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Finite-time boundedness and stabilisation of switched linear systems using event-triggered controllers

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Abstract: This study proposes a systematic control design approach to consider jointly the event-triggered communication mechanism and state-feedback control for switched linear systems. The systems determine the necessary samplings of the feedback signal by constructing predefined events that can reduce redundant signal transmission and updates. Specifically, the first main step in the design is to construct sufficient conditions for stability analysis in the form of linear matrix inequalities to utilise fully the idea of average dwell time. With the proposed event-triggering mechanism, the design renders the resulting switched closed-loop system finite-time bounded. Subsequently, the authors present the conditions for finding the parameter of the event-triggered sampling mechanism and the state-feedback sub-controller gains. Then, the results for the full state feedback control case are further extended to systems incorporating observer-based state-feedback control motivated by practical applications. For each case, an estimate of the positive lower bound on the inter-execution times is further derived to avoid Zeno behaviour. A numerical example is presented to illustrate the effectiveness of the proposed methods.

1 Introduction

In recent years, event-triggered control has received increasing attention in the active development of systems and control theory [1]. In traditional control systems, the controllers periodically sample their input signals at a fixed rate and the controllers or actuators update their readings periodically. Such practices are usually classified into time-triggered schemes. However, from the perspective of resource utilisation, the time-triggered scheme may be unnecessarily consuming energy for communication and computation, which is especially undesirable for systems under strict energy constraints or network bandwidth limitations. It is desirable to find a way to guide the system to sample and transmit signals according to the current needs of the system's performance. For this reason, an event-triggered communication scheme has been proposed for further reducing energy cost and improving the efficiency of resource allocation [2].

Compared with time-triggered control, the event-triggered control shows its remarkable advantages. In the early works [3, 4], the event-triggering scheme is typically implemented in the way that the controller is invoked when a pre-defined triggering signal is large enough and exceeds a certain threshold. The event-triggered communication scheme is applied broadly to various systems. A more formal stabilising event-triggered control method is discussed in [5], where a triggering scheme was presented based on the difference between the plant's current state and its previous sampled state. A new strategy for event-triggered state-feedback control is presented when there is disturbance in the control loop [6]. In [7], the periodic sampling-based event-triggering scheme was designed and a time-delay model was developed. To further improve the efficiency of event-triggered control, decentralised event-triggered control is introduced to update communication information in wireless actuator systems [8]. Due to the increasing popularity, recent years have seen a growing interest in event-triggered control for networked control systems [9–11] and multi-agent systems [12–14], and the event-triggering scheme has been developed to be robust against communication delays, packet losses and out of order packets, to name a few.

As an important class of hybrid systems [15, 16], switched systems have been a hot topic in the field of control theory and applications, and a number of important results have been reported, see a survey [17] and some recent results including stability and stabilisation issues [18–20], switched non-linear systems [21, 22], switched time-delay systems [23–25], filter design [26, 27], iterative learning control [28] and asymptotic stability under sampled-data and quantisation [29]. However, if an event-triggering scheme is introduced into switched systems, how to guarantee the desired performance of the closed-loop systems, though a fundamentally important problem, has not been fully addressed yet. The studies in [30, 31] have not looked into the relationship between the sub-system switching and event-triggered instants. The results in [32] are established under certain restrictive conditions and in some cases may fail to exclude the Zeno sampling behaviour.

Along this line, we study the problem of event-based state-feedback control for switched linear systems. The design allows us to take sub-controller gains into consideration together with the event-triggering rule such that the resulting switched closed-loop system is finite-time bounded. The finite-time stability or boundedness has important practical significance, since many engineering systems have time response constraints [33–35]. More exactly, the event-triggered sub-controller executes control tasks when an error norm as a function of the system state reaches the triggering threshold. By utilising the methods of multiple Lyapunov functions and average dwell time switching law, sufficient conditions for the stability analysis are constructed; in addition, the event-triggered control parameters are designed for the resulting switched closed-loop system by the linear matrix inequality (LMI) technique. Moreover, an estimation of the lower bound on the event-triggered intervals is given to show the prevention of Zeno behaviour. Since in many control applications, the full state information is not always available through measurement, the obtained results of full state feedback control are further extended to the event-triggered, observer-based, state-feedback control.

So the contribution of the paper is four-fold. First, general sufficient conditions are systematically given for finite-time boundedness of switched linear systems incorporating event-triggered sampling. Second, Zeno behaviour, one of the most nasty behaviours due to event triggering, is clearly prevented through the design process. Third, since the event-triggering signals and switching signals may interlace with each other, the influence of

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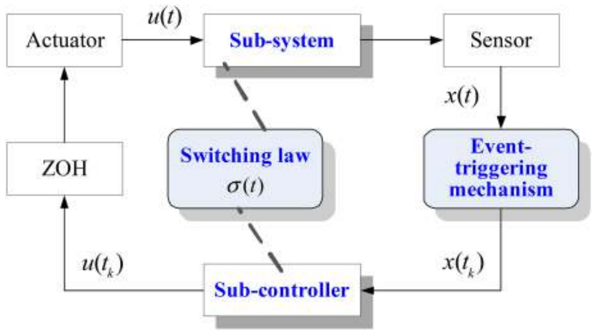

Fig. 1 *Event-triggered control framework of switched systems*

the coupling between the two signals on the analysis of stability and Zeno behaviour is clarified. Fourth, the event-triggered observer-based control design strategies are provided, which are more appealing in practice.

The structure of this paper is as follows. In Section 2, the problem statement and some preliminaries are described. In Section 3, the stability analysis and the control design for the event-triggered full state feedback control are developed. The results of the event-triggered observer-based state-feedback control are given in Section 4. To verify the effectiveness of the proposed event-triggered control methods, a numerical example is presented in Section 5. Section 6 concludes this paper.

Notation. In this study, the notations used are fairly standard. We denote by $\mathbb{R}$ the set of reals. We let $\mathbb{N}$ denote the set of natural numbers and define $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Given a vector $v \in \mathbb{R}^n$, $\| v \|$ is its Euclidean norm. Given a matrix M, M^{T} is its transpose and $\| M \|$ is its spectral norm. I represents the identity matrix. The notation $P > 0$(respectively, ≥ 0) means that P is real symmetric and positive definite (respectively, semi-positive definite). In symmetric block matrices, we use $*$ as an ellipsis for the terms that are introduced by symmetry. In addition, the upper Dini derivative will be used, which is defined as $D^{+}f(t) \triangleq \limsup_{h \to 0^{+}} (f(t+h) - f(t))/h$.

2 Problem statement and some preliminaries

Consider a switched continuous-time system of the form

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) &= C_{\sigma(t)}x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^l$ is the measured output, $\sigma(t): [0, +\infty) \to \underline{N} \triangleq \{1, 2, \ldots, N\}$ is the switching signal generated by a switching logic unit, and $\sigma(t) = i$ implies that the ith sub-system is active. Here A_i, B_i and C_i are real constant matrices of appropriate dimensions, and the pairs (A_i, B_i) and (C_i, A_i) are controllable and observable, respectively.

As shown in Fig. 1, for event-triggered control we need to design an event-triggering mechanism for monitoring the triggering condition and detecting when an event has occurred. Once such an event occurs, the current state of the system will be transmitted to the controller for updating the control input signal.

2.1 Event-triggering communication mechanisms

We will give in sequence two types of event-triggering mechanisms which are inspired and derived from one typical event-triggering mechanism in [6].

