathcal{E}}^{\alpha} \sim \mathcal{E}^{\alpha}$ we place no physical interpretation on $\underset{\sim}{\chi}^{\alpha}$. Thus on $\tilde{\mathcal{E}}^{\alpha}$ we will always have that,

$$\eta^{\alpha}(\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}^{\alpha}) = \underset{\sim}{0} \quad , \tag{34}$$

since $\underset{\sim}{x}^{\alpha} = \underset{\sim}{\chi}^{\alpha}$ on $\mathcal{E}^{\alpha}$ and $\eta^{\alpha} = 0$ on the relative complement $\tilde{\mathcal{E}}^{\alpha} \sim \mathcal{E}^{\alpha}$.

Introduce vector valued Lagrange multipliers $\underset{\sim}{\sigma}^{\alpha}$ and let $\mathcal{L} = \mathrm{II} + \mathcal{M}$ where

$$\mathcal{M} = - \sum_{\alpha=1}^{2} \int_{0}^{t} \int_{\tilde{\mathcal{E}}^{\alpha}} \underset{\sim}{\sigma}^{\alpha} \cdot \eta^{\alpha} (\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}^{\alpha}) \, d\tilde{\mathcal{E}}^{\alpha} dt . \tag{35}$$

We require that the variations of $\underset{\sim}{\chi}^{\alpha}$ satisfy the same conditions as before, but now for all $\tilde{\mathcal{E}}^{\alpha}$:

$$\left.\begin{array}{ll} \text{Case I:} & \delta \underset{\sim}{\chi} \overset{\text{def.}}{=} \delta \underset{\sim}{\chi}^{\alpha} \\ \text{Case II:} & \delta \chi(n) \overset{\text{def.}}{=} \delta \underset{\sim}{\chi}^{\alpha} \cdot \underset{\sim}{n} \end{array}\right\} \quad \text{on} \quad \tilde{\mathcal{E}}^{\alpha} \tag{36}$$

where $\mathbf{n}$ is a unit normal vector to $\tilde{\mathcal{E}}$. Computing the first variation of $\mathcal{L}$ we have the usual conditions emanating from II and

$$0 = -\sum_{\alpha=1}^{2} \int_0^t \int_{\tilde{\mathcal{E}}^\alpha} \Big\{ \delta\boldsymbol{\sigma}^\alpha \left(\eta^\alpha (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha)\right) + \delta\eta^\alpha \left(\boldsymbol{\sigma}^\alpha \cdot (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha)\right) + \delta\mathbf{x}^\alpha \cdot (\eta^\alpha \boldsymbol{\sigma}^\alpha - \mathbf{T}^\alpha) - \delta\boldsymbol{\chi}^\alpha \cdot (\eta^\alpha \boldsymbol{\sigma}^\alpha) \Big\} \, d\tilde{\mathcal{E}}^\alpha dt . \tag{37}$$

The first summand gives us (34) and we define

$$\mathcal{C}^\alpha = \{ \mathbf{X}^\alpha \in \tilde{\mathcal{E}}^\alpha : \mathbf{x}^\alpha(\mathbf{X}^\alpha) = \boldsymbol{\chi}^\alpha(\mathbf{X}^\alpha) \} . \tag{38}$$

The third summand defines $\eta^\alpha \boldsymbol{\sigma}^\alpha$ as the Piola - Kirchhoff traction vector. Note that this insures that $\mathbf{T} = \mathbf{0}$ on $\tilde{\mathcal{E}}^\alpha \sim \mathcal{C}^\alpha$ since $\eta^\alpha = 0$ there. The fourth summand gives us the appropriate Cauchy traction condition across $\mathcal{E}$ for each case of (36). The second summand is identically satisfied on $\mathcal{C}^\alpha$ since $\mathbf{x}^\alpha = \boldsymbol{\chi}^\alpha$. On $\tilde{\mathcal{E}}^\alpha \sim \mathcal{C}^\alpha$ it tells us that $\boldsymbol{\sigma}^\alpha$ is orthogonal to $\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha$, but this is of no physical interest.

Thus we can state the following theorems:

Theorem I': Let (1), (2), (5), (9), (12), (15), (31), (32) and $(36)_1$ hold. Then $\mathbf{x}$ is a solution to the no-slip contact problem (Case I), that is, (6), (7), (13), (14) and (17) also hold where $\mathcal{C}^\alpha$ is defined by (38), if $\delta\mathcal{L} = 0$ for arbitrary variations of $\mathbf{x}^\alpha$, $\boldsymbol{\chi}^\alpha$, η^α and $\boldsymbol{\sigma}^\alpha$, $\alpha = 1, 2$.

Theorem II': Let (1), (2), (5), (9), (12), (15), (31), (32) and $(36)_2$ hold. Then $\mathbf{x}$ is a solution to the sliding contact problem (Case II), that is, (3), (4), (8), (13), (14) and (17) hold where $\mathcal{C}^\alpha$ is defined by (38), if $\delta\mathcal{L} = 0$ for arbitrary variations of $\mathbf{x}^\alpha$, $\boldsymbol{\chi}^\alpha$, η^α and $\boldsymbol{\sigma}^\alpha$, $\alpha = 1, 2$.

The important feature of these theorems is that the regions $\tilde{\mathcal{E}}^-$ are fixed. Thus transversality conditions are absent, and the theorems may be applied to finite element formulations. In fact one would naturally take $\tilde{\mathcal{E}}^-$ to be a union of elements in $\mathcal{A}^-$, large enough to contain $\mathcal{C}^-$ throughout the motion.

Thus far our considerations have been quite general and, in fact, more general than would be required for the solution of particular classes of contact problems. In the next section we illustrate the many simplifications which can be made in the application of the preceeding theorems to a class of problems of wide practical interest.

5. Hertzian Contact Problems

We wish to characterize contact problems in which the contact surface is approximately planar and the bodies have undergone small deformations in the neighborhood of the contact surface.

Assume the following:

(1) $\underset{\sim}{n} \overset{\text{def}}{=} n_i \underset{\sim}{e}_i \approx \underset{\sim}{e}_3$ on $\mathcal{C}$, where the n_i indicate components with respect to the standard basis $\{\underset{\sim}{e}_i\}_1^3$ for $\mathbb{R}^3$, (see Fig. 5).

(2) $j^\alpha \approx 1$, $\alpha = 1,2$, thus $\underset{\sim}{t}^\alpha \approx \underset{\sim}{T}^\alpha$ on $\mathcal{C}^\alpha$.

Assumptions (1) and (2) together imply that,

$$t_3^\alpha \approx \underset{\sim}{t}^\alpha \cdot \underset{\sim}{n} \approx \underset{\sim}{T}^\alpha \cdot \underset{\sim}{n} \approx T_3^\alpha ,$$

and that,

$$(t_1^\alpha, t_2^\alpha, 0) \approx \underset{\sim}{t}^\alpha - (\underset{\sim}{t}^\alpha \cdot \underset{\sim}{n})\underset{\sim}{n} \approx \underset{\sim}{T}^\alpha - (\underset{\sim}{T}^\alpha \cdot \underset{\sim}{n})\underset{\sim}{n} \approx (T_1^\alpha, T_2^\alpha, 0) .$$

(3) Material points which eventually contact have, to the first order, the same initial coordinates z_1 and z_2. Explicitly we manifest this idea by requiring that the $\underset{\sim}{\chi}^\alpha$'s satisfy

$$\underset{\sim}{\chi}^1(z_1, z_2, X_3^1(z_1, z_2)) = \underset{\sim}{\chi}^2(z_1, z_2, X_3^2(z_1, z_2)) . \tag{39}$$

This is depicted in Fig. 6. Since X_3^α are given functions which define the surfaces $\mathcal{C}^\alpha$, it follows from (39) that,

$$\delta \underset{\sim}{\chi}^1 = \delta \underset{\sim}{\chi}^2 .$$

We term problems for which these assumptions hold Hertzian, since these assumptions are implicit in Hertz' classical theory [2] (see

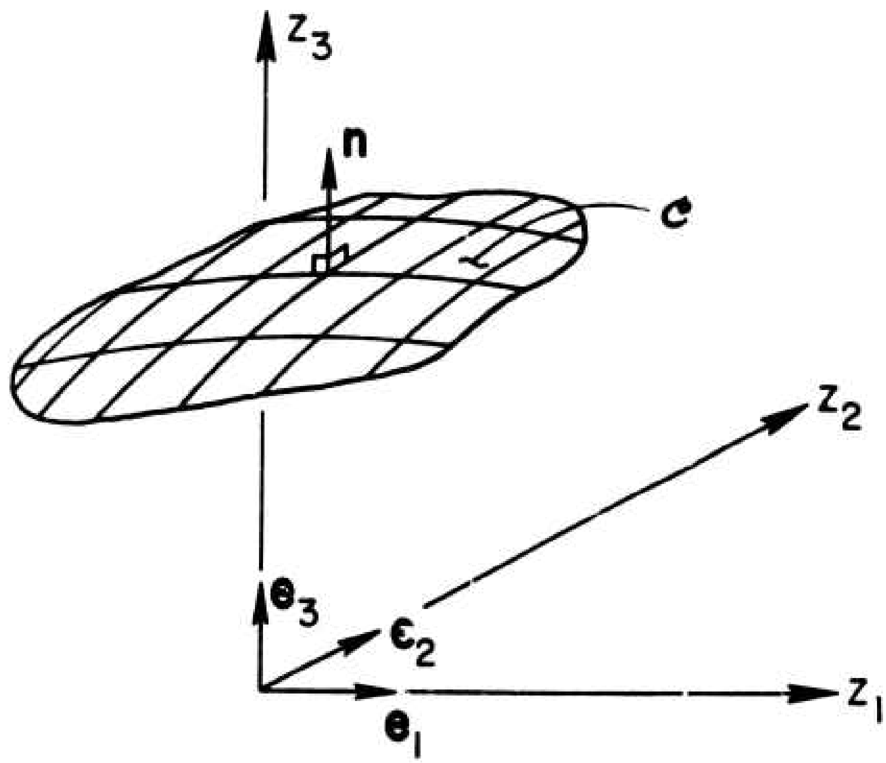


Figure 5

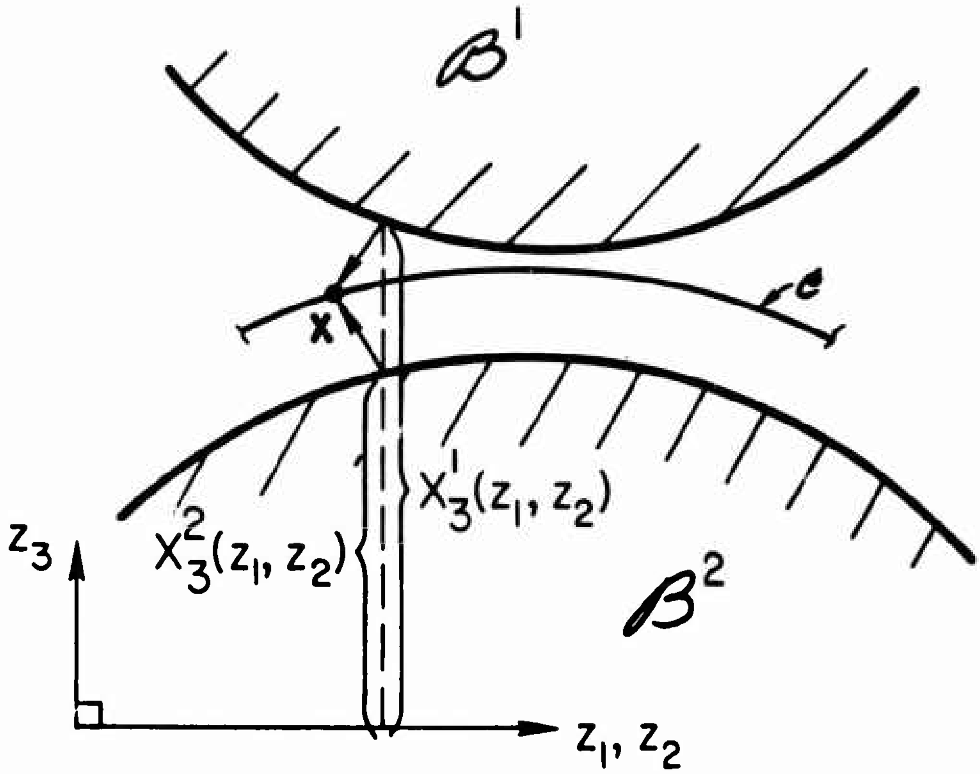


Figure 6

Goldsmith [3] for an excellent exposition of this work and also many applications of Hertz' theory to impact problems). It should be pointed out that the formulation we are about to give is still considerably more general than those to which Hertz' theory applies.

We now show how these assumptions allow us to make simplifications in the preceding theorems.

Theorems I and II:

Due to assumption (3) the term $\mathcal{K}$ can be replaced by an integral over a region in the z_1, z_2 -plane. This region, say c, is the projection of $\mathcal{C}$ onto the z_1, z_2 -plane, and due to assumption (2) it coincides, to the first order, with the projections of the $\mathcal{C}^\alpha$'s. Thus $\mathcal{K}$ can be written

$$\mathcal{K} = - \sum_{\alpha=1}^{2} \int_0^t \int_c \underset{\sim}{\tau}^\alpha \cdot (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha) \, dc \, dt \, . \tag{40}$$

Since, for Case I, we know that the momentum balance on $\mathcal{C}$ requires that

$$\underset{\sim}{\tau}^1 + \underset{\sim}{\tau}^2 = \underset{\sim}{0} \quad ,$$

we may make use of this relation immediately. Thus define

$$\underset{\sim}{\tau} = \underset{\sim}{\tau}^1 = -\underset{\sim}{\tau}^2 \quad ,$$

and substitute into (40). Employing (39), the integrand simplifies to

$$\underset{\sim}{\tau} \cdot (\underset{\sim}{x}^2 - \underset{\sim}{x}^1) \, . \tag{41}$$

The analog of (23) becomes

$$0 = \int_0^t \int_C \{ \delta \underline{\tau} \cdot (\underline{x}^2 - \underline{x}^1) +$$

$$+ \delta \underline{x}^1 \cdot (\underline{T}^1 - \underline{\tau}) + \delta \underline{x}^2 \cdot (\underline{T}^2 + \underline{\tau}) \} \, dc \, dt$$

$$+ \text{transversality condition.} \tag{42}$$

Thus the same conclusions of Theorem I can be drawn. However, from a numerical standpoint things are considerably different. First of all, since the $\underline{\chi}^\alpha$'s are absent in this formulation, we do not get a uniquely defined $\underline{t}$; $\underline{x}^1$ and $\underline{x}^2$ will not in general be the same pointwise. If the graph of $\underline{t}$ is important it could be constructed by averaging $\underline{x}^1$ and $\underline{x}^2$, which, if the solution is any good, should be reasonably close pointwise. On the other hand, the $\underline{\chi}^\alpha$'s being absent engenders a considerable saving in the number of equations to be solved and in their complexity.

The analogous case for Theorem II is constructed simply by setting

$$\tau_1 = \tau_2 = 0 \quad , \quad \tau \stackrel{\text{def.}}{=} \tau_3 \; .$$

Then the integrand of $\mathcal{K}$ becomes

$$\tau \, (x_3^2 - x_3^1) \tag{43}$$

and (42) reduces to

$$0 = \int_0^t \int_C \{ \delta \tau (x_3^2 - x_3^1) +$$

$$+ \delta x_3^1 (T_3^1 - \tau) + \delta x_3^2 (T_3^2 + \tau)$$

$$+ \delta x_\alpha^1 T_\alpha^1 + \delta x_\alpha^2 T_\alpha^2 \} \, dc \, dt \tag{44}$$

$$+ \text{transversality condition} \; .$$

Hence the conclusions of Theorem II hold.

Thus in the case of Hertzian contact we can add the simplifications manifested in (41) and (43) to the conditions of Theorems I and II, respectively, and still garner the same conclusions.

Theorems I' and II':

For these cases $\mathcal{M}$ can be written as an integral over $\tilde{\mathcal{C}}$, the projection of $\tilde{\mathcal{E}}$:

$$\mathcal{M} = - \sum_{\alpha=1}^{2} \int_{0}^{t} \int_{\tilde{\mathcal{C}}} \underset{\sim}{\sigma}^{\alpha} \cdot n^{\alpha} (\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}^{\alpha}) \, d\tilde{c} \, dt .$$

Due to the present geometric situation, it is appropriate to take

$$n^{1} = n^{2} ,$$

and thus define

$$n = n^{\alpha} , \quad \alpha = 1,2 .$$

Analagous to the considerations for Theorems I and II, the momentum balance across $\tilde{\mathcal{E}}$ motivates the simplification

$$\underset{\sim}{\sigma} \overset{\text{def.}}{=} \underset{\sim}{\sigma}^{1} = - \underset{\sim}{\sigma}^{2} .$$

With these and (39), the integrand of $\mathcal{M}$ can be written

$$\underset{\sim}{\sigma} \cdot n (\underset{\sim}{x}^{2} - \underset{\sim}{x}^{1}) .$$

A further simplification can be made by setting*

$$\sigma_{3} = - n .$$

This eliminates one unknown function and, as we shall see, has the effect of satisfying (5) naturally. Thus the integrand of $\mathcal{M}$ becomes

---

* This is a standard ploy of optimization theory, see p. 82, [4].

$$\sigma_\alpha \eta (x_\alpha^2 - x_\alpha^1) - (\eta)^2 (x_3^2 - x_3^1) , \tag{45}$$

and the analog of (23) is

$$\begin{aligned} 0 = \int_0^t \int_{\mathcal{C}} \{ & \delta\eta(\sigma_\alpha(x_\alpha^2 - x_\alpha^1) - 2\eta(x_3^2 - x_3^1)) \\ & + \delta\sigma_\alpha(\eta(x_\alpha^2 - x_\alpha^1)) \\ & + \delta x_\alpha^1 (T_\alpha^1 - \eta\sigma_\alpha) + \delta x_\alpha^2 (T_\alpha^2 + \eta\sigma_\alpha) \\ & + \delta x_3^1 (T_3^1 + (\eta)^2) + \delta x_3^2 (T_3^2 - (\eta)^2) \} \, d\mathcal{C} \, dt . \end{aligned} \tag{46}$$

Summand two tells us that either $\eta = 0$ or $x_\alpha^1 = x_\alpha^2$, on $\mathcal{C}$. Suppose $\eta \neq 0$, then $x_\alpha^1 = x_\alpha^2, \alpha = 1,2$. Summand one then gives us that $x_3^1 = x_3^2$ on $\mathcal{C}$. Thus we have

$$\eta (\underset{\sim}{x}^2 - \underset{\sim}{x}^1) = \underset{\sim}{0} , \qquad \text{on} \quad \mathcal{C} ,$$

as required, and $\mathcal{C}$ is defined as the subset of $\mathcal{C}$ where $\underset{\sim}{x}^1 = \underset{\sim}{x}^2$.

The last four summands give the momentum balance conditions, as usual, and, in addition, the last two summands imply that the normal tractions are compressive (since $(\eta)^2 \geq 0$). Thus we have the conclusions of Theorem I' and condition (5).

The analogous set up for Theorem II' is accomplished by setting $\sigma_\alpha = 0$ in (45) yielding

$$-(\eta)^2 (x_3^2 - x_3^1) \tag{47}$$

for the integrand of $\mathcal{M}$. With this Eq. (46) becomes

$$0 = \int_0^t \int_{\bar{c}} \{ -2\,\delta\eta(\eta(x_3^2 - x_3^1)) +$$

$$+ \delta x_\alpha^1 T_\alpha^1 + \delta x_\alpha^2 T_\alpha^2$$

$$+ \delta x_3^1 (T_3^1 + (\eta)^2) + \delta x_3^2 (T_3^2 - (\eta)^2) \} \, d\bar{c} \, dt$$

In this case we achieve the conclusion of Theorem II' and condition (5).

Thus to Theorems I' and II' we can delete condition (5), add the simplifications manifested in (45) and (47), and achieve the conclusions of Theorems I' and II', respectively, plus condition (5).

6. Contact Problems for One, Two and Three-dimensional Bodies

The previous work needs only trivial modification to be made applicable to contact problems involving bodies of different dimensions. There are many cases of considerable interest which fall into this category. For example, models consisting of a shell and a plate, or a solid and a plate, are useful for the study of head impact. The modifications necessary are essentially interpretative. An example illustrates this assertion.

Consider the frictionless Hertzian contact of a three-dimensional solid and a two-dimensional plate. Let $\mathcal{B}^1$ represent the solid and $\mathcal{B}^2$ the plate. In evaluating Π, the $\mathcal{B}^1$ part is as before while the $\mathcal{B}^2$ part would manifest the particular plate theory used. The contact term $\mathcal{K}$ (or $\mathcal{M}$) would be exactly as before. However note that $\mathcal{C}$ (or $\tilde{\mathcal{C}}$) is, in this case, also identifiable with part of the two-dimensional "volume" of the plate, rather than its boundary. Taking variations, everything is as before except that the term $\tau \delta x_3^2$ (or $-(\lambda)^2 \delta x_3^2$) contributes to the transverse momentum equation of the plate, rather than to its boundary conditions. The interpretation of τ (or $-(\lambda)^2$) is thus two-fold, i.e., it is the normal component of the traction vector with respect to $\mathcal{B}^1$, as before, and it is also the equivalent normal "body force" with respect to $\mathcal{B}^2$, manifested by the interaction with $\mathcal{B}^1$ (Fig. 7).

This interpretation is general, namely, for one and two-dimensional bodies the contact force is an equivalent "body force" which contributes to the momentum equations, rather than the boundary conditions. With this interpretation in mind, the construction of variational theorems, analogous to the ones constructed in Sections 2, 4 and 5, for the class of one, two and three-dimensional contact problems, is just a formal deductive exercise involving only appropriate definitions for Π.

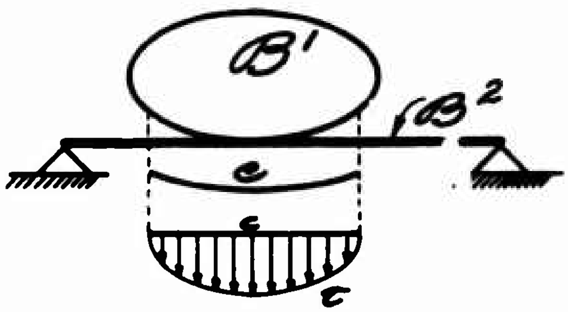


Figure 7

7. Impact

The previous sections deal with spatial aspects of contact problems. In this section we investigate temporal considerations, i.e., those phenomena which are unique to dynamic contact or impact. To manifest the problem encountered in such situations consider the following hypothetical situation. Assume that we are in the process of numerically solving some impact problem and suppose that it is discovered as we monitor the motion of the bodies that they impact somewhere in the time interval (t_1,t_2). At time t_1 we know the states of both bodies and we know that somewhere between t_1 and t_2 they have coalesced over a portion of their boundaries. Assume for the moment we know the geometry of the contact surface $\mathfrak{C}$. The question which arises then is what is the state of $\mathfrak{C}$ at time t_2, i.e., what are the velocity and traction vectors on $\mathfrak{C}$? It is necessary to know this information to carry forth the step forward time integration. The question though seems improperly posed without specifying considerable data about the nature of the impact. To get a handle on things, we will initially formulate a simple one-dimensional problem involving the impact of two elastic rods. Although this problem is trivial, it provides considerable insight into the general nature of impact of continuum bodies. Since we are interested in the state of $\mathfrak{C}$ (in this case a point) immediately after impact, whether the rods are finite or semi-infinite is immaterial.

