Finite-Time Bearing-Only Formation Tracking of Heterogeneous Mobile Robots with Collision Avoidance

Kefan Wu, Junyan Hu, *Member, IEEE*, Barry Lennox, *Senior Member, IEEE*, and Farshad Arvin, *Senior Member, IEEE*

***Abstract*—This brief proposes a bearing-only collision-free formation coordination strategy for networked heterogeneous robots, where each robot only measures the relative bearings of its neighbors to achieve cooperation. Different from many existing studies that can only guarantee global asymptotic stability (i.e., the formation can only be formed over an infinite settling period), a gradient-descent control protocol is designed to make the robots achieve a target formation within a given finite time. The stability of the multi-robot system is guaranteed via Lyapunov theory, and the convergence time can be defined by users. Moreover, we also present sufficient conditions for collision avoidance. Finally, a simulation case study is provided to verify the effectiveness of the proposed approach.**

***Index Terms*—Autonomous systems, bearing-only measurements, collision avoidance, multi-agent formation, mobile robots.**

I. Introduction

Inspired from natural swarms like fish schools and bird flocks, coordination algorithms of multi-robot teams have been explored in recent years, e.g., artificial pheromone system for swarm robotics [1], cooperative exploration in unknown environments [2], rendezvous of nonholonomic mobile robots [3], bio-inspired swarm shepherding strategies [4], motion tracking of mobile manipulators [5], [6], etc. Formation control is an emerging technique designed by the researchers, where the robots are coordinated to form a desired pattern around the target [7]. There are many potential real-world applications of formation control techniques, such as object transportation [8] and autonomous vehicle platooning [9].

With recent advancements in consensus theory and graph theory, distributed formation control of networked unmanned systems has become an emerging research topic in the area of robotics and control systems. Rao et al. [10] proposed a phase-based formation protocol for self-propelled vehicles. A two-layer formation-containment control framework was established in [11] for swarm systems. Liu et al. [12] studied collision-avoidance formation law for elliptical agents with dynamic mapping. However, in the aforementioned literature, numerous control protocols based on the condition that the distances or position among the agents are measurable, which requires high quality sensory system that is not always easy to be satisfied in GPS-denied environments. To deal with such limitations, the studies on bearing-only formation strategy have attracted much attention recently, where each robot can only detect the relative bearing information of its neighbors [13]. Compared to the position measurement, bearing-only method can minimize the requirements on the sensing ability.

This work was supported by the Engineering and Physical Sciences Research Council [grant numbers EP/R026084/1 and EP/P01366X/1] and EU Horizon-2020 [grant number 899520]. *(Corresponding author: Junyan Hu)*

The authors are with the Department of Electrical and Electronic Engineering, University of Manchester, Manchester, M13 9PL, U.K. (e-mail: junyan.hu@manchester.ac.uk; farshad.arvin@manchester.ac.uk)

In real-world applications, the relative bearing can be detected by vision-based localization systems and wireless sensor arrays. Hence, a bearing-only control protocol provides potential solutions to accomplish multi-robot cooperation tasks via on-board sensors. Zhao and Zelazo [14] proposed the bearing Laplacian matrix to verify the uniqueness of the target formation in higher dimension. Furthermore, Zhao et al. [15] extended the bearing-only protocols to deal with double-integrator and unicycle systems by gradient-descent approaches. However, in these two studies, only global asymptotic stability can be guaranteed, meaning that the desired formation can only be achieved over an infinite settling period. It is noticeable that convergence time is also a significant performance indicator in formation tasks. Hence, the finite-time control protocols have also been widely discussed in the literature. A finite-time consensus protocol was proposed for finite field networks in [16]. Zhang et al. [17] discussed finite-time formation control for multiple dynamic targets. A fixed-time observer-based control law for second-order systems was proposed in [18]. Several finite-time bearing-only formation designs were also analyzed in [19], [20]. However, the finite time is related to initial states and the control input may not be smooth because such controllers contain fractional power feedback and signum functions.

In this brief, we propose a finite-time bearing-only formation tracking protocol for heterogeneous multi-robot systems. Different from conventional position-based distributed control law, the coordination of each robot only depends on the relative bearings of its neighbors, which largely reduces the requirements on the sensing abilities. Furthermore, since the software and hardware of real robots may not be identical, robots with heterogeneous dynamics are also considered in the protocol design, which are more applicable in complicated formation tasks. The stability of the multi-robot system is guaranteed by Lyapunov theory and the convergence time to accomplish the target formation can be selected by users. Finally, we present sufficient condition to avoid the potential collisions among the robots.

