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Research Article

Finite Time Stability of Finance Systems with or without Market Confidence Using Less Control Input

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In this paper, we make an exploration of a technique to control a class of finance chaotic systems. This technique allows one to achieve the finite time stability of the finance system more effectively with less control input energy. First, the finite time stability of three dimension finance system without market confidence is analyzed by using a single controller. Then, two controllers are designed to stabilize the four-dimension finance system with market confidence. Moreover, the finite time stability of the three-dimension and four-dimension finance system with unknown parameter is also studied. Finally, simulation results are presented to show the chaotic behaviour of the finance systems, verify the effectiveness of the proposed control method, and illustrate its advantages compared with other methods.

1. Introduction

The complex nonlinear behavior and chaos phenomenon in economic system were first found in 1985 [1]. Since then, many research results of these phenomenon were presented through studying econometrics and financial models [2]. The authors in [3] proposed a nonlinear financial system using the method of systematic dynamics. Ma and Chen studied the bifurcation topological structure and the global complicated character of this financial system [4, 5]. The authors in [6] investigated the dynamical behavior of this system. Ma and Wang [7] studied the Hopf bifurcation and gave the verification for the topological horseshoe chaos in this finance system. Some extended forms of the financial system in [3] have been presented, such as fractional form [8–10] and delayed form [11, 12]. In fact, the most important factor influencing the economy is confidence, especially in times of economic crisis [13]. So [14] extended the above system to a four-dimensional form by considering market confidence into the system. All these extended forms of the financial system exhibit unstable and chaotic behaviors. From the economic perspective, the existence of the instable and complicated dynamic behavior means that there is an inherent indefiniteness in the financial system [15]. And the emergence of chaos makes it is almost impossible to predict the economic behaviors precisely. What is more, it most likely makes the system slide into huge financial losses or economic crisis. Therefore, it has important theoretical and practical value for the stable economic growth to study how to overcome this indefiniteness. In other words, we need to study how to realize effective control of the unstable and chaotic behaviors in the economic system.

In recent years, some interesting research results on the control of the financial systems in [3] or its extended forms have been presented. Based on nonlinear state feedback mechanism and multiobjective optimization framework, [16] proposed an active control policy design to achieve the stabilization of chaos in a fractional form of financial system. The active control strategy was also employed to synchronize two financial systems in [17]. In [18], some effective controllers were given for the synchronization control of the financial system by using feedback control method. Through designing adaptive sliding mode controller, [8] and [19] realized the chaos control of the three- and four-dimension fractional financial system, respectively. The authors in [20] modified the financial system and designed an adaptive controller to stabilize the chaos in financial system. In [21, 22], the similar adaptive technique was used to implement the synchronization of four-dimensional finance system. The hyperchaotic finance system was stabilized employing a controller which combines passive and feedback method in [23]. Unfortunately, in order to suppress instability or chaos in the financial system, all these methods need to add the controllers to all the state equations of the system. This would mean that the governments must take more radical policies to prevent a further economic slump in crisis.

It is well known that the radical solutions could be even more disruptive to the economy. Therefore, it is of great practical importance to suppress the unstable states or chaos in financial system with less control inputs. In [24], a fuzzy controller was designed to realize the stabilization of the fractional chaotic finance system. But the control signal never converges to zero in this method, which means that the control input applied on the finance systems will always exist. What is indisputable is that no one expected an endless financial regulation policy. In [25], a speed feedback controller is presented to achieve the control of chaotic finance system. But it needs a long time for stabilization. These two studies achieved the stable control of the financial system by applying a single controller exerted in one of the state equations.

It is more regrettable that all but [14] (which exerts four controllers to the system) of the above techniques just considered asymptotic stability of the financial system. However, there is no country around the world that wishes to come out of the financial crisis in a short period of time. So in this sense, finite time control, which can make the system states converge to an equilibrium point in a finite time [26], has more realistic significance for the economy to emerge from the crisis and embark on the track of recovery. Furthermore, studies have shown that the finite time control method has not only better robustness and disturbance rejection [27], but also the following advantages. First, finite time control method can integrate well with other methods. For example, [28] designed continuous state feedback controllers to solve the finite time synchronization control problem of chaotic system. Reference [29] presented adaptation laws and finite-time controller to stabilize nonlinear system, [30, 31] employed sliding mode controller to achieve finite time synchronization and control of the system. Combined with recurrent neural network, [32] realized finite time stability of the system. Second, finite time control method can stabilize the dynamic systems under various complicated condition effectively, such as the system subject to time delay [33], the time varying system [34, 35], the system with uncertain parameters [36], the stochastic nonlinear system [37], and nonlinear impulsive dynamical systems [38]. Due to the above-mentioned advantages, finite time control technique has been applied to many fields, such as designing consensus and collision avoidance algorithms of autonomous underwater vehicle [39], stabilizing the chaos in permanent magnet synchronous motor [40] and the centrifugal flywheel governor system [41], synchronizing multi-agent networks [42], and complex networks [43].

This paper will investigate the finite time control of the financial system with or without market confidence influence. According to previous analysis, it is considered much more significant to achieve stable control of the system by using as few controllers as possible. Unfortunately, nearly all the aforementioned results (about finite time control) exert control to all the state equations of the system. In [44], a single input controller was designed to stabilize the 3-dimensional (3D) chaotic system. Inspired by this result, we will stabilize the financial system in a finite time by a single controller. Then, we will develop this technique and apply it to stabilize the 4-dimensional (4D) financial system with market confidence. Furthermore, the application of this method in the stabilization of the finance system with unknown parameter will be investigated. The outline of the paper is as follows. After some preliminaries in Section 2, we will give in Section 3 the finite time stability theorem of the financial system without market confidence. Section 4 studies the finite time control of the system with market confidence. Some simulations are included in Section 5 to show the efficiency and superiority of this method. Finally, conclusions are presented in Section 6.

2. Preliminaries

This section mainly introduces the finance system and the finite time stability theory.

