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Finite-time asynchronous dissipative filtering of conic-type nonlinear Markov jump systems

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Abstract In the present study, the finite-time asynchronous dissipative filter design problem for the Markov jump systems with conic-type nonlinearity is studied. The hidden Markov model can describe the asynchronism embodied in the system modes and the filter modes reasonably. Moreover, a suitable Lyapunov-Krasovskii function is utilized and linear matrix inequalities are applied to obtain adequate conditions. These techniques guarantee the finite-time boundedness and strict dissipativity of the filtering error dynamic system. Furthermore, the design problems of the passive filter and the H_∞ filter are studied by adjusting the three parameters $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{V}$. Finally, the filter gains and the optimal index α^* are obtained and the correctness and feasibility of the designed approach are verified by a simulation example.

Keywords dissipative filtering, hidden Markov model, finite-time boundedness, Markov jump systems, conic-type nonlinearity

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1 Introduction

In the past few decades, Markov jump systems (MJSs) have attracted attention of many researchers [1–11]. Studies show that as a special stochastic hybrid system with specific forms of modes and states, MJSs can provide appropriate models for diverse applications. Accordingly, these systems have been widely applied in many areas, including the intelligent control [12], financial field [13], and flight control systems [14]. In real applications, the modes of MJSs jump with a transfer probability so that it is an enormous challenge to guarantee the synchronization between the system and the controller. In order to prevent this problem, the hidden Markov model (HMM) is normally applied in a nonsynchronous phenomenon with the known mode-dependent conditional probability different from the transition probability of systems [1, 15, 16]. Researchers [17–19] designed the asynchronous controllers with the HMM with H_∞ control, passive control and robust filtering considerations. The conic-type nonlinearity is a special type of nonlinear dynamics on a hypersphere, where the center and radius of the hypersphere are described by two linear systems. In the practical engineering, the conic-type nonlinearity is widely applied in different applications such as the stability analysis for a class of time-delayed MJSs [20], observer design for hidden MJSs [21], and sliding mode control problems [22]. It is worth noting that comprehensive investigations have been conducted so far on the conic-type nonlinear systems [23–25].

Reviewing the literature indicates that scholars have conducted numerous investigations about the filtering problems. More specifically, Kalman [26] first proposed the famous Kalman filtering theory in the 1960s. He [27] investigated the finite-time L_2-L_∞ filtering for T-S fuzzy jumping systems. Moreover, Yin et al. [28] designed the fuzzy model-based robust filter. Consequently, Hua et al. [29] designed the

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Table 1 Nomenclature table

Notation	Description
$\boldsymbol{E}\{\cdot\}$	The mathematical expectation operator
$\epsilon_{\max}(U)$	The maximum eigenvalue of U
$\epsilon_{\min}(U)$	The minimum eigenvalue of U
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	$n\times m$ real matrix
diag$\{A\ B\}$	Block-diagonal matrix of A and B
I	Unit matrix
A^{-1}	Matrix inverse
A^{T}	Matrix transpose
$*$	Symmetric matrix
$\mathsf{Her}(A)$	The sum of A and transposition of A

H_∞ filtering scheme for nonlinear Markovian jump systems. However, it is worth noting that the system modes and the filter modes are always asynchronous in real applications. Combining the HMM with the filter, the asynchronous filtering scheme can be used for real problems. For example, Wu et al. [30] designed an HMM-based L_2-L_∞ filter. Moreover, Zhang et al. [31] designed the H_∞ filter for jumping neural networks with the HMM.

On the other hand, the dissipativity [32] is a research hot point based on the input-output energy consideration, which includes many basic theories such as the circle criterion, Kalman-Yakubovich lemma, and passivity theorem. Studies show that the ability of a dissipative system to absorb the energy from the external environment is greater than its ability to supply such energy. Hill and Moylan [33] proved the stability problem for nonlinear dissipative and passive systems. Moreover, Wu et al. [34] and Dong et al. [35] designed the asynchronous dissipative controller for fuzzy MJSs. Liu et al. [36] achieved the mean-square asymptotic stability and strict dissipativity of MJSs by designing an asynchronous output feedback controller. Moreover, Feng and Lam [37] proposed a robust reliable dissipative filter for discrete delay singular systems. Dai et al. [38] considered the HMM-based dissipative filtering scheme for discrete-time Markov jumping systems. Studies show that applying the asynchronous dissipative filter design problem has gained remarkable achievements [39–41]. However, the dissipative filtering for MJSs with conic-type nonlinearity based on the HMM has not been investigated comprehensively. In order to resolve this shortcoming, it is intended to study this topic in the present study. To this end, a dissipative filter is designed for MJSs with conic-type nonlinearity based on the HMM. Moreover, the finite-time boundedness and strict dissipativity will be investigated by the Lyapunov function approach. The main contributions of the present study are as follows.

(1) For MJSs with conic-type nonlinearity, a finite-time dissipative filter is designed, which combines an HMM and a mode-dependent conditional probability matrix.

(2) Reasonable conditions are obtained through an appropriate Lyapunov function, which can prove the finite-time boundedness and strict dissipativity of the MJSs with conic-type nonlinearity.

(3) The filter gains and the optimal dissipative index α^* are obtained by solving a set of linear matrix inequalities (LMIs).

(4) By adjusting the $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{V}$, the passive filtering, H_∞ filtering and the relevant optimal index α^* are obtained, respectively.

Table 1 presents the notations used in this study.

2 Preliminaries

Consider a probability space $(\Omega, \mathcal{F}, \mathcal{P}_r)$. $\{r_t, t\geqslant 0\}$ is a random process, which presents the continuous-time discrete-state Markov stochastic process. Its value is in a finite range of $\mathcal{L}=\{1,2,\ldots,L\}$ and its transition rate matrix $\Pi=[\lambda_{sl}]$ is described as

$$\Pr\{r(t+\Delta t)=l|r(t)=s\}=\begin{cases}\lambda_{sl}\Delta t+o(\Delta t), & s\neq l,\\ 1+\lambda_{ss}\Delta t+o(\Delta), & s=l,\end{cases} \tag{1}$$

where the time interval Δt of the infinitesimal transition satisfies $\lim_{\Delta t\to 0}\frac{o(\Delta t)}{\Delta t}=0$. Moreover, the jump rate from mode s at time t to mode l at time $t+\Delta t$ is presented by λ_{sl}, where $\lambda_{sl}\geqslant 0$ and