II. Preliminaries and problem statement

A. Preliminaries

Consider n mobile robots (with n_l leaders and n_f followers) in $\mathbb{R}^d$ ($n \geq 2$, $d \geq 2$ and $n_l + n_f = n$). Let p_i be the position of ith robot. $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is the interaction topology among the robots, where $\mathcal{V} = \{v_1, \dots, v_n\}$ represents the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge set. The edge $(i, j) \in \mathcal{E}$ means that robot i can detect the relative bearing of robot j, and thus robot j is a neighbor of i. The set of neighbors of robot i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. It is obviously that $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$ if the interaction graph is undirected. $(\mathcal{G}, p)$ denotes the formation of $\mathcal{G}$ with its vertex i mapped to p_i for all $i \in \mathcal{V}$. Let $\mathcal{V}_l = \{1, \dots, n_l\}$ and $\mathcal{V}_f = \{n_l + 1, \dots, n\}$ denote the set of leaders and followers, respectively.

Suppose there are m undirected edges in $\mathcal{G}$ and each undirected edge can be given an arbitrary orientation. Then, we define the incidence matrix $H \in \mathbb{R}^{m \times n}$ for the oriented graph, where $[H]_{ki} = 1$ (or -1) if the node i is the head (or tail) node of the kth oriented edge, and $[H]_{ki} = 0$ otherwise. For an undirected topology, it shows that rank$(H) = n - 1$ and $H\mathbf{1}_n = 0$ [21].

Define the *edge vector* and *bearing vector* for edge (i, j), respectively, as

$$e_{ij} \triangleq p_j - p_i, \quad g_{ij} \triangleq \frac{e_{ij}}{\|e_{ij}\|} \tag{1}$$

where $\|\cdot\|$ is the spectral norm of a matrix or the Euclidean norm of a vector. The unit vector g_{ij} is the relative bearing of p_j with respect to p_i. Note that $e_{ij} + e_{ji} = g_{ij} + g_{ji} = 0$. For bearing vector g_{ij}, define

$$P_{g_{ij}} \triangleq I_d - g_{ij} g_{ij}^{\mathrm{T}} \in \mathbb{R}^{d \times d} \tag{2}$$

where $P_{g_{ij}}$ is an orthogonal projection matrix and $I_d \in \mathbb{R}^{d \times d}$ is the identity matrix. Note that $P_{g_{ij}} > 0$, $P_{g_{ij}} g_{ij} = 0$ and $P_{g_{ij}}^2 = P_{g_{ij}}$. As a result, $P_{g_{ij}} x = 0, \forall x \in \mathbb{R}^d \Leftrightarrow x$ is parallel to g_{ij}. $P_{g_{ij}}$ is important in bearing-based control and estimation problem [14]. Direct evaluation gives

$$\dot{g}_{ij} = \frac{P_{g_{ij}}}{\|e_{ij}\|} \dot{e}_{ij}. \tag{3}$$

Since $P_{g_{ij}} g_{ij} = 0$, we have $e_{ij}^\top \dot{g}_{ij} = g_{ij}^\top \dot{g}_{ij} = 0$.

Suppose the egde (i, j) corresponds to the kth directed edge in oriented graph where $k \in \{1, \cdots, m\}$. The edge and bearing vectors of the kth directed edge are defined as

$$e_k \triangleq e_{ij} = p_j - p_i, \quad g_k \triangleq g_{ij} = \frac{e_k}{\|e_k\|}. \tag{4}$$

Similarly, we have $e_k^\top \dot{g}_k = g_k^\top \dot{g}_k = 0$. According to definition of H, we also have $e = \bar{H}p$, where $e = \mathrm{col}(e_1, \cdots, e_m)$, $p = \mathrm{col}(p_1, \cdots, p_n)$, and $\bar{H} = H \bigotimes I_d$.