2.1. Finance Chaotic System

2.1.1. 3D Finance Chaotic System without Market Confidence. Considering the influences of production, money, security, and labor force, the authors of [45] used the system dynamic method to establish a financial model as

$$\begin{aligned} \dot{x} &= f_1\left(y - SV\right)x + f_2 z, \\ \dot{y} &= f_3\left(BEN - \alpha y - \beta x^2\right), \\ \dot{z} &= -f_4 z - f_5 x, \end{aligned} \tag{1}$$

where x denotes the interest rate, y is the investment demand, and z presents the price exponent. SV is the amount of saving, and BEN is the rate of return on investment. f_i, $i = 1, 2, \ldots, 5$, α, and β are constants.

For the financial model, the most important is not the value of parameters, but the relationship between the parameters and how relative changes of them affect the system behavior. By choosing the appropriate coordinate system and setting an appropriate dimension to state variables [4], a further simplified financial model is written as

$$\begin{aligned} \dot{x} &= z + \left(y - a\right)x, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \end{aligned} \tag{2}$$

where the parameter a is the saving, b is the per-investment cost, c is the elasticity of demands of commercials, and a, b, c are positive real constants.

Choosing the parameters $a = 2$, $b = 0.1$, $c = 1$, and the initial values $x_0 = 0.1$, $y_0 = 0.2$, $z_0 = 0.3$, one can obtain that the three lyapunov exponents of the finance system (2) are $0.134, 0.000, -0.514$, and the Lyapunov dimension is 2.261. The existence of topological horseshoe chaos has already been proved in the authors' previous work [7].

2.1.2. 4D Finance Chaotic System with Market Confidence. In financial market, the maintenance of confidence can stabilize the markets and promote financial growth. When financial crisis happens, market confidence will be shattered, and this in turn might lead to the market falling still further. The consensus view of economists is that, along with efforts to haul the world out of recession, one of the most important steps may be to restore the financial market confidence. For more accurate simulation of the dynamics of financial market during the financial crisis, the model should take the influence of market confidence into account. By considering market confidence into the financial system (2), [14] proposed a novel four-dimensional chaotic financial system as follows.

$$\begin{aligned} \dot{x} &= z + (y - a)x + m_1 w, \\ \dot{y} &= 1 - by - x^2 + m_2 w, \\ \dot{z} &= -x - cz + m_3 w, \\ \dot{w} &= -xyz, \end{aligned} \tag{3}$$

where x, y, z, a, b, c have the same meanings as those defined in system (2), w is the market confidence, and m_1, m_2, m_3 are the impact factors.

When the parameters are chosen as $a = 2, b = 5, c = 1.3$, $m_1 = 4.4, m_2 = 4.4, m_3 = 0.2$, and the initial values are taken as $x_0 = 1.5$, $y_0 = 2$, $z_0 = 0.3$, $w_0 = 0.1$, the four Lyapunov exponents of system (3) are $0.129, 0.000, -1.200, -7.061$, and the Lyapunov dimension is 2.1078. So the dynamics of finance system with market confidence is chaotic.

2.2. Finite Time Stability Theory. Finite time stabilization means that the state of the system can converge to the origin in a finite time. The definition of finite time stabilization and a lemma are given below.

Definition 1. Consider the following nonlinear system:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \tag{4}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the state of the system and $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x}))^{\mathrm{T}} : R^n \longrightarrow R^n$ is smooth function. Let $x_0 = 0$ be an equilibrium point of system (4). If there exists a positive constant T, such that

$$\lim_{t \longrightarrow T} \|\boldsymbol{x}(t)\| = 0, \tag{5}$$

and $\|\boldsymbol{x}(t)\| \equiv 0$ if $t \geq T$, then the stabilization of system (4) is achieved in a finite time.

Lemma 2 (see [27, 46]). *Suppose that there exists a continuous function $V(t) : D \longrightarrow R$ such that the following conditions hold:*

(i) $V(t)$ is positive definite,

(ii) There exist real numbers $c > 0$ and $\alpha \in (0, 1)$ and an open neighborhood $\Omega \subset D$ of the origin such that

$$\dot{V}(t) \leq -cV^{\alpha}(t) \quad \forall t \geq t_0, \; V(t_0) \geq 0. \tag{6}$$

Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$\begin{aligned} &V^{1-\alpha}(t) \leq V^{1-\alpha}(t_0) - c(1-\alpha)(t - t_0), \quad t_0 \leq t \leq t_1, \\ &V(t) \equiv 0, \quad \forall t \geq t_1, \end{aligned} \tag{7}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\alpha}(t_0)}{c(1-\alpha)}. \tag{8}$$

We refer the interested reader to [27, 46] for the proof of Lemma 2.

Based on above definition and lemma, this paper will design an adaptive control method with less control input to achieve the finite time stability of systems (2) and (3).

3. Finite Time Stability of 3D Financial System Using a Single Controller

By introducing the linear transformation $x_1(t) = x(t)$, $x_2(t) = y(t) - 1/b$, $x_3(t) = z(t)$, system (2) becomes

$$\begin{aligned} \dot{x}_1 &= \left(\frac{1}{b} - a\right)x_1 + x_3 + x_1 x_2, \\ \dot{x}_2 &= -bx_2 - x_1^2, \\ \dot{x}_3 &= -x_1 - cx_3. \end{aligned} \tag{9}$$

System (2) and system (9) are topologically equivalent.

System (9) can be written in compact form as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \tag{10}$$

where $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$, $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}}$. And it is easy to obtain that the origin $O(0, 0, 0)$ is an equilibrium point of system (9).

