Finite Gyroradius Stabilization of Ballooning Modes in a Toroidal Geometry

by

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Abstract

The stabilizing influence of finite-ion-gyroradius effects on magnetohydrodynamic ballooning modes for a simple model toroidal equilibrium is demonstrated.

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At present it is widely believed that the critical beta (ratio of plasma to magnetic pressure) in toroidal systems is determined by the onset conditions for high mode number pressure-driven perturbations in regions of unfavorable field line curvature. These "ballooning instabilities" have been extensively studied in the magnetohydrodynamic (MHD) limit[1-3] with the general conclusion that the most dangerous modes occur for large toroidal mode numbers ($n \gg 1$). However, in more recent work it has been emphasized that in this large-n regime kinetic modifications due, for example, to finite-ion-gyroradius effects could prove to be important.[4] In the present paper the primary purpose is to demonstrate that for a simple model toroidal equilibrium the finite-ion-gyroradius effects can exert a significant stabilizing influence on the ballooning modes.

The derivation of the appropriate set of kinetic eigenmode equations governing ballooning instabilities in general axisymmetric toroidal systems has been given in detail in Ref. 4. In the present study these equations are applied to the model equilibrium employed in a number of previous ideal MHD calculations.[1-3] The model essentially corresponds to a large-aspect-ratio tokamak with circular flux surfaces over which the poloidal magnetic field is uniform but the shear is nonuniform. In a more physically consistent sense, this model can also be taken to represent a low beta plasma with locally steep pressure gradients giving rise to the ballooning (destabilizing) forces and to the shear modulation over the surface.[5] For the large aspect ratio limit of interest here, trapped-particle and drift resonance effects are treated as being negligibly small. Hence, the general set of equations[4] reduces to the following simplified form of the governing kinetic ballooning mode equation,

$$\frac{d}{d\eta}\left[1 + (s\eta - \alpha\sin\eta)^2\right]\frac{d\phi}{d\eta} + \alpha\left[\cos\eta + \sin\eta(s\eta - \alpha\sin\eta)\right]\phi$$

$$+ \Omega(\Omega - B)\left[1 + (s\eta - \alpha\sin\eta)^2\right]\phi = 0 \qquad (1)$$

where the mean shear and pressure gradient parameters are respectively $s \equiv d(\ell n q) / d(\ell n r)$ and $\alpha \equiv -(2Rq^2/B^2)dP/dr$, and $B \equiv \omega_{*pi}/\omega_A$ represents the kinetic contribution with ω_{*pi} being the usual (pressure-driven ion diamagnetic drift frequency and $\omega_A \equiv v_A/qR$ being the Alfven frequency. Also, q is the familiar safety factor, P is the plasma pressure, R/r is the aspect ratio, and $-\infty < \eta < \infty$ is the range of the extended poloidal variable introduced in the ballooning representation.[1]

In analyzing Eq. (1) note that in the absence of the last term, the ideal MHD ballooning mode equation[1-3] governing marginal stability conditions is recovered. The appropriate boundary condition for this problem is $\phi \to 0$ as $|\eta| \to \infty$. Making the transformation, $\psi = A^{1/2}\phi$, Eq. (1) can be cast in the convenient form,

$$\left(\frac{d^2}{d\eta^2} + F\right)\psi = 0 \qquad (2)$$

where

$$F = \Omega(\Omega - B) + \frac{\alpha\cos\eta}{A} - \left[\frac{s - \alpha\cos\eta}{A}\right]^2 ,$$

$$A \equiv 1 + (s\eta - \alpha\sin\eta)^2 , \qquad B = (\alpha\Lambda)^{1/2}/2 , \qquad \Lambda \equiv b/\epsilon_p ,$$

$$1/\epsilon_p \equiv - Rd(\ell n P)/dr , \qquad b \equiv k_\perp^2\rho_i^2/2 , \text{ and } k_\perp = nq/r$$

with n being the toroidal mode number and ρ_i being the ion gyroradius. Note that both ϵ_p and b are assumed to be small in this calculation.

Equation (2) has been solved numerically for an appropriate range of parameters with the results summarized by Fig. 1. Here are plotted the marginal stability curves as functions of the shear parameter, s, and the pressure parameter, α, for various values of the finite gyroradius parameter, Λ. In the ideal MHD limit, $\Lambda = 0$, the previously reported results[2] (dashed curve) are reproduced. As Λ is increased, the stable region is correspondingly enlarged, thus demonstrating the stabilizing influence of the finite-ion-gyroradius effects. This trend, of course, becomes appreciable when b (a measure of the stabilizing gyroradius contributions) exceeds ϵ_p (a measure of the destabilizing pressure-driven forces). Note also that the so-called second (high beta) stable regime[2,5,6] becomes more accessible for finite Λ. Specifically, as the pressure gradient and/or beta is increased at a fixed value of shear, the plasma can pass more readily into the second stable regime.