Let $p^* = \mathrm{col}(p_1^*, \cdots, p_n^*)$ denote configuration of the target formation $(\mathcal{G}, p^*)$. We introduce the *bearing Laplacian matrix* $\mathcal{B} \in \mathbb{R}^{dn \times dn}$ to describe the properties of p^*. The block of $\mathcal{B}$ is shown as [14]

$$[\mathcal{B}]_{ij} = \begin{cases} \mathbf{0_{d \times d}}, & i \neq j,\ (i, j) \notin \mathcal{E}, \\ -P_{g_{ij}^*}, & i \neq j,\ (i, j) \in \mathcal{E}, \\ \sum_{k \in \mathcal{N}_i} P_{g_{ik}^*}, & i = j,\ i \in \mathcal{V}. \end{cases} \tag{5}$$

We can imply that $\mathcal{B}p = B\mathbf{1}_{dn} = 0$ and $\mathcal{B} \geq 0$. In leader-follower case, the partition $\mathcal{B}$ can be written as

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_{ll} & \mathcal{B}_{lf} \\ \mathcal{B}_{lf}^\top & \mathcal{B}_{ff} \end{bmatrix} \tag{6}$$

where $\mathcal{B}_{ll} \in \mathbb{R}^{dn_l \times dn_l}$ and $\mathcal{B}_{ff} \in \mathbb{R}^{dn_f \times dn_f}$. To guarantee the uniqueness of the target formation, we have the following lemma.

Lemma 1: [14] The target formation p^* can be uniquely determined by the positions of the stationary leaders $\{p_i^*\}_{i \in \mathcal{V}_l}$ and the bearing vectors $\{g_{ij}^*\}_{(i,j) \in \mathcal{E}}$ if and only if $\mathcal{B}_{ff}$ is non-singular.

B. Problem statement

We consider multiple leaders and followers in the multi-robot teams with single-integrator dynamics. Suppose the leaders are stationary, that is to say, $\dot{p}_i = 0$ for $i \in \mathcal{V}_l$. For heterogeneous follower robots, the dynamics can be written as

$$\dot{p}_i(t) = S_i u_i(t), \quad i \in \mathcal{V}_f. \tag{7}$$

where $u_i \in \mathbb{R}^d$ represents the control input of the ith follower, the diagonal matrix $S_i = \mathrm{diag}(s_{i1}, s_{i2}, \cdots, s_{id}) \in \mathbb{R}^{d \times d}$ is the matrix parameter of agent i to describe heterogeneous followers and all the diagonal entries of S_i are positive ($s_{im} > 0,\ \forall 1 \leq m \leq d$).

The main objective of this brief is shown as follows.

Problem 1: Given a finite time T. Design the control input for each heterogeneous follower agent $i \in \mathcal{V}_f$ by only utilizing the bearing vectors $\{g_{ij}(t)\}_{j \in \mathcal{N}_i}$ such that p converges to p^* as $t \to T$ and $p = p^*$ as $t \geq T$.

Assumption 1 : The target formation is unique, i.e., $\mathcal{B}_{ff} > 0$.

Remark 1: In order to transfer the Problem 1 into a stabilization problem of bearing vectors $\{g_{ij}^*\}_{(i,j) \in \mathcal{E}}$ in finite time, we should link the target formation with the bearing vectors $\{g_{ij}^*\}_{(i,j) \in \mathcal{E}}$. Hence, by Lemma 1, we have the above Assumption 1, which is commonly used in bearing-only control problems (e.g., [13]–[15]).

III. Main results

In this section, we consider bearing-only formation tracking problem based on gradient-descent method to deal with Problem 1. Firstly, we introduce a time-varying function $\mu(t) \geq 0$. Let $\mu(t) = 1$ for $t \geq T$. When $t \in [0, T)$, $\mu(t)$ is expressed as

$$\mu(t) = \left(\frac{T}{T - t} \right)^h \tag{8}$$

where h is a parameter selected by user.

Motivated by [13], [15], the control protocol of each follower can be designed as

$$u_i(t) = (a + b\frac{\dot{\mu}}{\mu}) U_i \sum_{j \in \mathcal{N}_i} (g_{ij}(t) - g_{ij}^*(t)), \quad i \in \mathcal{V}_f, \tag{9}$$

where $U_i = \mathrm{diag}(s_{i1}^{-1}, s_{i2}^{-1}, \cdots, s_{id}^{-1})$, a and b are two positive control gains, and we adopt the right derivative of $\mu(t)$ at $t = T$. $\mu(t)$ in the protocol is significant in finite-time analysis.

$\forall\ c \in \mathbb{R}^+$, we have $\mu^c(0) = 1$ and $\lim_{t\to T^-} \mu^c(t) = 0$,and $\mu^c(t)$ is monotonically decreasing on $[0, T)$.

Let $g = \text{col}(g_1, \cdots, g_m)$ and $g^* = \text{col}(g_1^*, \cdots, g_m^*)$, in order to analyze the finite-time convergence of the system by gradient-descent method, we introduce the following lemmas.