Consider the following subsystem of system (9):

$$\begin{aligned} \dot{x}_2 &= -bx_2 - x_1^2, \\ \dot{x}_3 &= -x_1 - cx_3. \end{aligned} \tag{11}$$

Obviously, the vector function $(f_2, f_3) = (-bx_2 - x_1^2, -x_1 - cx_3)$ is smooth in a neighborhood of $x_1 = 0$, and the system

$$\begin{aligned} \dot{x}_2 &= f_2(0, x_2, x_3), \\ \dot{x}_3 &= f_3(0, x_2, x_3) \end{aligned} \tag{12}$$

is stable about the origin $(x_2, x_3) = (0, 0)$ for all $(x_2, x_3) \in R^2$. Based on the above properties of system (9), this section will design a single controller u exerted to f_1 to achieve the finite time stability of the origin of system (9). The controlled system of the three-dimension financial system is given by

$$\begin{aligned}\dot{x}_1 &= f_1(\boldsymbol{x}) + u,\\ \dot{x}_2 &= f_2(\boldsymbol{x}),\\ \dot{x}_3 &= f_3(\boldsymbol{x}).\end{aligned} \quad (13)$$

Then we give the following finite time stability theorem for the controlled system (13).

Theorem 3. *The origin of system (9) can be stabilized within a finite time, if the controller u is designed as*

$$u = \begin{cases} px_1 - \xi \dfrac{(V_1(\boldsymbol{x}))^\alpha}{x_1}, & \text{if } x_1 \neq 0 \\ 0, & \text{if } x_1 = 0 \end{cases} \quad (14)$$

where $\xi > 0$, $\alpha \in (0,1)$ *and* $V_1(\boldsymbol{x}) = (1/2)(x_1)^2 + V_0(x_2, x_3)$, *in which* V_0 *is a positive definite function of* (x_2, x_3). p *is adapted under the following update law:*

$$\dot{p} = -\sigma \boldsymbol{x}^2, \quad (15)$$

where $\boldsymbol{x}^2 = x_1^2 + x_2^2 + x_3^2$ *and* σ *is a control parameter (*σ *can be arbitrary positive real number).*

Proof. Introduce a positive definite function as follows.

$$V(p, \boldsymbol{x}) = V_1(\boldsymbol{x}) + \frac{1}{2\sigma}(p + L)^2, \quad (16)$$

where L is a constant coefficient that needs to be determined.

By calculating the time derivative of $V(p, \boldsymbol{x})$ along the trajectory of system (13), we obtain

$$\begin{aligned}\dot{V}(p, \boldsymbol{x}) &= \frac{\partial V_1(\boldsymbol{x})}{\partial \boldsymbol{x}} \cdot (f_1(\boldsymbol{x}) + u, f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}} \\ &+ \frac{1}{\sigma}(p + L)\dot{p} = \frac{\partial V_1(\boldsymbol{x})}{\partial x_1} \cdot (f_1(\boldsymbol{x}) + u) \\ &+ \left(\frac{\partial V_0(x_2, x_3)}{\partial x_2}, \frac{\partial V_0(x_2, x_3)}{\partial x_3}\right) \cdot (f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}} \\ &- (p + L)\boldsymbol{x}^2 = x_1\left(f_1(\boldsymbol{x}) + px_1 - \xi\frac{(V_1(\boldsymbol{x}))^\alpha}{x_1}\right) \\ &+ \left(\frac{\partial V_0(x_2, x_3)}{\partial x_2}, \frac{\partial V_0(x_2, x_3)}{\partial x_3}\right) \\ &\cdot \left((f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}}\right. \\ &- \left.(f_2(0, x_2, x_3), f_3(0, x_2, x_3))^{\mathrm{T}}\right) \\ &+ \left(\frac{\partial V_0(x_2, x_3)}{\partial x_2}, \frac{\partial V_0(x_2, x_3)}{\partial x_3}\right) \\ &\cdot (f_2(0, x_2, x_3), f_3(0, x_2, x_3))^{\mathrm{T}} - (p + L)\boldsymbol{x}^2\end{aligned} \quad (17)$$

Since $f_1(\boldsymbol{x})$ is a Lipschitz continuous function, a positive real number c_1 can be found, such that

$$\|f_1(\boldsymbol{x})\| \leq c_1 \|\boldsymbol{x}\| \quad (18)$$

Obviously, the vector function $(f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))$ is smooth in the neighborhood of $x_1 = 0$, so there exists a real $c_2 > 0$ such that

$$\begin{aligned}&\left\|\left((f_2(x_1, x_2, x_3), f_3(x_1, x_2, x_3))^{\mathrm{T}}\right.\right. \\ &\quad - \left.\left.(f_2(0, x_2, x_3), f_3(0, x_2, x_3))^{\mathrm{T}}\right)\right\| \leq c_2\|x_1\| \\ &\leq c_2\|\boldsymbol{x}\|\end{aligned} \quad (19)$$

The system $\dot{x}_2 = f_2(0, x_2, x_3), \dot{x}_3 = f_3(0, x_2, x_3)$ can be written as

$$\begin{aligned}\dot{x}_2 &= -bx_2,\\ \dot{x}_3 &= -cx_3.\end{aligned} \quad (20)$$

For this system, choosing Lyapunov function as

$$V_0(x_2, x_3) = \frac{1}{2}(x_2^2 + x_3^2), \quad (21)$$

and taking the time derivative of V_0 along the solution of system (20), one has

$$\begin{aligned}\dot{V}_0(x_2, x_3) &= \left(\frac{\partial V_0(x_2, x_3)}{\partial x_2}, \frac{\partial V_0(x_2, x_3)}{\partial x_3}\right) \\ &\quad \cdot (f_2(0, x_2, x_3), f_3(0, x_2, x_3))^{\mathrm{T}} \\ &\leq -bx_2^2 - cx_3^2.\end{aligned} \quad (22)$$

So there is a real number $c_3 > 0$, such that

$$\dot{V}_0(x_2, x_3) \leq -c_3\|(x_2, x_3)\|^2. \quad (23)$$

It is obvious that

$$\left\|\left(\frac{\partial V_0(x_2, x_3)}{\partial x_2}, \frac{\partial V_0(x_2, x_3)}{\partial x_3}\right)\right\| = \|(x_2, x_3)\| \quad (24)$$