2.1.1 State-based event-triggering mechanism.:

The first adopted state-based event-triggering mechanism (SEM) is described by

$$t_{k+1} = \inf \{ t > t_k | \quad \| e_{\mathrm{ET}}(t) \| \geq \varepsilon_{\mathrm{sem}} \}, \tag{2}$$

where $e_{\mathrm{ET}}(t) = x(t_k) - x(t)$ is the error between the last transmitted system state and the current state, and the subscript ET indicates the first letter of the words ‘event’ and ‘triggering’. $\varepsilon_{\mathrm{sem}} > 0$ is a given threshold for event generation, and from (2), it can be deduced that $e_{\mathrm{ET}}^{\mathrm{T}}(t)e_{\mathrm{ET}}(t) \leq \bar{\varepsilon}_{\mathrm{sem}} = \varepsilon_{\mathrm{sem}}^2$. Intuitively, the parameter $\varepsilon_{\mathrm{sem}}$ will affect the update frequency of the control signal, and the smaller $\varepsilon_{\mathrm{sem}}$ is, the higher the update frequency will be.

For system (1), the full state feedback control $u(t) = K_{\sigma(t)}x(t)$ is first considered in this paper. In practice, the controllers are implemented by sampling the system state $x(t)$ using a sample-and-hold module at time instants $\{t_k\}_{k \in \mathbb{N}_0}$ and updating the control input as $u(t_k)$. We assume that the measuring of the state and the calculating and updating of control signal at each transmission instant are synchronised.

Then, the event-triggered controllers are given by

$$u(t) = K_{\sigma(t)}x(t_k), \quad t \in [t_k, t_{k+1}), \tag{3}$$

which implies that the controllers utilise the sampled $x(t_k)$ at the triggered instant t_k and the value of $x(t_k)$ will remain the same until the next instant t_{k+1}.

Furthermore, it is worth noting that the above event-triggering mechanism and controllers are based on state-feedback. In a practical system, since the full system state cannot always be directly measured and obtained, we will further study the problem of event-triggered observer-based state-feedback control for the switched continuous-time system (1).

2.1.2 Observed SEM.:

For system (1), the state observers are given by

$$\dot{\hat{x}}(t) = A_{\sigma(t)}\hat{x}(t) + B_{\sigma(t)}u(t) + L_{\sigma(t)}[y(t) - C_{\sigma(t)}\hat{x}(t)], \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state and $L_i \in \mathbb{R}^{n \times l}$ is the observer gain to be designed.

Correspondingly, the observed SEM (OSEM) is described by

$$t_{k+1} = \inf \{ t > t_k | \quad \| \hat{e}_{\mathrm{ET}}(t) \| \geq \varepsilon_{\mathrm{osem}} \}, \tag{5}$$

where $\hat{e}_{\mathrm{ET}}(t) = \hat{x}(t_k) - \hat{x}(t)$, $\varepsilon_{\mathrm{osem}} > 0$ is a given threshold and $\hat{e}_{\mathrm{ET}}^{\mathrm{T}}(t)\hat{e}_{\mathrm{ET}}(t) \leq \bar{\varepsilon}_{\mathrm{osem}} = \varepsilon_{\mathrm{osem}}^2$. Moreover, from the mechanism (5), the event-triggered observer-based controllers are given as (3) with the sampled estimated state $\hat{x}(t_k)$ replacing $x(t_k)$.

2.2 Problem statement

The following assumption, lemmas and definitions will be used in the rest of the paper.

Assumption 1: The matrices $C_{\sigma(t)}$ have a full row rank, i.e. rank $(C_{\sigma(t)}) = l$.

The singular value decomposition of the matrix C_i is $C_i = U_i[\Sigma_i \ 0]V_i$, where $\Sigma_i \in \mathbb{R}^{l \times l}$ is a diagonal matrix with positive diagonal elements in decreasing order, $U_i \in \mathbb{R}^{l \times l}$ and $V_i \in \mathbb{R}^{n \times n}$ are unitary matrices.

Lemma 1 [36]: For a given matrix C_i with a full row rank rank $(C_i) = l$, suppose that $M \in \mathbb{R}^{n \times n}$ is a symmetric matrix, then there exists a matrix $\hat{M} \in \mathbb{R}^{l \times l}$ such that $C_iM = \hat{M}C_i$, if and only if M has the following structure

$$M = V_i \begin{bmatrix} M_{11,i} & 0 \\ * & M_{22,i} \end{bmatrix} V_i^{\mathrm{T}},$$

where $M_{11,i} \in \mathbb{R}^{l \times l}$ and $M_{22,i} \in \mathbb{R}^{(n-l) \times (n-l)}$.

Lemma 2 [37]: Let L, E be real matrices of appropriate dimensions, then, for any scalar $\eta > 0$,

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$$L^{\mathrm{T}}E + E^{\mathrm{T}}L \le \eta L^{\mathrm{T}}L + \eta^{-1}E^{\mathrm{T}}E$$

Lemma 3 [38]: Assume $A \in \mathbb{R}^{n\times n}$ is Hurwitz, then there exists a positive scalar $c > 0$ such that

$$\| e^{At} \| \le c\, e^{((\lambda_{\max}(A))/2)t},$$

where $\lambda_{\max}(A) = \max_i \{\mathrm{Re}(\lambda_i(A))\}$.

Definition 1 [39]: For a switching signal $\sigma(t)$ and any $t_2 \ge t_1 \ge 0$, let $N_\sigma(t_1, t_2)$ be the number of switches of $\sigma(t)$ over the time interval (t_1, t_2). If $N_\sigma(t_1, t_2) \le N_0 + (t_2 - t_1)/\tau_a$ holds for $N_0 \ge 0, \tau_a > 0$, then τ_a is called the *average dwell time* and N_0 the *chatter bound*.

For SEM (2), from (1) and (3), the event-based switched closed-loop system between two consecutive instants t_k and t_{k+1} can be written as

$$\dot{x}(t) = \bar{A}_{\sigma(t)}x(t) + \bar{B}_{\sigma(t)}e_{\mathrm{ET}}(t), \tag{6}$$

where $\bar{A}_{\sigma(t)} = A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)}$ and $\bar{B}_{\sigma(t)} = B_{\sigma(t)}K_{\sigma(t)}$.

Without loss of generality, taking the state feedback case as an example, the boundedness definition is as follows.

Definition 2 [33]: System (6) is said to be *finite-time bounded* with respect to $(c_1, c_2, \bar{\varepsilon}_{\mathrm{sem}}, T_{\mathrm{f}}, R, \sigma(t))$ under switching signal $\sigma(t)$, if

$$\begin{aligned} &x^{\mathrm{T}}(0)Rx(0) \le c_1 \Rightarrow x^{\mathrm{T}}(t)Rx(t) \le c_2 \\ &\forall t \in [0, T_{\mathrm{f}}], \quad \forall e_{\mathrm{ET}}(t): e_{\mathrm{ET}}^{\mathrm{T}}(t)e_{\mathrm{ET}}(t) \le \bar{\varepsilon}_{\mathrm{sem}}, \end{aligned}$$

where $0 < c_1 < c_2$, $R > 0$.

We are interested in providing an event-based control method for switched linear systems, which consists of two problems:

i. Consider each type of the event-triggering mechanism, design the sub-controllers' gains and event generation threshold such that the resulting switched closed-loop system under an average dwell time (ADT) switching signal $\sigma(t)$ is finite-time bounded.
ii. In the resulting closed-loop system under each type of the event-triggering mechanism, provide a positive lower bound estimation on inter-execution times to exclude the Zeno behaviour.

3 Event-triggered full state feedback control

For problem (i), based on the multiple Lyapunov functions method and average dwell time technique, the following theorem shows that sufficient conditions can be established to guarantee finite-time boundedness of the switched closed-loop system (6) under SEM (2).