Lemma 2: ([15]) Assume that there is no collision among robots. We have

$$2\max_k \|e_k\| p^\top \bar{H}^\top (g - g^*) \geq p^\top \mathcal{B} p \tag{10}$$

Lemma 3: ([13]) Suppose $z : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is a continuously differentiable function, if

$$\dot{z}(t) \leq -\eta z - \xi \frac{\dot{\mu}}{\mu} z, \quad t \in [0, \infty) \tag{11}$$

where η and ξ are positive. Then, we conclude that $z(t) = 0$ if $t \geq T$ and

$$z(t) \leq e^{-\eta t} \mu^{-\xi} z(0), \quad t \in [0, T). \tag{12}$$

Lemma 4: if $\mathbf{a}$ and $\mathbf{b}$ are two unit vectors. Let $\alpha_1 \geq \alpha_2 \geq 0$, then

$$\|\alpha_1 \mathbf{a} - \alpha_2 \mathbf{b}\| \geq \alpha_2 \|\mathbf{a} - \mathbf{b}\|.$$

Proof: Let ϕ denote the angle between the unit vector $\mathbf{a}$ and $\mathbf{b}$, we have

$$\begin{aligned}
&\|\alpha_1 \mathbf{a} - \alpha_2 \mathbf{b}\|^2 - (\alpha_2 \|\mathbf{a} - \mathbf{b}\|)^2 \\
&= \alpha_1^2 - \alpha_2^2 - 2\alpha_1\alpha_2 \cos\phi + 2\alpha_2^2 \cos\phi \\
&= (\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - 2\alpha_2 \cos\phi) \\
&\geq 2\alpha_2(\alpha_1 - \alpha_2)(1 - \cos\phi) \geq 0.
\end{aligned}$$

This completes the proof. ■

Let $u = \text{col}(u_{n_l+1}, \cdots, u_n)$, $e^* = \text{col}(e_1^*, \cdots, e_m^*)$, $\delta_i = p_i - p_i^*$, and $\delta = \text{col}(\delta_1, \cdots, \delta_n)$. The distributed finite-time bearing-only controller design is shown in the following Theorem .

Theorem 1: Under Assumption 1, if

$$\|\delta(0)\| \leq \frac{1}{\sqrt{n}} \left(\min_{i,j\in\mathcal{V}} \|p_i^* - p_j^*\| - \gamma \right), \tag{13}$$

where $\gamma \in (0, \min_{i,j\in\mathcal{V}} \|p_i^* - p_j^*\|)$ is a constant, a collision-free path can be generated for each robot and problem 1 can be solved by the control protocol (9). Furthermore, let $\tilde{p}^* = p^* - \mathbf{1}_n \bigotimes \bar{p}$ and $\bar{p} = \sum_{i=1}^n p_i^*/n$ denote the centroid of the target formation, if

$$bh\lambda_{min}(\mathcal{B}_{ff}) > 2\|\bar{H}\|(\|\delta(0)\| + \|\tilde{p}^*\|), \tag{14}$$

the control input u is uniformly bounded and C^1 smooth for $t \in [0, \infty)$.

Proof: By implementing control protocol (9), the compact form of (7) can be expressed as

$$\dot{p} = (a + b\frac{\dot{\mu}}{\mu}) \begin{bmatrix} 0 & 0 \\ 0 & I_{dn_f} \end{bmatrix} \bar{H}^\top (g - g^*). \tag{15}$$

We choose the Lyapunov function as $V = \frac{1}{2}\|\delta\|^2$. The derivative of V along the system is

$$\begin{aligned}
\dot{V} &= \delta^\top \dot{p} \\
&= -(a + b\frac{\dot{\mu}}{\mu})\delta \begin{bmatrix} 0 & 0 \\ 0 & I_{dn_f} \end{bmatrix} \bar{H}^\top (g - g^*) \\
&= -(a + b\frac{\dot{\mu}}{\mu})\delta \bar{H}^\top (g - g^*) \\
&= -(a + b\frac{\dot{\mu}}{\mu})(p - p^*) \bar{H}^\top (g - g^*) \\
&= -(a + b\frac{\dot{\mu}}{\mu})[e^\top (g - g^*) - (e^*)^\top (g - g^*)] \\
&= -(a + b\frac{\dot{\mu}}{\mu}) \sum_{k=1}^m (\|e_k\|(1 - g_k^\top g_k^*) + \|e_k^*\|(1 - (g_k^*)^\top g_k)) \\
&\leq 0.
\end{aligned} \tag{16}$$

Hence, we can imply that for any $t \geq 0$, $\|\delta(t)\| \leq \|\delta(0)\|$.