Substituting inequalities (18), (19), (23), and (24) into the right hand of (34), we can get the following inequality:

$$\begin{aligned}\dot{V}(p, \boldsymbol{x}) &\leq c_1\|\boldsymbol{x}\|^2 + c_2\|\boldsymbol{x}\|\|(x_2, x_3)\| - c_3\|(x_2, x_3)\|^2 \\ &\quad - \xi(V_1(\boldsymbol{x}))^\alpha - L\|\boldsymbol{x}\|^2\end{aligned} \quad (25)$$

Taking $L \geq c_1 + c_2^2/4c_3$, we have

$$\dot{V}(p, \boldsymbol{x}) \leq -\xi(V_1(\boldsymbol{x}))^\alpha \quad (26)$$

From (33), it is easy to get that $V(p, \boldsymbol{x}) \geq V_1(\boldsymbol{x})$. So we can find a positive constant $l = \xi/c$ and one has

$$(V(p, \boldsymbol{x}))^\alpha \leq l(V_1(\boldsymbol{x}))^\alpha \quad (27)$$

Combining inequalities (26) and (27) yields

$$\dot{V}(p,\boldsymbol{x}) \le -c\left(V(p,\boldsymbol{x})\right)^{\alpha} \tag{28}$$

According to Lemma 2, under the adaptive law (30), the three-dimension financial system (9) can be stabilized by the controller (29) in finite time. The proof is then completed. □

In fact, the finance system is very complex and some system parameters are difficult to obtain. Therefore, to study the finite time control of finance system with unknown parameters is of great importance to realize the economical stable growth. When the parameter a of system (9) is unknown, we provide the following finite time stability theorem for the controlled system (13).

Theorem 4. *The origin of system (9) with unknown parameter a can be stabilized within a finite time, if the controller u is designed as*

$$u = \begin{cases} (p+\hat{a})x_1 - \xi\dfrac{(V_1(\boldsymbol{x}))^{\alpha}}{x_1}, & if\ x_1 \neq 0 \\ 0, & if\ x_1 = 0 \end{cases} \tag{29}$$

where $\xi > 0$, $\alpha \in (0,1)$ and $V_1(\boldsymbol{x}) = (1/2)(x_1)^2 + V_0(x_2,x_3)$, in which V_0 is a positive definite function of (x_2,x_3). p and $\hat{a}$ are adapted under the following update law, respectively:

$$\dot{p} = -\sigma\boldsymbol{x}^2, \tag{30}$$

$$\dot{\hat{a}} = -x_1^2, \tag{31}$$

where $\boldsymbol{x}^2 = x_1^2 + x_2^2 + x_3^2$, σ is an control parameter (σ can be arbitrary positive real number), and $\hat{a}$ is the estimation of the unknown parameter a.

Proof. Let $e_a = a - \hat{a}$; then

$$\dot{e_a} = -\dot{\hat{a}}. \tag{32}$$

Introduce a positive definite function

$$V(p,\boldsymbol{x}) = V_1(\boldsymbol{x}) + \frac{1}{2\sigma}(p+L)^2 + \frac{1}{2}e_a^2, \tag{33}$$

where L is a constant coefficient that needs to be determined.

The time derivative of $V(p,\boldsymbol{x})$ along the trajectories of systems (13) and (32) is

$$\begin{aligned}\dot{V}(p,\boldsymbol{x}) &= \frac{\partial V_1(\boldsymbol{x})}{\partial \boldsymbol{x}}\cdot\left(f_1(\boldsymbol{x})+u, f_2(\boldsymbol{x}), f_3(\boldsymbol{x})\right)^{\mathrm{T}} \\ &+ \frac{1}{\sigma}(p+L)\dot{p} + \dot{e}_a e_a = \frac{\partial V_1(\boldsymbol{x})}{\partial x_1}\cdot\left(\widetilde{f}_1(\boldsymbol{x}) - ax_1\right. \\ &\left.+ u\right) + \left(\frac{\partial V_0(x_2,x_3)}{\partial x_2}, \frac{\partial V_0(x_2,x_3)}{\partial x_3}\right) \\ &\cdot\left(f_2(\boldsymbol{x}), f_3(\boldsymbol{x})\right)^{\mathrm{T}} - (p+L)\boldsymbol{x}^2 + \dot{e}_a e_a \\ &= x_1\left(\widetilde{f}_1(\boldsymbol{x}) + px_1 - \xi\frac{(V_1(\boldsymbol{x}))^{\alpha}}{x_1}\right) \\ &+ \left(\frac{\partial V_0(x_2,x_3)}{\partial x_2}, \frac{\partial V_0(x_2,x_3)}{\partial x_3}\right) \\ &\cdot\left(\left(f_2(\boldsymbol{x}), f_3(\boldsymbol{x})\right)^{\mathrm{T}}\right. \\ &\left.- \left(f_2(0,x_2,x_3), f_3(0,x_2,x_3)\right)^{\mathrm{T}}\right) \\ &+ \left(\frac{\partial V_0(x_2,x_3)}{\partial x_2}, \frac{\partial V_0(x_2,x_3)}{\partial x_3}\right) \\ &\cdot\left(f_2(0,x_2,x_3), f_3(0,x_2,x_3)\right)^{\mathrm{T}} - (p+L)\boldsymbol{x}^2,\end{aligned} \tag{34}$$

where $\widetilde{f}_1(\boldsymbol{x}) = f_1(\boldsymbol{x}) + ax_1$. Obviously, $\widetilde{f}_1(\boldsymbol{x})$ also is a Lipschitz continuous function. The remaining proof of Theorem 4 is similar to that of Theorem 3. □

4. Finite Time Stability of 4D Financial System with Market Confidence Using Two Controllers

By applying linear transformation $y_1(t) = x(t)$, $y_2(t) = y(t) - 1/b$, $y_3(t) = z(t)$, $y_4(t) = w(t)$, system (3) becomes

$$\begin{aligned}\dot{y}_1 &= \left(\frac{1}{b} - a\right)y_1 + y_3 + y_1y_2 + m_1y_4, \\ \dot{y}_2 &= -by_2 - y_1^2 + m_2y_4, \\ \dot{y}_3 &= -y_1 - cy_3 + m_3y_4, \\ \dot{y}_4 &= -y_1y_2y_3 - \frac{1}{b}y_1y_3.\end{aligned} \tag{35}$$

The compact form of system (35) can be written as

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}), \tag{36}$$

where $\boldsymbol{y} = (y_1,y_2,y_3,y_4)^{\mathrm{T}}$, $\boldsymbol{g}(\boldsymbol{y}) = (g_1(\boldsymbol{y}), g_2(\boldsymbol{y}), g_3(\boldsymbol{y}), g_4(\boldsymbol{y}))^{\mathrm{T}}$. And $O(0,0,0,0)$ is an equilibrium point of system (35).