Theorem 1: For given positive constants c_1, c_2, η, α, $\bar{\varepsilon}_{\mathrm{sem}}$, T_{f}, and a constant symmetric matrix R, if there exist symmetric and positive definite matrices P_i with appropriate dimensions, rendering $\bar{P}_i = R^{-(1/2)}P_iR^{-(1/2)}$, $i \in \underline{N}$, such that

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^{\mathrm{T}} + \eta^{-1}\bar{B}_i\bar{B}_i^{\mathrm{T}} - \alpha\bar{P}_i < 0 \tag{7}$$

$$\frac{c_1}{\beta_1} + \frac{\tilde{\varepsilon}}{\alpha}(1 - e^{-\alpha T_{\mathrm{f}}}) < \beta_2^{-1}\mu^{-N_0}c_2\, e^{-\alpha T_{\mathrm{f}}}, \tag{8}$$

where $\tilde{\varepsilon} = \eta\bar{\varepsilon}_{\mathrm{sem}}$, then under SEM (2) the switched closed-loop system (6) is finite-time bounded with respect to $(c_1, c_2, \bar{\varepsilon}_{\mathrm{sem}}, T_{\mathrm{f}}, R, \sigma(t))$ for any ADT switching signal satisfying

$$\tau_a \ge \tau_a^* = \frac{T_{\mathrm{f}}\ln\mu}{\ln[(\beta_2^{-1}\mu^{-N_0}c_2)/((c_1/\beta_1) + (\tilde{\varepsilon}/\alpha)(1 - e^{-\alpha T_{\mathrm{f}}}))] - \alpha T_{\mathrm{f}}}, \tag{9}$$

where $\mu = (\beta_2/\beta_1)$, $\beta_1 = \min_{\forall i \in \underline{N}} \{\lambda_{\min}(P_i)\}$ and $\beta_2 = \max_{\forall i \in \underline{N}} \{\lambda_{\max}(P_i)\}$.

Before proving Theorem 1, we will show that the Zeno behaviour can be prevented, i.e. the following theorem will give a positive lower bound estimation on the inter-execution intervals under SEM (2).

Theorem 2: With any state-feedback gains $K_{\sigma(t)}$ and SEM (2), the inter-execution time $t_{k+1} - t_k$ is lower bounded by a positive constant $\bar{T}$ satisfying

$$\bar{T} = \frac{\varepsilon_{\mathrm{sem}}}{\phi_1 \| x(t_k) \| + (\phi_1 + \phi_2)\varepsilon_{\mathrm{sem}}}, \tag{10}$$

where $\phi_1 = \max_{\forall i \in \underline{N}} \{ \| \bar{A}_i \| \}$ and $\phi_2 = \max_{\forall i \in \underline{N}} \{ \| \bar{B}_i \| \}$.

Proof: Due to the introduction of the event-triggering scheme (2) in the switched system (1), the relationship between the sub-system's working interval $[l_q, l_{q+1})$ of the switched system and the inter-execution interval $[t_k, t_{k+1})$ of the event-triggering mechanism needs to be treated. Assume that the switched system is switched from sub-system i to j ($\forall i, j \in \underline{N}$) at the switching instant l_q, and $[l_q, l_{q+1})$ is the sustained working interval of sub-system j. To simplify notations, let $|.|$ represent $\| . \|$ and $e(t) \triangleq e_{\mathrm{ET}}(t)$. Moreover, in the following, the upper Dini derivative will be used.

Case 1: within any inter-execution interval $[t_k, t_{k+1}) \subset [0, T_{\mathrm{f}}]$, $k \in \mathbb{N}_0$, there is no switching. From system (6) and the definition of $e_{\mathrm{ET}}(t)$ in SEM (2), the following inequality is derived on $[t_k, t_{k+1}), k \in \mathbb{N}_0$, i.e.

$$\begin{aligned} D^+|e(t)| &= \limsup_{h \to 0^+} \frac{|e(t+h)| - |e(t)|}{h} \\ &\le \limsup_{h \to 0^+} \frac{|e(t+h) - e(t)|}{h} \\ &= \lim_{h \to 0^+} \frac{|e(t+h) - e(t)|}{h} \\ &= |\dot{e}(t)| \\ &= |\dot{x}(t)| \\ &\le |\bar{A}_{\sigma(t_k)}||x(t)| + |\bar{B}_{\sigma(t_k)}||e(t)| \\ &\le |\bar{A}_{\sigma(t_k)}||x(t_k)| + (|\bar{A}_{\sigma(t_k)}| + |\bar{B}_{\sigma(t_k)}|)|e(t)| \\ &\le |\bar{A}_{\sigma(t_k)}||x(t_k)| + (|\bar{A}_{\sigma(t_k)}| + |\bar{B}_{\sigma(t_k)}|)\varepsilon_{\mathrm{sem}} \end{aligned} \tag{11}$$

Letting $\psi_{\sigma(t)} = |\bar{A}_{\sigma(t)}|\,|x(t_k)| + (|\bar{A}_{\sigma(t)}| + |\bar{B}_{\sigma(t)}|)\,\varepsilon_{\mathrm{sem}}$, one has

$$\begin{aligned} \int_{t_k}^{t} D^+|e(s)|\mathrm{d}s &\le \int_{t_k}^{t} \psi_{\sigma(t_k)}\mathrm{d}s \\ &\le \int_{t_k}^{t} [\phi_1|x(t_k)| + (\phi_1 + \phi_2)\varepsilon_{\mathrm{sem}}]\mathrm{d}s \end{aligned}$$

Case 2: within any inter-execution interval $[t_k, t_{k+1})$, there are switchings, and we assume that $t_k \le l_q < l_{q+1} < l_{q+2} < \cdots < l_{q+n_1} \le t < l_{q+n_1+1} < \cdots < l_{q+n_1+n_2} \le t_{k+1}, \forall q + n_1 + n_2 \in \underline{N}, n_1, n_2 \in \mathbb{N}$. Then, one has

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$$
\begin{aligned}
&\int_{t_k}^{t} D^{+}|e(s)|\mathrm{d}s \\
&\le \int_{t_k}^{l_q} D^{+}|e(s)|\mathrm{d}s + \sum_{i=1}^{n_1}\int_{l_{q+i-1}}^{l_{q+i}} D^{+}|e(s)|\mathrm{d}s \\
&\quad + \int_{l_{q+n_1}}^{t} D^{+}|e(s)|\mathrm{d}s \\
&\le \int_{t_k}^{l_q} \psi_{\sigma(t_k)}\mathrm{d}s + \sum_{i=1}^{n_1}\int_{l_{q+i-1}}^{l_{q+i}} \psi_{\sigma(l_{q+i-1})}\mathrm{d}s \\
&\quad + \int_{l_{q+n_1}}^{t} \psi_{\sigma(l_{q+n_1})}\mathrm{d}s \\
&\le \int_{t_k}^{l_q} [\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}]\mathrm{d}s \\
&\quad + \sum_{i=1}^{n_1}\int_{l_{q+i-1}}^{l_{q+i}} [\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}]\mathrm{d}s \\
&\quad + \int_{l_{q+n_1}}^{t} [\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}]\mathrm{d}s \\
&\le \int_{t_k}^{t} [\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}]\mathrm{d}s
\end{aligned}
$$

which is consistent with that of Case 1.
Moreover, noticing the fact that $e(t_k) = 0$, one obtains

$$|e(t)| \le [\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}](t - t_k)$$

From this and SEM (2), letting

$$[\phi_1|x(t_k)| + (\phi_1+\phi_2)\varepsilon_{\mathrm{sem}}](t - t_k) = \varepsilon_{\mathrm{sem}}$$

hold for any $t_k \le t \le t_{k+1}$, then by denoting $\bar{T} = t - t_k$, a lower bound on the inter-execution interval can be obtained as (10). It can be shown from (10) that $\bar{T} > 0$ for any given event-triggered instant t_k. □

Remark 1: It is known that if the value of the triggering threshold $\varepsilon_{\mathrm{sem}}$ is chosen to be larger, it will take a longer time to reach the defined triggering condition. Also, if the threshold $\varepsilon_{\mathrm{sem}}$ is set to a small value, the triggering bound will become tighter, which will in turn induce more triggering. This fact can be verified by the estimation equation (10), from which it can be concluded that a larger event-triggering threshold $\varepsilon_{\mathrm{sem}}$ results in a larger inter-execution interval $\bar{T}$. In addition, it can also be verified that when the system state $x(t_k)$ converges to a small value, the triggering frequency will decrease and the inter-execution interval will become larger.