Assume that the pre-impact states of the two bodies are given by the following data:

$$\mathcal{B}^{\alpha} \cdot \quad v_{-}^{\alpha}, \quad (\partial x/\partial X)_{-}^{\alpha}, \quad P_{-}^{\alpha} \quad ; \alpha = 1,2 \qquad (48)$$

At impact the rods coalesce at $\mathcal{E}$, and for some finite time interval thereafter (at least) $x \in \mathcal{E}$ is persistent. At the moment of impact shock waves begin to propagate in each body. The space-time picture is depicted in Fig. 8. As discussed in section 1, since $\mathcal{E}$ is material and x is persistent, we have

$$v \overset{def}{=} v^1_+ = v^2_+ \quad , \qquad P \overset{def}{=} T^1_+ = -T^2_+ \quad , \tag{49*}$$

for the post-impact state (t_+). In addition to (49), the well known shock conditions must hold across the wave fronts:

$$[v^\alpha] + U^\alpha[(\partial x/\partial X)^\alpha] = 0 \ ,$$
$$\rho_0^\alpha U^\alpha [v^\alpha] = [P^\alpha] \ , \tag{50}$$

where U^α is the material velocity of the shock in $\mathcal{B}^\alpha$, and [ ] is the wave-front jump operator which assigns to a function the difference in its values behind and in front of the wave, i.e., $[f(X,t)] = f(X^-,t) - f(X^+,t)$ where X is a material point denoting the location of the wave-front. As can be deduced from Fig. 8, the states into which the shocks initially propagate are the pre-impact states given by (48), and the state at $\mathcal{E}$, immediately after the shocks pass, is given by the post-impact state (49). These observations in conjunction with (50) yield,

$$v^\alpha_- - v + U^\alpha\{(\partial x/\partial X)^\alpha_- - (\partial x/\partial X)^\alpha_+\} = 0 \ ,$$
$$\rho_0^\alpha U^\alpha (v^\alpha_- - v) + P^\alpha_- - P = 0 \ . \tag{51**}$$

*For convenience we choose the initial state to be the pre-impact state, thus we need not distinguish between Cauchy and Piola tractions.

**A consistency condition for these equations is that $v^1_- - v^2_- > 0$. Otherwise the impact would not occur.

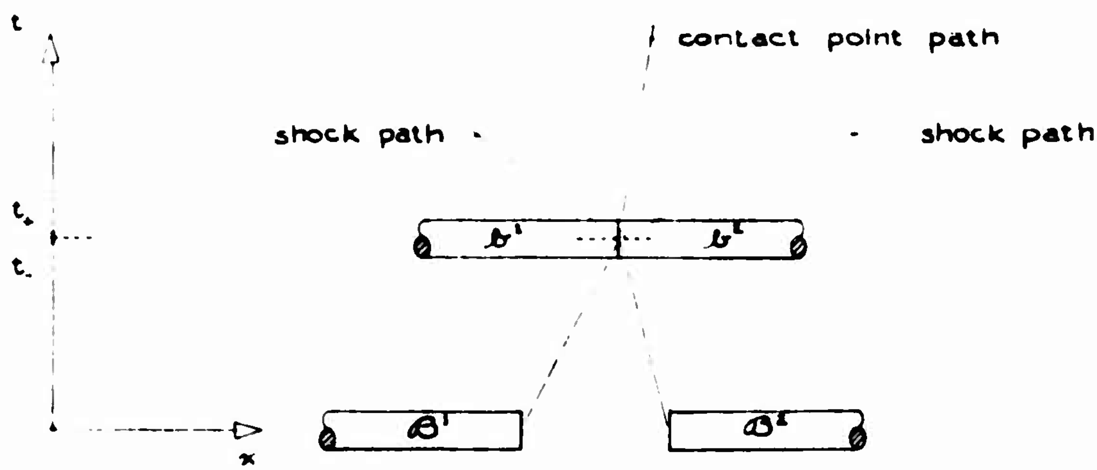


Figure 8

The four Eqs. (51) and constitutive equations relating P^α to $(\partial x/\partial X)^\alpha$ yield a formally deterministic system of six equations in the six unknowns $v, P, U^\alpha, (\partial x/\partial X)^\alpha$. Thus we see that the desired quantities v and P depend on the pre-impact states and material properties of both $\mathcal{B}^\alpha$. The precise form of this relationship depends upon the constitutive equations of the bodies. As a simple example, assume we have linear constitutive equations $P^\alpha = E^\alpha\{(\partial x/\partial X)^\alpha - 1\}$, E^α constant, and let the pre-impact state be given by

$$
\begin{aligned}
v_-^\alpha &= V^\alpha , \\
(\partial x/\partial X)_-^\alpha &= 1 , \\
P_-^\alpha &= 0 .
\end{aligned}
\tag{52}
$$

These conditions, when inserted in Eqs. (51), lead to:

$$
\begin{aligned}
v &= \frac{\rho_o^2 U^2 V^2 - \rho_o^1 U^1 V^1}{\rho_o^2 U^2 - \rho_o^1 U^1} , \\
P &= \frac{V^2 - V^1}{\left(\frac{U^2}{E^2} - \frac{U^1}{E^1}\right)} , \\
(U^\alpha)^2 &= E^\alpha / \rho_o^\alpha .
\end{aligned}
\tag{53}
$$

Note that the denominators in Eqs. $(53)_{1,2}$ present no problems since $\rho_o^\alpha > 0$, $E^\alpha > 0$ and $+1 = \operatorname{sgn} U^2 = -\operatorname{sgn} U^1$. This result is also appropriate whenever the intensity of the impact is small enough such that the non-linear constitutive equation can be replaced by its linear approximation about the pre-impact state. In this case E^α is a tangent modulus evaluated at the pre-impact strain $\{(\partial x/\partial X)_-^\alpha - 1\} = 0$. To further simplify, consider the case when both rods have identical properties (i.e., $\rho_o = \rho_o^\alpha$, $E = E^\alpha$, $\alpha = 1, 2$). Then

$$v = \frac{V^1 + V^2}{2} \quad ,$$

$$P = \rho U (V^2 - V^1)/2 \quad ,$$

$$(U)^2 = E/\rho_0 \ . \qquad (54)$$

In Eqs. (54) U is positive, and since consistency requires $V^1 - V^2 > 0$, P is compressive.

Thus for the one-dimensional case at least the problem of computing the post-impact state is easily achieved. The solution of (51) for the fully non-linear case can be automated as part of a numerical algorithm. Although this problem is trivial, it serves to indicate that the post-impact problem, the solution of which is essential in a numerical algorithm, is one of wave propagation.

In the analysis of higher dimensional bodies the solution of the post-impact problem becomes greatly complicated due to the geometric variety of impact conditions. However, considerable simplifications can be taken advantage of if one keeps in mind the nature of the discrete problem. For instance, if a certain portion of the boundaries of two bodies have coalesced in $\mathfrak{e}$, each interior point of $\mathfrak{e}$, at which the tangent plane is well defined, may be treated, to the first order, as a point on the mating surface of two impacting half-spaces. As long as time steps are kept small enough, the local behavior is well represented. The post-impact problem for the general case, analogous to (51), can be automated as part of the numerical algorithm, and for many simple cases can be solved explicitly.

With these notions in mind, let us return to the case of main interest in this report, namely three-dimensional continuum bodies. We shall consider only the case of a frictionless contact surface (Case II), and leave the solution of the post-impact problem for the no-slip case (Case I), which is more difficult, for future work. With the proper interpretations, the one-dimensional rod formulation (Eqs. (48-54)) suffices to completely characterize this case. This is so because no tangential motions or stresses may be communicated across a frictionless surface, and thus we need only consider the configuration of normal incidence. In this case the requisite constitutive functionals in (51) would be those relating P^{α}, the normal Piola stress, to the normal component of strain, holding all other components of strain fixed at the pre-impact values. For example, in the linear isotropic case, E^{α} (Young's modulus) in Eqs. (53,54) would be replaced by $\lambda^{\alpha}+2\mu^{\alpha}$ ($\lambda^{\alpha}, \mu^{\alpha}$ are the Lamé and shear moduli, respectively) and the propagation velocity would be that of dilatational waves.

PART II

A NUMERICAL SCHEME FOR ANALYSIS OF CONTACT-IMPACT PROBLEMS

8. Numerical Solution of Contact-Impact Problems

In performing numerical computations based on the above described variational formulation for contact-impact problems we have employed three distinct levels of approximation: (1) a spatial discretization of the bodies and contact surfaces, (2) a temporal discretization to determine the response of the discretized bodies, and (3) a numerical solution for the resulting system of nonlinear algebraic equations.

In the following sections we shall restrict our attention to the Hertzian contact problem described in Section 5. Significant numerical difficulties are encountered in the solution of impact problems; to complicate the problem further by introducing the additional steps necessary to determine the contact surface maps for the full kinematically nonlinear case is left for a future study. While this is a simple impact problem in terms of determining the contact surface and the full power of the preceding theory is neither necessary nor exploited in its solution, many of the features of the general problem are employed here.

9. Spatial Discretization of the Bodies and Contact Surface

The bodies $\mathcal{B}^1$ and $\mathcal{B}^2$ are discretized using standard finite element methods, (e.g., see [5]). In order to facilitate the computation of a discrete Hertzian contact surface the nodes of $\mathcal{B}^1$ are arranged so that they align with the nodes of $\mathcal{B}^2$. This is consistent with the notions of condition 3 of Section 5 and ensures that during determination of the approximation to the contact surface contiguous nodes of the two bodies will meet. Thus, the simulation of the contact surface is trivial. The development of a numerical model for Hertzian contact problems is based upon the form of Theorem II' which uses (47) for the integrand of $\mathcal{M}$. For numerical computations we introduce the displacement vector $\underset{\sim}{u}$ such that

$$\underset{\sim}{x} = \underset{\sim}{X} + \underset{\sim}{u} \quad . \tag{55}$$

For a compatible finite element displacement field the integrand of $\mathcal{M}$ can be approximated by taking $\eta^2(\underset{\sim}{z},t)$ as the product of $\mathcal{E}^2(t)$ and $\delta(\underset{\sim}{z}-\underset{\sim}{z}_i)$ (i.e., Dirac delta functions in space). This corresponds to taking η^2 as "concentrated nodal loads" which are the generalized forces of the contact pressure. With this discretization we can describe pseudo contact elements between each pair of candidate contact nodes. Let these nodes be denoted as $(\ )_i^\alpha$ and the generalized force as $(\mathcal{E}_i)^2$; then

$$\mathcal{M} = \int_0^t \sum_i (\mathcal{E}_i(t))^2 (u^2_{3i}(t) - u^1_{3i}(t) + X^2_{3i} - X^1_{3i})dt \tag{56}$$

where $\{i\}$ are the set of candidate contact nodes which span $\tilde{c}$; u^α_{3i} are the nodal displacements in the z_3 direction and X^α_{3i} are the nodal coordinates of the candidate contact nodes.*

*We assume here that 3 is the direction nominally normal to the contact surfaces, e.g., see Fig. 5.

Use of the finite element method in Theorem II' with $\mathcal{M}$ given by (56) produces a set of nonlinear second order ordinary differential equations which together with the impenetrability conditions define the discretized contact impact problem. These equations take the form:

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{u}} + \underset{\sim}{K}(\underset{\sim}{u}) = \underset{\sim}{R} \quad , \tag{57}$$

where $\underset{\sim}{M}$ is the usual finite element mass matrix, $\underset{\sim}{K}$ represents the elastic stiffness forces together with the contact terms, $\underset{\sim}{R}$ is the set of generalized forces resulting from boundary tractions and $\underset{\sim}{u}$ is the set of time dependent nodal displacements (which also include the $(\varepsilon_i)^2$). For inelastic materials Theorem II' can be extended by treating the first variation as a Galerkin method (principle of virtual work) and replacing the elastic constitutive model by more general theories, e.g., viscoelastic, elasto-plastic, viscoplastic, etc. In this case

$$\underset{\sim}{K}(\underset{\sim}{u}) \longrightarrow \underset{\sim}{K}(\underset{\sim}{u},\dot{\underset{\sim}{u}}) \tag{58}$$

in (57).

10. Temporal Discretization

A temporal discretization of the second order ordinary differential equations which result from a finite element spatial discretization of the contact-impact problem is accomplished herein by using the Newmark family of methods [6]. The Newmark family of methods is a one-step integration method with two free parameters which can be used to control stability and numerical damping. The method is essentially a difference method in time. The behavior of the method for linear elasto-dynamics problems is discussed in [6,7]. The algorithm is given by

$$\underset{\sim}{u}_{n+1} = \underset{\sim}{u}_{n} + \Delta t\, \dot{\underset{\sim}{u}}_{n} + \left(\tfrac{1}{2}-\beta\right)\Delta t^{2}\, \ddot{\underset{\sim}{u}}_{n} + \beta\, \Delta t^{2}\, \ddot{\underset{\sim}{u}}_{n+1} \ , \tag{59}$$

and $\dot{\underset{\sim}{u}}_{n+1} = \dot{\underset{\sim}{u}}_{n} + (1-\gamma)\Delta t\, \ddot{\underset{\sim}{u}}_{n} + \gamma\, \Delta t\, \ddot{\underset{\sim}{u}}_{n+1}$,

where $\underset{\sim}{u}_{n} = \underset{\sim}{u}(t_n)$, $\Delta t = t_{n+1} - t_n$, and β, γ are the two parameters. For linear problems $\gamma = .5 + \delta = .5$ produces no artificial viscosity and $\beta \geq \frac{1}{4}(1+\delta)^2$ produces unconditional stability (i.e., the method is stiffly stable). Such generalization is not possible for nonlinear problems and during solution it may be necessary to monitor the solution for any signs of instability. In (59) $\beta = 0$ produces an explicit method for $\underset{\sim}{u}_{n+1}$ and if $\underset{\sim}{M}$ is diagonal (lumped mass) with $\underset{\sim}{K}$ and $\underset{\sim}{R}$ independent of $\dot{\underset{\sim}{u}}$ the solution can be advanced without solving a large set of simultaneous equations; for all other cases the method is implicit and equations must be solved. Solution of $(59)_1$ for $\ddot{\underset{\sim}{u}}_{n+1}$ in terms of the solution at t_n and $\underset{\sim}{u}_{n+1}$ gives

$$\ddot{\underset{\sim}{u}}_{n+1} = \frac{1}{\beta\Delta t^{2}}\left(\underset{\sim}{u}_{n+1} - \underset{\sim}{u}_{n}\right) - \frac{1}{\beta\Delta t}\dot{\underset{\sim}{u}}_{n} - \left(\frac{1-2\beta}{2\beta}\right)\ddot{\underset{\sim}{u}}_{n} \tag{60}$$

which can also be used in $(59)_2$ to express the velocity in terms of the solution at t_n and $\underset{\sim}{u}_{n+1}$. Since in this process we divide by β and Δt it is no longer possible to consider zero β or zero time steps.

11. Solution of the Nonlinear Algebraic Problem

Use of the Newmark method in (57) (including (58)) yields the set of nonlinear algebraic equations:

$$\frac{1}{\beta\Delta t^2}\underset{\sim}{M}\,\underset{\sim}{u}_{n+1} + \underset{\sim}{K}(\underset{\sim}{u}_{n+1}, \underset{\sim}{u}_n, \dot{\underset{\sim}{u}}_n, \ddot{\underset{\sim}{u}}_n) = \underset{\sim}{R}_{n+1} + \underset{\sim}{M}\,\underset{\sim}{A}_n \quad , \tag{61}$$

where

$$\underset{\sim}{A}_n = \frac{1}{\beta\Delta t^2}\underset{\sim}{u}_n + \frac{1}{\beta\Delta t}\dot{\underset{\sim}{u}}_n + \left(\frac{1-2\beta}{2\beta}\right)\ddot{\underset{\sim}{u}}_n \quad .$$

A Newton-Raphson iterative solution to this set of equations can formally be constructed, giving:

$$\left(\frac{1}{\beta\Delta t^2}\underset{\sim}{M} + \partial_u\underset{\sim}{K} - \partial_u\underset{\sim}{R}\right)\Delta\underset{\sim}{u}^{(i)} = \underset{\sim}{R} - \underset{\sim}{K}(\underset{\sim}{u}^{(i)}_{n+1}, \underset{\sim}{u}_n, \dot{\underset{\sim}{u}}_n, \ddot{\underset{\sim}{u}}_n) - \underset{\sim}{M}\,\ddot{\underset{\sim}{u}}^{(i)}_{n+1} \quad , \tag{62}$$

where $\partial_u \underset{\sim}{R}$ is the effect of loads varying with the deformation and

$$(\partial_u \underset{\sim}{K})_{ij} = \partial K_i / \partial u_j \quad , \tag{63}$$

is the tangent stiffness matrix. The coefficient to $\Delta\underset{\sim}{u}^{(i)}$ is generally called the Jacobian matrix of the Newton-Raphson iteration. The solution is advanced by taking

$$\underset{\sim}{u}^{(i+1)}_{n+1} = \underset{\sim}{u}^{(i)}_{n+1} + \Delta\underset{\sim}{u}^{(i)} \quad , \tag{64}$$

and iterating until a norm of the solution satisfies

$$\|\Delta\underset{\sim}{u}^{(i)}\| \leq \epsilon\,\|\underset{\sim}{u}^{(i)}_{n+1}\| \quad , \tag{65}$$

where ϵ is some small positive error tolerance. In the work reported here the norm $\| \ \|$ is taken as the Euclidian norm

$$\| \underset{\sim}{x} \| = \left(\sum_i x_i^2 \right)^{1/2} , \tag{66}$$

and the load vector $\underset{\sim}{R}$ is assumed to be independent of $\underset{\sim}{u}$. For stable elastic materials the resulting tangent stiffness is then symmetric and positive definite, consequently, standard direct solution methods normally employed in the solution of linear finite element problems can be used. For inelastic materials or deformation dependent loads the tangent stiffness may be asymmetric. In these cases some special methods may be necessary to effect a solution.

12. Discretized Impact Conditions

In the previous numerical development $\tilde{c}$ has been defined by discrete points which correspond to nodes along the boundaries of $\mathcal{B}^1$ and $\mathcal{B}^2$. When, during the course of advancing the solution in time, any one of these points violates the impenetrability condition a re-solution must be obtained in which the $(\tilde{c})^2$ are now non-zero and the u_3^* satisfy the impenetrability condition. Some control and monitoring are required to effect this in a computer program. In addition to satisfying these conditions, the impact relations denoted in Section 7 must be invoked. In the present study these conditions are applied to the solution at the end of a time step in which points first go into contact. Accordingly we compute from (50)*

$$\dot{u}_+ = \frac{\rho_o^2 U^2 \dot{u}_-^2 - \rho_o^1 U^1 \dot{u}_-^1}{\rho_o^2 U^2 - \rho_o^1 U^1} , \qquad (67)$$

and assign this value to the appropriate node of $\mathcal{B}^1$ and $\mathcal{B}^2$.

To determine the solution vector $\underset{\sim}{u}$ at t_{n+1} we have solved the set of equations (61). As described above the shock conditions are then used to determine the value of the velocity at time t_{n+1} for all points which have come into contact during the time interval. In order to get a consistent solution at these points we must modify the accelerations and contact force to reflect the shock conditions. This is accomplished by re-solving the equilibrium conditions of $\mathcal{B}^1$ and $\mathcal{B}^2$ at point i. The expanded forms of the appropriate equations are:

*The $(\ )_-$ denotes a value which is computed before impact, whereas $(\ )_+$ denotes the value after impact.

$$M^1\ddot{u}^1_- + K^1(\underset{\sim}{u}) + (\varepsilon_i)^2_- = R^1 ,$$

(68)

and

$$M^2\ddot{u}^2_- + K^2(\underset{\sim}{u}) + (\varepsilon_i)^2_- = R^2 .$$

For nodes which have come into contact we must enforce the condition on acceleration

$$\ddot{u}^1_+ = \ddot{u}^2_+ = \ddot{u}_+ , \qquad (69)$$

and compute the contact force $(\varepsilon_i)^2{}_+$. The solution for these is obtained from

$$M^1\ddot{u}_+ + K^1(\underset{\sim}{u}) + (\varepsilon_i)^2_+ = R^1 ,$$

and

$$M^2\ddot{u}_+ + K^2(\underset{\sim}{u}) + (\varepsilon_i)^2_+ = R^2 .$$

These are two equations in two unknowns which can be solved for the $\ddot{u}_+$ and $(\varepsilon_i)^2_+$. If $K^\alpha(\underset{\sim}{u})$ is independent of velocity the stiffness forces and R^α will remain unchanged during the impact, hence we can solve the simpler problem

$$M^1\ddot{u}_+ + (\varepsilon_i)^2_+ = M^1\ddot{u}^1_- + (\varepsilon_i)^2_-$$

$$M^2\ddot{u}_+ - (\varepsilon_i)^2_- = M^2\ddot{u}^2_- - (\varepsilon_i)^2_-$$

whose solution is

$$\ddot{u}_+ = \frac{M^1\ddot{u}^1_- + M^2\ddot{u}^2_-}{M^1 + M^2} ,$$

and

$$2(\varepsilon_i)^2_+ = 2(\varepsilon_i)^2_- + M^1(\ddot{u}^1_- - \ddot{u}_+) - M^2(\ddot{u}^2_- - \ddot{u}_+) .$$

This completes the numerical specification of the solution at t_{n+1}; this solution process is now repeated for each of the succeeding time steps.