Consider the following subsystem of system (35):

$$\begin{aligned}\dot{y}_2 &= -by_2 - y_1^2 + m_2y_4, \\ \dot{y}_3 &= -y_1 - cy_3 + m_3y_4.\end{aligned} \tag{37}$$

Obviously, the vector function $(g_2,g_3) = (-by_2 - y_1^2 + m_2y_4, -y_1 - cy_3 + m_3y_4)$ is smooth in a neighborhood of $(y_1,y_4) = (0,0)$, and the system

$$\begin{aligned}\dot{y}_2 &= g_2(0,y_2,y_3,0), \\ \dot{y}_3 &= g_3(0,y_2,y_3,0)\end{aligned} \tag{38}$$

is stable about the origin $(y_2,y_3) = (0,0)$ for all $(y_2,y_3) \in R^2$. So we will design a double controller u_1, u_4 to achieve the finite time stability of the origin of system (35) in this section. The controlled system of the 4D financial system is given by

$$\begin{aligned}\dot{y}_1 &= g_1(\boldsymbol{y}) + u_1,\\ \dot{y}_2 &= g_2(\boldsymbol{y}),\\ \dot{y}_3 &= g_3(\boldsymbol{y}),\\ \dot{y}_4 &= g_4(\boldsymbol{y}) + u_4.\end{aligned} \tag{39}$$

Now, we propose the following finite time stability theorem for the controlled system (39).

Theorem 5. *The origin of system (35) can be stabilized within a finite time, if the controller u is designed as*

$$(u_1, u_4)^{\mathrm{T}} = \begin{cases} k(y_1, y_4)^{\mathrm{T}} - \xi' \dfrac{(V_1(\boldsymbol{y}))^{\alpha}(y_1, y_4)^{\mathrm{T}}}{\|(y_1, y_4)\|^2}, & \text{if } \|(y_1, y_4)\| \neq 0\\ 0, & \text{if } \|(y_1, y_4)\| = 0\end{cases} \tag{40}$$

where $\xi > 0,\ \alpha \in (0,1)$, *and* V_1 *is given as*

$$V_1(\boldsymbol{y}) = \frac{1}{2}\left((y_1, y_4)^{\mathrm{T}} \cdot (y_1, y_4)\right) + V_0(y_2, y_3), \tag{41}$$

in which V_0 *is a positive definite function of* (y_2, y_3), *and k is adapted adjustment according to the following update rule:*

$$\dot{k} = -\zeta \boldsymbol{y}^2, \tag{42}$$

where $\boldsymbol{y}^2 = y_1^2 + y_2^2 + y_3^2 + y_4^2$, ζ *is a control parameter* (ζ *can be arbitrary real number).*

Proof. Introduce a Lyapunov function with an undetermined constant coefficient L as follows:

$$V(k, \boldsymbol{y}) = V_1(\boldsymbol{y}) + \frac{1}{2\zeta}(k + L)^2, \tag{43}$$

By calculating the time derivative of $V(k, y)$ along the trajectory of system (39), we have

$$\begin{aligned}\dot{V}(k, \boldsymbol{y}) &= \frac{\partial V_1(\boldsymbol{y})}{\partial \boldsymbol{y}} \cdot (g_1(\boldsymbol{y}) + u_1, g_2(\boldsymbol{y}), g_3(\boldsymbol{y}), g_4(\boldsymbol{y}) + u_4)^{\mathrm{T}} + \frac{1}{\zeta}(k + L)\dot{k}\\ &= \left(\frac{\partial V_1(\boldsymbol{y})}{\partial y_1}, \frac{\partial V_1(\boldsymbol{y})}{\partial y_4}\right) \cdot (g_1(\boldsymbol{y}) + u_1, g_4(\boldsymbol{y}) + u_4)^{\mathrm{T}} + \left(\frac{\partial V_0(y_2, y_3)}{\partial y_2}, \frac{\partial V_0(y_2, y_3)}{\partial y_3}\right)\\ &\quad \cdot (g_2(\boldsymbol{y}), g_3(\boldsymbol{y}))^{\mathrm{T}} - (k + L)\boldsymbol{y}^2 = (y_1, y_4)\\ &\quad \cdot \left((g_1(\boldsymbol{y}), g_4(\boldsymbol{y}))^{\mathrm{T}} + k(y_1, y_4)^{\mathrm{T}} - \xi \frac{(V_1(\boldsymbol{y}))^{\alpha}(y_1, y_4)^{\mathrm{T}}}{\|(y_1, y_4)\|^2}\right)\\ &\quad + \left(\frac{\partial V_0(y_2, y_3)}{\partial y_2}, \frac{\partial V_0(y_2, y_3)}{\partial y_3}\right) \cdot \left((g_2(\boldsymbol{y}), g_3(\boldsymbol{y}))^{\mathrm{T}} - (g_2(0, y_2, y_3, 0), g_3(0, y_2, y_3, 0))^{\mathrm{T}}\right)\\ &\quad + \left(\frac{\partial V_0(y_2, y_3)}{\partial y_2}, \frac{\partial V_0(y_2, y_3)}{\partial y_3}\right) \cdot (g_2(0, y_2, y_3, 0), g_3(0, y_2, y_3, 0))^{\mathrm{T}} - (k + L)\boldsymbol{y}^2\end{aligned} \tag{44}$$