$\lambda_{ss} = -\Sigma_{s=1,s\neq l}^{L}\lambda_{sl}$. Consider the following conic-type nonlinear MJS:

$$\begin{cases} \dot{x}(t) = f(x(t), \omega(t)), \\ y(t) = C_{r(t)}x(t) + D_{r(t)}\omega(t), \\ z(t) = E_{r(t)}x(t), \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^q$ denote the state, measured output and the controlled output, respectively. Moreover, $\omega(t) \in \mathbb{R}^k$ is the external disturbance that belongs to $L_2[0, +\infty)$ and satisfies $\omega^{\mathrm{T}}(t)\omega(t) \leqslant \tilde{\omega}^2$. It should be indicated that $C_{r(t)}, D_{r(t)}, E_{r(t)}$ are known matrices with suitable dimensions. For the nonlinear function $f(x(t), \omega(t))$ which depends on $(x(t), \omega(t))$, it can be described by the following dynamic conic sector description:

$$\|f(x(t), \omega(t)) - [A_{r(t)}x(t) + B_{r(t)}\omega(t)]\| \leqslant 2\|A_{cr(t)}x(t) + B_{cr(t)}\omega(t)\|. \tag{3}$$

Remark 1. The conic-type nonlinear function $f(\cdot)$, which is defined on an n-dimensional hypersphere, can be described by inequality (3). Linear systems $A_{r(t)}x(t)+B_{r(t)}\omega(t)$ and $A_{cr(t)}x(t)+B_{cr(t)}\omega(t)$ describe the center and the radius of the hypersphere, respectively. It is worth noting that as a special type of nonlinear dynamics, the conic-type nonlinearity can represent many engineering nonlinear dynamics, including locally sinusoidal nonlinearity, saturation nonlinearity, dead zone nonlinearity and piecewise linear functions. More specifically, if the disturbance $\omega(t)$ does not exist, the Lipschitz nonlinearity can be obtained by (3).

When $r(t) = s$, substituting inequality (3) into MJS (2) results in the following expression:

$$\begin{cases} \dot{x}(t) = A_s x(t) + B_s\omega(t) + g_s(x(t), \omega(t)), \\ y(t) = C_s x(t) + D_s\omega(t), \\ z(t) = E_s x(t), \end{cases} \tag{4}$$

where $g_s(x(t), \omega(t)) = f(x(t), \omega(t)) - [A_s x(t) + B_s\omega(t)]$. On the other hand, the following expression can be obtained from inequality (3):

$$\|g_s(x(t), \omega(t))\|^2 \leqslant 2\|A_{cs}x(t) + B_{cs}\omega(t)\|^2. \tag{5}$$

In this case, the asynchronous filter for MJS is designed in the form below:

$$\begin{cases} \dot{x}_f(t) = A_{f\delta(t)}x_f(t) + B_{f\delta(t)}y(t), \\ z_f(t) = C_{f\delta(t)}x_f(t), \end{cases} \tag{6}$$

where $x_f(t) \in \mathbb{R}^n$, $z_f(t) \in \mathbb{R}^v$ and $y(t)$ denote the filtering state, filtering controlled output and the measured output, respectively. Moreover, $A_{f\delta(t)}, B_{f\delta(t)}$ and $C_{f\delta(t)}$ are the filtering parameters that should be designed. In (6), we adopt a variable $\delta(t)$ to denote the mode of the asynchronous filtering system. It shows that the actual system mode $r(t)$ can be observed/detected. Its value is within the range of $\mathcal{O} = \{1, 2, \dots, O\}$ and its conditional probability matrix $\Phi = [\phi_{sv}]$ is described as follows:

$$\Pr = \{\delta(t) = v | r(t) = s\} = \phi_{sv}, \tag{7}$$

where $\Sigma_{v=1}^{\mathcal{O}}\phi_{sv} = 1$. Meanwhile, the filtering system can be rewritten as

$$\begin{cases} \dot{x}_f(t) = A_{fv}x_f(t) + B_{fv}y(t), \\ z_f(t) = C_{fv}x_f(t). \end{cases} \tag{8}$$

The system (8) is substituted into MJS (4), and the state estimate error and the output estimate error are defined as $e(t) = x(t) - x_f(t)$ and $\tilde{z}(t) = z(t) - z_f(t)$, respectively. By defining $\tilde{x}(t) = \left[\begin{smallmatrix} x(t) \\ e(t) \end{smallmatrix}\right]$, the filtering error dynamic MJS can be rewritten as

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_{sv}\tilde{x}(t) + \tilde{B}_{sv}\omega(t) + \tilde{g}_s(x(t), \omega(t)), \\ \tilde{z}(t) = \tilde{C}_{sv}\tilde{x}(t), \end{cases} \tag{9}$$

where $\tilde{A}_{sv}(t) = \left[\begin{smallmatrix} A_s & 0 \\ A_s - B_{fv}C_s - A_{fv} & A_{fv} \end{smallmatrix}\right]$, $\tilde{B}_{sv}(t) = \left[\begin{smallmatrix} B_s \\ B_s - B_{fv}D_s \end{smallmatrix}\right]$, $\tilde{C}_{sv}(t) = \left[\begin{smallmatrix} E_s - C_{fv} & C_{fv} \end{smallmatrix}\right]$, $\tilde{g}_s(x(t), \omega(t)) = \left[\begin{smallmatrix} g_s(x(t),\, \omega(t)) \\ g_s(x(t),\, \omega(t)) \end{smallmatrix}\right]$.

Remark 2. Studies show that the mode operation of the Markov chain is of significant importance in real applications of MJSs. However, the mode $r(t)$ is not available for the filter. In other words, the real system mode is hidden to the filter, which causes some inaccuracies. Accordingly, the filter mode does not synchronize with the system mode. In the present study, $\delta(t)$ is regarded as the filter mode. The correlation between $\delta(t)$ and $r(t)$ is reflected by (7). Therefore, the filtering error dynamic MJS (9) can be regarded as a double random process. However, in the filter design, $A_{f\delta(t)}$, $B_{f\delta(t)}$ and $C_{f\delta(t)}$ only depend on $\delta(t)$, which can reflect the hidden model.

Based on the dissipative theory, the energy supply function for the filtering error dynamic MJS (9) can be described as

$$J(\tilde{z}(t), \omega(t), \mathcal{T}) = \int_0^{\mathcal{T}} \boldsymbol{E}\{S(\tilde{z}(t), \omega(t))\}\mathrm{d}t, \tag{10}$$

where $S(\tilde{z}(t), \omega(t)) = \tilde{z}^{\mathrm{T}}(t)\mathcal{U}\tilde{z}(t) + 2\tilde{z}^{\mathrm{T}}(t)\mathcal{G}\omega(t) + \omega^{\mathrm{T}}(t)\mathcal{V}\omega(t)$ is the supply rate. It should be indicated that real matrices $\mathcal{U}$, $\mathcal{G}$, $\mathcal{V}$ are known with $\mathcal{V} = \mathcal{V}^{\mathrm{T}}$, $\mathcal{U} = \mathcal{U}^{\mathrm{T}} < 0$, and $-\mathcal{U} = \underline{\mathcal{U}}^{\mathrm{T}}\underline{\mathcal{U}}$.