From (13), since

$$\begin{aligned}
\|p_i - p_j\| &= \|(p_i - p_i^*) - (p_j - p_j^*) + (p_i^* - p_j^*)\| \\
&\geq \|p_i^* - p_j^*\| - \|p_i - p_i^*\| - \|p_j - p_j^*\| \\
&\geq \|p_i^* - p_j^*\| - \sum_{m=1}^n \|p_m - p_m^*\| \\
&\geq \|p_i^* - p_j^*\| - \sqrt{n}\|p - p^*\| \\
&\geq \|p_i^* - p_j^*\| - \sqrt{n}\delta(0),
\end{aligned} \tag{17}$$

we have $\|p_i - p_j\| \geq \gamma$, $\forall t > 0$ and $\forall i, j \in \mathcal{V}$.

According to Lemma 2 and the fact $\mathcal{B}p^* = 0$ and $\delta = [0, \delta_f^\top]$, it follows from (16) that

$$\begin{aligned}
\dot{V} &\leq -(a + b\frac{\dot{\mu}}{\mu}) p \bar{H}^\top (g - g^*) \\
&\leq -(a + b\frac{\dot{\mu}}{\mu}) \frac{1}{2\max_k \|e_k\|} p^\top \mathcal{B} p \\
&= -(a + b\frac{\dot{\mu}}{\mu}) \frac{1}{2\max_k \|e_k\|} \delta^\top \mathcal{B} \delta \\
&= -(a + b\frac{\dot{\mu}}{\mu}) \frac{1}{2\max_k \|e_k\|} \delta_f^\top \mathcal{B} \delta_f \\
&\leq -(a + b\frac{\dot{\mu}}{\mu}) \frac{\lambda_{\min}(\mathcal{B}_{ff})}{2\max_k \|e_k\|} \|\delta\|^2.
\end{aligned} \tag{18}$$

Note that

$$\begin{aligned}
\max_k \|e_k\| \leq \|e\| = \|\bar{H}p\| &= \|\bar{H}(p - p^* + p^*)\| \\
&\leq \|\bar{H}\delta\| + \|\bar{H}p^*\| \\
&= \|\bar{H}\delta\| + \|\bar{H}\tilde{p}^*\| \\
&\leq \|\bar{H}\|(\|\delta\| + \|\tilde{p}^*\|) \\
&\leq \|\bar{H}\|(\|\delta(0)\| + \|\tilde{p}^*\|).
\end{aligned} \tag{19}$$

Combine (18) and (19), we obtain that

$$\begin{aligned}
\dot{V} &\leq -\underbrace{\frac{a\lambda_{\min}(\mathcal{B}_{ff})}{\|\bar{H}\|(\|\delta(0)\| + \|\tilde{p}^*\|)}}_{\bar{a}} V - \underbrace{\frac{b\lambda_{\min}(\mathcal{B}_{ff})}{\|\bar{H}\|(\|\delta(0)\| + \|\tilde{p}^*\|)}}_{\bar{b}} \frac{\dot{\mu}}{\mu} V \\
&= -\bar{a}V - \bar{b}\frac{\dot{\mu}}{\mu} V.
\end{aligned} \tag{20}$$

From Lemma 3, we have

$$\|\delta(t)\| \begin{cases} \leq e^{-\bar{a}t}\mu^{-\bar{b}}\|\delta(0)\|, & t \in [0, T) \\ \equiv 0, & t \in [T, \infty). \end{cases} \tag{21}$$

That is to say $p \to p^*$ in finite time T. Then, we will prove that u remains uniformly bounded and C^1 smooth.

By (15), we have

$$\|u\| \leq (a + b\frac{\dot{\mu}}{\mu})\|\bar{U}\bar{H}^\top\|\|(g - g^*)\|, \tag{22}$$

where $\bar{U} = diag(U_i)$. By Lemma 4, (13) and (17), we have

$$\begin{aligned} \|e - e^*\|^2 = &\sum_{i=1}^{m} \|g_i\|e_i\| - g_i^*\|e_i^*\|\|^2 \geq \gamma \sum_{i=1}^{m} \|g_i - g_i^*\|^2 \\ &\geq m\gamma\|g - g^*\|^2, \end{aligned} \tag{23}$$

then it follows

$$\|g - g^*\|^2 \leq \frac{1}{m\gamma}\|e - e^*\|^2 \leq \frac{1}{m\gamma}\|\bar{H}\|^2\|\delta(t)\|^2. \tag{24}$$