Proof of Theorem 1: Choose the Lyapunov function candidate as

$$V(x(t)) = V_{\sigma(t)}(x(t)) = x^{\mathrm{T}}(t)\bar{P}_{\sigma(t)}^{-1}x(t)$$

Case 1: for any $t \in [l_q, l_{q+1})$, if it falls within the triggered inter-execution interval, i.e. $t_k \le l_q$ and $t_{k+1} \ge l_{q+1}$. Then, from Lemma 2 and conditions (2) and (7), the time derivative of $V(x(t))$ for $t \in [l_q, l_{q+1})$ along the trajectory of system (6) yields

$$
\begin{aligned}
\dot{V}(x(t)) &= \dot{x}^{\mathrm{T}}(t)\bar{P}_{\sigma(l_q)}^{-1}x(t) + x^{\mathrm{T}}(t)\bar{P}_{\sigma(l_q)}^{-1}\dot{x}(t) \\
&\le x^{\mathrm{T}}(t)[\bar{A}_{\sigma(l_q)}^{\mathrm{T}}\bar{P}_{\sigma(l_q)}^{-1} + \bar{P}_{\sigma(l_q)}^{-1}\bar{A}_{\sigma(l_q)} \\
&\quad +\eta^{-1}\bar{P}_{\sigma(l_q)}^{-1}\bar{B}_{\sigma(l_q)}\bar{B}_{\sigma(l_q)}^{\mathrm{T}}\bar{P}_{\sigma(l_q)}^{-1}]x(t) \\
&\quad +\eta e_{\mathrm{ET}}^{\mathrm{T}}(t)e_{\mathrm{ET}}(t) \\
&\le \alpha V_{\sigma(l_q)}(x(t)) + \tilde{\varepsilon}
\end{aligned} \tag{12}
$$

Integrating both sides of (12) from l_q to t gives

$$V(x(t)) \le e^{\alpha(t-l_q)}V_{\sigma(l_q)}(x(l_q)) + \tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s \tag{13}$$

Case 2: for any $t \in [l_q, l_{q+1})$, if the triggered inter-execution interval falls within the working interval of the sub-system, one has that during a switching sub-system's working period, the event-triggering mechanism is triggered and the control signal is updated (possibly for multiple times), e.g. $t_k < l_q \le t_{k+1} \le t_{k+2} \le \cdots t_{k+m} < l_{q+1}, \forall m \in \mathbb{N}$. Also, in each subinterval, one can obtain the same result with that in (12).
Moreover, integrating both sides of (12) over each subinterval leads to

$$
V(x(t)) \le \begin{cases}
e^{\alpha(t-l_q)}V_{\sigma(l_q)}(x(l_q)) \\
+\tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s, t \in [l_q, t_{k+1}) \\
e^{\alpha(t-t_{k+1})}V_{\sigma(t_{k+1})}(x(t_{k+1})) \\
+\tilde{\varepsilon}\int_{t_{k+1}}^{t} e^{\alpha(t-s)}\mathrm{d}s, t \in [t_{k+1}, t_{k+2}) \\
\vdots \\
e^{\alpha(t-t_{k+m})}V_{\sigma(t_{k+m})}(x(t_{k+m})) \\
+\tilde{\varepsilon}\int_{t_{k+m}}^{t} e^{\alpha(t-s)}\mathrm{d}s, t \in [t_{k+m}, l_{q+1})
\end{cases} \tag{14}
$$

Note that the function $e_{\mathrm{ET}}(t)$ is piecewise continuous on an interval $[l_q, l_{q+1})$, and it has been shown in Theorem 2 that there exists a positive lower bound on the inter-execution intervals, i.e. $e_{\mathrm{ET}}(t)$ is continuous except for possibly finite jump discontinuities on the interval $[l_q, l_{q+1})$. Moreover, according to the definition of the error function $e_{\mathrm{ET}}(t)$ in (2), it is bounded on $[l_q, l_{q+1})$. Then from (6) and the definition of $V(x(t))$, both $x(t)$ and $V(x(t))$ are continuous functions in the variable t on the time interval $[l_q, l_{q+1})$ [40]. From this together with the fact $\sigma(l_q) = \sigma(t_{k+1}) = \cdots = \sigma(t_{k+m})$, one can obtain the same result from (14) as in (13) for Case 1.
Moreover, from the definitions of β_1 and β_2, for $\forall x(t) \in \mathbb{R}^n$ and $\forall i, j \in \underline{N}$, one has

$$
\begin{aligned}
x^{\mathrm{T}}(t)P_i^{-1}x(t) &\le \lambda_{\max}(P_i^{-1})x^{\mathrm{T}}(t)x(t) \le \beta_1^{-1}x^{\mathrm{T}}(t)x(t) \\
x^{\mathrm{T}}(t)P_j^{-1}x(t) &\ge \lambda_{\min}(P_j^{-1})x^{\mathrm{T}}(t)x(t) \ge \beta_2^{-1}x^{\mathrm{T}}(t)x(t)
\end{aligned}
$$

It follows from the above conditions that $x^{\mathrm{T}}(t)P_i^{-1}x(t) \le \mu x^{\mathrm{T}}(t)P_j^{-1}x(t)$. This further yields $x^{\mathrm{T}}(t)\bar{P}_i^{-1}x(t) \le \mu x^{\mathrm{T}}(t)\bar{P}_j^{-1}x(t)$ which is the relationship of the Lyapunov function between any adjacent switching sub-systems. Moreover, note that l_q and l_q^- are the moments that before and after a switch. Then, one can have

$$V_{\sigma(l_q)}(x(l_q)) \le \mu V_{\sigma(l_q^-)}(x(l_q^-)) \tag{15}$$

For any $t \in [0, T_{\mathrm{f}}]$, let $0 = t_0 < l_1 < l_2 < \cdots < l_q = l_{N_\sigma(0,t)} < t$ denote the switching instants of $\sigma(t)$ on the interval $[0, t]$, which also implies that $N_\sigma(0, t) \le N_\sigma(0, T_{\mathrm{f}})$. It then follows from (13) and (15) that

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$$
\begin{aligned}
V(x(t)) &\le \mu e^{\alpha(t-l_q)} V_{\sigma(l_q^-)}(x(l_q^-)) + \tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s \\
&\le \mu e^{\alpha(t-l_q)}[e^{\alpha(l_q-l_{q-1})}V_{\sigma(l_{q-1})}(x(l_{q-1})) \\
&\quad +\tilde{\varepsilon}\int_{l_{q-1}}^{l_q} e^{\alpha(l_q-s)}\mathrm{d}s] + \tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s \\
&\le \mu^2 e^{\alpha(t-l_{q-2})}V_{\sigma(l_{q-2})}(x(l_{q-2})) \\
&\quad +\mu^2\tilde{\varepsilon}\int_{l_{q-2}}^{l_{q-1}} e^{\alpha(t-s)}\mathrm{d}s + \mu\tilde{\varepsilon}\int_{l_{q-1}}^{l_q} e^{\alpha(t-s)}\mathrm{d}s \\
&\quad +\tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s \\
&\quad \cdots \\
&\le \mu^q e^{\alpha(t-t_0)}V_{\sigma(t_0)}(x(t_0)) + \mu^q\tilde{\varepsilon}\int_{t_0}^{l_1} e^{\alpha(t-s)}\mathrm{d}s \\
&\quad +\cdots + \mu\tilde{\varepsilon}\int_{l_{q-1}}^{l_q} e^{\alpha(t-s)}\mathrm{d}s + \tilde{\varepsilon}\int_{l_q}^{t} e^{\alpha(t-s)}\mathrm{d}s \\
&\le \mu^{N_\sigma(0,T_\mathrm{f})}e^{\alpha T_\mathrm{f}}V_{\sigma(0)}(x(0)) \\
&\quad +\mu^{N_\sigma(0,T_\mathrm{f})}\tilde{\varepsilon}\int_{0}^{T_\mathrm{f}} e^{\alpha(T_\mathrm{f}-s)}\mathrm{d}s
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
V(x(t)) &\le \mu^{N_\sigma(0,T_\mathrm{f})}e^{\alpha T_\mathrm{f}}\left[V_{\sigma(0)}(x(0)) + \frac{\tilde{\varepsilon}}{\alpha}(1-e^{-\alpha T_\mathrm{f}})\right] \\
&\le \mu^{N_0+(T_\mathrm{f}/\tau_a)}e^{\alpha T_\mathrm{f}}\left[V_{\sigma(0)}(x(0)) + \frac{\tilde{\varepsilon}}{\alpha}(1-e^{-\alpha T_\mathrm{f}})\right]
\end{aligned}
\tag{16}
$$