Definition 1. Given a time interval $[0, \mathcal{T}]$, positive scalars a_1, a_2 with $a_2 > a_1$ and a weighting matrix $S > 0$, the filtering error dynamic MJS (9) with $\int_0^{\mathcal{T}} \omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t \leqslant d$ $(d \geqslant 0)$ is stochastically finite-time bounded (FTB) respect to $(a_1, a_2, \mathcal{T}, S, d)$ if the following condition is satisfied [20]:

$$x^{\mathrm{T}}(0)Sx(0) \leqslant a_1 \Rightarrow \boldsymbol{E}\{x^{\mathrm{T}}(t)Sx(t) < a_2\}, \quad \forall t \in \{0, \mathcal{T}\}. \tag{11}$$

Remark 3. The FTB concept can be converted to the finite-time stability [42] if the parameter d is set to zero. It is worth noting that the concepts of FTB and Lyapunov stability are different. In fact, the Lyapunov stability mainly focuses on the steady state performance, while the FTB mainly analyzes the boundedness of the transient states. In the present study, it is intended to verify the finite-time boundedness of the designed filter instead of the Lyapunov stability.

Definition 2. For zero initial condition, the filtering error dynamic MJS (9) is strictly ($\mathcal{U}$, $\mathcal{G}$, $\mathcal{V}$)-α-dissipative, if the given scalars $\alpha > 0$ and $\mathcal{T} > 0$ satisfy the following inequality [34]:

$$J(\tilde{z}(t), \omega(t), \mathcal{T}) > \alpha \int_0^{\mathcal{T}} \omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t. \tag{12}$$

Lemma 1. Given two real matrices X and Y with suitable dimensions, a constant $\epsilon > 0$, and vectors $x, y \in \mathbb{R}^n$, the following inequality holds [20]:

$$2x^{\mathrm{T}}XYy \leqslant \epsilon^{-1}x^{\mathrm{T}}X^{\mathrm{T}}Xx + \epsilon y^{\mathrm{T}}Y^{\mathrm{T}}Yy. \tag{13}$$

3 Results and discussion

In this section, it is intended to prove that the filtering error dynamic MJS (9) is FTB. To this end, sufficient conditions are given by the following theorems.

Theorem 1. The filtering error dynamic MJS (9) is stochastically FTB respect to $(a_1, a_2, \mathcal{T}, S, d)$ under the given scalars $\gamma_s > 0$. In this case, for any $s \in \mathcal{L}$ and $v \in \mathcal{O}$, there are a set of mode-dependent scalars $\sigma_s > 0$ and positive definite symmetric matrices $P_s > 0$ satisfying the following matrix inequalities:

$$\Psi < 0, \tag{14}$$

$$S < P_s < \sigma_s S, \tag{15}$$

$$\mathrm{e}^{\gamma_s \mathcal{T}}\sigma_s a_1 + \frac{d}{\gamma_s}(1 - \mathrm{e}^{\gamma_s \mathcal{T}}) < a_2, \tag{16}$$

where

$$\Psi = \begin{bmatrix} \mathcal{A}_{slv} - \gamma_s P_s & \mathcal{B}_{sv} & \mathcal{C}_s & A_{cs}^{\mathrm{T}} \\ * & \mathcal{D}_{slv} - \gamma_s P_s & \mathcal{C}_s - \mathcal{F}_{sv} & 0 \\ * & * & -I & B_{cs}^{\mathrm{T}} \\ * & * & * & -\frac{1}{2}\varepsilon^{-1} I \end{bmatrix},$$

$\mathcal{A}_{slv} = \sum_{l=1}^{L} \lambda_{sl} P_l + \mathsf{Her}(A_s^{\mathrm{T}} P_s) + \varepsilon^{-1} P_s P_s$, $\mathcal{B}_{sv} = \sum_{v=1}^{\mathcal{O}} \phi_{sv}(A_s^{\mathrm{T}} P_s + C_s^{\mathrm{T}} B_{fv}^{\mathrm{T}} P_s - A_{fv}^{\mathrm{T}} P_s)$, $\mathcal{C}_s = P_s B_s$, $\mathcal{D}_{slv} = \sum_{l=1}^{L} \lambda_{sl} P_l + \Sigma_{v=1}^{\mathcal{O}} \phi_{sv}(\mathsf{Her}(A_{fv}^{\mathrm{T}} P_s)) + \varepsilon^{-1} P_s P_s$, $\mathcal{F}_{sv} = \sum_{v=1}^{\mathcal{O}} \phi_{sv}(P_s B_{fv} D_s)$.
Proof. A stochastic Lyapunov function candidate is selected as

$$V(\tilde{x}(t)) = \tilde{x}^{\mathrm{T}}(t) P_s \tilde{x}(t). \tag{17}$$

For this candidate, the weak infinitesimal generator of $V(\tilde{x}(t))$ can be described as

$$\begin{aligned} \Lambda V(\tilde{x}(t)) &= \tilde{x}^{\mathrm{T}}(t) \left(\sum_{l=1}^{L} \lambda_{sl} P_l \right) \tilde{x}(t) + 2 \sum_{v=1}^{\mathcal{O}} \phi_{sv} [\dot{\tilde{x}}^{\mathrm{T}}(t) P_s \tilde{x}(t)] \\ &= \tilde{x}^{\mathrm{T}}(t) \left(\sum_{l=1}^{L} \lambda_{sl} P_l \right) \tilde{x}(t) + 2 \sum_{v=1}^{\mathcal{O}} \phi_{sv} \{ \tilde{x}^{\mathrm{T}}(t) \tilde{A}_{sv}^{\mathrm{T}} P_s \tilde{x}(t) + \omega^{\mathrm{T}}(t) \tilde{B}_{sv}^{\mathrm{T}} P_s \tilde{x}(t) + \tilde{g}_s^{\mathrm{T}} P_s \tilde{x}(t) \}. \end{aligned} \tag{18}$$

Considering inequality (5) and Lemma 1, the following inequality can be obtained:

$$\begin{aligned} 2\tilde{g}_s^{\mathrm{T}} P_s \tilde{x}(t) &\leqslant \epsilon^{-1} \tilde{x}^{\mathrm{T}}(t) P_s P_s \tilde{x}(t) + \epsilon \tilde{g}_s^{\mathrm{T}} \tilde{g}_s \\ &\leqslant \epsilon^{-1} \tilde{x}^{\mathrm{T}}(t) P_s P_s \tilde{x}(t) + 2\epsilon [A_{cs} x(t) + B_{cs} \omega(t)]^{\mathrm{T}} [A_{cs} x(t) + B_{cs} \omega(t)]. \end{aligned} \tag{19}$$