At this point it is important to compare the solution procedure for impact of a continuum discretized by a finite element method with the solution procedure for a physically discrete body, i.e., a body composed of mass points joined by massless elastic springs. Both problems may be described by algebraic equations of the form of (57). The impenetrability condition is also identical. The impact conditions, however, are different. For the discretized continuum the procedure is described above. The study of the impact of mass points is considered in elementary mechanics books, e.g. [8]. The impact of two mass points is described by impulsive motion such that at t_- the velocities of the two mass points are V_-^1 and V_-^2; after impact at time t_+, the two points have velocities V_+^1 and V_+^2. The two points will not in general stay in contact (i.e., $V_+^1 \neq V_+^2$) but will rebound. The conditions used to compute the V_+^1 and V_+^2 are:

Balance of Momentum*

$$M^1 \{V^1\} + M^2 \{V^2\} = 0 , \tag{71}$$

and use of an equation involving the "coefficient of restitution", e :

$$\frac{V_+^2 - V_+^1}{V_-^1 - V_-^2} = e . \tag{72}$$

For $e=1$ energy is conserved whereas for $e=0$ the points "stick" and energy is dissipated. We must comment in passing that (72) is the energy

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* $\{f(t)\} = f(t_+) - f(t_-)$.

equation in disguise. To see this we can write the jump conditions for energy as

$$\frac{1}{2}M^1\{(V^1)^2\} + \frac{1}{2}M^2\{(V^2)^2\} = \{\nu\} \quad . \tag{73}$$

The term $\{\nu\}$ can exist only if other energies are dissipated during the jump. We rewrite (73) by using

$$\frac{1}{2}\{(V^1)^2\} = [V^1]\langle V^1\rangle \quad ,$$

where

$$\langle V^1\rangle = \frac{1}{2}(V_+^1 + V_-^1) \quad . \tag{74}$$

Use of the momentum equation (71) then gives, after dividing by $M^1\{V^1\}$

$$\langle V^1\rangle - \langle V^2\rangle = \frac{\{\nu\}}{M^1\{V^1\}} \quad ,$$

or after recollecting terms and dividing by $(V_-^1 - V_-^2)$ we obtain:

$$\frac{V_+^2 - V_+^1}{V_-^1 - V_-^2} = 1 - \frac{\{\nu\}}{M^1\{V^1\}(V_-^1 - V_-^2)} \quad . \tag{75}$$

The significance of the coefficient of restitution then is associated with the right hand side of (75).

It is clear from the above developments that the numerical simulation of the discretized continuum and the physically discrete system involve two distinct methods for treating the impact conditions. It is imperative then to associate the correct method for the problem at hand. In the present study we are interested in the impact of continua, and in this case we shall employ the discrete shock condition to effect the solution. This a priori assumes that the response we are computing involves a time scale associated with wave propagation problems. Consequently, we cannot expect the computation procedure for advancing the solution in time to be accurate if we take time steps greatly in excess of transit times through each body. In this context it may be important to consider an "explicit" time integration procedure in future work. The stability restrictions may be too severe to make this feasible.

PART III

FEAP 74 - A COMPUTER PROGRAM FOR SOLUTION OF CONTACT-IMPACT PROBLEMS

13. Development of a Contact-Impact Model for FEAP

In order to incorporate an ability to compute solutions to contact-impact problems using a finite element method as described above it is necessary to have available a computer program which can solve the nonlinear equations of motion given by (61). The computer program FEAP is a general program to solve finite element problems. The program has a capability of solving both quasistatic and dynamic problems and can incorporate several types of elements simultaneously. The nonlinear capabilities required for the solution of contact-impact problems have been incorporated into FEAP and currently includes the user options (see Input Instructions):

(1) Selection of quasistatic or dynamic option: The dynamic option will integrate the equations of motion using the one-step Newmark method to advance the solution in time. Quasistatic analysis is accommodated by any one-step algorithm. The algorithm employed is incorporated into each element routine and thus id defined by the developer of each element. Impact problems require description of the contact surface and wave speeds.

(2) Selection of the nonlinear method to advance the solution: Options include:

(a) No iterations in each time step. Unbalanced forces at each time are added to the next time step.

(b) Iterations in each time step to achieve a balance of force within each time step. In this option the user can select to reform the Jacobian matrix for each iteration or only at the first iteration in each time step.

In the impact problems solved to date it has been necessary to use the general form of the Newton-Raphson algorithm. This includes a complete formating and factoring of the Jacobian matrix for each iteration of each time step in the analysis. If the method described herein is to become computationally effective improvements in the computer program are paramount. Undoubtedly the most important aspect in reducing computer times is to introduce a substructuring system so that the highly nonlinear equations in the vicinity of the contact surface can be isolated from the remainder of the bodies. This will normally involve only a small number of equations in the total system of (62). The solution of a large finite element problem will generally concentrate the computer solution time in the forming and factoring of the tangent stiffness matrix. The fewer times that it is necessary to perform this costly step the more efficient tne solution algorithm. Substructuring can be used then to restrict the part of the equations which must be formed and factored often, and thus greatly reduce the computer costs in analyzing impact problems.

The version of FEAP which can currently be used to analyze contact-impact problems includes, in addition to the nonlinear Newton-Raphson iterative algorithm, a new special contact-impact element and a new subroutine to describe impact surfaces and the discrete shock conditions described in Section 12. These are described in the following sections.

14. <u>Contact Element for Hertzian Contact</u>

The contact-impact element which has been developed is called ELMT05 and can be used along any coordinate direction. As developed it cannot be used along normals which are in non-coordinate directions. The development of the contact element assumes that within the framework of linear elasticity theory a node on ω^1 will impact on a node of ω^2. In using this contact element we shall assume that the contact surface on ω^2 is located at larger coordinate values than the contact surface of ω^1. The contact element is described by three nodes. Node 1 is associated with ω^1, Node 3 is associated with ω^2, and Node 2 is used as storage for the contact force $(\mathcal{E})^2$. The user can select the direction of contact motion by specifying the degree of freedom of the nodal unknowns to which the contact is to be measured; this must agree with the physical direction of the element (see Fig. 9). The degree of freedom for the contact element is specified during the MATERIAL data input and consists of a single card in I5 format. The element nodes are described along with all other elements according to Section 4 of the Input Instructions. The Node order as shown in Fig. 9 must be observed.

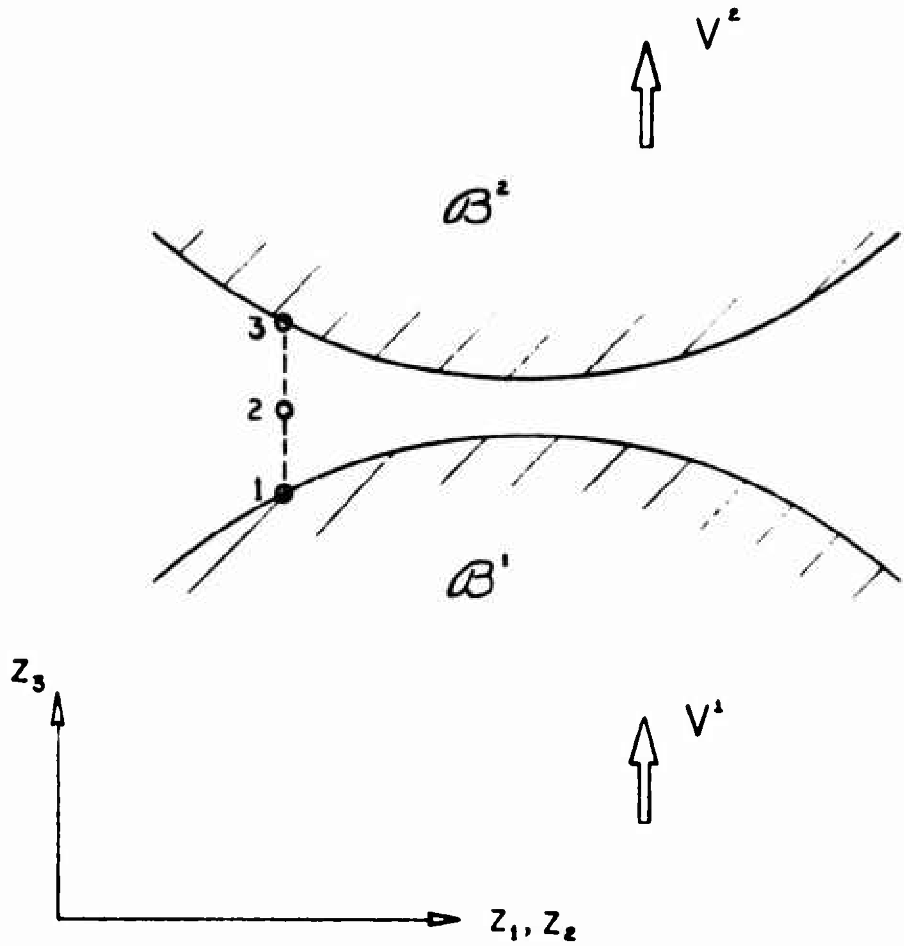

Figure 9

15. Impact Surface Description

The definition of the impact surface includes a list of all elements on the contact surface together with the degree of freedom describing the direction of contact motion (as described above). In addition, the product of mass density and wave velocity (always a positive number) for each body is input. This assumes, currently, that (1) each contact surface belongs to a linear material, and (2) the same material exists along all of the contact surface. This data need be prescribed only for impact problems, quasistatic contact problems do not require this data since no velocity or acceleration computations are performed in this class of problems. Data to be input for the impact surface is given in Table I.

Table I - Impact Surface Data

CARD 1) (6X,A6)

COL. 7 to 12		Must contain CONTAC

CARD 2) (2F10.0)

COL. 1 to 10	ρU	of body 1
COL. 11 to 20	ρU	of body 2

CARD 3) (I5)

COL. 1 to 5		NLIST, number of elements on contact surface

CARD 4) (2I5)

Repeat NLIST times

COL. 1 to 5		Contact element number
COL. 6 to 10		Degree of freedom of this contact element

16. Example Problems

Two example problems are included to illustrate the characteristics of the methodology and the associated computer program described above for Hertzian contact problems. The first problem is a quasistatic contact problem which is used to demonstrate the ability of the computer program to compute an evolving contact surface. The second problem will demonstrate the ability of the program to properly model the temporal response of an impact problem.

To model a problem in which a contact surface will change under different load levels we consider two beams with an initial parabolic curvature. A symmetric configuration is analyzed and the resulting finite element model is shown in Fig. 10. Each element is nominally one unit by one unit. The gap at the load end is initially 0.5 units. The material properties used are E = 500 and ν = 0. The load P is applied as shown and allowed to increase linearly in time. The problem then is to determine the contact surface at various load levels. In order to eliminate a singularity in the system of equations it was necessary to permanently attach the two nodes at the symmetry axis of the contact surface. All other nodes along the boundaries between the two bodies are assumed to be possible cont-ct points and contact elements are assigned between vertical nodal pairs. The load was varied from 0.2 to 0.8 in increments of 0.1 and the computed contact surface and forces were computed. These are given in Table II. The deformed shape at a load of 0.4 is also shown in Fig. 10 as a dotted form. The attached node at the center has influenced the solution at loads above 0.3 since the contact pressure there is tensile (negative). The force is small and should not greatly affect the actual contact region computed. As the load increases the

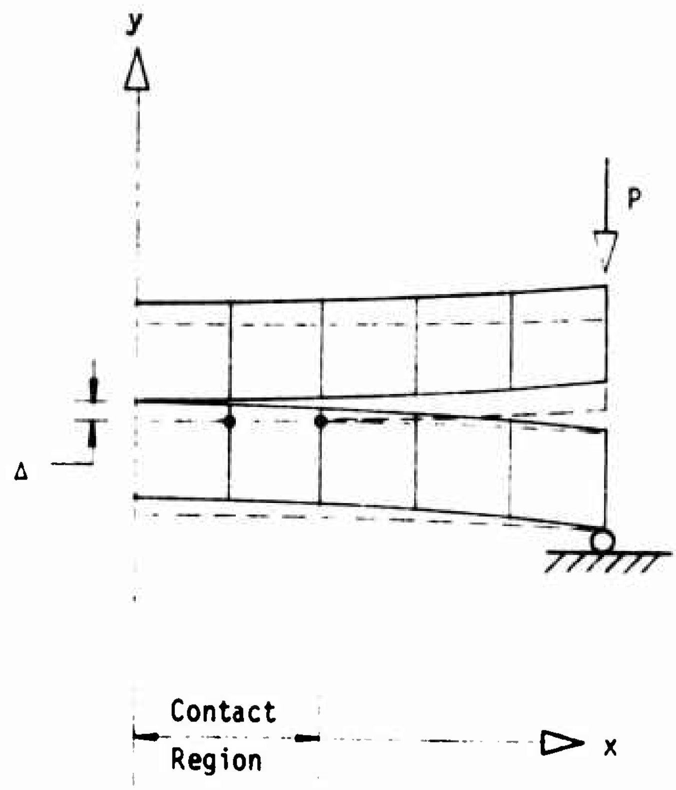

Figure 10

contact surface moves toward the load. This is conceptually correct since if the beams were modeled according to Euler-Bernoulli theory the contact force would be a point load which gradually moves from the center to the outer edge according to the relationship (using the above values for sizes and material properties)

$$X = 5\left(1 - \frac{1}{6P}\right) \quad .$$

This relation predicts that the contact point will be non-zero only after P exceeds 1/6. The finite element model is in qualitative agreement with this beam theory, but since shear deformations are included the finite element solution gives a distributed load on the contact surface. It is interesting to also note that the contact force over the center of the beams is zero, just as in the beam theory.

Table II - Contact Forces

LOAD	X-COORDINATE 0	1	2	3	4	5	BEAM THEORY-X
0.2	0.2	-	-	-	-	-	0.83
0.3	.07	.23	-	-	-	-	2.22
0.4	-.01	.07	.34	-	-	-	2.92
0.5	-0.00	.02	.23	.25	-	-	3.33
0.6	-.01	-	.09	.51	.01	-	3.61
0.7	-.01	-	.07	.45	.19	-	3.81
0.8	-.01	-	.05	.38	.38	-	3.96

This problem demonstrates that the computer program can model the evolution of a contact surface. Of particular importance is to note that as the load increases the program can both attach and detach a contact point. This is an essential requirement for the analysis of impact problems as is shown in the next problem.

As a simple example we consider the impact against a rigid wall of a finite, linear elastic rod traveling at constant velocity. The rod has a modulus of elasticity E of 100, and a mass density ρ of 0.1. The arrival velocity is taken to be 0.1 (units may be assigned in any convenient system). The rod is taken to be 10 units long and is divided into 10 elements plus one contact element as shown in Fig. 11. At time zero the rod is just arriving at the wall. The exact solution predicts a contact duration of 0.2 time units. This corresponds to the time required for a wave to travel from the contact point to the left end and back to the contact point at which time the rod will part from the wall. The problem was analyzed using FEAP with time steps of 0.01 unit (transit time across an element) and the rod remains in contact until time 0.20 units and has rebounded at time 0.21. Thus the program can predict accurately the contact duration of the rod. The finite element solution obtained is compared with the exact solution in Fig. 12. The agreement of stresses and contact force is good. The largest discrepancy exists in defining the shock front, which is "smeared" by the finite element method and ordinary differential equation solution method used here. This is the same type of solutions which are commonly obtained with numerical solutions of this type even without impact. Solutions such as the impact shocks generated are probably one of the most difficult responses to accurately calculate by a finite element method.

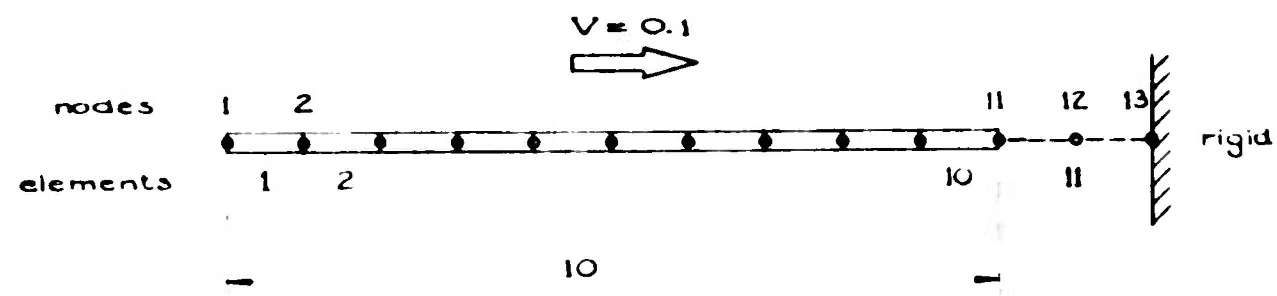

Figure 11

Comparison of Numerical Data with Exact Solution for Bar Impacting Rigid Wall

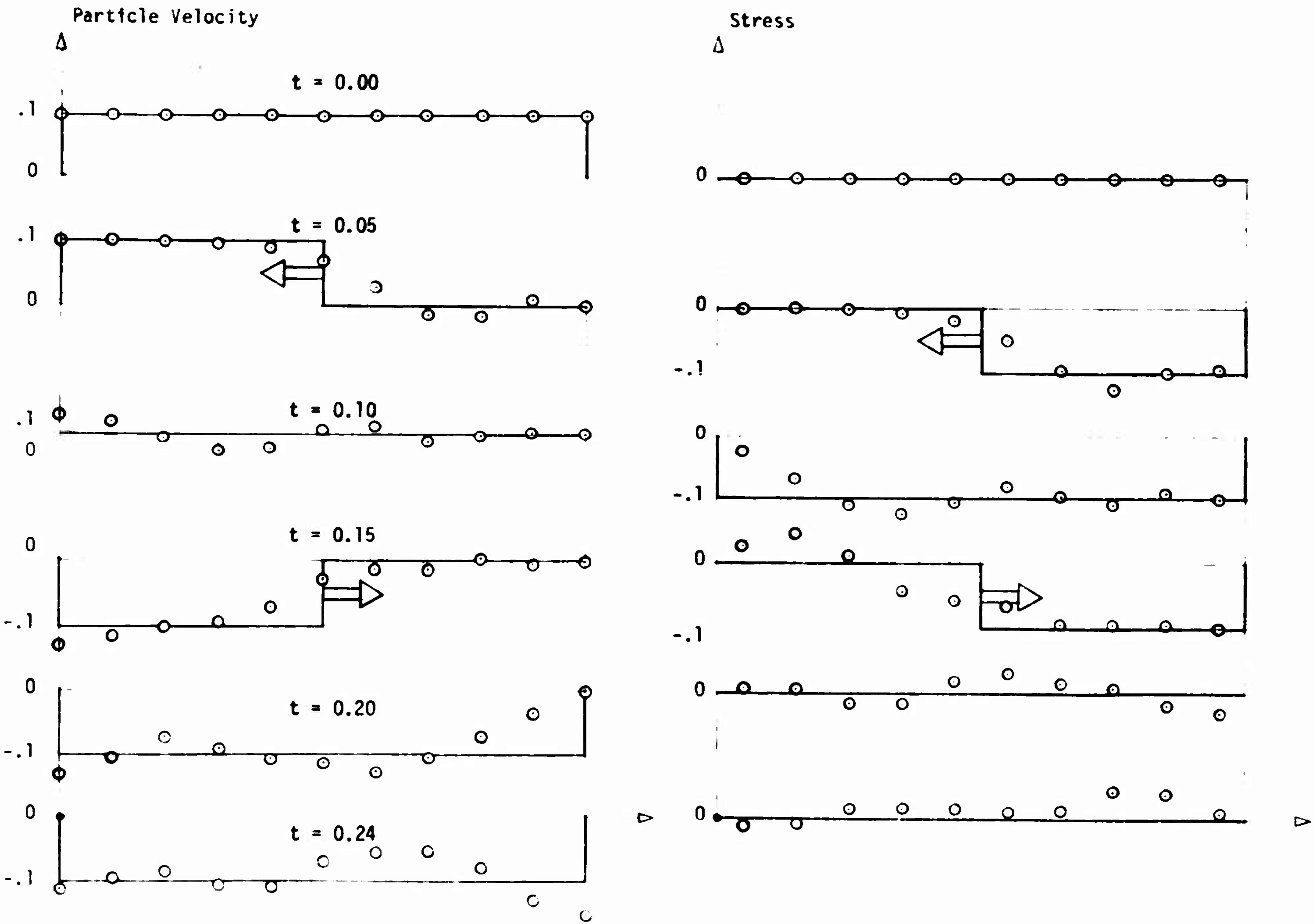

Figure 12

17. Closure and Recommendations for Future Work

In the preceding sections we have presented a theory for contact-impact problems together with the numerical development of a Hertzian contact-impact model. The computer program FEAP 74 has been modified to include the model and has successfully solved a contact problem and an impact problem. The work reported herein must be considered initiatory; the general theories and their numerical implementation have not been completed. The problem is of such a complicated nature and the literature existing prior to this study was so meager that we consider it fitting to document the work completed thus far.

We have attempted to qualify each stage of the development throughout the report, however, it may be fitting to reiterate future work which we consider to be essential for numerical models to be effective and efficient tools for predictive analyses.

(1) The restriction of Hertzian type contact must be removed. This involves the non-trivial task of finding appropriate numerical methods to handle the $\underset{\sim}{\chi}^{\alpha}$ maps.

(2) Improved methods for solving the set of nonlinear algebraic equations must be found. We have suggested two methods which should be considered: (a) Substructure the problem about the contact regime so that a more efficient forming and factoring of the tangent stiffness can be performed; and (b) Since the impact problem is a wave propagation problem an explicit time integration of the equations of motion should be explored. In complex situations the explicit integration method may have severe stability limitations which could make it unacceptable.

(3) Methods of utilizing the shock conditions need to be explored further. We have noted some peculiar anomalies when the bodies separate. These appear to be caused by a shock like separation phenomena.

(4) When the wave propagation property of the impact problem is ignored by taking time steps greatly in excess of the transit times in a body the computed response is meaningless. Under such situations the bodies rebound within a single time step. Currently the rebound velocity is much too large. When the shock conditions are used for a class of problems where the response desired is in the target instead of in the impactor, it may be expedient to take a large time step. Methods should be explored to accomplish this capability.

The above recommendations for future work should in no way minimize what has been accomplished by the present study. For the first time a contact-impact theory in the form of a variational problem has been presented in a general form. This formulation was motivated by the fact that numerical solutions would be obtained by a finite element method. In addition the necessary foundation for the numerical solution has been thought out and within this context a computer program has been developed for Hertzian contact-impact problems.

The implementations considered here have produced results which are hopeful signs for the eventual success of the more general impact problems.

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APPENDICES

A. Input Instructions for Contact/Impact Problems

In order to analyze contact/impact problems in FEAP, users must prepare the data for a time dependent analysis. This will include the following Data Type Identification Cards (see Section 1, Appendix B):

FEAP 74

MATERIAL

NODAL

ELEMENT

CONTACT (for impact problems only)

loadings

INITIAL CONDITIONS (if non-zero)

and

VISCOE (for quasi-static contact problems)

or

IMPLICIT (for impact problems)

In performing the necessary solution to (62) the full Newton-Raphson method must be employed; this is controlled by the data in col. 76-80 of the first card following the VISCOE or IMPLICIT card, and consists of a negative number (negative uses full Newton-Raphson iteration with the absolute value of the number giving the number of iterations to be performed before going to the next time step). As an example of the required input data Table A shows the input data used for the impact problem reported in Section 16.