In summary, for the model toroidal equilibrium considered, the inclusion of finite gyroradius effects is found to exert a significant stabilizing influence on ballooning modes and is suggestive that the corresponding critical beta limit could also be improved. It should be emphasized, however, that an appreciable improvement in critical beta (β_c) would be possible <u>only if</u> the ideal MHD calculations of this quantity at infinite-n were found to be much lower than that at moderate values of n; i.e., $\beta_c(n \to \infty) \ll \beta_c(n \sim 10)$. As demonstrated in recent finite-n MHD calculations,[7] this can indeed be the case for certain types of realistic equilibria. More general kinetically-modified ballooning mode calculations, which are properly interfaced with actual self-consistent equilibria, are currently in progress.

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Figure Captions

Fig. 1. Stability boundaries for ballooning modes as a function of the shear parameter, $s = rq'/q$, and the pressure parameter, $\alpha = -(2Rq^2/B^2)\, dP/dr = 2\beta q^2/\epsilon_p$, at various values of the finite gyroradius parameter, $\Lambda = b/\epsilon_p$.

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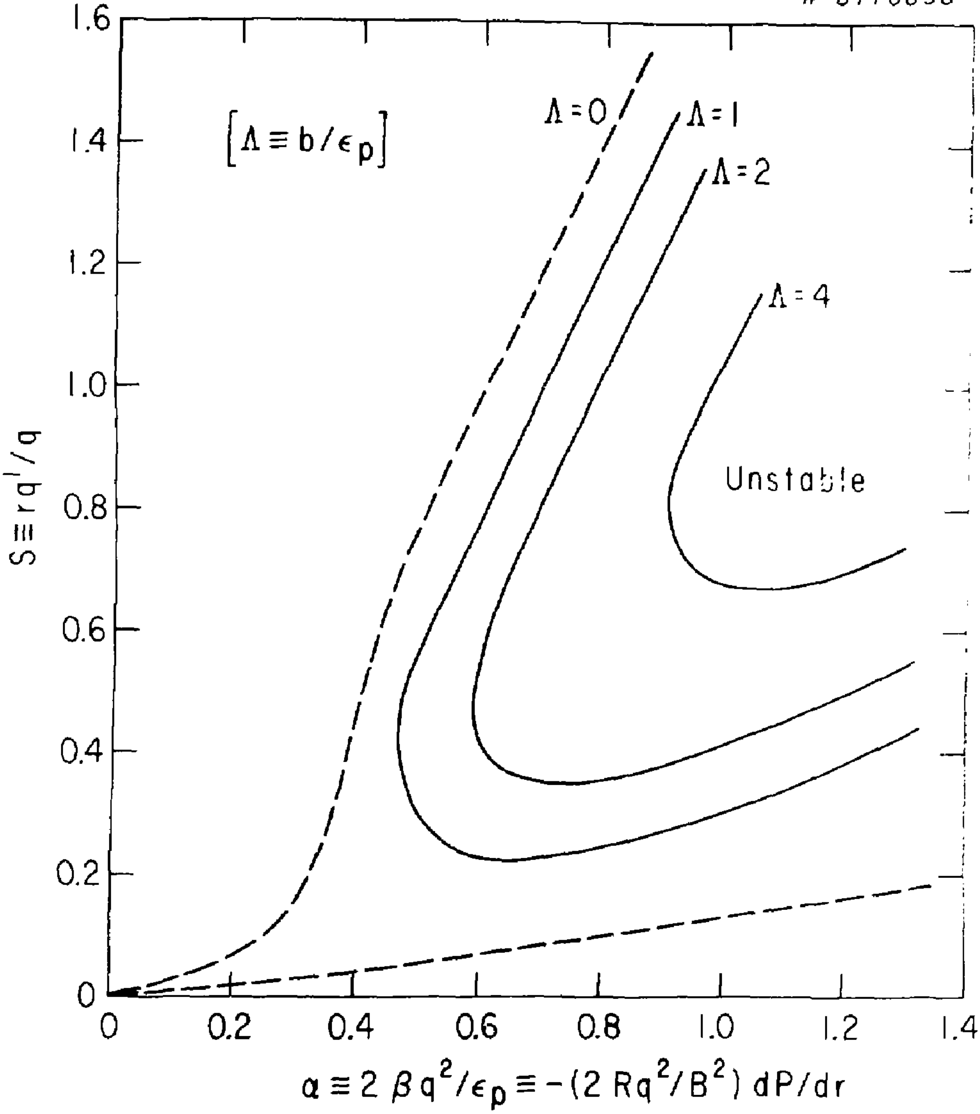

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