For any $\gamma_s > 0$, the following equation is defined:

$$J_1(t) = \boldsymbol{E}\{\Lambda V(\tilde{x}(t)) - \gamma_s V(\tilde{x}(t)) - \omega^{\mathrm{T}}(t)\omega(t)\}. \tag{20}$$

Considering (18)–(20), the following expressions can be obtained:

$$\begin{aligned} J_1(t) &= \boldsymbol{E}\{\Lambda V(\tilde{x}(t)) - \gamma_s V(\tilde{x}(t)) - \omega^{\mathrm{T}}(t)\omega(t)\} \\ &\leqslant \eta^{\mathrm{T}}(t) \Psi_1 \eta(t) + 2\epsilon [A_{cs} x(t) + B_{cs} \omega(t)]^{\mathrm{T}} [A_{cs} x(t) + B_{cs} \omega(t)], \end{aligned} \tag{21}$$

where $\eta^{\mathrm{T}}(t) = [\tilde{x}^{\mathrm{T}}(t) \quad \omega^{\mathrm{T}}(t)]$, $\Psi_1 = \left[\begin{smallmatrix} \mathcal{M} - \gamma_s P_s & \Sigma_{v=1}^{\mathcal{O}} \phi_{sv} [\tilde{B}_{sv}^{\mathrm{T}} P_s] \\ * & -I \end{smallmatrix}\right]$, $\mathcal{M} = \Sigma_{l=1}^{L} \lambda_{sl} P_l + \Sigma_{v=1}^{\mathcal{O}} \phi_{sv} [\mathsf{Her}(\tilde{A}_{sv}^{\mathrm{T}} P_s)] + \epsilon^{-1} P_s P_s$. By substituting $\tilde{A}_{sv}^{\mathrm{T}}$ and $\tilde{B}_{sv}^{\mathrm{T}}$ into the filtering error dynamic MJS (9), the following expressions can be obtained:

$$J_1 \leqslant \xi^{\mathrm{T}}(t) \Psi_2 \xi(t) + 2\epsilon [A_{cs} x(t) + B_{cs} \omega(t)]^{\mathrm{T}} [A_{cs} x(t) + B_{cs} \omega(t)],$$

where $\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t) \quad \omega^{\mathrm{T}}(t)]$,

$$\Psi_2 = \begin{bmatrix} \mathcal{A}_{slv} - \gamma_s P_s & \mathcal{B}_{sv} & \mathcal{C}_s \\ * & \mathcal{D}_{slv} - \gamma_s P_s & \mathcal{C}_s - \mathcal{F}_{sv} \\ * & * & -I \end{bmatrix},$$

$\mathcal{A}_{slv} = \Sigma_{l=1}^{L} \lambda_{sl} P_l + \mathsf{Her}(A_s^{\mathrm{T}} P_s) + \varepsilon^{-1} P_s P_s$, $\mathcal{B}_{sv} = \Sigma_v^{\mathcal{O}} \phi_{sv}(A_s^{\mathrm{T}} P_s + C_s^{\mathrm{T}} B_{fv}^{\mathrm{T}} P_s - A_{fv}^{\mathrm{T}} P_s)$, $\mathcal{C}_s = P_s B_s$, $\mathcal{D}_{slv} = \Sigma_{l=1}^{L} \lambda_{sl} P_l + \Sigma_{v=1}^{\mathcal{O}} \phi_{sv}(\mathsf{Her}(A_{fv}^{\mathrm{T}} P_s)) + \varepsilon^{-1} P_s P_s$, $\mathcal{F}_{sv} = \Sigma_{v=1}^{\mathcal{O}} \phi_{sv}(P_s B_{fv} D_s)$. Meanwhile, when the Schur complement is applied for inequality (14), we have $J_1 < 0$. Then, the following inequality is mathematically expressed:

$$\boldsymbol{E}\{\Lambda V(\tilde{x}(t))\} < \gamma_s V(\tilde{x}(t)) + \omega^{\mathrm{T}}(t)\omega(t). \tag{22}$$

By multiplying the abovementioned inequality by $\mathrm{e}^{-\gamma_s t}$ and taking integration from 0 to t, the following expression is obtained:

$$\mathrm{e}^{-\gamma_s t} \boldsymbol{E}\{V(\tilde{x}(t))\} - \boldsymbol{E}\{V(0)\} < \int_0^t \mathrm{e}^{-\gamma_s t} \omega^{\mathrm{T}}(t)\omega(t) \mathrm{d}t. \tag{23}$$

Since $\gamma_s > 0$ and $t \in [0, \mathcal{T}]$, inequality (23) can be rewritten in the form below:

$$\begin{aligned}\boldsymbol{E}\{V(\tilde{x}(t))\} &= \boldsymbol{E}\{\tilde{x}^{\mathrm{T}}(t)P_s\tilde{x}(t)\} \\ &< \mathrm{e}^{\gamma_s t}\boldsymbol{E}\{V(0)\} + e^{\gamma_s t}d\int_0^t \mathrm{e}^{-\gamma_s\tau}\mathrm{d}\tau \\ &< \mathrm{e}^{\gamma_s t}\left[\tilde{x}^{\mathrm{T}}(0)P_s\tilde{x}(0) + \frac{d}{\gamma_s}(1 - \mathrm{e}^{-\gamma_s t})\right] \\ &\leqslant \mathrm{e}^{\gamma_s \mathcal{T}}\left[\tilde{x}^{\mathrm{T}}(0)P_s\tilde{x}(0) + \frac{d}{\gamma_s}(1 - \mathrm{e}^{-\gamma_s \mathcal{T}})\right]. \end{aligned} \tag{24}$$

Then, it is found that the following correlation holds:

$$\boldsymbol{E}\{\tilde{x}^{\mathrm{T}}(t)S\tilde{x}(t)\} < \frac{\mathrm{e}^{\gamma_s\mathcal{T}}\epsilon_{\max}(S^{-\frac{1}{2}}P_sS^{-\frac{1}{2}}\tilde{x}^{\mathrm{T}}(0)S\tilde{x}(0)) + \frac{d}{\gamma_s}(1 - \mathrm{e}^{-\gamma_s\mathcal{T}})}{\epsilon_{\min}(S^{-\frac{1}{2}}P_sS^{-\frac{1}{2}})}. \tag{25}$$

Based on inequality (15), it can be proved that $\epsilon_{\max}(S^{-\frac{1}{2}}P_sS^{-\frac{1}{2}}) < \sigma_s$ and $\epsilon_{\min}(S^{-\frac{1}{2}}P_sS^{-\frac{1}{2}}) > 1$. Accordingly, $\boldsymbol{E}\{\tilde{x}^{\mathrm{T}}(t)S\tilde{x}(t) < a_2\}$. Consequently, the proof is completed.

