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Finite generalized Hall planes and their collineation groups

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The study of finite translation planes is central to the study of finite projective planes. Almost all the known finite planes are either translation planes or their duals or appear in a derivable chain of planes based on a dual translation plane. The recent work of Ostrom and others has made it clear that the class of finite translation planes is quite large and by no means completely determined as yet. It is thus important that various types of finite translation plane be subjected to detailed study. Such studies must inevitably lead to a more complete theory of translation planes and a more intimate understanding of related classes of planes.

A chief concern of much recent work in finite projective plane theory has been the structure of collineation groups, because of the close connection between the existence of geometrical configurations and the existence of collineations. Thus it would seem worthwhile in studying particular classes of planes to give emphasis to their collineation groups.

This thesis is a study of a particular class of finite translation planes, namely generalized Hall planes, with emphasis on the collineations which they admit. A projective plane π is a generalized Hall plane with respect to the line l_∞ and the Baer subplane π_0 if and only if π is a translation plane with translation line l_∞, l_∞ is a line of π_0 and there is a group of collineations $G(\pi_0)$ of π which fixes π_0 pointwise and which is transitive on the points of $l_\infty \backslash (\pi_0 \cap l_\infty)$. The class of finite

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generalized Hall planes is quite wide and it is possible to construct a classification, into six disjoint sub-classes, in terms of the central collineations which they possess. This is achieved in Chapter IV.

The main problem to which attention is directed is the development of the theory of generalized Hall planes sufficiently far as to be able to determine the collineation groups of many of the known examples. Many types of translation plane have a fixed point (for example, semi-field planes) or a fixed pair of points (for example, most generalized André planes) on the translation line. It is not possible to establish either of these results for generalized Hall planes in general. However, in Chapter III we show that a finite generalized Hall plane of order $q^2 > 16$ has a set of points of order $q + 1$ on the translation line (in fact, $\pi_0 \cap l_\infty$ which is fixed by all collineations. This is the key to further results about collineation groups and also to the determination of many collineation groups.

A number of classes of generalized Hall planes are considered at various points and some of these have their full collineation group determined in Chapter V. Several of these classes of planes are due to the author.

The Hall planes were the first class of translation planes constructed with a collineation group $G(\pi_0)$, and it is not surprising that they can be usefully employed as a touchstone with which to compare other classes. A number of characterizations of the Hall planes within the class of generalized Hall planes are included in this thesis. For example, the following characterization of the Hall planes appears in Chapter IV. A finite generalized Hall plane is a Hall plane if and only if every point of $\pi_0 \cap l_\infty$ is the centre of a non-trivial central collineation.

In some cases it is possible, given a generalized Hall plane, to construct a chain of planes containing a plane which is not a translation plane nor a dual translation plane. Such planes are a certain type of semi-translation plane. This situation is investigated in Chapter VI and the knowledge of collineation groups of generalized Hall planes obtained in previous chapters is applied to determine the full collineation group of some of these semi-translation planes.