Combined (24) with (22), we have

$$\|g - g^*\| \begin{cases} \leq \sqrt{\frac{1}{m\gamma}}\|H\|\mu^{-\bar{b}}e^{-\bar{a}t}\|\delta(0)\|, & t \in [0, T) \\ \equiv 0, & t \in [T, \infty), \end{cases} \tag{25}$$

and

$$\|\frac{\dot{\mu}}{\mu}(g - g^*)\| \begin{cases} \leq \sqrt{\frac{1}{m\gamma}}\|H\|\frac{h}{T}\mu^{-(\bar{b}-\frac{1}{h})}e^{-\bar{a}t}\|\delta(0)\|, & \\ & t \in [0, T) \\ \equiv 0, & t \in [T, \infty), \end{cases} \tag{26}$$

from (14), we have $\bar{b} - \frac{1}{h} > 0$, so we can obtain

$$\lim_{t \to T^-} \|\frac{\dot{\mu}}{\mu}(g - g^*)\| = 0. \tag{27}$$

By (21), (22), (25), (26), and (27), it can be concluded that

$$\lim_{t \to T^-} \|u\| = 0. \tag{28}$$

That is to say u is uniformly bounded and continuous on $[0, \infty)$.

Next, we focus on the derivative of u. Since

$$\begin{aligned} \frac{du}{dt} =& \frac{bh}{T^2}\mu^{\frac{2}{h}}\bar{U}\bar{H}^\top(g - g^*) + (a + b\frac{\dot{\mu}}{\mu})\bar{U}\bar{H}^\top\dot{g} \\ =& \frac{bh}{T^2}\mu^{\frac{2}{h}}\bar{U}\bar{H}^\top(g - g^*) + (a + b\frac{\dot{\mu}}{\mu})\bar{U}\bar{H}^\top P\bar{H}\dot{p} \\ =& [\frac{bh}{T^2}\mu^{\frac{2}{h}}\bar{U}\bar{H}^\top + (a + b\frac{\dot{\mu}}{\mu})^2\bar{U}\bar{H}^\top P\bar{H}\bar{H}^\top](g - g^*) \end{aligned} \tag{29}$$

where $P = \text{diag}(\frac{P_{g_k}}{\|e_k\|})$. It is easily to see that $\frac{du}{dt}$ is continuous on $[0, T)$ and (T, ∞). Furthermore $\|P\|$ is bounded, so there exist $\Lambda > 0$ such that $\|\bar{U}\|\|\bar{H}^\top\|^2\|\bar{H}\|\|P\| < \Lambda$. Hence, from (29), we have

$$\begin{aligned} \left\|\frac{du}{dt}\right\| \leq& \frac{bh}{T^2}\mu^{\frac{2}{h}}\|\bar{U}\bar{H}^\top\|\|g - g^*\| + \Lambda(a + b\frac{\dot{\mu}}{\mu})^2\|g - g^*\| \\ =& [\Lambda a^2 + 2ab\Lambda\mu^{\frac{1}{h}} + (\Lambda b^2 + \frac{bh}{T^2}\|\bar{U}\bar{H}^\top\|)\mu^{\frac{2}{h}}]\|g - g^*\| \end{aligned} \tag{30}$$

From (14), we have $\bar{b} - (2/h) > 0$, similar to the analysis of (26), (27) and (30), we can imply that

$$\lim_{t \to T^-} \|\frac{du}{dt}\| = 0. \tag{31}$$

That is to say du/dt is uniformly bounded and continuous on $[0, \infty)$. So it can be concluded that the control input u is uniformly bounded and C^1 smooth for $t \in [0, \infty)$. ■

Remark 2: We utilize the forward difference method to deal with (15) with our controller. Since there is only one loop in the forward difference algorithm and the iterations of this algorithm is proportional to the finite-time T that is selected by the user, the computational complexity of this algorithm is $O(T)$. Hence, the proposed algorithm can be realized in real-time because it can ensure that the execution time increases linearly with the finite-time T.

Remark 3: The collision avoidance is considered in our protocol. We can observe that condition (13) is the sufficient condition to avoid collision from (17). If we select the initial positions of the follower robots properly to make the initial error satisfy (13), the collision will not appear in the process of tracking. When the formation size in a real-world implementation becomes very large, some of the robots may have occlusion problems when using vision systems to determine their relative orientation. However, since the proposed controller is distributed, the interaction topology of the robot network can be changed to ensure that each robot is able to detect at least one neighbor and thus the formation can still be achieved.