Theorem 2. The filtering error dynamic MJS (9) is stochastically FTB with respect to $(a_1, a_2, \mathcal{T}, S, d)$, and it is strictly $(\mathcal{U}, \mathcal{G}, \mathcal{V})$-dissipative under the given scalars $\gamma_s > 0$, if for any $s \in \mathcal{L}$ and $v \in \mathcal{O}$, there exist a set of mode-dependent scalars $\sigma_s > 0$ and positive definite symmetric matrices $P_s > 0$ satisfying (14)–(16) and the following matrix inequality:

$$\Xi < 0, \tag{26}$$

where

$$\Xi = \begin{bmatrix} \mathcal{A}_{slv} - \mathcal{H}_{sv} & \mathcal{B}_{sv} - \mathcal{I}_{sv} & \mathcal{C}_s - \mathcal{J}_{sv} & A_{cs}^{\mathrm{T}} \\ * & \mathcal{D}_{slv} - \mathcal{K}_{fv} & \mathcal{C}_s - \mathcal{F}_{sv} - C_{fv}^{\mathrm{T}}\mathcal{G} & 0 \\ * & * & \alpha I - \mathcal{V} & B_{cs}^{\mathrm{T}} \\ * & * & * & -\frac{1}{2}\varepsilon^{-1}I \end{bmatrix},$$

$\mathcal{A}_{slv} = \Sigma_{l=1}^{L}\lambda_{sl}P_l + \mathsf{Her}(A_s^{\mathrm{T}}P_s) + \varepsilon^{-1}P_sP_s$, $\mathcal{B}_{sv} = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(A_s^{\mathrm{T}}P_s + C_s^{\mathrm{T}}B_{fv}^{\mathrm{T}}P_s - A_{fv}^{\mathrm{T}}P_s)$, $\mathcal{C}_s = P_sB_s$, $\mathcal{D}_{slv} = \Sigma_{l=1}^{L}\lambda_{sl}P_l + \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(\mathsf{Her}(A_{fv}^{\mathrm{T}}P_s)) + \varepsilon^{-1}P_sP_s$, $\mathcal{F}_{sv} = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(P_sB_{fv}D_s)$, $\mathcal{H}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{U}(E_s - C_{fv})$, $\mathcal{I}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{U}C_{fv}$, $\mathcal{J}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{G}$, $\mathcal{K}_{fv} = C_{fv}^{\mathrm{T}}\mathcal{U}C_{fv}$.

Proof. In order to prove this theorem, an index function is initially defined as

$$J_2(t) = \boldsymbol{E}\{\Lambda V(\tilde{x}(t))\} - S(\tilde{z}(t), \omega(t)) + \alpha\omega^{\mathrm{T}}(t)\omega(t). \tag{27}$$

Based on (18) and (27), the defined function can be rewritten as

$$\begin{aligned} J_2(t) &= \boldsymbol{E}\{\Lambda V(\tilde{x}(t))\} - S(\tilde{z}(t), \omega(t)) + \alpha\omega^{\mathrm{T}}(t)\omega(t) \\ &\leqslant \eta^{\mathrm{T}}(t)\Xi_1\eta(t) + 2\epsilon[A_{cs}x(t) + B_{cs}\omega(t)]^{\mathrm{T}}[A_{cs}x(t) + B_{cs}\omega(t)] \\ &\quad - \tilde{x}^{\mathrm{T}}(t)\tilde{C}_{sv}^{\mathrm{T}}\mathcal{U}\tilde{C}_{sv}\tilde{x}(t) - 2\tilde{x}^{\mathrm{T}}(t)\tilde{C}_{sv}^{\mathrm{T}}\mathcal{G}\omega(t) - \omega^{\mathrm{T}}(t)\mathcal{V}\omega(t), \end{aligned} \tag{28}$$

where $\Xi_1 = [\begin{smallmatrix} \mathcal{M} & \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}[\tilde{B}_{sv}^{\mathrm{T}}P_s] \\ * & \alpha I \end{smallmatrix}]$. By substituting $\tilde{A}_{sv}^{\mathrm{T}}$, $\tilde{B}_{sv}^{\mathrm{T}}$ and $\tilde{C}_{sv}^{\mathrm{T}}$ into the filtering error dynamic MJS (9), it can be proved that $J_2 \leqslant \xi^{\mathrm{T}}(t)\Xi_2\xi(t) + 2\epsilon[A_{cs}x(t) + B_{cs}\omega(t)]^{\mathrm{T}}[A_{cs}x(t) + B_{cs}\omega(t)]$, where $\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t) \quad \omega^{\mathrm{T}}(t)]$,

$$\Xi_2 = \begin{bmatrix} \mathcal{A}_{slv} - \mathcal{H}_{sv} & \mathcal{B}_{sv} - \mathcal{I}_{sv} & \mathcal{C}_s - \mathcal{J}_{sv} \\ * & \mathcal{D}_{slv} - \mathcal{K}_{fv} & \mathcal{C}_s - \mathcal{F}_{sv} - C_{fv}^{\mathrm{T}}\mathcal{G} \\ * & * & \alpha I - \mathcal{V} \end{bmatrix},$$

$\mathcal{A}_{slv} = \Sigma_{l=1}^{L}\lambda_{sl}P_l + \mathsf{Her}(A_s^{\mathrm{T}}P_s) + \varepsilon^{-1}P_sP_s$, $\mathcal{B}_{sv} = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(A_s^{\mathrm{T}}P_s + C_s^{\mathrm{T}}B_{fv}^{\mathrm{T}}P_s - A_{fv}^{\mathrm{T}}P_s)$, $\mathcal{C}_s = P_sB_s$, $\mathcal{D}_{slv} = \Sigma_{l=1}^{L}\lambda_{sl}P_l + \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(\mathsf{Her}(A_{fv}^{\mathrm{T}}P_s)) + \varepsilon^{-1}P_sP_s$, $\mathcal{F}_{sv} = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(P_sB_{fv}D_s)$, $\mathcal{H}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{U}(E_s - C_{fv})$,

$\mathcal{I}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{U}C_{fv}$, $\mathcal{J}_{sv} = (E_s - C_{fv})^{\mathrm{T}}\mathcal{G}$, $\mathcal{K}_{fv} = C_{fv}^{\mathrm{T}}\mathcal{U}C_{fv}$. Meanwhile, when the Schur complement is implemented in inequality (26), it is found that $J_2 < 0$. Then the following inequality is obtained by integrating $J_2 < 0$ with zero initial conditions:

$$\boldsymbol{E}\left\{V(\tilde{x}(t)) - \int_0^{\mathcal{T}} S(\tilde{z}(t), \omega(t))\mathrm{d}t + \int_0^{\mathcal{T}} \alpha\omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t\right\} < 0. \tag{29}$$

Since $V(\tilde{x}(t)) > 0$, the following inequality is obtained:

$$\int_0^{\mathcal{T}} \boldsymbol{E}\{S(\tilde{z}(t), \omega(t))\}\mathrm{d}t > \alpha \int_0^{\mathcal{T}} \omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t. \tag{30}$$

Comparing the obtained inequality with expressions (10) and (12), the strict dissipativity of the filtering error dynamic MJS (9) is obtained. Consequently, the proof is completed.