14.19.14 OUTPUT

```
    FEAP74 * * ONE DIMENSIONAL BAR CONTACT PROBLEM
   1      X
   1      U
   3
  13 NODAL
   1                  0.
  11                  10.
  13          111111  10.
  11 ELEMENTS
   1    1              1  1
  1   2
  11    2
 11  12  13
   3 INITIAL CONDITIONS
    DISPLACEMENT
    VELOCITY
    ACCELERATIONS
   1    1              .1
  11    0              .1


   2 MATERIALS
   1 ELM09
  100.      1.         .01
   2 ELM05
   1
    CONTACT
   1        1.0        1.0E+30
  11    1

   1 IMPLICIT
  1.0       5    1    1   13    1   11    0    0   0.25                -3
  .01      25    1    1   13    1   11    0    0   0.25                -3
    STOP
  .01      10    1    1   13    9   11    0    0   0.25                -3
```

B. Input Instructions for FEAP 74

The input instructions for the description of a finite element mesh, together with the initial and boundary conditions, is described by subroutine MANUAL listed on the following pages. The input of the contact surface for impact problems is described in Table I of this report.

The description of material properties for the contact element is described in Section 14. For material properties for other elements in FEAP special input instructions must be supplied.

```
      SUBROUTINE MANUAL                                                 MAN   1C
C                                                                       MAN   2C
C**** USER INSTRUCTIONS AND INPUT FORMATS FOR FEAP-74  ****************  MAN   3C
C                                                                       MAN   4C
C                                                                       MAN   5C
C     FINITE ELEMENT ANALYSIS PROGRAM                                   MAN   6C
C                                                                       MAN   7C
C      ****** I N D E X ******                                          MAN   8C
C                                                                       MAN   9C
C  SECTION    WORD      C O N T E N T                                   MAN  10C
C                                                                       MAN  11C
C    1.0                DATA DIRECTORY WORDS                            MAN  12C
C    2.0      FEAP74    PROBLEM INITIATION AND CONTROL CARDS            MAN  13C
C    2.1      REMARK    REMARK CARD                                     MAN  14C
C    2.2      TITLE     TITLE CHANGE                                    MAN  15C
C    2.3      STOP      EXECUTION TERMINATION                           MAN  16C
C    3.0      MATERI    MATERIAL CHARACTERIZATION                       MAN  17C
C    4.0      NODAL     SEQUENTIAL NODAL GENERATION                     MAN  18C
C    4.1      GENERA    NON-SEQUENTIAL NODAL GENERATION                 MAN  19C
C    4.2      BOUNDA    BOUNDARY CODE INPUT                             MAN  20C
C    4.3      POLAR     POLAR TO CARTESIAN CONVERSION                   MAN  21C
C    5.0      ELEMEN    ELEMENT GENERATION                              MAN  22C
C    5.1      BLOCK     BLOCK MESH GENERATION                           MAN  23C
C    6.0      VECTOR    USER VECTOR INPUT                               MAN  24C
C    6.1      INITIA    INITIAL CONDITION                               MAN  25C
C    7.0      FORCE     GENERALIZED NODAL FORCE                         MAN  26C
C    7.1      BLOADS    SURFACE LOADS                                   MAN  27C
C    7.2      ELOADS    ELEMENT LOADS                                   MAN  28C
C    7.3                PROPORTIONAL LOADS                              MAN  29C
C    8.0      SOLVE     INITIATION OF STATIC SOLUTION                   MAN  30C
C             RESOLV    STATIC SOLUTION                                 MAN  31C
C    8.1      EXPLIC    EXPLICIT DYNAMIC INTEGRATION                    MAN  32C
C    8.2      IMPLIC    IMPLICIT INTEGRATION                            MAN  33C
C             VISCOE    IMPLICIT TIME INTEGRATION                       MAN  34C
C    8.3      MESH      MESH CHECK                                      MAN  34C
C             PLOT      PLOT MESH                                       MAN  34C
C    8.4      FOURIE    FOURIER SERIES HARMONICS                        MAN  35C
C    9.0                OUTPUT CONTROL                                  MAN  35C
C                                                                       MAN  36C
C        FEAP74 IS A GENERAL (F)INITE (E)LEMENT (A)NALYSIS (P)ROGRAM     MAN  37C
C        WHICH FURNISHES TO THE USER MESH INPUT/OUTPUT, ELEMENT ASSEMBLY MAN  38C
C        AND SOLUTION OF EQUATIONS (LINEAR, IMPLICIT AND EXPLICIT TIME   MAN  39C
C        DEPENDENT, NONLINEAR), PRESCRIBED GENERALIZED NODAL FORCES,     MAN  40C
C        PRESCRIBED NODAL AND ELEMENT DATA, AND OUTPUT OF THE            MAN  41C
C        GENERALIZED DISPLACEMENTS AND FORCES.  ELEMENT MATRICES FOR TWO MAN  42C
C        AND THREE DIMENSIONAL LINEAR ELASTICITY, SHELLS, PLATES, AND    MAN  43C
C        FIELD (LAPLACE EQUATION) PROBLEMS ARE AVAILABLE.  ALTERNATIVELY MAN  44C
C        USERS MAY SUPPLY THEIR OWN ELEMENT LIBRARY BY PROVIDING A       MAN  45C
C        SUBROUTINE CALLED ELMTNN.  WHERE NN IS A TWO DIGIT NUMBER       MAN  46C
C        (01-10), IDENTIFYING THE ELEMENT SUBROUTINE. EACH ELEMENT       MAN  47C
C        SUBROUTINE HAS AT LEAST FOUR BASIC FUNCTIONS WHICH ARE          MAN  48C
C        DELINEATED BY A SWITCHING PARAMETER. ISW. IN THE SUBROUTINE.    MAN  49C
C                                                                       MAN  50C
```

```
C        ELMTNN(N.MA.NDIM.NDF.NEL.NEL1.NSTF.NSIZV.NVEC.MCT.DM.D.XYZ.          MAN  51C
C        IX.F.FORCE.ESTIF.U.VECT.ISW)                                          MAN  52C
C                                                                              MAN  53C
C                       N IS ELEMENT NUMBER.                                   MAN  54C
C                       MA IS THE MATERIAL NUMBER.                             MAN  55C
C                       NDIM IS SPATIAL DIMENSION. 1.2.OR 3.                   MAN  56C
C                       NDF IS NUMBER OF DEGREES OF FREEDOM PER NODE.          MAN  57C
C                       NEL IS THE NUMBER OF EXTERNAL NODES PER ELEMENT        MAN  58C
C                       NEL1 IS DIMENSION OF ELEMENT PROPERTY ARRAY.           MAN  59C
C                       NSTF IS THE SIZE OF THE ELEMENT STIFFNESS.             MAN  60C
C                       NSIZV IS THE SIZE OF UTILITY VECTORS.                  MAN  61C
C                       NVEC IS THE NUMBER OF UTILITY VECTORS.                 MAN  62C
C                       MCT IS A PRINTER LINE COUNTER.                         MAN  63C
C                       DM IS A PARAMETER FOR MATERIAL IDENTIFICATION.         MAN  64C
C                       D(L.1) IS MATERIAL PROPERTY MATRIX (63 CELLS).         MAN  65C
C                       XYZ(NDIM.1) ARE NODAL COORDINATES.                     MAN  66C
C                       IX(NEL1.1) ARE ELEMENT  PROPERTIES. NODES. ETC.        MAN  67C
C                       F(NDF.1) ARE NODAL GENERALIZED FORCES.                 MAN  68C
C                       FORCE(NSTF.2) IS ELEMENT FORCE VECTOR TO BE            MAN  69C
C                       COMPUTED. COLUMN 2 IS LUMPED MASS                      MAN  70C
C                       ESTIF(NSTF.NSTF) IS ELEMENT MATRIX TO BE               MAN  71C
C                       COMPUTED.                                              MAN  72C
C                       VECT(NSIZV.1) ARE PRESCRIBED NODAL OR ELEMENT          MAN  73C
C                       QUANTITIES. TEMPERATURES ETC.                          MAN  74C
C                       U(NDF.1) IS SOLUTION VECTOR.                           MAN  75C
C                       ISW IS SWITCHING PARAMETER.                            MAN  76C
C                                                                              MAN  77C
C                           ISW=1. ** MATERIAL CHARACTERIZATION**              MAN  78C
C                           ISW=2. ** CHECK ELEMENT FOR POSITIVE AREA *        MAN  79C
C                           ISW=3. ** ELEMENT STIFFNESS AND                    MAN  80C
C                                     LOAD COMPUTATION**                       MAN  80C
C                           ISW=4. ** ELEMENT STRESSES AND PRINTOUT**          MAN  81C
C                           ISW=5. ** ELEMENT LOAD COMPUTATION ONLY**          MAN  82C
C                           ISW=6. ** NONLINEAR GENERALIZED FORCES**           MAN  83C
C                           OTHER ISW MAY BE USED FOR SPECIAL PURPOSES.        MAN  84C
C                                                                              MAN  85C
C        USERS CAN GENERATE SURFACE LOADINGS BY PROVIDING SLDNN                MAN  86C
C        SUBROUTINES (WHERE NN IS A TWO DIGIT NUMBER BETWEEN 01 AND 05)        MAN  87C
C        THAT SPECIFY THE LOAD ROUTINE.  THE SUBROUTINE IS ACCESSED BY         MAN  88C
C        THE CALL TO                                                           MAN  89C
C                                                                              MAN  90C
C        SLDNN(NDIM.NDF.NDP.NPRES.IPRES.PP.XYZ.FS)                             MAN  91C
C                                                                              MAN  92C
C                       WHERE IN ADDITION TO QUANTITIES DEFINED ABOVE          MAN  93C
C                       FOR ELMTNN.                                            MAN  94C
C                       NDP IS THE DIMENSION OF LOADED SURFACE                 MAN  95C
C                       NPRES IS NUMBER OF LOADED NODES (MAX 8)                MAN  96C
C                       IPRES(8) ARE NODE NUMBERS OF LOADED NODES.             MAN  97C
C                       PP(8) ARE LOAD VALUES AT CORRESPONDING IPRES           MAN  98C
C                           NODES.                                             MAN  99C
C                       FS(6.8) ARE THE COMPUTED GENERALIZED (NODAL)           MAN 100C
C                           FORCES FOR EACH DEGREE OF FREEDOM AT EACH          MAN 101C
C                           IPRES NODE.                                        MAN 102C
C                                                                              MAN 103C
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C        SEE SECTION 7.1 FOR DATA INPUT DETAILS.                              MAN1040
C                                                                             MAN1050
C                                                                             MAN1060
C        INTEGRATION TABLE IS ACCESSED BY THE CALL                            MAN1070
C                                                                             MAN1080
C          CALL INTEGL(LIM,NC1,NDIM,LINT,STUW)                                MAN1090
C                                                                             MAN1100
C                       STUW(4,M) INTEGRATION POINTS AND WEIGHTS.             MAN1110
C                       **NOTE** M MUST BE SET EXPLICITLY AND BE LARGER       MAN1120
C                       THAN OR EQUAL TO LINT.                                MAN1130
C                       LINT - RETURNS WITH NUMBER INTEGRATION POINTS.        MAN1140
C                       NC1 = 0  RETURNS GAUSS POINTS AND WEIGHTS IN          MAN1150
C                            STUW.                                            MAN1160
C                            LIM = 1 TO 5 IS NUMBER OF GAUSS POINTS/DIR-      MAN1170
C                            ECTION.                                          MAN1180
C                       NC1 = 1  RETURNS A SPECIAL 3-D GAUSS FORMULA.         MAN1190
C                            SET LIM = 1 FOR 6 PT. CUBIC ACCURACY             MAN1200
C                            SET LIM = 2 FOR 14 PT. QUINTIC ACCURACY.         MAN1210
C                       NC1 = 2  RETURNS TRIANGULAR INTEGRATION FORMULA       MAN1220
C                            SET LIM = 1 FOR 1 PT. LINEAR ACCURACY.           MAN1230
C                            SET LIM = 2 FOR 3 PT. QUADRATIC ACCURACY.        MAN1240
C                            SET LIM = 3 FOR 7 PT. QUARTIC ACCURACY.          MAN1250
C                                                                             MAN1260
C                                                                             MAN1270
C      1.) DATA TYPE IDENTIFICATION CARDS (I5,1X,12A6).                       MAN1280
C                                                                             MAN1290
C        EACH DATA SEGMENT IS PRECEDED BY A CARD WHICH IDENTIFIES THE         MAN1300
C        TYPE OF DATA AND LIMITS ON THE AMOUNT OF DATA WHICH IMMEDIATELY      MAN1310
C        FOLLOWS THE CARD.  EXCEPT AS NOTED THE DATA SEGMENTS MAY APPEAR      MAN1320
C        IN ANY ORDER.  THE IDENTITY CARDS MAY ALSO AID THE USER IN           MAN1330
C        INTERPRETTING THE INPUT DATA CARDS.  AS SUPPLIED THERE ARE           MAN1340
C        TWENTY-FIVE DIFFERENT DATA IDENTIFICATION CARDS.  THESE ARE          MAN1350
C                                                                             MAN1360
C        COL  7 TO 12       IDENTITY(RESTRICTIONS)                            MAN1370
C                                                                             MAN1380
C        FEAP74             START OF EACH PROBLEM (MUST PRECEDE ALL OTHER     MAN1390
C                           DATA).                                            MAN1400
C        TITLE              CHANGE OUTPUT PAGE HEADINGS                       MAN1410
C        REMARK             COMMENTS ON OUTPUT                                MAN1420
C        MATERI             MATERIAL CHARACTERIZATION.                        MAN1430
C        NODAL              NODAL CARDS                                       MAN1440
C        POLAR              POLAR CONVERSION. (PRECEDE BY NODAL,GENERA. OR    MAN1450
C                           BLOCK )                                           MAN1460
C        ELEMEN             ELEMENT CONNECTION CARDS.                         MAN1470
C        GENERA             GENERATE NODES IN A LINEAR PATH BY ANY INCREMNT   MAN1480
C        BLOCK              GENERATE ALL MESH DATA (BOTH NODAL AND ELEMENT)   MAN1490
C                           FOR A 2,(OR 3) DIMENSIONAL REGION WHOSE BOUNDARY  MAN1500
C                           MAY BE DEFINED BY 4(8)OR 8(20)COLLOCATED POINTS   MAN1510
C        BOUNDA             BOUNDARY CODE PRESCRIPTION (PRECEDE BY NODAL OR   MAN1520
C                           GENERATE OR BLOCK)                                MAN1530
C        VECTOR             PRESCRIBED NODAL OR ELEMENT DATA (PRECEDE BY      MAN1540
C                           NODAL, OR GENERA AND ELEMEN, OR BLOCK)            MAN1550
C        FORCE              NODAL GENERALIZED FORCES (PRECEDE BY NODAL OR     MAN1560
C                           GENERA OR BLOCK).                                 MAN1570
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C        BLOADS          SURFACE LOADINGS (SAME AS FORCE).                     MAN1580
C        ELOADS          ELEMENT LOADINGS (SAME AS FORCE).                     MAN1590
C        MESH            CHECK CONSISTENCY OF MESH ONLY (SAME AS SOLVE)        MAN1600
C        PLOT            PLOT MESH (SAME AS SOLVE)                             MAN1610
C        INITIA          INITIAL CONDITION PRESCRIPTION FOR DYNAMIC            MAN1620
C                        ANALYSIS (PRECEDE BY NODAL, GENERA OR BLOCK)          MAN1630
C        SOLVE           COMPLETE FORMULATION AND SOLUTION FROM ELEMENTS       MAN1640
C                        (PRECEDE BY MATERI,NODAL OR GENERA, AND ELEMEN        MAN1650
C                        OR PRECEDE BY MATERI AND BLOCK )                      MAN1660
C        RESOLV          USE PREVIOUS PROBLEM DESCRIPTION WITH NEW LOAD        MAN1670
C                        ONLY (PRECEDE BY SOLVE AND NEW LOADING CARDS).        MAN1680
C        EXPLIC          DYNAMIC SOLUTION BY EXPLICIT INTEGRATION. (SAME       MAN1690
C                        AS SOLVE)                                             MAN1700
C        IMPLIC          IMPLICIT INTEGRATION OF DYNAMIC PROBLEMS              MAN1710
C                        (PRECEDE BY SAME DATA AS FOR SOLVE)                   MAN1720
C        VISCOE          QUASI-STATIC LINEAR VISCOELASTIC INTEGRATION          MAN1730
C                        (PRECEDE BY SAME DATA AS FOR SOLVE)                   MAN1740
C        FOURIE          FOURIER COMPOSITION (SAME AS SOLVE)                   MAN1750
C        ADDUP           ACCUMULATE FOURIER SOLUTION (AFTER FOURIE)            MAN1760
C        STOP            NORMAL EXIT (MUST FOLLOW ALL DATA)                    MAN1770
C                                                                              MAN1780
C        ***NOTE*** EACH IDENTIFIER IS PUNCHED STARTING IN COL 7 (LEFT         MAN1790
C            JUSTIFIED).                                                       MAN1800
C                                                                              MAN1810
C      EXCESS CARDS MAY EXIST BETWEEN EACH SECTION OF DATA. HOWEVER.           MAN1820
C      THE DATA TO BE USED MUST IMMEDIATELY FOLLOW THE TYPE CARD AND           MAN1830
C      MUST BE IN PROPER ORDER.  NO PARTICULAR ORDER OF THE TYPE               MAN1840
C      CARDS IS NECESSARY EXCEPT THAT THE FEAP74 CARD MUST ALWAYS BE           MAN1850
C      THE FIRST CARD IN EACH SET OF DATA. AND RESTRICTIONS MUST BE            MAN1860
C      OBSERVED.                                                               MAN1870
C                                                                              MAN1880
C                                                                              MAN1890
C     2.) PROBLEM INITIATION AND CONTROL CARDS                                 MAN1900
C                                                                              MAN1910
C        CARD 1. (6X,12A6)                                                     MAN1920
C                                                                              MAN1930
C        COL  7 TO 12    MUST CONTAIN WORD FEAP74                              MAN1940
C        COL 13 TO 78    OUTPUT PAGE HEADER                                    MAN1950
C                                                                              MAN1960
C        CARD 2. (I5,1X,3A6)                                                   MAN1970
C                                                                              MAN1980
C        COL  1 TO 5     NDIM - SPATIAL DIMENSION OF PROBLEM (1 TO 3)          MAN1990
C        COL  7 TO 12    NAMES TO BE PRINTED AS OUTPUT HEADERS TO              MAN2000
C        COL 13 TO 18    COORDINATES - IF BLANK SET TO 1,2,3 AS NEEDED.        MAN2010
C        COL 19 TO 24                                                          MAN2020
C                                                                              MAN2030
C        CARD 3. (I5,1X,6A6)                                                   MAN2040
C                                                                              MAN2050
C        COL  1 TO  5    NDF - NUMBER OF UNKNOWNS PER NODE (1 TO 6)            MAN2060
C        COL  7 TO 12    NAMES TO BE PRINTED AS OUTPUT HEADERS OF THE          MAN2070
C        COL 13 TO 18    GENERALIZED DISPLACEMENTS AND FORCES - IF             MAN2080
C          ........      BLANK SET TO 1,2,3,4,5,6 AS NECESSARY                 MAN2090
C        COL 37 TO 42                                                          MAN2100
C                                                                              MAN2110
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C        CARD 4. (6I5.5F10.0)                                                  MAN2120
C                                                                              MAN2130
C        COL  1 TO 5     NEN - MAXIMUM NUMBER OF NODES CONNECTED TO ANY        MAN2140
C                        ELEMENT (1 TO 20).                                    MAN2150
C        COL  6 TO 10    NEXTRA - INCREASES ELEMENT MATRIX SIZE FROM           MAN2160
C                        NDF*NEN TO NDF*NEN + NEXTRA                           MAN2170
C        COL 11 TO 15    IREC - COMPUTE GENERALIZED FORCE CHECK IF             MAN2180
C                        NONZERO (FOR TIME INVARIANT ANALYSIS ONLY)            MAN2190
C        COL 16 TO 20    MBAN - MAXIMUM EXPECTED BANDWIDTH. DEFAULT IS         MAN2200
C                        SET TO 100. USED AS AN ERROR CHECK TO PREVENT         MAN2210
C                        RUNNING WITH AN OBVIOUS ERROR.                        MAN2220
C        COL 21 TO 25    IBUF - BUFFER SIZE FOR STORAGE OF HISTORY             MAN2230
C                        EFFECTS IN TIME DEPENDENT ANALYSIS. DEFAULT IS        MAN2240
C                        IBUF = ISZDT/20                                       MAN2250
C        COL 26 TO 30    NC1  - USER INTEGER CONSTANT                          MAN2260
C        COL 31 TO 40    CON1 - USER DEFINED CONSTANT                          MAN2270
C        COL 41 TO 50    CON2 - USER DEFINED CONSTANT                          MAN2280
C        COL 51 TO 60    CON3(1) - USER DEFINED CONSTANT                       MAN2290
C        COL 61 TO 70    CON3(2) - USER DEFINED CONSTANT                       MAN2300
C        COL 71 TO 80    CON3(3) - USER DEFINED CONSTANT                       MAN2310
C                                                                              MAN2320
C      2.1)  *REMARK* USER COMMENTS ON OUTPUT. (6X.12A6)                       MAN2330
C                                                                              MAN2340
C        SUBSEQUENT CARDS                                                      MAN2350
C                                                                              MAN2360
C        COL  7 TO 12    MUST CONTAIN REMARK                                   MAN2370
C        COL 13 TO 78    STATEMENTS TO BE OUTPUT . USE AS MANY REMARK          MAN2380
C                        CARDS AS DESIRED. INSERT BEFORE ANY TYPE CARD.        MAN2390
C                                                                              MAN2400
C      2.2) TITLE CHANGE ON OUTPUT (6X.12A6)                                   MAN2410
C                                                                              MAN2420
C        COL  7 TO 12    MUST CONTAIN TITLE                                    MAN2430
C        COL 13 TO 78    NEW TITLE DESCRIPTOR                                  MAN2440
C                                                                              MAN2450
C      2.3) EXECUTION TERMINATION (6X.A4)                                      MAN2460
C                                                                              MAN2470
C        COL  7 TO 10    MUST CONTAIN STOP. INSERT AFTER LAST PROBLEM.         MAN2480
C                                                                              MAN2490
C                                                                              MAN2500
C     3.) MATERIAL CHARACTERIZATION (I5.1X.12A6)                               MAN2510
C                                                                              MAN2520
C        COL  1 TO  5    NUMMAT - NUMBER OF DIFFERENT MATERIAL CHARACT-        MAN2530