Remark 4: $\frac{\dot{\mu}}{\mu}$ plays a key role in finite-time formation task. From (26) and (30), we explore the connection between $\|u\|$ and $\|\delta\|$ and it can be seen that the control input u is bounded and C^1 smooth if condition (14) is satisfied. Condition (14) also reveals the lower boundary of bh, i.e., the increase of $\frac{\dot{\mu}}{\mu}$ is slower than the decrease of $\|\delta\|$ with large b and h. Furthermore, different from [18]–[20], the user-specified finite-time do not rely on the initial position because there is no signum functions or fractional power feedback in the controller.

IV. SIMULATION RESULTS

In this section, a simulation case study performed in Matlab is presented to validate the feasibility of the control protocol (9). Four omnidirectional mobile robots (i.e., two fixed leaders and two followers) with single-integrator dynamics are used in the task. For two heterogeneous followers, we set the parameters as $S_1 = I_d$ and $S_2 = 2I_d$. All the robots are expected to form a square shape target formation using bearing-only measurements. For the parameters, we set $a = 2$, $b = 5$, $h = 6$, and $T = 50$. In Fig. 1, the initial positions of the leaders (shown as green star and blue star) are $(1, 0)$ and $(5, 0)$, respectively. For the followers (marked by pink and yellow nodes), we choose their positions as $(-1.5, -2)$ and $(7.5, -1.5)$, which satisfy the conditions in Theorem 1. The formations of the robots at $t = 0$ s, $t = 10$ s, and $t = 50$ s are linked by blue dash lines, purple dash lines, and red solid lines, respectively. The pink and the yellow dotted lines are the trajectories of the followers from $t = 0$ s to

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2021.3066555, IEEE Transactions on Circuits and Systems II: Express Briefs

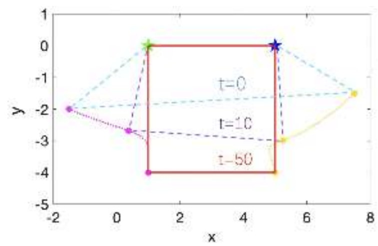

Fig. 1. Positions of the robots at different time instants.

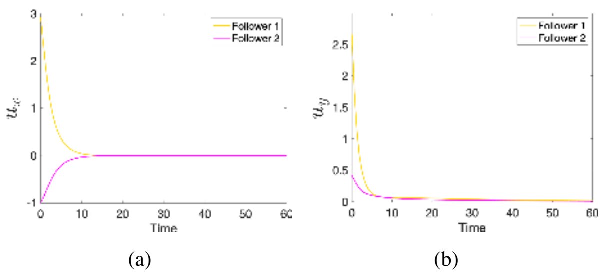

Fig. 2. Control inputs of the followers. (a) Along the X-axis. (b) Along the Y-axis.

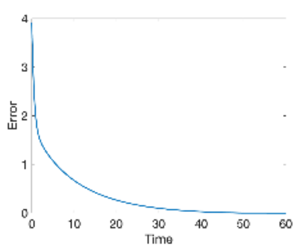

Fig. 3. Time variation of the formation tracking error $\|p - p^*\|$.

$t = 50$ s. Control inputs of the followers along the X and Y axes are illustrated in Fig. 2(a) and Fig. 2(b), respectively. It can be seen that the designed inputs are uniformly bounded and C^1 smooth as proved in Theorem 1. Fig. 3 shows that the formation tracking error $\|p - p^*\|$ reaches zero at $t = 50$ s. From the observed results, all the robots can form the target square formation within the given finite time T using bearing-only measurements.

V. Conclusion

This brief considered the bearing-only formation tracking problem in networked heterogeneous robots. A gradient-descent control law was firstly proposed to track the target formation within a given finite time. Instead of using the relative distance or position information as analyzed in the previous studies, each robot only has to detect the relative bearings of its neighbors. Heterogeneous dynamics of the robots were also considered in the protocol design, which were more applicable in real-world formation tasks involving different robotic platforms. Furthermore, the sufficient conditions for collision avoidance using the proposed method were also presented. Finally, the simulation case study showed the feasibility of the proposed control law. In future work, related to the protocol design, a robust method, such as [22], will be exploited to deal with external disturbances.

References

[1] S. Na, Y. Qiu, A. E. Turgut, J. Ulrich, T. Krajník, S. Yue, B. Lennox, and F. Arvin, "Bio-inspired artificial pheromone system for swarm robotics applications," *Adaptive Behavior*, 1059712320918936, 2020.