Remark 4. Generally, there are two special cases of dissipative filtering, called the passive filtering and H_∞ filtering. It is proved that the filtering error dynamic MJS (9) is strictly dissipative in Theorem 2. Meanwhile, it is proved that the filtering error dynamic MJS (9) is passive or achieves a given H_∞ performance by adjusting $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{V}$ [34, 39].

Then, the following two special cases of dissipative filtering are obtained accordingly.

(1) Passive filtering: If parameters are set to $\mathcal{U} = 0$, $\mathcal{G} = 1$, $\mathcal{V} = 2\alpha$, the filtering error dynamic MJS (9) is passive.

(2) H_∞ filtering: If parameters are set to $\mathcal{U} = -I$, $\mathcal{G} = 0$, $\mathcal{V} = \alpha + \alpha^2$, the filtering error dynamic MJS (9) achieves the given H_∞ performance.

The correctness and feasibility of dissipative filtering and the two special cases will be discussed in Section 4.

In this section, inequalities in Theorems 1 and 2 should be transformed to a solvable form. Meanwhile, the filter gains and the optimal index α^* are obtained through the LMI tools. The adequate conditions are given in Theorem 3.

Theorem 3. The filtering error dynamic MJS (9) is stochastically FTB with respect to $(a_1, a_2, \mathcal{T}, S, d)$ and it is strictly $(\mathcal{U}, \mathcal{G}, \mathcal{V})$-dissipative under the given scalars $\gamma_s > 0$ and matrices $\mathcal{U}$, $\mathcal{G}$, $\mathcal{V}$, if for any $s \in \mathcal{L}$ and $v \in \mathcal{O}$, there are a set of mode-dependent scalars $\sigma_s > 0$, M, N and positive definite symmetric matrices $P_s > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ * & \Upsilon_3 \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & \Gamma_3 \end{bmatrix} < 0, \tag{32}$$

where

$$\Upsilon_1 = \begin{bmatrix} \chi_1 & \chi_2 & P_sB_s & A_{cs}^{\mathrm{T}} \\ * & \chi_3 & \chi_4 & 0 \\ * & * & -I & B_{cs}^{\mathrm{T}} \\ * & * & * & -\frac{1}{2}\epsilon^{-1}I \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} P_s & 0 & \mathfrak{G}_1 & 0 \\ 0 & P_s & 0 & \mathfrak{G}_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_3 = \mathrm{diag}\{-\epsilon I \quad -\epsilon I \quad \mathfrak{G}_2 \quad \mathfrak{G}_2\},$$

$\chi_1 = (\lambda_{ss} - \gamma_s)P_s + \mathsf{Her}(A_s^{\mathrm{T}}P_s)$, $\chi_2 = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(A_s^{\mathrm{T}}P_s + C_s^{\mathrm{T}}N_{sv}^{\mathrm{T}} - M_{sv}^{\mathrm{T}})$, $\chi_3 = (\lambda_{ss} - \gamma_s)P_s + \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}\mathsf{Her}(M_{sv}^{\mathrm{T}})$, $\chi_4 = P_sB_s - \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}N_{sv}D_s$,

$$\Gamma_1 = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & A_{cs}^{\mathrm{T}} \\ * & \theta_4 & \theta_5 & 0 \\ * & * & \alpha I - \mathcal{V} & B_{cs}^{\mathrm{T}} \\ * & * & * & -\frac{1}{2}\epsilon^{-1}I \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} P_s & 0 & \mathfrak{G}_3 & \mathfrak{G}_1 & 0 \\ 0 & P_s & \mathfrak{G}_4 & 0 & \mathfrak{G}_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_3 = \mathrm{diag}\{-\epsilon I \quad -\epsilon I \quad -I \quad \mathfrak{G}_2 \quad \mathfrak{G}_2\},$$

$\theta_1 = \lambda_{ss}P_s + \mathsf{Her}(A_s^{\rm T}P_s)$, $\theta_2 = \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}(A_s^{\rm T}P_s + C_s^{\rm T}N_{sv}^{\rm T} - M_{sv}^{\rm T})$, $\theta_3 = P_sB_s - (E_s - C_{fv})^{\rm T}\mathcal{G}$, $\theta_4 = \lambda_{ss}P_s + \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}\mathsf{Her}(M_{sv}^{\rm T})$, $\theta_5 = P_sB_s - \Sigma_{v=1}^{\mathcal{O}}\phi_{sv}N_{sv}D_s - C_{fv}^{\rm T}\mathcal{G}$, $\mathfrak{G}_1 = [\sqrt{\lambda_{s1}}I, \ldots, \sqrt{\lambda_{ss-1}}I, \sqrt{\lambda_{ss+1}}I, \ldots, \sqrt{\lambda_{sl}}I]$, $\mathfrak{G}_2 = -\mathrm{diag}\{P_1, \ldots, P_{s-1}, P_{s+1}, \ldots, P_l\}$, $\mathfrak{G}_3 = (E_s - C_{fv})^{\rm T}\underline{\mathcal{U}}^{\rm T}$, $\mathfrak{G}_4 = C_{fv}^{\rm T}\underline{\mathcal{U}}^{\rm T}$. Moreover, the filter parameters can be expressed as $A_{fv} = P_s^{-1}M_{sv}$, $B_{fv} = P_s^{-1}N_{sv}$.

Proof. Since inequalities (14) and (26) cannot be solved directly, we let $M_{sv} = P_sA_{fv}$ and $N_{sv} = P_sB_{fv}$. Then, the Schur complement is applied to get inequalities (31) and (32) by $\mathcal{U} = \mathcal{U}^{\rm T} < 0$ and $-\mathcal{U} = \underline{\mathcal{U}}^{\rm T}\underline{\mathcal{U}}$. Consequently, the proof is completed.