On the other hand, for any $i \in \underline{N}$, one has

$$
\begin{aligned}
V(x(t)) &= x^\mathrm{T}(t)\bar{P}_i^{-1}x(t) = x^\mathrm{T}(t)R^{1/2}P_i^{-1}R^{1/2}x(t) \\
&\ge \beta_2^{-1}x^\mathrm{T}(t)Rx(t)
\end{aligned}
\tag{17}
$$

and

$$
\begin{aligned}
V(x(0)) &= x^\mathrm{T}(0)\bar{P}_i^{-1}x(0) = x^\mathrm{T}(0)R^{1/2}P_i^{-1}R^{1/2}x(0) \\
&\le \beta_1^{-1}x^\mathrm{T}(0)Rx(0)
\end{aligned}
\tag{18}
$$

Thus, using conditions (16)–(18) together with $x^\mathrm{T}(0)Rx(0) \le c_1$ gives

$$
x^\mathrm{T}(t)Rx(t) \le \beta_2 e^{\alpha T_\mathrm{f}}\mu^{N_0+(T_\mathrm{f}/\tau_a)}\left[\frac{c_1}{\beta_1} + \frac{\tilde{\varepsilon}}{\alpha}(1-e^{-\alpha T_\mathrm{f}})\right]
\tag{19}
$$

Then, when $\mu = 1$, the finite-time boundedness of switched closed-loop system (6), i.e. $x^\mathrm{T}(t)Rx(t) \le c_2$, can be directly ensured by condition (8). When $\mu > 1$, it first follows from (8) that $\ln[\frac{\beta_2^{-1}\mu^{-N_0}c_2}{(c_1/\beta_1)+(\tilde{\varepsilon}/\alpha)(1-e^{-\alpha T_\mathrm{f}})}] - \alpha T_\mathrm{f} > 0$. Hence, using the average dwell time condition (9) can lead to $x^\mathrm{T}(t)Rx(t) \le c_2$ from (19). Finally, one can conclude that the switched closed-loop system (6) is finite-time bounded from Definition 2. □

Then, in the following theorem we give the sufficient design conditions under SEM (2).

Theorem 3: For given positive constants c_1, c_2, η, α, T_f, and a constant matrix R, if there exist a positive constant $\bar{\varepsilon}_\mathrm{sem}$, matrices Y_i and symmetric and positive definite matrices P_i with appropriate dimensions, rendering $\bar{P}_i = R^{-(1/2)}P_iR^{-(1/2)}$, $i \in \underline{N}$, such that (8)

$$
\begin{bmatrix} \Xi_1 & B_iY_i \\ * & -\eta\bar{P}_i \end{bmatrix} < 0
\tag{20}
$$

$$
I - \bar{P}_i \le 0,
\tag{21}
$$

where $\Xi_1 = A_i\bar{P}_i + \bar{P}_iA_i^\mathrm{T} + B_iY_i + Y_i^\mathrm{T}B_i^\mathrm{T} - \alpha\bar{P}_i$, then, there exists a set of state-feedback controller gains $K_i = Y_i\bar{P}_i^{-1}$ and event-triggering threshold $\varepsilon_\mathrm{sem} = |\sqrt{\bar{\varepsilon}_\mathrm{sem}}|$ such that the switched closed-loop system (6) is finite-time bounded with respect to $(c_1, c_2, \bar{\varepsilon}_\mathrm{sem}, T_\mathrm{f}, R, \sigma(t))$, under SEM (2) and any ADT switching signal $\sigma(t)$ satisfying (9).

Proof: It follows from (7) of Theorem 1 that

$$
\begin{aligned}
&(A_i + B_iK_i)\bar{P}_i + \bar{P}_i(A_i + B_iK_i)^\mathrm{T} + \eta^{-1}B_iK_iK_i^\mathrm{T}B_i^\mathrm{T} \\
&-\alpha\bar{P}_i < 0
\end{aligned}
$$

With the change of variable $Y_i = K_i\bar{P}_i$ and condition (21), the above condition is reduced to

$$
\begin{aligned}
&A_i\bar{P}_i + \bar{P}_iA_i^\mathrm{T} + B_iY_i + Y_i^\mathrm{T}B_i^\mathrm{T} + \eta^{-1}B_iY_i\bar{P}_i^{-1}Y_i^\mathrm{T}B_i^\mathrm{T} \\
&-\alpha\bar{P}_i < 0
\end{aligned}
\tag{22}
$$

Then, using Schur complement formula, condition (22) is equivalent to (20). □

Remark 2: From Theorem 3, one can have a set of feasible solutions of $\bar{\varepsilon}_\mathrm{sem}$, Y_i and P_i by solving the conditions (8), (20) and (21), and then obtain the event-triggering thresholds by $\varepsilon_\mathrm{sem} = |\sqrt{\bar{\varepsilon}_\mathrm{sem}}|$. Moreover, the proposed sufficient conditions also allow one to give the event-triggering threshold ε_sem (or $\bar{\varepsilon}_\mathrm{sem}$) beforehand instead of as a variable, according to the users' design requirements.

Remark 3: When using the theorem conditions, the initial and final bounded range parameters c_1, c_2, the parameter R and the finite time interval T_f are selected first, depending on the desired requirement. Afterwards, choose other appropriate parameter values η and α, and then try to solve the LMI conditions.

Remark 4: Theorems 1 and 3 give a state-based design results based on the SEM (2). However, if we re-formulate the mechanism (2) as $t_{k+1} = \inf\{t > t_k | \parallel K_{\sigma(t)}e_\mathrm{ET}(t) \parallel \ge \varepsilon_\mathrm{sem}\}$, i.e. using an event-triggering with respect to the control signal, not the state, it would allow to eliminate the term KK^T in $\bar{B}_i\bar{B}_i^\mathrm{T}$ of (7) and, thus, get rid of condition (21).

Remark 5: Note that the lower bound estimation given by (10) depends on the sampled state value $|x(t_k)|$ for each inter-execution interval $t_{k+1} - t_k$. Now, we give another method for computing a unified lower bound on the inter-execution intervals.
From (11), one has

$$
\begin{aligned}
D^+|e(t)| &\le |\bar{A}_{\sigma(t_k)}||x(t)| + |\bar{B}_{\sigma(t_k)}||e(t)| \\
&\le |\bar{A}_{\sigma(t_k)}||x(t)| + |\bar{B}_{\sigma(t_k)}|\varepsilon_\mathrm{sem}
\end{aligned}
$$

Moreover, from Theorems 1 or 3 the following facts hold, i.e.

$$
\lambda_{\min}(R)x^\mathrm{T}(t)x(t) \le x^\mathrm{T}(t)Rx(t) \le c_2
$$

Then one can have

$$
|x(t)| \le \bar{c} = \sqrt{\frac{c_2}{\lambda_{\min}(R)}}
\tag{23}
$$

It follows from (23) that

$$
D^+|e(t)| \le |\bar{A}_{\sigma(t_k)}|\bar{c} + |\bar{B}_{\sigma(t_k)}|\varepsilon_\mathrm{sem}
\tag{24}
$$

Thus, another positive lower bounded on the inter-execution intervals can be obtained as

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$$\bar{T} = \frac{\varepsilon_{\text{sem}}}{\phi_1 \bar{c} + \phi_2 \varepsilon_{\text{sem}}} \tag{25}$$

4 Event-triggered observer-based state-feedback control

In this section, we extend the results of the previous section to the observer-based state-feedback case.