C                        ERIZATIONS TO FOLLOW.                                 MAN2540
C        COL  7 TO 12    MUST CONTAIN WORD MATERI                              MAN2550
C                                                                              MAN2560
C        THE FOLLOWING CARDS ARE SUPPLIED FOR EACH MATERIAL TO BE CHARAC       MAN2570
C        TERIZED (MUST BE EXACTLY NUMMAT SETS OF CARDS)                        MAN2580
C                                                                              MAN2590
C        CARD 1.  ELEMENT SELECTOR CARD (I5.1X.A5.11A6)                        MAN2600
C                                                                              MAN2610
C        COL  1 TO  5    MATERIAL NUMBER (1 TO NUMMAT                          MAN2620
C        COL  7 TO 11    ELMNN - WHERE NN IS NUMBER OF ELEMENT CLASS (01       MAN2630
C                        TO 10) TO WHICH THE CHARACTERIZATION BELONGS.         MAN2640
C        COL 12 TO 77    ALPHANUMERIC INFORMATION TO BE OUTPUT.                MAN2650
```

```
C
C        CARD 2.1. ETC. ** USER DEFINED FOR EACH ELEMENT TYPE PROVIDED.
C
C        EXCESS BLANK CARDS ARE PERMISSIBLE BETWEEN EACH MATERIAL SET.
C
C     4.) NODAL CARDS (I5.1X.A6)
C
C        COL  1 TO  5     NUMNP - NUMBER OF NODAL POINTS
C        COL  7 TO 12     MUST CONTAIN NODAL
C
C        SUBSEQUENT CARDS  LAST NODAL CARD MUST NOT BE GENERATED.
C        (I5.I15.3F10.0)
C
C        COL  1 TO  5     NODE NUMBER
C        COL 15           1 IF 1 DISPLACEMENT IS SPECIFIED
C        COL 16           1 IF 2 DISPLACEMENT IS SPECIFIED
C        COL 17           1 IF 3 DISPLACEMENT IS SPECIFIED
C        COL 18           1 IF 4 DISPLACEMENT IS SPECIFIED
C        COL 19           1 IF 5 DISPLACEMENT IS SPECIFIED
C        COL 20           1 IF 6 DISPLACEMENT IS SPECIFIED
C        COL 21 TO 30     1 COORDINATE VALUE
C        COL 31 TO 40     2 COORDINATE VALUE * AS REQUIRED
C        COL 41 TO 50     3 COORDINATE VALUE
C
C     NODAL CARDS MUST BE IN ORDER. MISSING NODES ARE INTERPOLATED
C     LINEARLY FROM INPUT NODES. IF SUCCEEDING CARDS HAVE IDENTICAL
C     BOUNDARY CODES. THIS BOUNDARY CODE WILL BE ASSIGNED TO THE INTER-
C     VENING NODES. IN ALL OTHER CASES THE BOUNDARY CODE IS SET TO ZERO
C     *TERMINATE ON NODE NUMNP OR A BLANK CARD*
C
C     4.1) NON SEQUENTIAL NODAL GENERATOR OPTION. (I5.1X.A6)
C
C        COL  1 TO  5     NUMBER OF NODAL POINTS
C        COL  7 TO 12     MUST CONTAIN GENERA
C
C        SUBSEQUENT CARDS (2I5.I10.3F10.0)
C
C        COL  1 TO  5     NODE-NUMBER
C        COL  6 TO 10     NODE-NUMBER-INCREMENT WHICH WILL BE SUCESSIVELY
C                         ADDED TO NODE-NUMBER UNTIL SUM IS GREATER THAN
C                         NODE-NUMBER ON FOLLOWING CARD (ALGEBRAIC).
C        COL 15 TO 20     BOUNDARY CODE. SAME AS INPUT FOR NODAL.
C                         IF SUCCEEDING CARDS HAVE IDENTICAL BOUNDARY
C                         CODES. THIS BOUNDARY CODE WILL BE ASSIGNED TO
C                         THE INTERVENING NODES. IN ALL OTHER CASES THE
C                         BOUNDARY CODE IS SET TO ZERO.
C        COL 21 TO 30     1 COORDINATE VALUE  *
C        COL 31 TO 40     2 COORDINATE VALUE  * AS REQUIRED *
C        COL 41 TO 50     3 COORDINATE VALUE  *
C
C        *TERMINATE WITH BLANK CARD *
C
C     4.2  BOUNDARY CODE PATCH-UP OPTION. (6X.A6)
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C        COL   7 TO 12    MUST CONTAIN BOUNDA                                 MAN318C
C                                                                             MAN319C
C        SUBSEQUENT CARDS. (8I5)                                              MAN320C
C                                                                             MAN321C
C        COL   1 TO  5    N. NODE NUMBER TO HAVE REDEFINED BOUNDARY CODE.     MAN322C
C        COL   6 TO 10    NX. GENERATOR INCREMENT TO BE ADDED ALGEBRAI-       MAN323C
C                         CALLY TO N. UNTIL SUM EXCEEDS (MAX OR MIN) THE      MAN324C
C                         N OF THE FOLLOWING CARD.                            MAN325C
C        COL  11 TO 15    IBC(I), (I=1,2...NDF) CODE FOR SPECIFYING FORCE     MAN326C
C        COL  16 TO 20    OR DISPLACEMENT BOUNDARY CONDITIONS.                MAN327C
C        COL  ...                                                             MAN328C
C                              IBC(I) .EQ. 0. FORCE SPECIFIED.                MAN329C
C                              IBC(I) .GT. 0. DISPLACEMENT SPECIFIED, NO      MAN330C
C                              INTERVENING GENERATION.                        MAN331C
C                              IBC(I) .LT. 0. DISPLACEMENT SPECIFIED.         MAN332C
C                              GENERATE BETWEEN MISSING NODES IN ALGEBRAIC    MAN333C
C                              INCREMENTS OF NX.                              MAN334C
C                                                                             MAN335C
C        * TERMINATE WITH A BLANK CARD. *                                     MAN336C
C                                                                             MAN337C
C       4.3) POLAR OR CYLINDRICAL COORDINATE CONVERSION TO CARTESIAN          MAN338C
C            COORDINATES (6X.A6)                                              MAN339C
C                                                                             MAN340C
C        COL   7 TO 12    MUST CONTAIN POLAR     (LEFT JUSTIFIED)             MAN341C
C                                                                             MAN342C
C        CARD 1.   (3I5.5X.2F10.0)                                            MAN343C
C                                                                             MAN344C
C        COL   1 TO  5    N1. FIRST NODE TO BE CONVERTED                      MAN345C
C        COL   6 TO 10    N2. LAST  NODE TO BE CONVERTED                      MAN346C
C        COL  11 TO 15    N3. INCREMENT ADDED (ALGEBRAICALLY). N1 TO N2       MAN347C
C        COL  21 TO 30    X0. ORIGIN OF POLAR X-COORDINATE                    MAN348C
C        COL  31 TO 40    Y0. ORIGIN OF POLAR Y-COORDINATE                    MAN349C
C                                                                             MAN350C
C      5.) ELEMENT CARDS (I5.1X.A6)                                           MAN351C
C                                                                             MAN352C
C        COL   1 TO  5    NUMEL - NUMBER OF ELEMENTS                          MAN353C
C        COL   7 TO 12    MUST CONTAIN ELEMEN                                 MAN354C
C                                                                             MAN355C
C        SUBSEQUENT CARDS (4I5.20I3/20I4)                                     MAN356C
C                                                                             MAN357C
C        CARD 1.                                                              MAN358C
C                                                                             MAN359C
C        COL   1 TO  5    ELEMENT NUMBER                                      MAN360C
C        COL   6 TO 10    MATERIAL NUMBER                                     MAN361C
C        COL  11 TO 15    NUMBER OF SUBSEQUENT ELEMENTS USING SAME            MAN362C
C                         STIFFNESS MATRIX * SAVES RECOMPUTATION OF           MAN363C
C                         SIMILAR MATRICES.  ELEMENT MUST ALSO HAVE           MAN364C
C                         SAME ELEMENT FORCE VECTOR * IF THESE ARE            MAN365C
C                         IN THE STIFFNESS SUBROUTINE *                       MAN366C
C        COL  16 TO 20    PRINT ELEMENT MATRIX IF NONZERO.                    MAN367C
C        COL  21 TO 2?    INDC(1) ELEMENT INCREMENT ARRAY ON NODE 1.          MAN368C
C        COL  2? TO 26    INDC(2)   * IF NOT INPUT IS SET AUTOMATICALLY       MAN369C
C                         UP TO                                               MAN370C
C        COL  ?? TO ??    I ? ??    FOR SERENDIPITY ELEMENTS * SEE REPORT     MAN371C
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C                                                                               MAN372C
C        CARD 2.                                                                MAN373C
C                                                                               MAN374C
C        COL  1 TO  4    NODE 1                                                 MAN375C
C        COL  5 TO  8    NODE 2                                                 MAN376C
C        COL  9 TO 12    NODE 3          IX(NEL1,1) ARRAY                       MAN377C
C                             CONTINUE IN I4 FORMAT TO A MAXIMUM                MAN378C
C        COL 77 TO 80    NODE 20                                                MAN379C
C                                                                               MAN380C
C        ELEMENT CARDS MUST BE IN ORDER. MISSING ELEMENTS ARE GENERATED         MAN382C
C        BY INCREMENTING NODES.                                                 MAN383C
C        LAST ELEMENT CARD MUST NOT BE GENERATED.                               MAN384C
C        * TERMINATE ON ELEMENT NUMEL OR A BLANK CARD *                         MAN385C
C                                                                               MAN386C
C      5.1) BLOCK GENERATOR. GENERATES ALL MESH DATA.(6X,A6)                    MAN387C
C                                                                               MAN388C
C        COL  1 TO  5    NUMBER OF NODAL POINTS TO BE GENERATED.                MAN389C
C        COL  7 TO 12    MUST CONTAIN BLOCK                                     MAN390C
C                                                                               MAN391C
C        SUBSEQUENT CARDS (10I5/6(4X,I6)/(10X,3F10.0))                          MAN392C
C                                                                               MAN393C
C        CARD 1.                                                                MAN394C
C                                                                               MAN395C
C        COL  1 TO  5    NN. NUMBER OF POINTS REQUIRED TO DEFINE                MAN396C
C                        BOUNDARY OF REGION. FOR 2-DIM., NN=4 OR 8.             MAN397C
C                        FOR 3-DIM., NN= 8 OR 20.                               MAN398C
C        COL  6 TO 10    NUMBER OF ELEMENTS IN R-DIRECTION.                     MAN399C
C        COL 11 TO 15    NUMBER OF ELEMENTS IN S-DIRECTION.                     MAN400C
C        COL 16 TO 20    NUMBER OF ELEMENTS IN T-DIRECTION.                     MAN401C
C        COL 21 TO 25    INITIAL NODE NUMBER. DEFAULT = 1.                      MAN402C
C        COL 26 TO 30    INITIAL ELEMENT NUMBER. DEFAULT = 1.                   MAN403C
C        COL 31 TO 35    MATERIAL NUMBER OVER REGION. DEFAULT = 1               MAN404C
C        COL 36 TO 40    BOUNDARY CODE SKIP. A NON-ZERO ENTRY WILL OMIT         MAN405C
C                        SETTING ALL INTERIOR BOUNDARY CODES TO ZERO.           MAN406C
C        COL 41 TO 45    IREUSE - REUSE ELEMENT STIFFNESS OPTION - USES         MAN407C
C                        EACH ELEMENT STIFFNESS IREUSE TIMES BEFORE             MAN408C
C                        GENERATING A NEW ELEMENT STIFFNESS MATRIX.             MAN409C
C        COL 46 TO 50    ELEMENT STIFFNESS-PRINT. A NON-ZERO ENTRY  WILL        MAN410C
C                        CAUSE PRINT-OUT OF FIRST ELEMENT.                      MAN411C
C        COL 51 TO 55    IMSH - IF NONZERO PRINT COMPUTED NODES.                MAN412C
C        COL 56 TO 60    IELM - IF NONZERO PRINT COMPUTED ELEMENTS.             MAN413C
C                                                                               MAN414C
C        CARD 2.  (BOUNDARY CODE AS DEFINED IN NODAL CARD.)                     MAN415C
C                                                                               MAN416C
C        COL  5 TO 10    BOUNDARY CODE OVER FACE -R.                            MAN417C
C        COL 15 TO 20    BOUNDARY CODE OVER FACE +R.                            MAN418C
C        COL 25 TO 30    BOUNDARY CODE OVER FACE -S.                            MAN419C
C        COL 35 TO 40    BOUNDARY CODE OVER FACE +S.                            MAN420C
C        COL 45 TO 50    BOUNDARY CODE OVER FACE -T.                            MAN421C
C        COL 55 TO 60    BOUNDARY CODE OVER FACE +T.                            MAN422C
C                                                                               MAN423C
C                                                                               MAN424C
C                                                                               MAN425C
C                                                                               MAN426C
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C         COL 21 TO 30     2-COORDINATE OF BOUNDARY-DEFINING-POINT.                  MAN427C
C         COL 31 TO 40     3-COORDINATE OF BOUNDARY-DEFINING-POINT.                  MAN428C
C                                                                                    MAN429C
C         NOTES. 1. BLOCK GENERATES ONLY 4 PT. QUADRILATERALS OR 8 PT.               MAN430C
C                   BRICKS.                                                          MAN431C
C                2. INPUT OF CARDS 3.) FOLLOW ORDER RULES FOR ELEMENT                MAN432C
C                   INPUT (SEE SECTION 5.)).                                         MAN433C
C                3. R-S-T ARE LOCAL COORDINATES.                                     MAN433C
C                   I.E. (-1 .LE. R.S.T .LE. 1). WHERE R IS DIRECTED                 MAN433C
C                   FROM NODE 1 TO 2. S IS IN PLANE OF FIRST THREE NODES             MAN433C
C                   AND T IS NORMAL TO R-S PLANE.                                    
C                                                                                    MAN435C
C     6.)  VECTOR CARDS. I.E. USER DEFINED INPUT (I5.1X.A6)                          MAN436C
C                                                                                    MAN437C
C         COL  1 TO  5     NVEC. NUMBER OF DIFFERENT VECTORS (7 MAX)                 MAN438C
C         COL  7 TO 12     MUST CONTAIN VECTOR                                       MAN439C
C                                                                                    MAN440C
C         SUBSEQUENT CARDS                                                           MAN441C
C                                                                                    MAN442C
C         CARD 1.  (2I5)                                                             MAN443C
C                                                                                    MAN444C
C         COL  1 TO  5     NSIZV. VECTOR LENGTH.COMMON TO ALL NVEC VECTORS           MAN445C
C         COL  6 TO 10     IPICK. CODED PARAMETER.                                   MAN446C
C                                                                                    MAN447C
C                          IPICK = 0. VECTORS ASSOCIATED WITH NODES                  MAN448C
C                          IPICK = 1. VECTORS ASSOCIATED WITH DEG.FREEDOM            MAN449C
C                          IPICK = 2. VECTORS ASSOCIATED WITH ELEMENTS               MAN450C
C                                                                                    MAN451C
C         CARD 2.  (6X. 2A6) REPEAT NVEC TIMES                                       MAN452C
C                                                                                    MAN453C
C         COL  7 TO 18      DESCRIPTIVE TITLE FOR VECTOR                             MAN454C
C                                                                                    MAN455C
C         CARD 3.  (2I5.7F10.0)                                                      MAN456C
C                                                                                    MAN457C
C         COL  1 TO  5     POSITION NUMBER OF VECTOR ELEMENT. 1 TO NSIZV             MAN458C
C         COL  6 TO 10     GENERATOR INCREMENT                                       MAN459C
C         COL 11 TO 20     VECTOR ELEMENT VALUE OF VECTOR 1                          MAN460C
C         COL 21 TO 30     VECTOR ELEMENT VALUE OF VECTOR 2                          MAN461C
C         COL  .......     AS REQUIRED FOR NVEC VECTORS                              MAN462C
C                                                                                    MAN463C
C         LINEAR INTERPOLATION IS PERFORMED ON ALL VECTORS BETWEEN                   MAN464C
C         NON-CONSECUTIVE POSITION NUMBERS SPECIFIED IN COL 1 TO 5                   MAN465C
C         IF INCREMENT IS NONZERO.                                                   MAN466C
C         IF DESCRIPTIVE TITLES OF ALL VECTORS ARE BLANK CARDS.                      MAN467C
C         PRINTING OF THE VECTOR VALUES IS SUPPRESSED.                               MAN468C
C                                                                                    MAN469C
C         * TERMINATE ON BLANK CARD *                                                MAN470C
C                                                                                    MAN471C
C     6.)   INITIAL CONDITIONS FOR TIME DEPENDENT ANALYSIS.                          MAN472C
C                                                                                    MAN473C
C         COL  1 TO  5     NITIA. NUMBER OF INITIAL CONDITION VECTORS                MAN474C
C         COL  7 TO 1.     MUST CONTAIN INITIA                                       MAN475C
C                                                                                    MAN476C
C         SUBSEQUENT CARDS                                                           MAN477C
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C                                                                              MAN478C
C       CG     (6X,2A6) REPEAT NICD TIMES                                      MAN479C
C                                                                              MAN480C
C       COL  7 TO 19    DESCRIPTIVE TITLE FOR INITIAL CONDITIONS               MAN481C
C                                                                              MAN482C
C       CARD 2.  (2I5,7F10.0)                                                  MAN483C
C                                                                              MAN484C
C       COL  1 TO  5    POSITION NUMBER, AS IN VECTOR CARDS FOR IPICK=1        MAN485C
C       COL  6 TO 10    GENERATOR INCREMENT                                    MAN486C
C       COL 11 TO 20    INITIAL CONDITION 1                                    MAN487C
C       COL 21 TO 30    INITIAL CONDITION 2                                    MAN488C
C       COL ........    AS REQUIRED FOR NICD INITIAL CONDITIONS                MAN489C
C                                                                              MAN490C
C       INTERPOLATION BETWEEN INPUT VALUES AS DESCRIBED IN VECTOR INPUT        MAN491C
C       **** NOTE **** IF MISSING THE INITIAL CONDITIONS ARE SET ZERO          MAN492C
C                                                                              MAN493C
C     7.) FORCE CARDS (I5,1X,A6)                                               MAN494C
C                                                                              MAN495C
C       COL  1 TO  5    LAST NODE TO WHICH A FORCE IS TO BE SPECIFIED          MAN496C
C       COL  7 TO 12    MUST CONTAIN FORCE                                     MAN497C
C                                                                              MAN498C
C       SUBSEQUENT CARDS (I5,5X,6F10.0)                                        MAN499C
C                                                                              MAN500C
C       THE FOLLOWING VALUES ARE EACH INTERPRETTED AS FORCES IF THE            MAN501C
C       CORRESPONDING BOUNDARY CODE IS A 0 *ZERO* AND AS A DISPLACEMENT        MAN502C
C       IF THE CORRESPONDING BOUNDARY CODE IS 1 *ONE*.                         MAN503C
C                                                                              MAN504C
C       COL  1 TO  5    NODE TO WHICH FORCE OR DISPLACEMENT IS APPLIED         MAN505C
C       COL 11 TO 20    VALUE OF 1 FORCE DISPLACEMENT                          MAN506C
C       COL 21 TO 30    VALUE OF 2 FORCE DISPLACEMENT    * AS *                MAN507C
C       COL 31 TO 40    VALUE OF 3 FORCE DISPLACEMENT * REQUIRED *             MAN508C
C       COL 41 TO 50    VALUE OF 4 FORCE DISPLACEMENT                          MAN509C
C       COL 51 TO 60    VALUE OF 5 FORCE DISPLACEMENT                          MAN510C
C       COL 61 TO 70    VALUE OF 6 FORCE DISPLACEMENT                          MAN511C
C                                                                              MAN512C
C                                                                              MAN513C
C      7.1) SURFACE LOAD CARDS (I5,1X,A6)                                      MAN514C
C                                                                              MAN515C
C       COL  1 TO  5    NUMBER OF LOADED FACE CARDS                            MAN516C
C       COL  7 TO 12    MUST CONTAIN BLOADS                                    MAN517C
C                                                                              MAN518C
C       CARD 1. (I5,1X,A5,I4,8I5,8I3)                                          MAN519C
C                                                                              MAN520C
C       COL  1 TO  5    DIMENSION OF LOADING SURFACE. (1 OR 2).                MAN521C
C       COL  7 TO 11    SLD(NN). ALPHA-NUMERIC NAME OF SURFACE LOADING         MAN522C
C                       SUBROUTINE (NN IS BETWEEN 1 AND 5)                     MAN523C
C       COL 12 TO 15    NET.NUMBER OF ADDITIONAL ELEMENT LOAD                  MAN524C
C                       SURFACES TO BE GENERATED FROM CURRENT MODEL.           MAN525C
C       COL 16 TO 20    IFFES(N). NODE NUMBERS DEFINING LOADING SURFACE        MAN526C
C       COL 21 TO 25    OF CURRENT ELEMENT.                                    MAN527C
C       COL ...         (IDENTIF FROM 2 TO 8 AS REQUIRED)                      MAN528C
C       COL [illegible]                                                        MAN[illegible]
C       COL [illegible] INCREMENT VALUE ADDED TO IFFES(N) TO [illegible]       MAN[illegible]
C       [illegible]     [illegible] NUMBERS OF A SURFACE [illegible]           MAN[illegible]
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C        COL ...          (IDENTIFY FROM 2 TO 8 AS REQUIRED)                   MAN532C
C                                                                               MAN533C
C        CARD 2. (8F10.0)                                                       MAN534C
C                                                                               MAN535C
C        COL  1 TO 80     LOAD AT NODES GIVEN ON PREVIOUS CARD *                MAN536C
C                         MUST CORRESPOND IN SEQUENCE TO THE NODE NUMBERS       MAN537C
C                                                                               MAN538C
C                                                                               MAN539C