4 Simulation

In order to evaluate the correctness and feasibility of the designed approach, a two-jumping-mode MJS with the parameter listed below is studied in this section:

$$A_1 = \begin{bmatrix} -6.5 & -0.2 \\ 3 & -0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.4 & -1.1 \\ 0.8 & -0.9 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0.5 \\ -0.2 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & -0.5 \\ -0.2 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 2 & 0.3 \\ -1 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 2 & -0.1 \\ -1 & 0.1 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.5 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad A_{c1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{c2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_{c1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{c2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \omega(t) = \begin{bmatrix} \mathrm{e}^{-2.12t} \times \sin(0.05t) \\ \mathrm{e}^{-2.12t} \times \sin(0.05t) \end{bmatrix}.$$

In this case, the conic-type nonlinearity can be expressed as

$$g_s(x(t), \omega(t)) = \begin{bmatrix} 0.01 \times (|x_1 + 0.1| + |x_1 - 0.1|) \\ 0.01 \times (|x_1 + 0.1| + |x_1 - 0.1|) \end{bmatrix}.$$

The transition rate Π_{sl} and HMM conditional probability Φ are defined as $\Pi_{sl} = \left[\begin{smallmatrix} -4 & 4 \\ 5 & -5 \end{smallmatrix}\right]$, $\Phi = \left[\begin{smallmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{smallmatrix}\right]$. Moreover, the corresponding dissipative parameters are $\mathcal{U} = \left[\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}\right]$, $\mathcal{G} = 0.4$, $\mathcal{V} = 1.4$. By solving LMIs (15)-(16) and (31)-(32), the optimal dissipative index $\alpha^* = 0.1205$ and the filter gains can be obtained in the form below:

$$A_{f1} = \begin{bmatrix} -3.4661 & 0.7684 \\ 0.9328 & -1.3705 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} 0.0011 & -0.0003 \\ -0.0008 & -0.0018 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.1265 & 0.1484 \\ 0.1584 & 0.4189 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} 0.0006 & 0.0011 \\ 0.0010 & 0.0022 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} 0.1870 & -0.0302 \\ -0.0302 & 0.0887 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} 0.1651 & -0.0188 \\ -0.0188 & 0.0327 \end{bmatrix}.$$

Figure 1 illustrates the simulation results of the system and the filter modes. Figures 2 and 3 show the trajectories of state errors and output errors, respectively.

Then, the influence is analyzed with different Φ values. Table 2 indicates that three cases are considered. More specifically, Cases I–III represent the synchronous case, the partially asynchronous case, and the asynchronous case, respectively.

In Case I, the optimal synchronous dissipative performance index is obtained as $\alpha^* = 0.1581$. Moreover, the optimal partially asynchronous dissipative performance index in Case II is obtained as $\alpha^* = 0.1206$. It is found that the optimal asynchronous dissipative performance index in Case III is $\alpha^* = 0.1205$. It is observed that the optimal dissipative performance index α^* decreases as the asynchronous degree increases.

The abovementioned analysis indicates that the dissipative filtering includes the passive filtering and the H_∞ filtering if three parameters $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{V}$ are adjusted as the following.

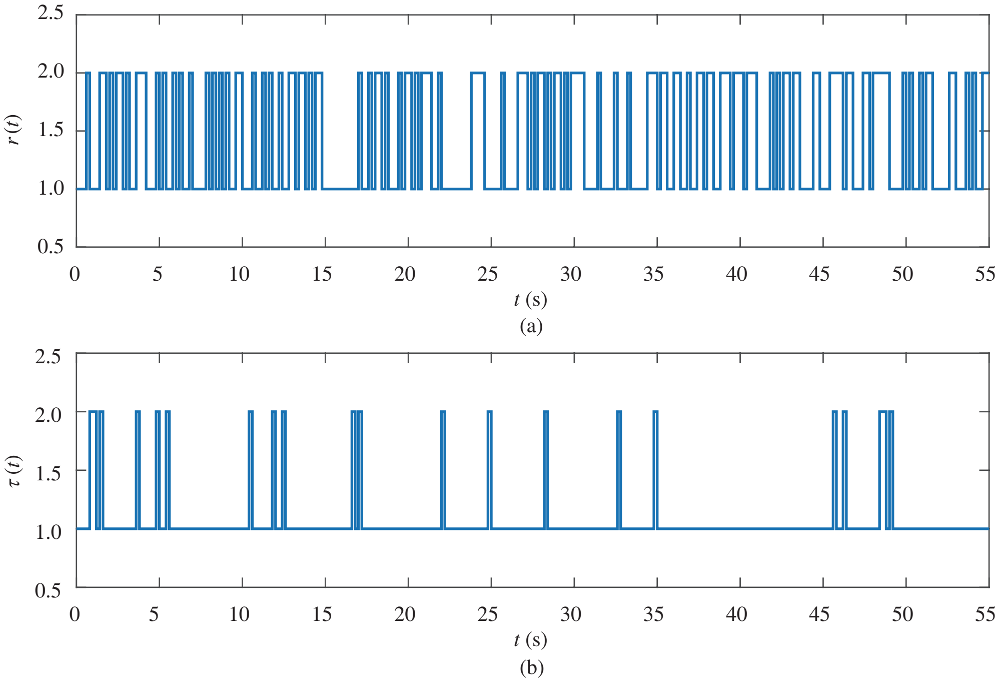

Figure 1 (Color online) (a) The system mode; (b) the filter mode.

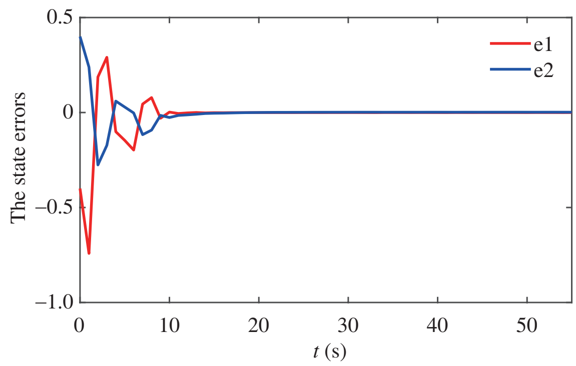

Figure 2 (Color online) Trajectories of state errors in the dissipative filtering case.

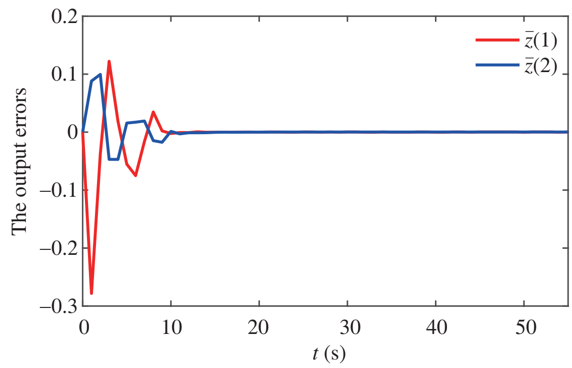

Figure 3 (Color online) Trajectories of output errors in the dissipative filtering case.