The dynamic equation of the error state $\tilde{x}(t) = x(t) - \hat{x}(t)$ between the actual system (1) and the observers (4) is constructed as

$$\dot{\tilde{x}}(t) = [A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}]\tilde{x}(t) \tag{26}$$

Applying the OSEM (5) into the state observers (4) gives

$$\begin{aligned} \dot{\hat{x}}(t) = {} & [A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}]\hat{x}(t) + L_{\sigma(t)} C_{\sigma(t)} \tilde{x}(t) \\ & + B_{\sigma(t)} K_{\sigma(t)} \hat{e}_{\text{ET}}(t) \end{aligned} \tag{27}$$

for the time period $t \in [t_k, t_{k+1})$. Then, the augmented switched closed-loop system by combining (26) with (27) is obtained as

$$\dot{\xi}(t) = \hat{A}_{\sigma(t)} \xi(t) + \hat{B}_{\sigma(t)} \hat{e}_{\text{ET}}(t), \tag{28}$$

where $\xi(t) = [\hat{x}^{\text{T}}(t)\ \tilde{x}^{\text{T}}(t)]^{\text{T}}$ is the augmented state and the parameter matrices are

$$\hat{A}_{\sigma(t)} = \begin{bmatrix} A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)} & L_{\sigma(t)} C_{\sigma(t)} \\ 0 & A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)} \end{bmatrix}$$

$$\hat{B}_{\sigma(t)} = \begin{bmatrix} B_{\sigma(t)} K_{\sigma(t)} \\ 0 \end{bmatrix}$$

In parallel with the state-feedback case, sufficient conditions of the finite-time boundedness for switched linear systems under OSEM (5) can be derived similarly, which is omitted here. In the following, a positive lower bound estimation on the inter-execution intervals is first presented.

Corollary 1: With any state-feedback gains $K_{\sigma(t)}$, observer gains $L_{\sigma(t)}$ and OSEM (5), the inter-execution time $t_{k+1} - t_k$ is lower bounded by a positive constant $\bar{T}$ satisfying

$$\bar{T} = \frac{\varepsilon_{\text{osem}}}{(\phi_2 + \phi_3) \parallel \hat{x}(t_k) \parallel + \phi_3 \varepsilon_{\text{osem}} + \Xi_1}, \tag{29}$$

where $\phi_2 = \max_{\forall i \in \underline{N}} \{ \parallel \bar{B}_i \parallel \}$, $\phi_3 = \max_{\forall i \in \underline{N}} \{ \parallel A_i \parallel \}$, $\phi_4 = \max_{\forall i \in \underline{N}} \{ \parallel L_i C_i \parallel \}$, $\phi_5 = \max_{\forall i \in \underline{N}} \{\lambda_{\max}(A_i - L_i C_i)\}/2$ and $\Xi_1 = c e^{\phi_5 t_k} \phi_4 \parallel \tilde{x}(0) \parallel$.

Proof: By using similar arguments as in the proof of Theorem 2, the proof can be completed. To simplify notations, let $|\,.\,|$ represent $\parallel . \parallel$ and $\hat{e}(t) \triangleq \hat{e}_{\text{ET}}(t)$. The following derivation is partially inspired by Zhang and Feng [41]. As Case 1, for instance, from system (27) and the definition of $\hat{e}_{\text{ET}}(t)$ in OSEM (5), the following inequality is derived.

$$\begin{aligned} & D^+ |\hat{e}(t)| \\ & \leq |\dot{\hat{x}}(t)| \\ & \leq |A_{\sigma(t_k)} \hat{x}(t) + B_{\sigma(t_k)} K_{\sigma(t_k)} \hat{x}(t_k) \\ & \quad + L_{\sigma(t_k)} C_{\sigma(t_k)} \tilde{x}(t)| \\ & \leq |A_{\sigma(t_k)}| \, |\hat{x}(t)| + |B_{\sigma(t_k)} K_{\sigma(t_k)}| \, |\hat{x}(t_k)| \\ & \quad + |L_{\sigma(t_k)} C_{\sigma(t_k)}| \, |e^{[A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}]t} \tilde{x}(0)| \\ & \leq |A_{\sigma(t_k)}| (|\hat{x}(t_k)| + \varepsilon_{\text{osem}}) + |B_{\sigma(t_k)} K_{\sigma(t_k)}| \, |\hat{x}(t_k)| \\ & \quad + |L_{\sigma(t_k)} C_{\sigma(t_k)}| \, |e^{[A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}]t}| \, |\tilde{x}(0)|, \end{aligned} \tag{30}$$

where $\tilde{x}(t) = e^{[A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}]t} \tilde{x}(0)$ is obtained from (26), $\tilde{x}(0)$ is the initial estimate error, and it is natural to assume that $|\tilde{x}(0)|$ is bounded.

Moreover, from Lemma 3 and the fact that $A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}$ is Hurwitz, one can obtain

$$\begin{aligned} & D^+ |\hat{e}(t)| \\ & \leq |A_{\sigma(t_k)}| (|\hat{x}(t_k)| + \varepsilon_{\text{osem}}) + |B_{\sigma(t_k)} K_{\sigma(t_k)}| \, |\hat{x}(t_k)| \\ & \quad + c e^{((\lambda_{\max}(A_{\sigma(t)} - L_{\sigma(t)} C_{\sigma(t)}))/2) t_k} |L_{\sigma(t_k)} C_{\sigma(t_k)}| \, |\tilde{x}(0)| \end{aligned}$$

Then, similarly, by denoting $\bar{T} = t - t_k$, a strictly positive lower bound on the inter-execution intervals can be obtained as (29), for any given event-triggered instant t_k.□

Corollary 2: For given positive constants c_{o_1}, c_{o_2}, η, α, T_{f}, and a constant matrix R, if there exist a positive constant $\bar{\varepsilon}_{\text{osem}}$, matrices Y_{1i} and Y_{2i}, and symmetric and positive definite matrices P_i with appropriate dimensions, rendering $\bar{P}_i = R^{-(1/2)} P_i R^{-(1/2)}$, $i \in \underline{N}$, such that (8)

$$\begin{bmatrix} \Xi_5 & Y_{2i} C_i & B_i Y_{1i} \\ * & \Xi_6 & 0 \\ * & * & -\eta \bar{P}_{1i} \end{bmatrix} < 0 \tag{31}$$

$$I - \bar{P}_{1i} \leq 0, \tag{32}$$

where

$$\begin{aligned} \Xi_5 &= A_i \bar{P}_{1i} + \bar{P}_{1i} A_i^{\text{T}} + B_i Y_{1i} + Y_{1i}^{\text{T}} B_i^{\text{T}} - \alpha \bar{P}_{1i} \\ \Xi_6 &= A_i \bar{P}_{2i} + \bar{P}_{2i} A_i^{\text{T}} - Y_{2i} C_i - C_i^{\text{T}} Y_{2i}^{\text{T}} - \alpha \bar{P}_{2i} \\ \bar{P}_i &= \begin{bmatrix} \bar{P}_{1i} & 0 \\ * & \bar{P}_{2i} \end{bmatrix} \\ \bar{P}_{2i} &= V_i \begin{bmatrix} \bar{P}_{11,i} & 0 \\ * & \bar{P}_{22,i} \end{bmatrix} V_i^{\text{T}} \\ \tilde{\varepsilon} &= \eta \bar{\varepsilon}_{\text{osem}} \end{aligned}$$

then, the augmented switched closed-loop system (28) is finite-time bounded with respect to $(c_{o_1}, c_{o_2}, \bar{\varepsilon}_{\text{osem}}, T_{\text{f}}, R, \sigma(t))$, under OSEM (5) and any ADT switching signal $\sigma(t)$ satisfying (9). Furthermore, the state feedback controller gains are $K_i = Y_{1i} \bar{P}_{1i}^{-1}$, the state observer gains are $L_i = Y_{2i} \hat{\bar{P}}_{2i}^{-1} = Y_{2i} U_i \Sigma_i \bar{P}_{11,i} \Sigma_i^{-1} U_i^{\text{T}}$, and the event-triggering threshold is $\varepsilon_{\text{osem}} = |\sqrt{\bar{\varepsilon}_{\text{osem}}}|$, where $\hat{\bar{P}}_{2i}$ satisfies $C_i \bar{P}_{2i} = \hat{\bar{P}}_{2i} C_i$.