Since $g_1(\boldsymbol{y}), g_4(\boldsymbol{y})$ are Lipschitz continuous, and the vector function $(g_2(\boldsymbol{y}), g_3(\boldsymbol{y}))$ is smooth in the neighborhood of $(y_1, y_4) = (0, 0)$, there exists real $l_1, l_2 > 0$, such that

$$\|(g_1(\boldsymbol{y}), g_4(\boldsymbol{y}))\| \leq l_1 \|\boldsymbol{y}\| \tag{45}$$

$$\begin{aligned}&\left\|(g_2(\boldsymbol{y}), g_3(\boldsymbol{y}))^{\mathrm{T}} - (g_2(0, y_2, y_3, 0), g_3(0, y_2, y_3, 0))^{\mathrm{T}}\right\|\\ &\quad \leq l_2 \|(y_2, y_3)\| \leq l_2 \|\boldsymbol{y}\|\end{aligned} \tag{46}$$

For the system $\dot{y}_2 = g_2(0, y_2, y_3, 0), \dot{y}_3 = g_3(0, y_2, y_3, 0)$, choosing Lyapunov function as $V_0(y_2, y_3) = (1/2)(y_2^2 + y_3^2)$, and taking the time derivative of V_0 along the solution this system, one has

$$\begin{aligned}\dot{V}_0(y_2, y_3) &= \left(\frac{\partial V_0(y_2, y_3)}{\partial y_2}, \frac{\partial V_0(y_2, y_3)}{\partial y_3}\right) \cdot (g_2(0, y_2, y_3, 0), g_3(0, y_2, y_3, 0))^{\mathrm{T}}\\ &\leq -by_2^2 - cy_3^2 \leq -l_3 \|(y_2, y_3)\|^2,\end{aligned} \tag{47}$$

where $l_3 > 0$ is a real number.

Combining (44), (45), (46), and (47) gives

$$\begin{aligned}\dot{V}(k, \boldsymbol{y}) &\leq l_1 \|\boldsymbol{y}\|^2 + l_2 \|\boldsymbol{y}\| \|(y_2, y_3)\| - l_3 \|(y_2, y_3)\|^2\\ &\quad - \xi (V_1(\boldsymbol{y}))^{\alpha} - L \|\boldsymbol{y}\|^2\end{aligned} \tag{48}$$

Similar to the proof of Theorem 3, we get

$$\dot{V}(k, \boldsymbol{y}) \leq -c (V(k, \boldsymbol{y}))^{\alpha} \tag{49}$$

According to Lemma 2, the four-dimension financial system (35) can be stabilized by the controller (40) under the adaptive law (42) in finite time. □

Remark 6. Similar to Theorem 4, when the parameter a is unknown, system (35) also can be stabilized within a finite time by simply substituting the controller (40) in Theorem 5 with

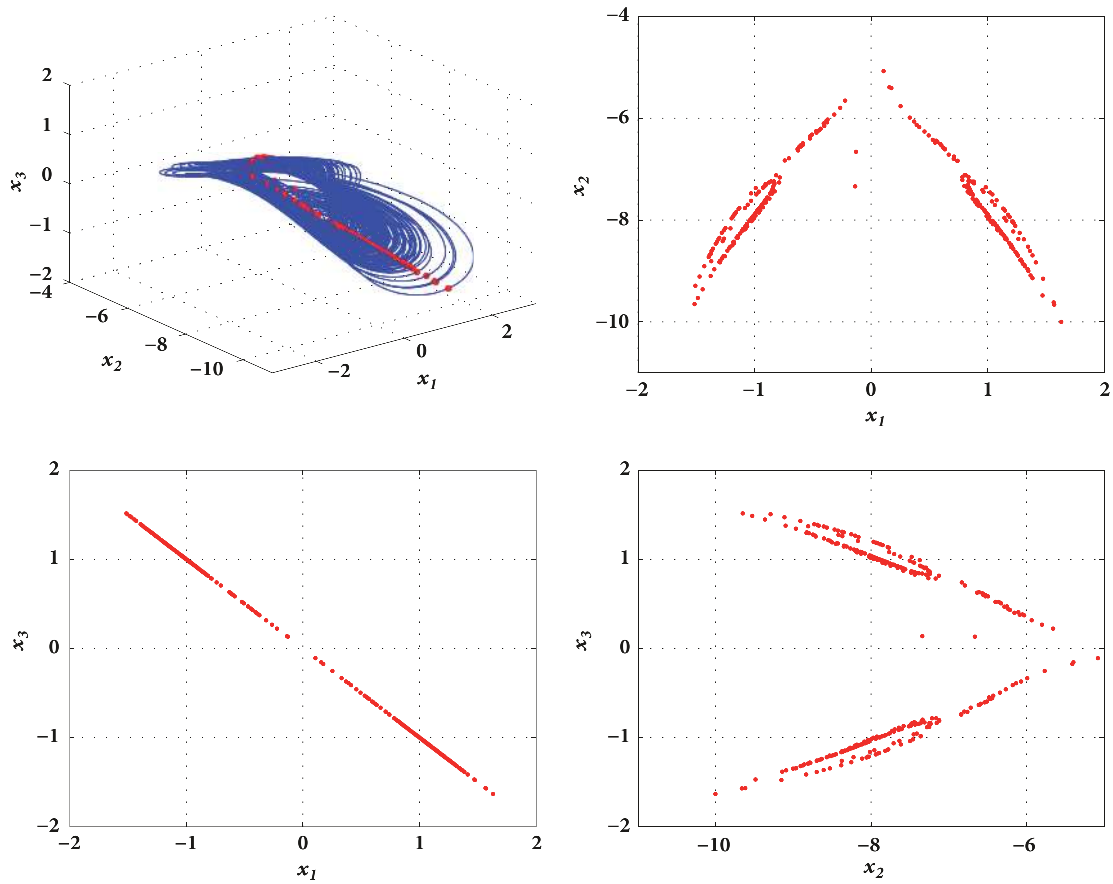


FIGURE 1: The chaotic attractor and projects of poincare section of 3D finance system (9).