C                                                                               MAN540C
C      7.2) ELEMENT LOAD CARDS (I5,1X,A6 )                                      MAN541C
C                                                                               MAN542C
C        COL  1 TO  5     NLD. NUMBER OF ELEMENT LOAD CARDS.                    MAN543C
C        COL  7 TO 11     MUST CONTAIN ELOADS                                   MAN544C
C                                                                               MAN545C
C        SUBSEQUENT CARDS (I5,1X,A5,I4,I5,6F10.0)                               MAN546C
C                                                                               MAN547C
C        COL  1 TO  5     IEL. INITIAL ELEMENT OF A GENERATED SEQUENCE.         MAN548C
C        COL  7 TO 11     ELM(NN). ALPHA-NUMERIC NAME OF ELEMENT                MAN549C
C                         SUBROUTINE WHERE ELEMENT LOADS ARE COMPUTED.          MAN550C
C                         USED AS CHECK TO INSURE IEL, ETC. ARE PROPER          MAN551C
C                         ELEMENTS.                                             MAN552C
C        COL 12 TO 15     INC. INCREMENT NUMBER IN A GENERATED SEQUENCE,        MAN553C
C                         (DEFAULT = 1).                                        MAN554C
C        COL 16 TO 20     JEL. TERMINAL ELEMENT NUMBER IN A GENERATED           MAN555C
C                         SEQUENCE. IF JEL = 0, ONLY IEL IS COUNTED.            MAN556C
C        COL 21 TO 80     PR-USER DEFINED VALUES FOR DETERMINING BODY           MAN557C
C                         LOADS IN THE ISW=5 PORTION OF ELM(NN).                MAN558C
C                                                                               MAN559C
C        NOTE. USER MUST PROVIDE COMPUTATION OF LOADS IN ELMTNN.                MAN560C
C        PR IS TRANSFERED TO SUBROUTINE ELMTNN IN THE U VECTOR,                 MAN561C
C        WHEN ISW =5, ONLY.                                                     MAN562C
C                                                                               MAN563C
C      7.3) PROPORTIONAL LOADS FOR TIME DEPENDENT ANALYSIS                      MAN564C
C                                                                               MAN565C
C        TRANSFER TO THIS OPTION OCCURS ONLY FOR TIME ANALYSES.                 MAN566C
C                                                                               MAN567C
C        ONE CARD FOR EACH PROPORTIONAL LOAD REQUIRED                           MAN568C
C                                                                               MAN569C
C        COL  1 TO  5     PROPORTIONAL LOAD TYPE, 1,2 OR 3                      MAN570C
C        COL  6 TO 10     K. TABLE CONSTANT                                     MAN571C
C        COL 11 TO 20     TMIN. SMALLEST TIME LOADING IS VALID                  MAN572C
C        COL 21 TO 30     TMAX. LARGEST TIME LOADING IS VALID                   MAN573C
C        COL 31 TO 40     A0                                                    MAN574C
C        COL 41 TO 50     A1                                                    MAN575C
C        COL 51 TO 60     A2                                                    MAN576C
C        COL 61 TO 70     A3                                                    MAN577C
C        COL 71 TO 80     A4                                                    MAN578C
C                                                                               MAN579C
C        LOAD TYPE 1.   T = TIME                                                MAN580C
C                                                                               MAN581C
C            PROP = A0 + A1*T + A2*T*T + A3*T*T*T + A4*T*T*T*T                  MAN582C
C                                                                               MAN583C
C        LOAD TYPE 2.                                                           MAN584C
C                                                                               MAN585C
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C           PROP = A0*(SIN(A1*T))**K + A2*(COS(A3*T))**M + A5                   MAN5860
C                                                                                MAN5870
C        LOAD TYPE 3.                                                            MAN5880
C                                                                                MAN5890
C           PROP - USER DEFINED FUNCTION FROM SUBROUTINE                         MAN5900
C                  EXPRLD(PROP.T.A)  WHERE A IS AN ARRAY WITH                    MAN5910
C                  INFORMATION FOR COLUMNS 6-80 OF DATA CARDS.                   MAN5920
C                                                                                MAN5930
C        **NOTE** PROPORTIONAL LOADS CAN BE ACCUMULATED FROM DIFFERENT           MAN5940
C                 TYPES AT THE SAME TIME.                                        MAN5950
C                                                                                MAN5960
C     8.0 INITIATION OF TIME INDEPENDENT SOLUTION (I5.1X,A6)                     MAN5970
C                                                                                MAN5980
C        COL  1 TO  5     IOUT. OUTPUT CONTROL CODE.                             MAN5990
C                                                                                MAN6000
C                         IOUT .EQ. 0. ALL STRESSES AND DISP. PRINTED            MAN6010
C                         IOUT .NE. 0. SELECTED PRINTOUT. MORE DATA INPUT        MAN6020
C                         SEE SECTION 9 FOR DATA PREPARATION.                    MAN6030
C                                                                                MAN6040
C        COL  7 TO 12     MUST CONTAIN  SOLVE  *INDICATES ALL DATA INPUT*        MAN6050
C                         COMPLETE FORMULATION AND SOLUTION OF EQUATIONS.        MAN6060
C        COL  7 TO 12     MUST CONTAIN  RESOLV TO OBTAIN SUBSEQUENT              MAN6070
C                         SOLUTIONS WHERE BOUNDARY CODES DO NOT CHANGE           MAN6080
C                         AND ALL PRESCRIBED DISPLACEMENTS ARE ZERO.             MAN6090
C                                                                                MAN6100
C      8.1  INITIATION OF DYNAMIC SOLUTION BY EXPLICIT INTEGRATION.              MAN6110
C                                                                                MAN6120
C        COL  1 TO  5     IPRT.  OUTPUT CONTROL FOR DISPLACEMENT AND             MAN6130
C                         STRESS PRINTOUT. SEE SECT. 9 FOR DATA INPUT.           MAN6140
C                         IOUT = 1 - MIN(1.IPRT)                                 MAN6140
C                         IF IOUT .NE. 0. THE SPATIAL CONTROL DATA               MAN6150
C                         COMES AT THE END OF THE DYNAMIC SEGMENT.               MAN6160
C        COL  7 TO 12     MUST CONTAIN EXPLIC                                    MAN6170
C                                                                                MAN6180
C        SUBSEQUENT CARDS (2I5.2F10.0.2I5)                                       MAN6190
C                                                                                MAN6200
C        COL  1 TO  5     NUMBER OF TIME STEPS                                   MAN6210
C        COL  6 TO 10     PRINT INTERVAL                                         MAN6220
C        COL 11 TO 20     TIME INCREMENT                                         MAN6230
C        COL 21 TO 30     NEWMARK DELTA-DAMPING TERM (GAMMA - .5)                MAN6240
C        COL 31 TO 35     NUMBER OF TIME EVOLUTION ELEMENT VARIABLE PLOTS        MAN6250
C        COL 36 TO 40     NPROP. NUMBER OF PROPORTIONAL LOADS TO BE INPUT        MAN6260
C        COL 41 TO 45     NFORC. LAST NODE ON WHICH A FORCE IS CHANGED           MAN6270
C                         DURING EACH TIME STEP.                                 MAN6280
C        COL 46 TO 50     KKK. STABILITY CHECK OVERRIDE ** CAUTION USE           MAN6290
C                         ONLY WHEN A BETTER ESTIMATE OF THE STABLE TIME         MAN6300
C                         STEP IS AVAILABLE THAN CAN BE PERFORMED BY CODE        MAN6310
C                         KKK ZERO. USES INTERNAL STABILITY CHECK.               MAN6320
C                         KKK NONZERO. DISREGARDS STABILITY CHECK.               MAN6330
C        COL 51 TO 55     NONL.   0 FOR LINEAR                                   MAN633
C                                 1 FOR NON LINEAR                               MAN633C
C        COL 56 TO 60     LPF.    0 PRINTS LOADS                                 MAN633C
C                                 1 SUPPRESS PRINT                               MAN633C
C                                                                                MAN634C
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C        SUBSEQUENT CARDS (3I5) ONE FOR EACH STRESS PLOT.                                MAN635C
C                                                                                        MAN636C
C        COL  1 TO  5    ELEMENT NUMBER CONTAINING STRESS TO BE PLOTTED.                 MAN637C
C        COL  6 TO 10    LOCAL COORDINATE POINT CODE, 1 TO 9, AS                         MAN638C
C                        PATTERNED AFTER, COL 11 TO COL 19, IN SECT. 9.                  MAN639C
C        COL 11 TO 15    PLOT COMPONENT CODE, 1 TO 6 FOR SIGMA(I,J),                     MAN640C
C                        I.E., SIGMA(1,1)=1, SIGMA(1,2)=2, SIGMA(1,3)=3,                 MAN641C
C                        SIGMA(2,2) = 4, SIGMA(2,3) = 5, SIGMA(3,3) = 6.                 MAN642C
C                                                                                        MAN643C
C        IF(NPROP.NE.0) READ PROPORTIONAL LOAD CARDS, SEE SECT. 7.3                      MAN644C
C                                                                                        MAN645C
C        IF(NFORC.NE.0) READ FORCE CARDS AT EACH TIME STEP. IF OUTPUT IS                 MAN646C
C        LIMITED BY IOUT NONZERO, THE FIRST FORCE CARD SET PRECEDES                      MAN647C
C        OUTPUT CARDS AND THE REMAINDER FOLLOW THE OUTPUT CARDS NO BLANK                 MAN648C
C        CARDS MAY BE USED BETWEEN SETS OF CARDS OTHER THAN THE USUAL                    MAN649C
C        BLANK TERMINATOR CARD FOR FORCE INPUT CARDS.                                    MAN650C
C                                                                                        MAN651C
C        IF(IOUT.NE.0)   DATA FOR SPATIAL PRINTOUT CONTROL, SEE SECT.9.                  MAN652C
C                                                                                        MAN653C
C                                                                                        MAN654C
C       SPECIAL COMMENTS FOR DYNAMIC OPTION                                              MAN655C
C                                                                                        MAN656C
C       (1) ONLY COLUMNS 1 TO 66 ARE AVAILABLE FOR PAGE HEADING.                         MAN657C
C       (2) MAXIMUM ADVANTAGE OF ELEMENT REUSE OPTION SHOULD BE TAKEN.                   MAN658C
C       (3) INITIAL CONDITIONS FOR DISPLACEMENT AND VELOCITY VECTORS,                    MAN659C
C           AS WELL AS STORAGE FOR ACCELERATION VECTOR, MAY BE MADE                      MAN660C
C           THROUGH INPUT OF AN INITIAL CONDITION CARD SET, WITHOUT                      MAN661C
C           SPECIFIED INITIAL CONDITIONS THEY ARE AUTOMATICALLY SET ZERO.                MAN662C
C       (4) SPATIAL LOADING IS INPUT THROUGH FORCE OR BOUNDARY                           MAN663C
C           PRESSURE CARDS.                                                              MAN664C
C       (5) EXTREME CAUTION ON ORDER OF DATA CARDS MUST BE OBSERVED.  NO                 MAN665C
C           EXTRA CARDS ARE PERMITTED AND STRICT COUNTS ARE OBSERVED                     MAN666C
C           EXCEPT FOR THE NUMBER OF FORCE CARDS USED IN EACH TIME STEP.                 MAN667C
C                                                                                        MAN668C
C       8.2) INITIATION OF IMPLICIT TIME INTEGRATIONS (I5,1X,A6)                         MAN669C
C                                                                                        MAN670C
C        COL  1 TO  5    NSEQ, NUMBER OF TIME SEQUENCES                                  MAN671C
C        COL  7 TO 12    MUST CONTAIN VISCOE FOR LINEAR VISCOELASTIC                     MAN672C
C                        QUASI-STATIC PROBLEMS (ONE INITIAL CONDITION                    MAN673C
C                        ONLY MUST BE USED)                                              MAN674C
C        COL  7 TO 12    MUST CONTAIN IMPLIC FOR DYNAMIC IMPLICIT                        MAN675C
C                        INTEGRATIONS (THREE INITIAL CONDITIONS ARE                      MAN676C
C                        REQUIRED, MORE CAN BE SPECIFIED WITHOUT ERROR)                  MAN677C
C                                                                                        MAN688C
C        SUBSEQUENT CARDS, ONE SET FOR EACH TIME SEQUENCE                                MAN689C
C                                                                                        MAN690C
C        CARD 1, (F10.0,8I5,2F10.0,2I5)                                                  MAN691C
C                                                                                        MAN692C
C        COL  1 TO 10    DT, TIME INCREMENT (NONZERO FOR IMPLIC)                         MAN693C
C        COL 11 TO 15    NTS, NUMBER OF TIME STEPS IN SEQUENCE                           MAN694C
C        COL 16 TO 20    INT, PRINT INTERVAL (DEFAULT 1)                                 MAN695C
C        COL 21 TO 25    NNI, FIRST NODE PRINTED                                         MAN696C
C        COL 26 TO 30    NNE, LAST  NODE PRINTED                                         MAN697C
C        COL 31 TO 35    NEI, FIRST ELEMENT STRESS TO BE PRINTED                         MAN698C
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C        COL 36 TO 40    NEF. LAST  ELEMENT STRESS TO BE PRINTED             MAN6990
C        COL 41 TO 45    NPROP. NUMBER OF PROPORTIONAL LOADS IN SEQUENCE     MAN7000
C        COL 46 TO 50    NFORC. LAST NODE FOR GENERALIZED FORCES TO BE       MAN7010
C                        INPUT FOR EACH TIME IN SEQUENCE (SEE SECT.7.        MAN7020
C                        FOR DATA PREPARATION FORMATS)                       MAN7030
C        COL 51 TO 60    BETA. NEWMARK INTEGRATION PARAMETER (IMPLIC)        MAN7040
C                        DEFAULT VALUE IS 0.25                               MAN7050
C        COL 61 TO 70    DEL = GAMMA - 0.5. NEWMARK INTEGRATION              MAN7060
C                        PARAMETER (IMPLIC)                                  MAN7070
C        COL 71 TO 75    NUMPLT. NUMBER OF STRESS PLOTS REQUESTED            MAN7080
C                        (INPUT ON FIRST TIME SEQUENCE ONLY)                 MAN7090
C        COL 76 TO 80    NITS. NUMBER OF ITERATIONS PER TIME STEP            MAN7100
C                        (TO ACHIEVE EQUILIBRIUM IN NON LINEAR PROBLEMS)     MAN7110
C                        ZERO OR BLANK FOR LINEAR PROBLEMS.                  MAN7120
C                        POSITIVE - REFORM THE NEWTON JACOBIAN MATRIX        MAN7130
C                                   AT START OF EACH TIME STEP ONLY.         MAN7140
C                        NEGATIVE - REFORM THE NEWTON JACOBIAN MATRIX        MAN7150
C                                   AT EACH ITERATION IN EACH TIME STEP.     MAN7160
C                        IF CONVERGENCE OCCURS BEFORE ABS(NITS)              MAN7170
C                        ITERATIONS, THE PROGRAM WILL PROCEED TO THE         MAN7180
C                        NEXT TIME STEP.                                     MAN7190
C                        NOTE * * BECAUSE OF THE CONVERGENCE TEST USED,      MAN7200
C                                 A MINIMUM OF TWO ITERATIONS MUST BE        MAN7210
C                                 PERFORMED BEFORE THE CHECK CAN BE          MAN7220
C                                 IMPOSED.  THUS, USE OF NITS CAN BE         MAN7230
C                                 INEFFICIENT IF LINEAR PROBLEMS ARE         MAN7240
C                                 BEING SOLVED.                              MAN7250
C                                                                            MAN7260
C        CARD 2. (2I5,7F10.0)                                                MAN7270
C                                                                            MAN7280
C        ONE FOR EACH NPROP. SEE SECT.7.3 FOR DATA PREPARATION               MAN7290
C                                                                            MAN7300
C                                                                            MAN7310
C        SUBSEQUENT CARDS FOR EACH TIME STEP IN THE SEQUENCE                 MAN7320
C                                                                            MAN7330
C        FORCE CARDS.  SEE SECTION 7. FOR DATA PREPARATION FORMATS.          MAN7340
C                                                                            MAN7350
C      8.3) MESH CHECKS AND PLOTS                                            MAN7360
C                                                                            MAN7360
C        COL  7 TO 10    MUST CONTAIN MESH.                                  MAN7360
C                        PROGRAM WILL CHECK THE PREVIOUSLY INPUTTED MESH     MAN7360
C                        FOR ERRORS, BUT DOES NOT PROCEED TO SOLVE.          MAN7360
C        COL  7 TO 10    MUST CONTAIN PLOT.                                  MAN7360
C                        PERFORMS SAME CHECKS AS FOR MESH AND PLOTS THE      MAN7360
C                        MESH. (GDS OR NOVA PLOT ROUTINES AVAILABLE).        MAN7360
C                                                                            MAN7360
C     8.4) INPUT OF FOURIER SERIES HARMONICS                                 MAN7360
C                                                                            MAN7360
C        COL  1 TO  5    IOUT. (SEE 8.)                                      MAN7360
C        COL  7 TO 12    FOURIE                                              MAN7360
C                                                                            MAN7360
C        CARD 1.  (I5,5X,6F10.0)                                             MAN7360
C                                                                            MAN7360
C        COL  1 TO 10    HARMONIC NUMBER                                     MAN7360
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C        COL 11 TO 20     1 - FORCE/DISPL. MULTIPLIER                       MAN7360
C        COL 21 TO 30     2 - FORCE/DISPL. MULTIPLIER                       MAN7360
C        COL 31 TO 40     3 - FORCE DISPL. MULTIPLIER                       MAN7360
C        COL 41 TO 50     4 - FORCE USER MULTIPLIER                         MAN7360
C        COL 51 TO 60     5 - FORCE USER MULTIPLIER                         MAN7360
C        COL 61 TO 70     6 - FORCE USER MULTIPLIER                         MAN7360
C        COL 71 TO 80     USER CONSTANT.                                    MAN7360
C                                                                           MAN7360
C                                                                           MAN7360
C      9.) OUTPUT CONTROL FOR LIMITED PRINTS                                MAN7370
C                                                                           MAN7380
C        DISPLACEMENT OUTPUT CONTROL. IF IOUT .NE. 0.                       MAN7390
C                                                                           MAN7400
C        CARD 1.  (I5)                                                      MAN7410
C                                                                           MAN7420
C        COL  1 TO  5     NUMDIS - NUMBER OF DISPLACEMENT PRINT CARDS       MAN7430
C                                                                           MAN7440
C        SUBSEQUENT CARDS (2I5) SKIP IF NUMDIS = 0                          MAN7450
C                                                                           MAN7460
C        COL  1 TO  5     NODAL NUMBER TO BE OUTPUT.                        MAN7470
C        COL  6 TO 10     HIGHER NODE NUMBER OF A GENERATED SEQUENCE,       MAN7480
C                         IF ZERO JUST FIRST NODE IS COUNTED.               MAN7490
C        COL 11 TO 15     INCREMENT TO GENERATOR. DEFAULT = 1               MAN7500
C                         *** REPEAT UNTIL.NUMDIS CARDS HAVE BEEN READ      MAN7510
C                                                                           MAN7520
C        STRESS OUTPUT CONTROL. IF IOUT .NE. 0.                             MAN4530
C                                                                           MAN7540
C        CARD 1.  (I5.5X,9I1)                                               MAN7550
C                                                                           MAN7560
C        COL  1 TO  5     NUMSTR - NUMBER OF STRESS OUTPUT CARDS            MAN7570
C        COL 11 TO 19     NSIG(9) - PRINT PATTERN WITHIN AN ELEMENT.        MAN7580
C                         LOCAL POINTS OF EACH ELEMENT CAN BE               MAN7590
C                         SUPRESSED BY NON-ZERO ENTRIES AS FOLLOWS.         MAN7600
C                                                                E.G.       MAN7610
C              COL  11    SUPPRESS PRINT AT LOCAL POINT 1, ( 0, 0, 0)       MAN7620
C              COL  12    SUPPRESS PRINT AT LOCAL POINT 2, (-1, 0, 0)       MAN7630
C              COL  13    SUPPRESS PRINT AT LOCAL POINT 3, ( 1, 0, 0)       MAN7640
C              COL  14    SUPPRESS PRINT AT LOCAL POINT 4, ( 0,-1, 0)       MAN7650
C              COL  15    SUPPRESS PRINT AT LOCAL POINT 5, ( 0, 1, 0)       MAN7660
C              COL  16    SUPPRESS PRINT AT LOCAL POINT 6, ( 0, 0,-1)       MAN7670
C              COL  17    SUPPRESS PRINT AT LOCAL POINT 7, ( 0, 0, 1)       MAN7680
C              COL  18    SUPPRESS PRINT AT LOCAL POINT 8                   MAN7690
C              COL  19    SUPRESS PRINT AT LOCAL POINT 9                    MAN7700
C                                                                           MAN7710
C        SUBSEQUENT CARDS (2I5) SKIP IF NUMSTR = 0                          MAN7720
C                                                                           MAN7730
C        COL  1 TO  5     ELEMENT NUMBER TO BE PRINTED.                     MAN7740
C        COL  6 TO 10     HIGHER ELEMENT NUMBER OF A GENERATED SEQUENCE,    MAN7750
C                         IF ZERO ONLY FIRST ELEMENT IS COUNTED.            MAN7760
C        COL 11 TO 15     INCREMENT TO GENERATOR. DEFAULT = 1               MAN7770
C                         *** REPEAT UNTIL.NUMSTR CARDS HAVE BEEN READ      MAN7780
C                                                                           MAN7790
C                                                                           MAN7800
C*************************************************************************  MAN7810
```