Table 2 Φ values for three various cases

Case I: synchronous	Case II: partially asynchronous	Case III: asynchronous
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0.25 & 0.75 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{bmatrix}$

(1) For the passive filtering, parameters are set to $\mathcal{U} = 0$, $\mathcal{G} = 1$, $\mathcal{V} = 2\alpha$. Under these circumstances, the filtering error dynamic MJS (9) is passive. By solving LMIs (31)-(32), the optimal passive index $\alpha^* = 0.6961$ and the passive filter gains are obtained as

$$A_{f1} = \begin{bmatrix} -3.4400 & 0.7805 \\ 0.9507 & -1.3433 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} -0.0021 & 0.0012 \\ 0.0024 & 0.0059 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.1163 & 0.1334 \\ 0.1462 & 0.3925 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} -0.0012 & -0.0028 \\ -0.0028 & -0.0056 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} 0.1093 & 0.0277 \\ 0.0277 & 0.0011 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} 0.1078 & 0.0292 \\ 0.0292 & -0.0082 \end{bmatrix}.$$

Figures 4 and 5 show the trajectories of state errors and output errors obtained in the passive filtering

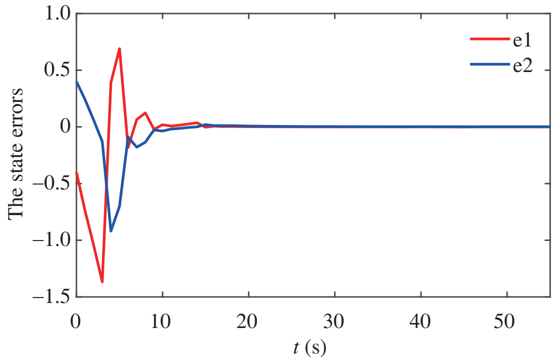

Figure 4 (Color online) Trajectories of state errors in the passive filtering case.

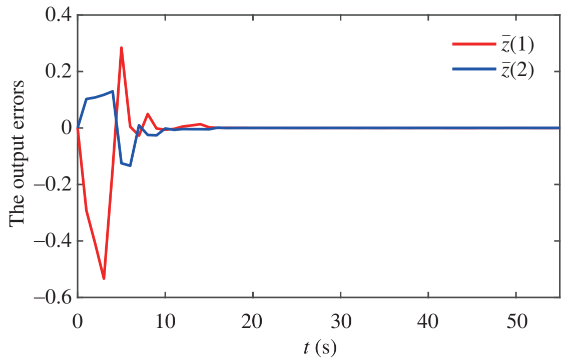

Figure 5 (Color online) Trajectories of output errors in the passive filtering case.

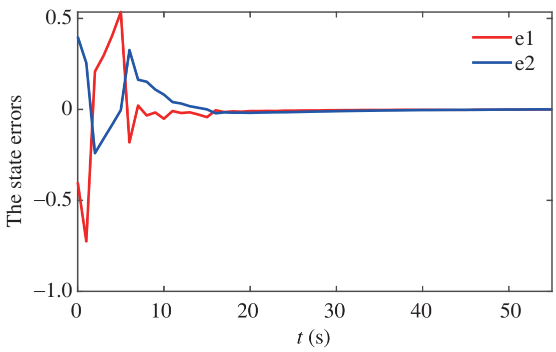

Figure 6 (Color online) Trajectories of state errors in the H_∞ filtering case.

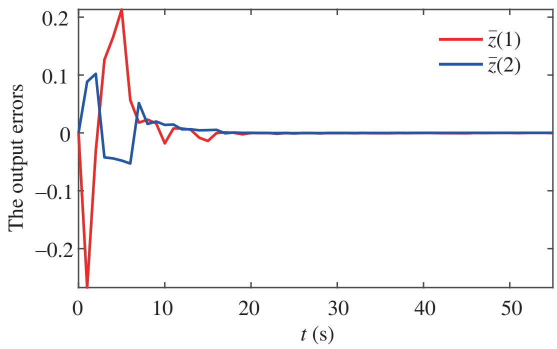

Figure 7 (Color online) Trajectories of output errors in the H_∞ filtering case.

case, respectively. It is observed that the trajectories of state errors and output errors tend to zero.

(2) The filtering error dynamic MJS (9) achieves the given H_∞ performance by letting $\mathcal{U} = -I$, $\mathcal{G} = 0$, $\mathcal{V} = \alpha + \alpha^2$. By solving LMIs (31)-(32), the optimal H_∞ index $\alpha^* = 1.6778$ and the H_∞ filter gains are obtained as

$$A_{f1} = \begin{bmatrix} -3.0837 & 0.9477 \\ 1.0343 & -0.7542 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} 0.00019 & 0.00006 \\ -0.00005 & -0.00007 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.0452 & -0.0027 \\ 0.0132 & 0.1163 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} -0.00016 & -0.00004 \\ -0.00003 & -0.00003 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} 0.2716 & -0.0557 \\ -0.0557 & 0.0595 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} 0.2716 & -0.0557 \\ -0.0557 & 0.0595 \end{bmatrix}.$$

Figures 6 and 7 illustrate the trajectories of state errors and output errors in the H_∞ filtering case, respectively. It is observed that the trajectories of state errors and output errors incline to zero. The simulation results in Figures 2–7 demonstrate that the designed filters, including the dissipative filter, the passive filter, and the H_∞ filter are feasible and applicable.

Remark 5. More recently, the HMM-based H_∞ filter for MJSs was designed [43–46]. Scholars [39–41] investigated the asynchronous dissipative filter of fuzzy MJSs. Comparing the obtained results with those reported for the H_∞ filter proves that the designed asynchronous dissipative filter can be effectively applied for nonlinear MJSs with finite-time boundedness, i.e., the conic-type nonlinear MJSs.

5 Conclusion

In the present study, the asynchronous dissipative filtering of MJSs with conic-type nonlinearity is designed. Moreover, an HMM is introduced to illustrate the nonsynchronous embodied in the system modes and the filter modes. Meanwhile, the stochastic finite-time boundedness and the strict dissipativity of the filter for the MJSs with conic-type nonlinearity have been verified by the proposed adequate conditions. The filter gains and the optimal index α^* are obtained by solving a set of LMIs. Finally, the correctness and feasibility of the designed approach are demonstrated by a given simulation example.

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