$$(u_1, u_4)^{\mathrm{T}} = \begin{cases} k\left(y_1, y_4\right)^{\mathrm{T}} - \xi' \dfrac{\left(V_1\left(\mathbf{y}\right)\right)^{\alpha}\left(y_1, y_4\right)^{\mathrm{T}}}{\left\|\left(y_1, y_4\right)\right\|^2} + \left(\hat{a} y_1, 0\right)^{\mathrm{T}}, & \text{if } \left\|\left(y_1, y_4\right)\right\| \neq 0 \\ 0, & \text{if } \left\|\left(y_1, y_4\right)\right\| = 0 \end{cases} \tag{50}$$

and designing the parameter updated law as

$$\dot{\hat{a}} = -y_1^2 \tag{51}$$

5. Simulation Results

In this section, we will give some numerical simulations to show the chaotic behaviors of the finance systems (9) and (35). Furthermore, other simulation results are employed to illustrate the effectiveness and advantage of the proposed control method.

5.1. 3D Finance System. When $a = 2, b = 0.1, c = 1$, and the Poincare plane is defined as $x_1 = -x_3$, the chaotic attractor and projects of poincare section of system (9) on coordinate planes are shown in Figure 1.

The existence of chaos makes it very difficult to predict and analyze the economic trends. In the following subsections, the effectiveness of the controller in Theorems 3 and 4 will be shown through simulations, and some explanations from the view of economics will be given.

5.1.1. Finite Time Control with Known Parameters. To confirm the validity of the control method proposed in Theorem 3, we carry out numerical simulations using the following initial conditions $[x_1(0), x_2(0), x_3(0)] = [8, 5, 3]$, $p(0) = -1$, and the control parameters $\xi = 0.000001, \alpha = 1/2, \sigma = 5$. As can be clearly seen from Figure 2, the trajectory of the controlled system (13) converges to origin. This means that the chaos behavior in the finance system will be eliminated, and the stabilized economic performance can be achieved. And more significantly, this result provides an effective economic adjustment measure by only applying the interest rate management tool.

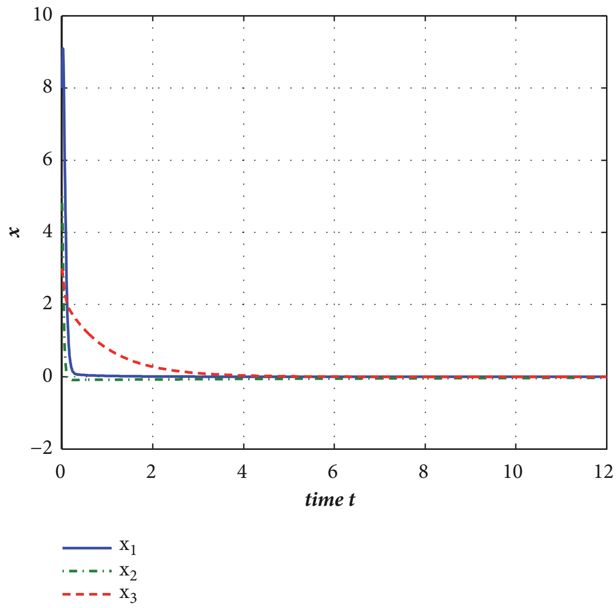


Figure 2: Stabilization of the 3D finance system (9).

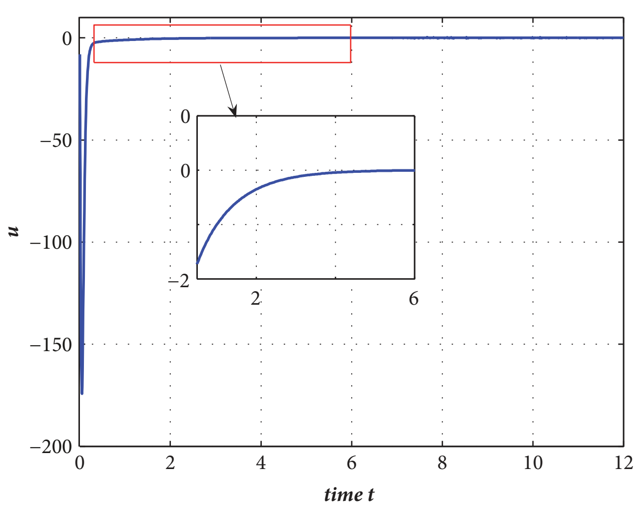


Figure 3: The trajectory of the controller u.

From Figure 3, one can see that the trajectory of the controller u also converges to zero within a short period of time, which means that the financial regulation policies can be removed when the finance system has stabilized in finite time.

Remark 7. For the autonomous system $\dot{\boldsymbol{x}} = A\boldsymbol{x} + f(\boldsymbol{x})$, where $\boldsymbol{x} = (x_1, x_2, ..., x_n)^{\mathrm{T}}$, $A = (a_{ij})_{n\times n}$, $\boldsymbol{f}(\boldsymbol{x})$ is a nonlinear function. In [25], the speed feedback control method was designed as $-k\dot{x}_i$ feedback on the right side of the equation of x_j of $k > 0, i \neq j$. k is a control parameter that satisfies Routh-Hurwitz criterion. Obviously, this method provides six operational approaches to control a three-dimensional system. However, stabilization of the 3D finance system is only achieved when $-k\dot{x}_1$ is fed back on x_3 (a proof is provided in Appendix). According to the analysis in Appendix, the parameters are taken as $a = 2, b = 0.3, c = 0.5$. When the initial conditions $[x_1(0), x_2(0), x_3(0)] = [8, 5, 3]$, the finite time control result with $p(0) = -1, \xi = 0.000001, \alpha = 1/2, \sigma = 3$ and speed feedback control result with $k = 3$ are shown in Figures 4(a) and 4(b), respectively. Combining with Figure 5, we can see clearly that the finite time controller provides more effective vibration reduction and converges more quickly with smaller control inputs than the speed feedback controller. This means that the economic control measures designed according to the control method presented in this paper can stabilize the economic system by using smaller regulation in a shorter time than that of speed feedback control.