C. Listing of the Contact/Impact Subroutines Added to FEAP 74

The listings for the subroutines which are added to FEAP 74 for the contact/impact theory described herein are given below.

```
              SUBROUTINE CONTAC(ISW,IX,NEL1,NDF,NUMNP,DU,U,UD,UDD)
         C******************* CONTAC ***** 04/29/74 ********************************
000014        LOGICAL CFLAG,FLAGC
000014        DIMENSION IX(NEL1,NUMNP),DU(NDF,NUMNP),U(NDF,NUMNP),UD(NDF,NUMNP),
             C          UDD(NDF,NUMNP)
000014        DIMENSION RMP(10)
000014        COMMON/CNTACT/FLAGC,CFLAG,LIST,ICLIST(10),ICDEG(10),RU1,RU2,
             C     RM(2,10)
000014        COMMON/LABELS/HEAD(12),O,IPG,XHED(3),UHED(6),XH,FH,UH,NSTR,FLAG(7)
000014        COMMON/TAPES/ ITP5,ITP6
000014        DATA CFLAG,FLAGC/.FALSE.,.FALSE.
000014        GO TO (1,2,3),ISW
000022   1    READ(ITP5,1000) LIST,RU1,RU2
000034        RP = RU1+RU2
000036        IF(RP.EQ.0.0) RP = 1.
000044        READ(ITP5,1001) (ICLIST(L),ICDEG(L),L=1,LIST)
000070        WRITE(ITP6,2000) O,HEAD,IPG,RU1,RU2,(ICLIST(L),ICDEG(L),L=1,LIST)
000131        IPG = IPG + 1
000133        FLAGC = .TRUE.
000134        RETURN
000134   2     DO 200 L = 1,LIST
000141        N = ICLIST(L)
000143        IDEG = ICDEG(L)
000144        I = IX(1,N)
000150        RM(1,L) = DU(IDEG,I)
000155        I = IX(3,N)
000161   200  RM(2,L) = DU(IDEG,I)
000171        WRITE(ITP6,2002)
000175        DO 220 L = 1,LIST
000202        N = ICLIST(L)
000204        RMP(L) = RM(1,L) + RM(2,L)
000210        IF(RM(1,L).EQ.0.0.OR.RM(2,L).EQ.0.0) RMP(L) = RMP(L)*1.E+30
000222   220  WRITE(ITP6,2001) N,(RM(I,L),I=1,2),RMP(L)
000256        DO 210 N = 1,NUMNP
000260        DO 210 I = 1,NDF
000261   210  DU(I,N) = 0.0
000272        RETURN
000272   3    DO 300 L = 1,LIST
000274        N=ICLIST(L)
000276        IDEG = ICDEG(L)
000277        K = IX(2,N)
000303        UD(IDEG,K) = 0.0
000310        UDD(IDEG,K) = 0.0
000313        IF(U(IDEG,K)) 300,300,310
000317   310  I = IX(1,N)
000324        J = IX(3,N)
000330        CU =(RU1*U(IDEG,I)+RU2*U(IDEG,J))/RP
000342        [illegible]
000347        [illegible]
000352        [illegible] RM(1,L)*[illegible](IDEG,I)+RM(2,L)*[illegible](IDEG,J)[illegible]/RMP(L)
[illegible]   IF[illegible]IDEG,K)-RM(2,L)*(UD(IDEG,J)-CU)
[illegible]   IF(RM[illegible](IDEG,K)=[illegible]+RM(1,L)*(UDD(IDEG,J)-CU[illegible]
[illegible]
```

```
   451          UDD(IDEG,J) = CU
 .0454    300   CONTINUE

000457          RETURN

000457   1000   FORMAT(I5,5X,2F10.0)
000457   1001   FORMAT(2I5)
000457   2000   FORMAT(A1,12A6,30X,4HPAGE,I4//5X,25HCONTACT SHEET DESCRIPTION//
               C      10X,13HBODY 1 RO*U =,E13.5/
               C      10X,13HBODY 2 RO*U =,E13.5//
               X   11X,7HELEMENT,1X,9HDIRECTION (I15,I10))
000457   2001   FORMAT(I10,6E13.4)
000457   2002   FORMAT( 10H   ELEMENT,13H  BODY 1 MASS,13H  BODY 2 MASS)
000457          END
```

```
                SUBROUTINE ELMT05(N,MA,NDIM,NDF,NEL,NEL1,NSTF,NSIZV,NVEC,MCT,DM,D,      ELM  1C
               X XYZ,IX,F,FORCE,ESTIF,U,VECT,ISW)
          C****************** ELMT05 ***** 04/29 74 *************************************
000027          LOGICAL NPR
000027          DIMENSION ESTIF(NSTF,NSTF),FORCE(NSTF,2)
000027          DIMENSION U(NDF,1),IX(NEL1,1),PLOT(8),THED(8)
000027          COMMON/LOCALS/ DUL(6,20),UL(6,20),UDL(6,20),UDDL(6,20)
000027          COMMON/PRTPLT/ NSIG(9),NPLT(9,2),NT,NSTEP,NUMPLT,NEDATA(20,3),NPR
000027          COMMON/SHAP/ XJAC,SHAPE(4,20),SG(3,3),SH(3,3),X(3,20)
000027          COMMON/TAPES/ ITP5,ITP6
000027          COMMON/TIMHIS/ TIME,DT,DTP,NH,ISZH,C0,C1,C2,C3,C4,C5,C6,NCT
000027          EQUIVALENCE (PLOT,TAU),(PLOT(2),ETA)
000027          DATA THED/3HTAU,3HETA,2HU1,3HUD1,4HUDD1,2HU3,3HUD3,4HUDD3/
000027          GO TO (1,2,3,3,5,3), ISW
000041  1       READ(ITP5,1000) IDEG,NPS
000051          IDEG = MAX0(IDEG,1)
000055          WRITE(ITP6,2000) IDEG,NPS
000064          TOL = 1.E-06
000066          I = NDF  + IDEG
000073          J = I + NDF
000074          RI = 0.5
000075          RJ = 0.5
000076          C6 = 0.
000077          NV = NPS
000100          NA = NPS + NPS
000101          RETURN
000102  2       DM = X(IDEG,3) - X(IDEG,1)
000106          DM = DM + 1.E-25
000110          RETURN
000110  3       TAU = UL(IDEG,2)
000112          DD = (X(IDEG,3)-X(IDEG,1)) + (UL(IDEG,3)-UL(IDEG,1))
000120          ETA = 0.
000121          IF(DD.LT.TOL) ETA = 1.
000125          IF(TAU.LT.0.0  ) ETA = 0.0
000130          IF(ISW.GT.3) GO TO 4
000134          ESTIF(IDEG,I) = ETA
000142          ESTIF(I,IDEG) = ETA
000145          ESTIF(I,J) = -ETA
000151          ESTIF(J,I) = -ETA
000155          ESTIF(I,I) = 1.0 - ETA
000163          RETURN
000163  4        IF(ISW.EQ.4) GO TO 6
000166          FORCE(I,1) = ETA*DD
000174          IF(ETA.EQ.0.0) FORCE(I,1) = - TAU
000203          TAU = AMAX1(0.0,TAU)
000207          FORCE(IDEG,1) = -TAU
000215          FORCE(J,1) = TAU
000220  6       IF(.NOT.NPR) WRITE(ITP6,2001) N,DM,MA,TAU,ETA
000243          IF(NUMPLT.LE.0) RETURN
000246          PLOT(3) = UL(IDEG,1)
000250          PLOT(4) = UDL(IDEG,1)
                PLOT(5) = UDDL(IDEG,1)
                PLOT(6) = UL(IDEG,3)
```

```
 .256          PLOT(7) = UDL(IDEG.3)
 .0260         PLOT(8) = UDDL(IDEG.3)
000262         DO 60 K = 1.NUMPLT
000263         KK = NPLT(K.2)
000265         IF(NPLT(K.1).GT.0) CALL FLDATA(NDIM.NPLT(K.1).THED(KK).X(1.2).
             C    PLOT(KK))
000306   60    CONTINUE
000311   5     RETURN
000312   1000  FORMAT(2I5)
000312   2000  FORMAT(5X.21HPOINT CONTACT ELEMENT.
             C          5X.27HCONTACT DEGREE OF FREEDOM = I3.5X.5HNPS = I5)
000312   2001  FORMAT(5X.7HELEMENT.I5.5X.A5.5X.8HMATERIAL.I3.5X.5HTAU =.E15.5.5X.
             X    5HETA =.F3.1)
000312         END
```

```
                 SUBROUTINE ELMT09(N.MA.NDIM.NDF.NELM.NEL1.NSTF.NSIZV.NVEC.MCT.DM.         ELM  1C
                X D.XYZ.IX.F.FORCE.ESTIF.U.VECT.ISW)                                       ELM  2C
000027           LOGICAL NOPRNT.NPR.NPL
000027           DIMENSION ESTIF(NSTF.NSTF).D(3.21.1).IX(NEL1.1).U(NDF.1).                 ELM  4C
                X V(3.20).DU(3).XX(3).FORCE(NSTF.2)                                        ELM  5C
000027           COMMON GAUS  LIM.SGAUSS(5.5).WGAUSS(5.5)                                  ELM  3C
000027           COMMON/LABELS/HEAD(12).O.IPG.XHED(3).UHED(6).XH.FH.UH.NSTP.FLAG(7)
000027           COMMON/LOCALS/ DUL(6.20).UL(6.20).UDL(6.20).UDDL(6.20)
000027           COMMON/PRTPLT/ NSIG(9).NPLT(9.2).NT.NSTEP.NUMPLT.NEDATA(20.3).NPP
000027           COMMON/SHAP/ XJAC.SHAPE(4.20).SG(3.3).SK(3.3).X(3.20)
000027           COMMON TAPES  ITP5.ITP6                                                   ELM 10C
000027           COMMON/TIMHIS/ TIME.DT.DTP.NH.ISZH.C0.C1.C2.C3.C4.C5.C6.NCT
000027           DATA SH.EH.BL/6HSTRESS.6HSTRAIN.6H                                        ELM 11C
000027           GO TO (1.2.3.4.5.4) . ISW
000041   1       CONTINUE                                                                  ELM 13C
       C....     BAR STIFFNESS CHARACTERIZATION                                            ELM 14C
000041           READ(ITP5.1000) E.AA.RO
000053           WRITE(ITP6.2100) E.AA.RO
000065           D(1.1.MA) = E                                                             ELM 17C
000075           D(1.2.MA)=AA
000100           D(1.3.MA)=RO
000103           C6 = 0.
000104           RETURN                                                                    ELM 20C
000104   2       CONTINUE                                                                  ELM 21C
000104           RETURN                                                                    ELM 22C
       C....     COMPUTATION OF BAR STIFFNESS NO BENDING * ISOPARAMETRIC 1ST AND 2N        ELM 23C
000105   3       CONTINUE                                                                  ELM 24C
       C....     CONSTRUCT STIFFNESS AT INTEGRATION POINTS                                 ELM 25C
000105           EA = D(1.1.MA)*D(1.2.MA)                                                  ELM 26C
000113           RA = D(1.3.MA)*D(1.2.MA)                                                  ELM 27C
000116           DO 260 II = 1.NELM                                                        ELM 28C
000120           SS = SGAUSS(II.NELM)                                                      ELM 29C
000123           CALL LINE(SS.NDIM.NELM)                                                   ELM 30C
000125           WW=WGAUSS(II.NELM)
000134           DVOL=EA*WW/(XJAC*XJAC*XJAC)
       C....     COMPUTE A LUMPED MASS MATRIX                                              ELM 32C
000137           IU = 0                                                                    ELM 33C
000140           RAA=RA*XJAC*WW
000142           DO 217 I = 1.NELM                                                         ELM 34C
000143           AA=RAA*SHAPE(2.I)
000146           DO 215 KK = 1.NDIM                                                        ELM 35C
000150   215     FORCE(IU+KK.2) = FORCE(IU+KK.2) + AA                                      ELM 37C
000152   217     IU = IU + NDF                                                             ELM 38C
000166           DO 250 KK = 1.NDIM                                                        ELM 39C
000167           DO 250 LL = KK.NDIM                                                       ELM 40C
000170           SGD=SG(KK.LL)*DVOL
000174           I1 = KK                                                                   ELM 41C
000175           DO 240 I = 1.NELM                                                         ELM 42C
000177           SHI=SHAPE(1.I)*SGD
000200           L = I                                                                     ELM 43C
                 IF(KK.EQ.LL) L = I                                                        ELM 44C
                 J1 = LL + NDF*(L-1)                                                       ELM 45C
                 DO 230 J = L.NELM                                                         ELM 46C
```

```
  .14          ESTIF(I1.J1)=ESTIF(I1.J1)+SHI*SHAPE(I.J)
 .J225   230   J1 = J1 + NDF                                                ELM 480
 .00231  240   I1 = I1 + NDF                                                ELM 490
000234   250   CONTINUE                                                     ELM 50
000241   260   CONTINUE                                                     ELM 51
         C.... CONSTRUCT SYMMETRIC PART OF STIFFNESS MATRIX                 ELM 52
000243         KK = NELM*NDF                                                ELM 530
000245         LL = KK - 1                                                  ELM 540
000247         DO 360 I=1.LL                                                ELM 550
000250         K = I + 1                                                    ELM 56
000252         DO 360 J = K.KK                                              ELM 570
000254         ESTIF(I.J) = ESTIF(I.J) + ESTIF(J.I)                         ELM 580
000266   360   ESTIF(J.I) = ESTIF(I.J)                                      ELM 590
000302         IF(C6.EQ.0.0) RETURN
000304         DO 370 I = 1.NSTF
000306   370   ESTIF(I.I) = ESTIF(I.I) + C6*FORCE(I.2)
000324         RETURN                                                       ELM 600
000325   4     CONTINUE                                                     ELM 610
000325         E = D(1.1.MA)                                                ELM 640
000331         NELP = NELM
000332         DO 400 NN = 1.NELP
000334         NOPRNT=NPR
000336         IF(NSIG(NN).GT.0) NOPRNT=.TRUE.
000341         NPL=.TRUE.
000342         IF(NPLT(NN.1).GT.0) NPL = .FALSE.
000345         SS = SGAUSS(NN.NELP)
000351         CALL LINE(SS.NDIM.NELM)                                      ELM 710
         C.... COMPUTE STRESS AND STRAIN                                    ELM 720
000353         DO 100 KK = 1.NDIM                                           ELM 730
000360         XX(KK) = 0.0                                                 ELM 740
000361         DU(KK) = 0.                                                  ELM 750
000362         DO 100 LL = 1.NELM                                           ELM 760
000364         XX(KK) = XX(KK) + X(KK.LL)*SHAPE(2.LL)                       ELM 770
000372   100   DU(KK) = DU(KK) +UL(KK.LL)*SHAPE(1.LL)
000404         EPS = 0.                                                     ELM 790
000405         DO 200 KK = 1.NDIM                                           ELM 800
000406   200   EPS = EPS + SK(KK.1)*DU(KK)/XJAC/XJAC                        ELM 810
000416         SIG = E*EPS                                                  ELM 820
000420         IF(NOPRNT) GO TO 350
000421         MCT = MCT - 1                                                ELM 830
000423         IF(MCT.GT.0) GO TO 300                                       ELM 840
000426         MCT=50
000426         WRITE(ITP6.2000) HEAD.TIME.IPG.( XHED(I).XH.I=1.NDIM).BL.SH.BL.EH
000465         IPG = IPG + 1                                                ELM 870
000467    300  WRITE(ITP6.2001) N.MA.DM.(XX(I).I=1.NDIM).SIG.EPS            ELM 880
000523   350   IF(.NOT.NPL) CALL PLDATA(NDIM.NPLT(NN.1).5HAXIAL.XX.SIG)
000542         IF(ISW.EQ.4) GO TO 400
000545         WT = WGAUSS(NN.NELM)*D(1.2.MA)
000553         WW = WT*D(1.3.MA)*XJAC
000557         IU = 0
000560         DO 400 I = 1.NELM
000561         H  = SHAPE
000565         ER = SHAPE(1.I)*SIG*WT
```

```
000573   615    FORCE(KK+IU.I) = FORCE(KK+IU.I) - AA*UDDL(KK.I) - BB*S[illegible]
000615   600    IU = IU + NDF
000621   400    CONTINUE                                                             ELM 31[illegible]
000624   5      RETURN
000625   1000   FORMAT(3F10.0)                                                       ELM [illegible]
000625   2000   FORMAT(1H1.12A6.E13.5.12X.4HPAGE.I3//10H    ELEMENT.[illegible]8HMATE[illegible]
               X 3X.5(2A5))
000625   2001   FORMAT(I10.I5.5X.A5.5E12.4)                                          ELM 96[illegible]
000625   2100   FORMAT(3[illegible]HOLINEAR ELASTIC MATERIAL. BAR ELEMENT//
               X        5X.3HE =.E15.5.5X.6HAREA =.E15.5.5X.9HDENSITY =E15.5/)       ELM 97[illegible]
000625   4000   FORMAT(A5.I5.1P3E12.5.24X.I10)                                       ELM 98[illegible]
000625          END                                                                  ELM 99[illegible]
```

```
              SUBROUTINE LINE(S,NDIM,NEL)
        C***** LINE          ****** 1/02/73 ***************************************
        C....  SHAPE FUNCTION ROUTINE FOR 1ST AND 2ND ORDER ISOPARAMETRIC LINES
000006        COMMON SHAPE/XJAC,SHAPE(1,20),SG(3,3),SK(3,3),X(3,20)
        C....  FORM SHAPE FUNCTIONS
000006        SHAPE(1,1) = -0.5
000007        SHAPE(2,1) = 0.5*(1.0-S)
000012        SHAPE(1,NEL) = 0.5
000014        SHAPE(2,NEL) = 0.5*(1.0+S)
000016        IF(NEL.EQ.2) GO TO 350
000020        SHAPE(1,2) = -2.0*S
000021        SHAPE(2,2) = 1.0-S*S
000024        K = NEL-1
000026        DO 100 I = 1,NEL,K
000027        SHAPE(1,I) = SHAPE(1,I) - 0.5*SHAPE(1,2)
000033  100   SHAPE(2,I) = SHAPE(2,I) - 0.5*SHAPE(2,2)
        C....  FORM JACOBIAN MATRIX AND DETERMINANT
000040  350   DO 360 I = 1,3
000042  360   SK(I,1) = 0.0
000046        DO 400 I = 1,NDIM
000047        DO 400 J = 1,NEL
000050  400   SK(I,1) = SK(I,1) + X(I,J)*SHAPE(1,J)
000063        XJAC = 0.0
000064        DO 500 I = 1,NDIM
000065        XJAC = XJAC + SK(I,1)*SK(I,1)
000070        DO 500 J = 1,NDIM
000071  500   SG(I,J) = SK(I,1)*SK(J,1)
000103        XJAC = SQRT(XJAC)
000105        RETURN
000105        END
```

```
                SUBROUTINE NLNORM(ISW,MDEG,NN,U,DU,DF,F,PROP,NCT,NTT,IDEST,IBLK)
          C***** NLNORM    ****** 12/14/73   ***************************************
000017          DIMENSION U(1),DU(MDEG),DF(1),F(MDEG),IDEST(1)
000017          COMMON NORMS  UNORM,DNORM,ANORM,CS,CSP,DP,DNP,IFL,XFLAG,ERR
000017          GO TO (100,200,200,100), ISW
000026      100 CS=1.0
000027          CSP=1.0
000030          DP=0.75
000032          IFL=0
000032          XFLAG=.FALSE.
000033          ERR=1.0E-3
000035          RETURN
000036      200 UNORM = 0.
000037          DNORM = 0.
000037          ANORM=0.
000041          DO 500 N = 1,MDEG
000042          DUN = DU(N)
000044          UN = U(N)
000046          IF(ISW.EQ.3) UN = UN + DF(N) + DUN
000053          ANORM=ANORM+UN*DUN
000056          UNORM=UNORM+UN*UN
000060      500 DNORM=DNORM+DUN*DUN
000065          UNORM=SQRT(UNORM)
000067          DNORM=SQRT(DNORM)
000070          IF(NN.EQ.1) DNP=UNORM
000077          CS=1.0
000100          IF(UNORM*DNORM.NE.0.) CS=ANORM/(UNORM*DNORM)
000105          IF(IFL.EQ.1) DP=0.75
000111          IF(IFL.EQ.1.AND.XFLAG) CSP=CS
000120          IFL=0
000121          IF(DNORM.LE.0.5*UNORM) GO TO 550
000124          IFL=1
000125          DP=0.5*UNORM/DNORM
000127      550 IF(DP*DNORM.GT.DNP) DP=DNP/DNORM
000134          IF(DP.EQ.0.0) DP = 1.0
000136          IF(CS*CSP.LT.0.) DP=DP/2.
000142          IF(CS*CSP.GT.0.) DP=1.25*DP
000146          DNP=DP*DNORM
000150          CSP=CS
000150          IF(ISW.EQ.2) RETURN
000153          DO 600 N = 1,MDEG
000155          DF(N) = DF(N) + DU(N)
000160          IF(NCT.EQ.0) DF(N) = DU(N)
000164      600 DU(N) = DF(N)
000171          RETURN
000171          END
```

```
              SUBROUTINE TSOLVE(NUMNP.NUMEL.NUMMAT.NDIM.NDF.NEN.NEL1.NSTF.NVEC.
             1   NSIZV.NICD.NSICD.IBLK.ISZA.NEQB.MAXBAN.NDES.ID1.MB.IBUF.DS.
             2   TYPE.D.ICOD.XYZ.F.IX.IDEST.VECT.FORCE.ESTIF.LD.A.DU.MAXB.H.U.DF.
             3   NSEQ.TYME)
       C***** TSOLVE     ****** 03 06/74   ******************************************
000053        LOGICAL CFLAG.FLAGC
000053        LOGICAL NPR VFLAG.DFLAG
000053        REAL LABL(12)
000053        DIMENSION TYPE(1).ICOD(1).XYZ(NDIM.1).F(NDF.1).IX(NEL1.1).
             1   IDEST(NDF.1).VECT(NSIZV.1).FORCE(NSTF.2).ESTIF(NSTF.NSTF).
             2   A(NEQB.1).DU(NDF.1).H(IBUF).U(NDF.1).DF(1).DS(MB).TYME(7).LD(1)
000053        COMMON/CNTACT FLAGC.CFLAG.LIST.ICLIST(10).ICDEG(10).RU1.RU2.
             C      RM(2.10)
000053        COMMON/LABELS/HEAD(12).O.IPG.XHED(3).UHED(6).XH.FH.UH.NSTR.FLAG(7)
000053        COMMON/LOCALS/ DUL(6.20).UL(6.20).UDL(6.20).UDDL(6.20)
000053        COMMON/NORMS/ UNORM.DNORM.ANORM.CS.CSP.DP.DNP IFL.XFLAG.EPR
000053        COMMON/PRTPLT/ NSIG(9).NPLT(9.2).NT.NSTEP.NUMPLT.NEDATA(20.3).NPR
000053        COMMON/SHAP/ XJAC.SHAPE(4.20).SG(3.3).SK(3.3).X(3.20)
000053        COMMON/TAPES/ ITP5.ITP6
000053        COMMON/TIMHIS/ TIME.DT.DTP.NH.ISZH.C0.C1.C2.C3.C4.C5.C6.NCT
000053        COMMON/VTAPE/ ITP13.ITP14.ITPD.ITUP
000053        DATA AWORD1.AWORD2 8H  F13.4..8H  E13.4./.AWORD3/8H  F12.2./
000053        DATA VH.AH.AWORD4.AWORD5 6H VEL. .6H ACCEL .6H(/12X..8H9E13.4))/
000053        DATA LABL(1) 8H (I12.
000053        DATA CFLAG.FLAGC/.FALSE...FALSE./
       C....  SUBROUTINE TO PERFORM IMPLICIT INTEGRATION WITH 1 OR 3 INIT. COND.
000053        DO 2 N=1.NEN
000054        DO 2 J=1.NDF
000055        DUL(J.N)=0.
000060        UL(J.N) = 0.0
000063        UDL(J.N)=0.
000065 2      UDDL(J.N) = 0.0
000073        DO 3 I=2.7
000074 3      NSIG(I)=1
000077        IF(IBLK.EQ.0) GO TO 4
000101        KU = NUMNP NICD
000104        REWIND 7
000106        WRITE(7) DS
       C....  SET THE OUTPUT LABELS FOR DISPLACEMENT PRINTS
000117 4      HEAD(12) = 6H TIME=
000124        IFLG =-NSICD
000126        MDEG = NSICD
000127        MDEG2 = MDEG + MDEG
000130        I = 2
000131        DO 5 N = 1.NDIM
000133        LABL(I) = AWORD1
000134 5      I = I + 1
000137        L = NDIM + 1
000140        DO 7 N = L.3
000141        LABL(I) = AWORD2
000143 7      I = I + 1
000146        LABL(I) = AWORD4
000150        LABL(I+1) = AWORD5
```

```
000152          DTP = 0.                                                          TSO 44C
000153          TIME = 0.                                                         TSO 45C
000154          MB = MAXBAN + 1                                                   TSO 46C
000156          NE1 = NEN + 1                                                     TSO 47C
      C.... INITIALIZE INCREMENTAL FORCE VECTOR                                   TSO 48C
000157          DO 6 N = 1,NSICD                                                  TSO 49C
000160   6      DF(N) = 0.0                                                       TSO 50C
000165          DO 10 N = 1,NUMEL                                                 TSO 51C
000166   10     IX(NE1,N) = 0                                                     TSO 52C
      C.... PUT INITIAL DATA ON TAPE                                              TSO 53C
000177          ISZH = IBUF                                                       TSO 54C
000201          ITRD = ITP13                                                      TSO 55C
000202          ITWR = ITP14                                                      TSO 56C
000204          REWIND ITRD                                                       TSO 57C
000206          IF(IBLK.GT.0) WRITE(ITRD) ((U(I,J),I=1,NDF),J=1,KU)               TSO 58C
000241          DO 11 N = 1,ISZH                                                  TSO 59C
000243   11     H(N) = 0.                                                         TSO 60C
000247          NTB= (NH+ISZH-1)/ISZH                                             TSO 61C
000253          WRITE(ITRD) H                                                     TSO 62C
000264          IF(NTB.LE.1) GO TO 13                                             TSO 63C
000272          NT = NTB*2                                                        TSO 64C
000273          DO 12 N = 1,NT                                                    TSO 65C
000275   12     WRITE(ITRD) H                                                     TSO 66C
000314   13       REWIND ITRD                                                     TSO 67C
000316          IF(IBLK.GT.0) READ(ITRD) DM                                       TSO 68C
000331          NEP = NUMNP + NUMNP                                               TSO 69C
000332          C6 = 0.0                                                          TSO 70C
000333          NSTEP = 0                                                         TSO 71C
000334          NST = 0                                                           TSO 72C
000335          NUMPLT=0                                                          TSO 73C
000336          PROP = 0.                                                         TSO 74C
000337          CFLAG = .FALSE.
000340          DO 900 M = 1,NSEQ                                                 TSO 75C