[2] J. Hu, H. Niu, J. Carrasco, B. Lennox, and F. Arvin, "Voronoi-based multi-robot autonomous exploration in unknown environments via deep reinforcement learning," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 14413–14423, 2020.

[3] P. Lu, S. Baldi, G. Chen, and W. Yu, "Neuro-adaptive cooperative tracking rendezvous of nonholonomic mobile robots," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3167–3171, 2020.

[4] J. Hu, A. E. Turgut, T. Krajník, B. Lennox, and F. Arvin, "Occlusion-based coordination protocol design for autonomous robotic shepherdingtasks," *IEEE Transactions on Cognitive and Developmental Systems*, 2020, DOI:10.1109/TCDS.2020.3018549.

[5] A. H. Khan, S. Li, D. Chen, and L. Liao, "Tracking control of redundant mobile manipulator: An RNN based metaheuristic approach," *Neurocomputing*, vol. 400, pp. 272–284, 2020.

[6] A. H. Khan, S. Li, and X. Luo, "Obstacle avoidance and tracking control of redundant robotic manipulator: An RNN-based metaheuristic approach," *IEEE Transactions on Industrial Informatics*, vol. 16, no. 7, pp. 4670–4680, 2019.

[7] P. Lu, H. Wang, F. Zhang, W. Yu, and G. Chen, "Formation control of nonholonomic mobile robots using distributed estimators," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3162–3166, 2020.

[8] H. Bai and J. T. Wen, "Cooperative load transport: A formation-control perspective," *IEEE Transactions on Robotics*, vol. 26, no. 4, pp. 742–750, 2010.

[9] J. Hu, P. Bhowmick, F. Arvin, A. Lanzon, and B. Lennox, "Cooperative control of heterogeneous connected vehicle platoons: An adaptive leaderfollowing approach," *IEEE Robotics and Automation Letters*, vol. 5, no. 2, pp. 977–984, 2020.

[10] P. Rao and X. Li, "Cooperative formation of self-propelled vehicles with directed communications," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 2, pp. 315–319, 2020.

[11] J. Hu, P. Bhowmick, and A. Lanzon, "Two-layer distributed formation-containment control strategy for linear swarm systems: Algorithm and experiments," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 16, pp. 6433–6453, 2020.

[12] Y. Liu, P. Shi, and C.-C. Lim, "Collision-free formation control for multi-agent systems with dynamic mapping," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 10, pp. 1984–1988, 2020.

[13] Z. Li, H. Tnunay, S. Zhao, W. Meng, S. Q. Xie, and Z. Ding, "Bearing-only formation control with prespecified convergence time," *IEEE Transactions on Cybernetics*, 2020.

[14] S. Zhao and D. Zelazo, "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions," *Automatica*, vol. 69, pp. 334–341, 2016.

[15] S. Zhao, Z. Li, and Z. Ding, "Bearing-only formation tracking control of multiagent systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 11, pp. 4541–4554, 2019.

[16] Y. Li, H. Li, and S. Wang, "Finite-time consensus of finite field networks with stochastic time delays," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3128–3132, 2020.

[17] W. Zhang, Y. Zhao, W. He, and G. Wen, "Time-varying formation tracking for multiple dynamic targets: Finite-and fixed-time convergence," *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2020.

[18] B. Tian, H. Lu, Z. Zuo, and W. Yang, "Fixed-time leader–follower output feedback consensus for second-order multiagent systems," *IEEE Transactions on Cybernetics*, vol. 49, no. 4, pp. 1545–1550, 2018.

[19] S. Zhao, F. Lin, K. Peng, B. M. Chen, and T. H. Lee, "Finite-time stabilisation of cyclic formations using bearing-only measurements," *International Journal of Control*, vol. 87, no. 4, pp. 715–727, 2014.

[20] Q. Van Tran, M. H. Trinh, D. Zelazo, D. Mukherjee, and H.-S. Ahn, "Finite-time bearing-only formation control via distributed global orientation estimation," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 2, pp. 702–712, 2018.

[21] C. Godsil and G. F. Royle, *Algebraic graph theory*. Springer Science & Business Media, 2013, vol. 207.

[22] Y.-B. Bae, H.-S. Ahn, and Y.-H. Lim, "Leader-follower bearing-based formation system with exogenous disturbance," in *IEEE International Symposium on Applied Computational Intelligence and Informatics*, 2019, pp. 39–44.