Proof: First, choose the Lyapunov function candidate for the augmented switched closed-loop system (28) as

$$V(\xi(t)) = V_{\sigma(t)}(\xi(t)) = \xi^{\text{T}}(t) \bar{P}_{\sigma(t)}^{-1} \xi(t),$$

where $\bar{P}_i$ is defined below (32).

Then, using the conditions (7) and (8) in Theorem 1 with the replacements $\bar{A}_i \to \hat{A}_i$, $\bar{B}_i \to \hat{B}_i$ and $\varepsilon_{\text{sem}} \to \varepsilon_{\text{osem}}$, the sufficient analysis conditions can be similarly deduced to guarantee the finite-time boundedness of switched closed-loop system (28) under OSEM (5). In this case, (7) can be written as

$$\hat{A}_i \bar{P}_i + \bar{P}_i \hat{A}_i^{\text{T}} + \eta^{-1} \hat{B}_i \hat{B}_i^{\text{T}} - \alpha \bar{P}_i < 0 \tag{33}$$

Substituting the specific expressions $\hat{A}_i$ and $\hat{B}_i$ into (33) gives

$$\begin{bmatrix} \Xi_7 & L_i C_i \bar{P}_{2i} \\ * & \Xi_8 \end{bmatrix} < 0, \tag{34}$$

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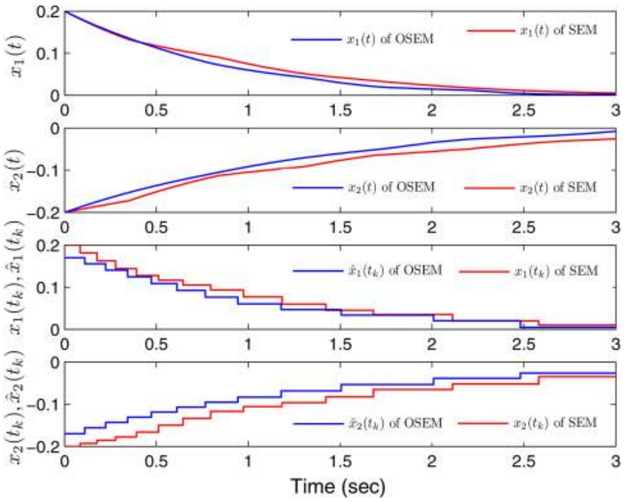

Fig. 2 *State response (upper two sub-figures) and event-triggered updated state (lower two sub-figures) under SEM (2) and OSEM (5), respectively. State-feedback control with SEM (red line) and observer-based state-feedback control with OSEM (blue line)*

where

$$\Xi_7 = A_i\bar{P}_{1i} + \bar{P}_{1i}A_i^{\mathrm{T}} + B_iK_i\bar{P}_{1i} + \bar{P}_{1i}K_i^{\mathrm{T}}B_i^{\mathrm{T}} + B_iK_iK_i^{\mathrm{T}}B_i^{\mathrm{T}} - \alpha\bar{P}_{1i}$$

$$\Xi_8 = A_i\bar{P}_{2i} + \bar{P}_{2i}A_i^{\mathrm{T}} - L_iC_i\bar{P}_{2i} - \bar{P}_{2i}C_i^{\mathrm{T}}L_i^{\mathrm{T}} - \alpha\bar{P}_{2i}$$

According to the structure of the defined matrix $\bar{P}_{2i}$ (below (32)) and Lemma 1, there exists a matrix $\hat{P}_{2i}$ satisfying the condition $C_i\bar{P}_{2i} = \hat{P}_{2i}C_i$. Furthermore, using the changes of variables $Y_{1i} = K_i\bar{P}_{1i}$ and $Y_{2i} = L_i\hat{P}_{2i}$, Schur complement and condition (32), one can obtain the sufficient design condition (31) from (34). □

Remark 6: Corollary 1 provides a theoretical proof to ensure that the Zeno behaviour does not occur. Note that the lower bound estimation given by (29) depends on the sampled state value $|\hat{x}(t_k)|$ for each inter-execution interval $t_{k+1} - t_k$. Here, a computation method is given using another estimation of a unified lower bound on the inter-execution intervals.
Since from Corollary 2

$$\lambda_{\min}(R)\xi^{\mathrm{T}}(t)\xi(t) \le \xi^{\mathrm{T}}(t)R\xi(t) \le c_{o_2},$$

one can obtain

$$|\hat{x}(t)|, |\tilde{x}(t)| \le \bar{c}_o,$$

where $\bar{c}_o = \sqrt{(c_{o_2}/\lambda_{\min}(R))}$. In view of this, it follows from (27) and (30) that

$$D^+|\hat{e}(t)| \le (\phi_1 + \phi_4)\bar{c}_o + \phi_2\varepsilon_{\mathrm{osem}} \tag{35}$$

Then, a positive lower bound on the inter-execution intervals can be computed as

$$\bar{T} = \frac{\varepsilon_{\mathrm{osem}}}{(\phi_1 + \phi_4)\bar{c}_o + \phi_2\varepsilon_{\mathrm{osem}}} \tag{36}$$

5 Simulation and comparison results

In this section, two numerical examples are provided. The first one is to illustrate the effectiveness of the proposed event-triggered control methods (both SEM and OSEM). Also, the second one is to illustrate the effectiveness of the proposed event-triggered control compared with time-triggered control.

Example 1: Consider the switched linear system given by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \quad x(0) = [0.2 \ \ -0.2]^{\mathrm{T}}$$

with

$$A_1 = \begin{bmatrix} -1.0 & 0.3 \\ 0 & 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 1.3 \end{bmatrix}, \quad C_1 = [1.3 \ \ 1.0]$$

$$A_2 = \begin{bmatrix} 0.4 & 0.4 \\ 0 & -1.0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.2 \\ 0.3 \end{bmatrix}, \quad C_2 = [1.5 \ \ 0.1]$$

Since the system matrices A_1 and A_2 are not Hurwitz, both subsystems 1 and 2 are unstable. The simulations are conducted for the SEM and OSEM, respectively.
First, we consider the finite-time boundedness under SEM and state-feedback control, i.e. the co-design of SEM (2) and the state-feedback gains in (3). Choose $c_1 = 1$, $c_2 = 10$, $\eta = 1.6$, $\alpha = 0.0001$, $T_{\mathrm{f}} = 3$ and $R = I$, after solving the LMI conditions (8), (20) and (21) in Theorem 3, we obtain a set of solutions for the event-triggering threshold and state-feedback controller gains, which are $\varepsilon_{\mathrm{sem}} = 0.02$ and

$$K_1 = -[0.6531 \ \ 0.1009], \quad K_2 = [0.0019 \ \ -0.6951]$$

Then using (9), the average dwell time for guaranteeing the finite-time boundedness of the resulting switched closed-loop system (6) is $\tau_a = 0.7 > \tau_a^* = 0.2359$.
On the other hand, we consider the event-triggered observer-based state-feedback control, i.e. the co-design of OSEM (5), the state-feedback gains in (3) and the observer gains in (4). The values of the corresponding parameters are chosen as $c_{o_1} = 1$, $c_{o_2} = 10$, $\eta = 3.8$, $\alpha = 0.0002$, $T_{\mathrm{f}} = 3$ and $R = I$. Solving the LMI conditions (8), (31) and (32) in Corollary 2 gives a set of feasible solutions, which are $\varepsilon_{\mathrm{osem}} = 0.02$ and

$$L_1 = [1.9729 \ \ 0.3970]^{\mathrm{T}}, \quad L_2 = [-0.3043 \ \ 2.6662]^{\mathrm{T}}$$
$$K_1 = -[1.0912 \ \ 0.1112], \quad K_2 = -[0.0167 \ \ 1.1159]$$