000341          READ(ITP5,1000) DT,NTS,INT,NNI,NNF,NEI,NEF,NPROP,NFORC,C0,DM,NT,NL TSO 76C
000376          IF(NICD. GT.1) C6= DM                                             TSO 77C
000406          IF(NL.NE.0) NST= NL                                               TSO 78C
000410          NTT = IABS(NST)                                                   TSO 79C
000412          DFLAG = .FALSE.                                                   TSO 80C
000413          IF (NTT. NE. 0 )        DFLAG = .TRUE.                            TSO 81C
000415          NSTEP=NSTEP+NTS                                                   TSO 82C
000417          IF(M.EQ.1) NUMPLT=NT                                              TSO 83C
000422          IF(INT.LE.0) INT = 1                                              TSO 84C
000425          WRITE(ITP6,2000) O,HEAD,TIME,IPG DT,NTS,INT,NNI,NNF,NEI,NEF       TSO 85C
000457          IF(DFLAG) WRITE(ITP6,2032) NST                                    TSO 86C
000476          IPG = IPG + 1                                                     TSO 87C
000500          IF(NICD.EQ.1) GO TO 1                                             TSO 88C
000501          IF(DT.EQ.0.0) GO TO 901                                           TSO 89C
000502          CALL UPDATE(3)                                                    TSO 90C
000504   1      IF(NPROP.GT.0) PROP = PROPLD(TIME,NPROP)                          TSO 91C
000517          IF(NFORC.GT.0.AND.NPROP.GT.0) WRITE(ITP6,4031)                    TSO 92C
      C.... READ STRESS PLOT INFORMATION                                          TSO 93C
 0536           IF(NUMPLT.LE.0.OR.M.GT.1) GO TO 502                               TSO 94C
   47           REWIND 12                                                         TSO 95C
    1           WRITE(ITP6,2005)                                                  TSO 9
```

```
 555           DO 501 N=1.NUMPLT                                                  TSO 97C
 00562         READ(ITP5.1006) (NEDATA(N .I).I=1.3)                               TSO 98C
 000600  501   WRITE(ITP6.2006) N.(NEDATA(N .I).I=1.3)                            TSO 99C
 000632        CALL PLZERO                                                        TSO100C
 000633  500   VFLAG = .FALSE.                                                    TSO101C
 000634        DO 600 NT = 1.NTS                                                  TSO102C
 000641        PPROP = PROP                                                       TSO103C
 000643        PROP = 1.0                                                         TSO104C
 000644        IF(DFLAG)CALL NLNORM(4.MDEG.NT.U.DF.DU.F.PPROP.NCT.NTT.IDEST.IBLK) TSO105C
 000667        IF(NPROP.GT.0) PROP = PROPLD(TIME+DT.0)                            TSO106C
 000701        IF(NFORC.GT.0) CALL RESET(-NFORC.NUMNP.NDF.F)                      TSO107C
 000713        NCT = 0                                                            TSO108C
 000714  14    REWIND ITWR                                                        TSO109C
 000716        IF(IBLK.EQ.0) GO TO 15                                             TSO110C
 000720        IF(.NOT.VFLAG) GO TO 20                                            TSO111C
 000721        DO 21 I = 1.NDEG                                                   TSO112C
 000723  21    DF(I) = 0.                                                         TSO113C
 000730        GO TO 20                                                           TSO114C
 000730  15    DO 19 N = 1.NUMNP                                                  TSO115C
 000732        DO 18 K = 1.NDF                                                    TSO116C
 000733        J = IDEST(K.N)                                                     TSO117C
 000740        IF(J.EQ.0) GO TO 18                                                TSO118C
 000741        IF(VFLAG) GO TO 17                                                 TSO119C
 000743        DO 16 I = 1.MAXBAN                                                 TSO120C
 000744  16    A(J.I) = 0.0                                                       TSO121C
 000756  17    DF(J) = F(K.N)*PROP                                                TSO122C
 000766  18    CONTINUE                                                           TSO123C
 000771  19    CONTINUE                                                           TSO124C
 000773  20    NH = 1                                                             TSO125C
 000774        IF(NTB.GT.1) READ(ITRD) H                                          TSO126C
 001614        IF(NCT.GT.0) GO TO 44                                              TSO127C
         C.... OUTPUT THE SOLUTION VECTOR FOR THE CURRENT TIME                    TSO128C
 001017        IF(NNI.EQ.0.OR.((NT-1)/INT)*INT.NE.NT-1) GO TO 40                  TSO129C
 001032        MCT = 0                                                            TSO130C
 001033        DO 30 N = NNI.NNF                                                  TSO131C
 001034        MCT = MCT - 1                                                      TSO132C
 001036        IF(MCT.GT.0) GO TO 31
 001040        IF(NICD.EQ.1) WRITE(ITP6.2001) O.HEAD.TIME.IPG.PPROP.M.NT.
              X   (XHED(I).XH.I=1.NDIM).(UHED(I).UH.I=1.NDF)
 001121        IF(NICD.NE.1) WRITE(ITP6.2001) O.HEAD.TIME.IPG.PPROP.M.NT.
              X   (XHED(I).XH.I=1.NDIM).(UHED(I).UH.I=1.NDF).(UHED(I).VH.I=1.NDF)
              X  .(UHED(I).AH.I=1.NDF)
 001235        IPG = IPG + 1                                                      TSO136C
 001237        MCT = 50                                                           TSO137C
 001240  31    IF(NICD.EQ.1)WRITE(ITP6.LABL)N.(XYZ(I.N).I=1.NDIM).(U(I.N)
              X   .I=1.NDF)
 001311        IF(NICD.NE.1)WRITE(ITP6.LABL)N.(XYZ(I.N).I=1.NDIM).(U(I.N).I=1.NDF
              X ) .(U(I.NUMNP+N).I=1.NDF).(U(I.NEP+N).I=1.NDF)
 001420  30    CONTINUE
         C.... UPDATE (1) * * FOR DYNAMIC SOLUTIONS ONLY                          TSO139C
 001423  40    IF(NICD.GT.1) CALL UPDATE(1.MDEG.NICD.U.U(1.NUMNP+1).U(1.NEP+1),   TSO140C
              X   DU.F.DF.IDEST.PROP)                                             TSO141C
 001457        IF(IBLK.GT.0) WRITE(ITWR) ((U(I.J).I=1.NDF). J=1.KU )              TSO142C
 001512  44    MP = 0                                                             TSO143C
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001513          TEMP = 0.                                                      TS0144C
001514          DO 400 N = 1,NUMEL                                             TS0145C
001515          NPR = .TRUE.                                                   TS0146C
001516          DO 43 I = 1,9                                                  TS0147C
001520          NPLT(I,1) = 0                                                  TS0148C
001521    43    NPLT(I,2) = 0                                                  TS0149C
001524          IF(NCT.GT.0) GO TO 46                                          TS0150C
001526          IF(N.GE.NEI.AND.N.LE.NEF) NPR = .FALSE.                        TS0151C
001540          IF(NUMPLT.LE.0) GO TO 46                                       TS0152C
001542          DO 45 I = 1,NUMPLT                                             TS0153C
001543          IF(NEDATA(I,1).NE.N) GO TO 45                                  TS0154C
001546          J = NEDATA(I,2)                                                TS0155C
001547          NPLT(J,1) = I                                                  TS0156C
001551          NPLT(J,2) = NEDATA(I,3)                                        TS0157C
001553    45    CONTINUE                                                       TS0158C
001556    46    MA = MOD(IX(NEL1,N),100)                                       TS0159C
001567          IF(MR.LE.0) MRR= IX(NEL1,N)/1000                               TS0160C
001600          IF(MR.LE.0) MR = MRR                                           TS0161C
001603          DO 60 I = 1,NSTF                                               TS0162C
001605          FORCE(I,1) = 0.                                                TS0163C
001612          FORCE(I,2)=0.                                                  TS0164C
001616          LD(I) = 0                                                      TS0165C
001620          IF(MR.NE.MRR.OR.VFLAG) GO TO 60                                TS0166C
001625          DO 50 J = 1,NSTF                                               TS0167C
001627    50    ESTIF(I,J) = 0.0                                               TS0168C
001640    60    CONTINUE                                                       TS0169C
001643          L = 0                                                          TS0170C
001644          DO 110 I = 1,NEN                                               TS0171C
001645          K = IX(I,N)                                                    TS0172C
001653          DO 90 J = 1,NDIM                                               TS0173C
001654    90    X(J,I) = 0.                                                    TS0174C
001661          IF(K.EQ.0)  GO TO 120                                          TS0175C
001662          NEL = I                                                        TS0176C
001663          DO 100 J = 1,NDIM                                              TS0177C
001665   100    X(J,I) = XYZ(J,K)                                              TS0178C
001701          DO 110 J = 1,NDF                                               TS0179C
001702          L = L + 1                                                      TS0180C
001704          DUL(J,I) = DU(J,K)                                             TS0181C
001713          UL(J,I) = U(J,K)                                               TS0182C
001722          IF(NCT.GT. 0) UL(J,I) =UL(J,I) +DUL(J,I)                       TS0183C
001731          IF(NICD.EQ.1) GO TO 110                                        TS0184C
001734          UDL(J,I) = U(J,K+NUMNP)                                        TS0185C
001744          UDDL(J,I) = U(J,K+NEP)                                         TS0186C
001754          IF(NCT.GT.0) UDDL(J,I)= UDDL(J,I) +CS*DUL(J,I)                 TS0187C
001765   110    LD(L) = IDEST(J,K)                                             TS0188C
002001   120    DM = TYPE(MA)                                                  TS0189C
002004          IF(DM.NE.TEMP) MCT = 0                                         TS0190C
002007          TEMP = DM                                                      TS0191C
002010          CALL TICTOC(TYME,5)                                            TS0192C
        C.....  COMPUTE ELEMENT STRESSES AND UPDATE FORCES                     TS0193C
002013          CALL ELMLIB(N,MA,NDIM,NDF,NEL,NEL1,NSTF,NSIZV,NVEC,MCT,DM,D,XYZ,  TS0194C
               X IM,H,FORCE,ESTIF,U,VECT,5)                                    TS0195C
  047           CALL TICTOC(TYME,5)                                            TS0196C
        C.....  FORM STIFFNESSIF NEEDED FOR THE NEXT TIME STPEP                TS0197C
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002052          IF((.NOT.VFLAG.AND.MR.EQ.MPR).OR.(VFLAG.AND.IX(NE1.N).EQ.1))       TS0198C
               XCALL ELMLIB(N.MA.NDIM.NDF.NEL.NEL1.NSTF.NSIZV.NVEC.MCT.IM.D.XYZ.   TS0199C
               X IX.H.FORCE.ESTIF.U.VECT.3)                                         TS0200C
002131          IF(MOD(IX(NEL1.N),1000)/100.GT.0.AND..NOT.VFLAG)                    TS0201C
               X CALL PPTMAT(HEAD.IPG.N.NSTF.ESTIF.FORCE.LD.NSTF.0)                 TS0202C
         C.... MODIFY FOR THE DISPLACEMENT B.C.                                     TS0203C
002165          CALL MODIFY(NDF.NEL.NEL1.NE1.IBLK.NSTF.PROP.IX.ICOD.F.FORCE.ESTIF.  TS0204C
               X    N)                                                              TS0205C
002205          IF(VFLAG) GO TO 300                                                 TS0206C
002212          IF(IBLK.EQ.0) CALL COREIN(A.DF.NEOB.ESTIF.FORCE.LD.NSTF)            TS0207C
002231          IF(IBLK.GT.0) WRITE(7) ESTIF.FORCE.LD                               TS0208C
002263          IF(.NOT.FLAGC.OR.CFLAG) GO TO 400
002266          L = 1
002267          DO 130 I = 1.NEL
002270          K = IX(I.N)
002276          DO 130 J = 1.NDF
002277          DU(J.K) = DU(J.K) + FORCE(L.2)
002311   130    L = L + 1
002317          GO TO 400                                                           TS0209C
002320   300    CONTINUE                                                            TS0210C
         C.... ADD THE FORCE TO THE SOLUTION FOR A RESOLVE                          TS0211C
002320          DO 310 K = 1.NSTF                                                   TS0212C
002322          J = LD(K)                                                           TS0213C
002325          IF(J.GT.0) DF(J) = DF(J) + FORCE(K.1)                               TS0214C
002335   310    CONTINUE                                                            TS0215C
002340   400    CALL TICTOC(TYME.3)                                                 TS0216C
002351          IF(FLAGC.AND..NOT.CFLAG) CALL CONTAC(2.IX.NEL1.NDF.NUMNP.DU)
002364          CFLAG = .TRUE.
002365          IF(NTB.GT.1) WRITE(ITWR) H                                          TS0217C
002405          IF(IBLK.GT.0) WRITE(ITWR) (IX(NE1.N).N=1.NUMEL)                     TS0219C
002436          IF(.NOT.VFLAG) CALL SOLVEQ(NUMNP.NUMEL.NDF.ID1.MB.MAXBAN.9.NSTF.    TS0220C
               1    ISZA.NEOB.IBLK.A.DU.DF.IDEST.FORCE.ESTIF.LD.MAXB.NDEG)          TS0221C
002476          IF(VFLAG) CALL RESVEQ(NUMNP.NDF.MB.MAXBAN.ISZA.NEOB.IBLK.A.DF.DF.   TS0222C
               1    IDEST.IDEST.MAXB.IFLG)                                          TS0223C
002524          CALL TICTOC(TYME.4)                                                 TS0224C
         C.... UPDATE THE SOLUTION                                                  TS0225C
002527          IF(NT.EQ.NTS.AND.M.EQ.NSEQ) GO TO 960                               TS0226C
002542          IF(NCT.GT.0) GO TO 410                                              TS0227C
002544          DTP = DT                                                            TS0228C
002545          I = ITRD                                                            TS0229C
002546          ITRD = ITWR                                                         TS0230C
002547          ITWR = I                                                            TS0231C
002550   410    IF(IBLK.GT.0) BACKSPACE ITRD                                        TS0232C
002554          IF(IBLK.GT.0) READ(ITRD) (IX(NE1.N).N=1.NUMEL)                      TS0233C
002605          REWIND ITRD                                                         TS0234C
002607          IF(IBLK.GT.0) READ(ITRD) ((U(I.J).I=1.NDF).J=1.KU)                  TS0235C
002642          IF(DFLAG) CALL NUNORM(3.NDEG.NT.U.DF.DU.F.PROP.NCT.NTT.IDEST.IBLK)  TS0236C
002666          VFLAG = .TRUE.                                                      TS0237C
002667          IF(NCT.LT.0) VFLAG = .FALSE.                                        TS0238C
[illegible]     NCT = NCT + 1                                                       TS0239C
[illegible]     IF(.NOT.IFLAG) GO TO 700
[illegible]     WRITE(6.2000) NCT.UNORM.ENORM.ANORM
[illegible]     IF(NCT.LT.NTT.AND.ENORM.GT.NORM*ERR) GO TO 14                      TS0240C
[illegible] 700 IF [illegible] AG) [illegible]=.FALSE.
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         C.... UPDATE (2)                                                          TS02450
002733         CALL UPDATE(2,MDEG,NICD,U,U(1,NUMNP+1),U(1,NEP+1),DU,F,DF,IDEST,    TS02460
              X   FROP)                                                            TS02470
002751         IF(FLAGC) CALL CONTAC(3,IX,NEL1,NDF,NUMNP,DU,U,U(1,NUMNP+1),
              X             U(1,NEP+1))
003016         MP = MP - 1                                                         TS02480
003020   800   TIME = TIME + DT                                                    TS02490
003024   900   CONTINUE                                                            TS02500
003027         IF(NUMPLT.GT.0) CALL PLOTGO(NDIM,NEL,DM,U)                          TS02510
003040         CALL TICTOC (TIME,6)                                                TS02520
003043         WRITE(ITP6,2033) TYME                                               TS02530
003052         RETURN                                                              TS02540
003053   901   WRITE(ITP6,2030)                                                    TS02550
003057         IPG = 0                                                             TS02560
003060         RETURN                                                              TS02570
         C.... FORMATS                                                             TS02580
003061  1000   FORMAT(F10.0,8I5,2F10.0,2I5)                                        TS02590
003061  1006   FORMAT(3I5)                                                         TS02600
003061  2000   FORMAT(A1,12A6,E13.5,17X,4HPAGE,I4//5X,23HTIME DEPENDENT SOLUTION   TS02610
              X//10X,17HTIME INCREMENT  = 1PE15.4/                                 TS02620
              X  10X,17HNUMBER OF STEPS = I5/                                      TS02630
              X  10X,17HPRINT INTERVAL  = I5/                                      TS02640
              X  10X,14HPRINT NODES    ,I5,3H TO,I5/                               TS02650
              X  10X,14HPRINT ELEMENTS,I5,3H TO,I5)                                TS02660
003061  2001   FORMAT(A1,12A6,E13.5,17X,4HPAGE,I4//5X,12HNODAL VALUES,5X,          TS02670
              X    27HPROPORTIONAL LOAD FACTOR = ,F8.3,5X, 8HSEQUENCE,I4,5X,
              X    9HTIME STEP,I4//12H NODAL POINT,9(1X,2A6),(12X,9(1X,2A6)) )
003061  2002   FORMAT(10X,17HITERATION LIMIT =,I5/1X)                              TS02700
003061  2005   FORMAT(4X,50H DESCRIPTION OF STRESS EVOLUTION PLOTS TO BE MADE.  /  TS02710
              X  5X,10H PLOT NO.  5X,8H ELEMENT ,7X, 9H XYZ-CODE,6X,9H SIG-CODE/)  TS02720
003061  2006   FORMAT(4(7X,I5,3X))                                                 TS02730
003061  2030   FORMAT(5X,53H**FATAL ERROR 33** TIME STEP ZERO FOR DYNAMIC PROBLEM  TS02740
              X 1)                                                                 TS02750
003061  2033   FORMAT      22X,12HELAPSED TIME //10X,25HINPUT PROPERTIES AND MESH  TS02760
              .F10.3 10X,25HCHECK AND PLOT INPUT DATA,F10.3 10X,14HFORM STIFFNESS  TS02770
              .F21.3 10X,21HSOLUTION OF EQUATIONS,F14.3/10X,15HOUTPUT STRESSES,    TS02780
              .F20.3 10X,27HIMPLICIT ALGORITHM    DISPL.,F8.3/10X,19HTOTAL TIME/IN TS02790
              PLICIT,F16.3 1)                                                      TS02800
003061  2001   FORMAT(5X,89H WARNING  IHK  BOTH THE PROPORTIONAL LOADING AND FOR   TS02810
              CE ARE BEING RESET ON EACH TIME STEP )                               TS02820
003061  2002   FORMAT 5 NCT I5,  UNORM E15.5  DNORM E15.5  ANORM E15.5)            TS02420
003061         END
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14.11.57 OUTPUT
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                  SUBROUTINE UPDATE(ISW,NDEG,NICD,U,UD,UDD,DU,F,DF,IDEST,PROP)          UPD   1C
             C***** UPDATE    ****** 12/14/73  ****************************************  UPD   2C
000016            DIMENSION U(1),DU(1),F(1),DF(1),IDEST(1),UD(1),UDD(1)                 UPD   3C
000016            COMMON/TAPES/ ITP5,ITP6                                               UPD   4C
000016            COMMON/TIMHIS/ TIME,DT,DTP,NH,ISZH,C0,C1,C2,C3,C4,C5,C6,NCT           UPD   5C
000016            GO TO (100,300,500), ISW                                              UPD   6C
             C.... UPDATE (1) * * PREUPDATE OF ACCELERATIONS FOR DYNAMIC SOLUTIONS.     UPD   7C
000024       100  DO 200 N = 1,NDEG                                                     UPD   8C
000026       200  UDD(N) = C4*UDD(N) - C5*UD(N)                                         UPD   9C
000035            RETURN                                                                UPD  10C
             C.... UPDATE (2) * * UPDATE THE SOLUTION FOR GENL. DISPL. VEL. ACCL.       UPD  11C
000035       300  DO 400 N = 1,NDFG                                                     UPD  12C
000037            DU(N) = DF(N)                                                         UPD  13C
000043            IF(IDEST(N).EQ.0) DU(N) = F(N)*PROP - U(N)                            UPD  14C
000053            TEMP = DU(N)                                                          UPD  15C
000056            IF(NICD.EQ.1) GO TO 400                                               UPD  16C
000060            P = UDD(N)                                                            UPD  17C
000061            UD(N) = C1*UD(N) + C2*P + C3*TEMP                                     UPD  18C
000067            UDD(N) = P + C6*TEMP                                                  UPD  19C
000073       400  U(N) = U(N) + TEMP                                                    UPD  20C
000100            RETURN                                                                UPD  21C
             C.... UPDATE(3) * * SET INTEGRATION CONSTANTS                              UPD  22C
000100       500  C6 = C6 + 0.5                                                         UPD  23C
000102            IF(C0.EQ.0.0) C0 = 0.25                                               UPD  24C
000104            WRITE(ITP6,2002) C0,C6                                                UPD  25C
000114            C5 = 1./C0/DT                                                         UPD  26C
000116            C4 = 1. - 0.5/C0                                                      UPD  27C
000121            C3 = C6*C5                                                            UPD  28C
000123            C2 = (1. - C6/C0/2.)*DT/C4                                            UPD  29C
000127            C1 = 1. - C6/C0 + C2*C5                                               UPD  30C
000134            C6 = C5/DT                                                            UPD  31C
000137            RETURN                                                                UPD  32C
000137      2002  FORMAT(10X,30HNEWMARK INTEGRATION PARAMETERS/                         UPD  33C
                 C10X17HBETA VALUE      = F6.3/                                         UPD  34C
                 C10X17HGAMMA VALUE     = F6.3)                                         UPD  35C
000137            END
```