From (9), the average dwell time is obtained as $\tau_a = 0.7 > \tau_a^* = 0.5226$. The initial state of the observers is set as $\hat{x}(0) = [0.17 \ \ -0.17]^{\mathrm{T}}$.
The simulation results about the switched closed-loop system's state response together with the event-triggered updated state for both cases are shown in Fig. 2. For the event-triggered state-feedback control with SEM, the evolution of the error norm $\| e_{\mathrm{ET}}(t) \|$ and the inter-execution intervals are shown in Fig. 3. For the event-triggered observer-based state-feedback control with OSEM, the corresponding results are shown in Fig. 4. Also, the switching signal ($N_0 = 2$) used by both cases is shown in Fig. 5. As shown in Fig. 2, the system state is finite-time bounded with either the event-triggered state-feedback controllers or the event-triggered observer-based state-feedback controllers, and the performances of both are similar. More importantly, as can be seen in Figs. 3 and 4, the triggering frequency always decreases with the convergence of the system state, and the Zeno behaviour is excluded in both cases.

Example 2: Consider the switched linear system given by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), x(0) = [0.1 \ \ -0.1]^{\mathrm{T}}$$

with

$$A_1 = \begin{bmatrix} 0 & -0.3 \\ 0.6 & 3.0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 1.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -1.2 \\ 0.4 & -4.0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.0 \\ 0.3 \end{bmatrix}$$

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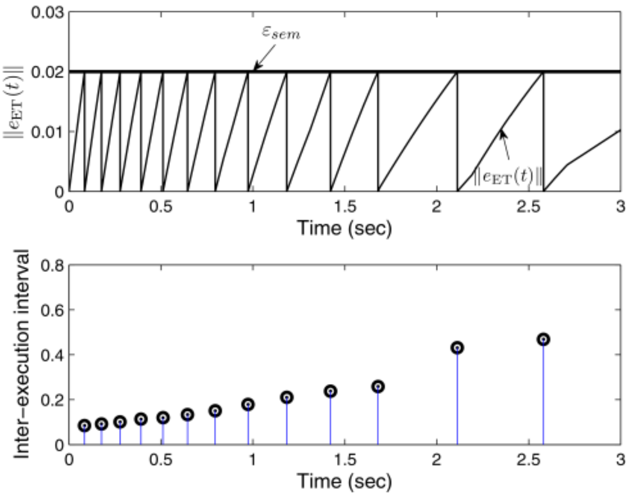

Fig. 3 *Evolution of* $\| e_{\mathrm{ET}}(t) \|$ *(upper figure) and inter-execution intervals (lower figure) under SEM (2) (state-feedback control)*

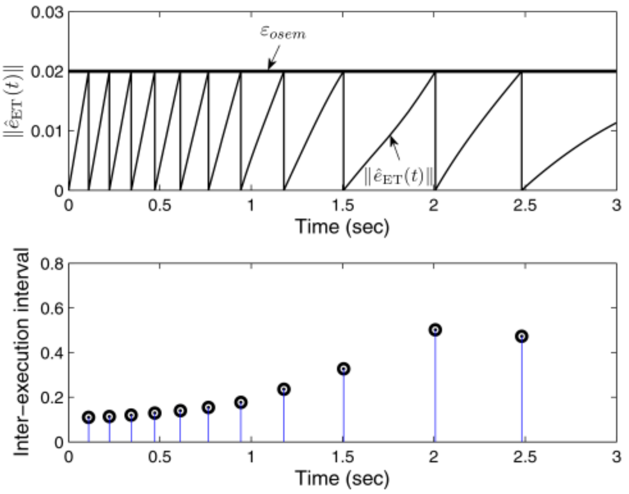

Fig. 4 *Evolution of* $\| \hat{e}_{\mathrm{ET}}(t) \|$ *(upper figure) and inter-execution intervals (lower figure) under OSEM (5) (observer-based state-feedback control)*

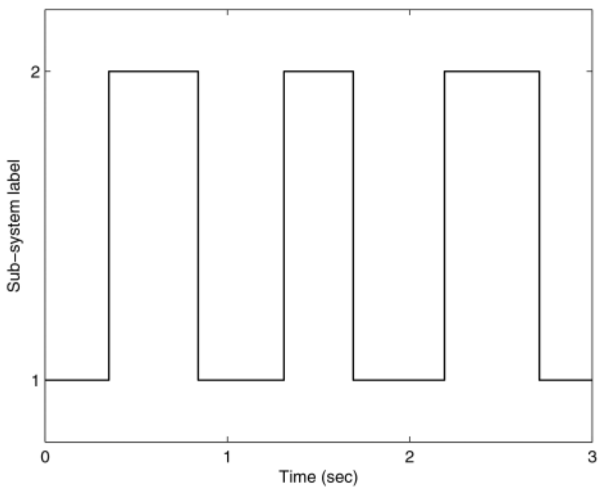

Fig. 5 *Switching signal*

In this example, we make a comparison between the proposed event-triggered control (take SEM as an example) and the time-triggered control. The sampling period of time-triggered control is selected as the average inter-execution time in the same line [42]. The parameters c_1, c_2, T_f and R are chosen the same as Example 1. The comparison results are given in Table 1, in which ν_{ET} and ν_{TT} denote the maximum value of $x^{\mathrm{T}}(t)Rx(t)$ for event-triggered control and time-triggered control, respectively. Clearly, the event-triggered control has a smaller maximum value of $x^{\mathrm{T}}(t)Rx(t)$ than the corresponding time-triggered control for the same given initial condition. Thus, it is inferred directly that in the sense of a uniform average sampling period, for a given initial condition, the proposed event-triggered control can more effectively configure the system resources to get relatively good performance than the time-triggered control.

Table 1 Comparison of the proposed event-triggered control (SEM) and time-triggered control under different thresholds $\varepsilon_{\mathrm{sem}}$

Selected $\varepsilon_{\mathrm{sem}}$	0.05	0.10	0.15
maximum interval	0.9680	1.2792	1.8475
minimum interval	0.1151	0.3338	0.2967
lower bound estimation	0.0033	0.0081	0.0120
average inter-execution interval	0.5000	0.6000	0.7500
maximum value ν_{TT} ($\times 10^{-3}$)	4.3847	78.1000	83.2000
maximum value ν_{ET} ($\times 10^{-3}$)	3.0470	4.5513	14.9000

6 Conclusions

By implementing an event-triggering mechanism with fixed threshold in switched linear systems, we are interested in the finite-time boundedness problem of the system via state-feedback control. A design method has been developed for designing the event-triggering mechanism and sub-controllers. Different from the traditional time-triggering scheme in switched systems, the sub-controllers are triggered and updated only if the state signal-based error norm reaches a pre-defined threshold. As a basic and important type of control system, state-feedback control design has been considered in the study. Moreover, the multiple Lyapunov functions approach and LMI technique have been adopted to construct the sufficient conditions for the design, which can guarantee the finite-time boundedness of the resulting switched closed-loop system. In addition, a positive lower bound on inter-execution intervals has been presented to avoid Zeno behaviour. Moreover, motivated by the application, the results obtained in the full state feedback have been extended to the observer-based state-feedback control. The simulation has shown that the transmission frequency of the feedback signal could be reduced to a certain level and the finite-time boundedness of the closed-loop system can be ensured. We are working towards applying some of the ideas to switching systems that contain non-linear components.

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