Remark 8. Reference [6] presented the adaptive method of system (9). According to this method the controlled system is designed as $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}) - \tilde{k}x_1, f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}}, \tilde{k} = \beta x_1^2$. In addition, linear feedback control method designed as $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}) - \theta x_1, f_2(\boldsymbol{x}), f_3(\boldsymbol{x}))^{\mathrm{T}}$ is also used to compare with the presented method. When $a = 2$, $b = 0.1$, $c = 1$, $[x_1(0), x_2(0), x_3(0)] = [8, 5, 3]$, $p(0) = -1$, $\xi = 0.000001, \alpha = 1/2, \sigma = 5$ and $\beta = 10, \theta = 8$. The control results and the controllers varying with time are demonstrated in Figures 6 and 7, respectively. It can be seen from the simulation results that the control input of the presented method has a weak advantage over adaptive and linear feedback method. However, the presented method exhibits a much better speed of convergence than the other two methods. From the view of economics, the shorter the adjustment time, the smaller the influence of the economic chaos, which reveals that the presented method has distinct advantages compared to other two methods.

5.1.2. Finite Time Control with Unknown Parameters. To verify the effectiveness of the control method in Theorem 4, we carry out numerical simulations under the initial conditions $[x_1(0), x_2(0), x_3(0)] = [8, 5, 3]$, $p(0) = -1$, and the control parameters $\xi = 0.000001, \alpha = 1/2, \sigma = 4.235$. The initial estimated value of parameter a is $\hat{a}(0) = -5$. As can be clearly seen from Figure 8, the trajectory of the controlled system converges to origin, and the estimated value of a converges to its real value. These results clearly show that the effective control of economic crisis can be realized even if some financial data is unknown.

5.2. 4D Finance System. When $a = 2, b = 5, m_1 = 4.4, m_2 = 4.4, m_3 = 0.2$, the simulation results exhibit that the system (35) enters chaos state through period-doubling bifurcation with the parameter c changing from 1.4 to 1.3. For example, when $c = 1.4$, system (35) exhibits a single limit cycle as illustrated in Figure 9(a). And $c = 1.35$, a double limit cycle as Figure 9(b); $c = 1.33$, a 4-period limit cycle as Figure 9(c); $c = 1.32$, an 8-period limit cycle as Figure 9(d); $c = 1.319$, a multiperiod limit cycle as Figure 9(e). When $c = 1.31$, chaos attractor with largest Lyapunov exponent 0.087 occurs in the system as shown in Figure 9(f).

When the parameters and the Poincare section are taken as $a = 2, b = 5, c = 1.3, m1 = 4.4, m2 = 4.4, m3 = 0.2$, and

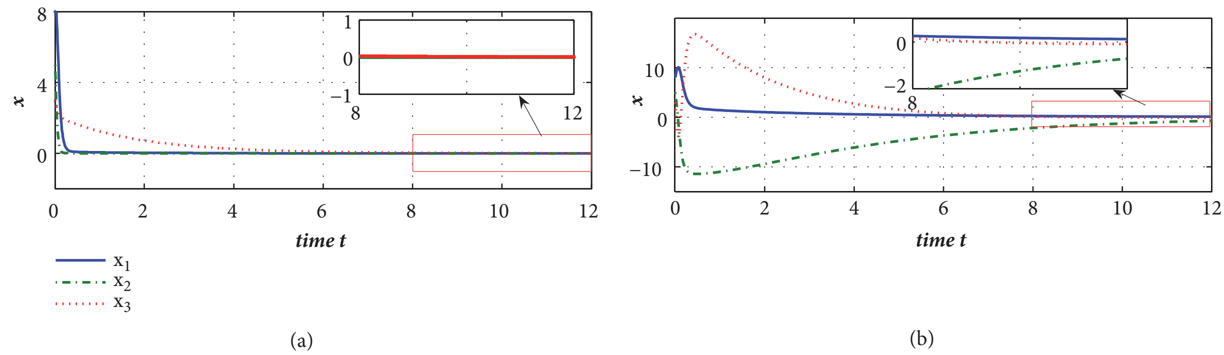


FIGURE 4: The control results of the finite time method and speed feedback method of system (9): (a) finite time method; (b) speed feedback method.

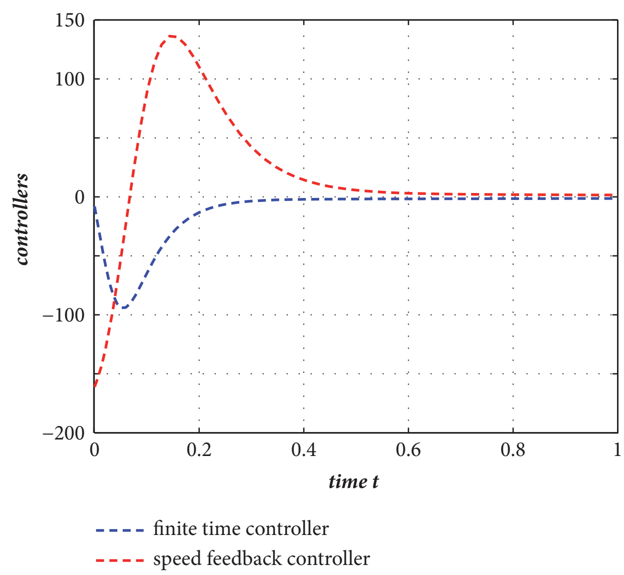


FIGURE 5: Controllers of the finite time method and speed feedback method of system (9).

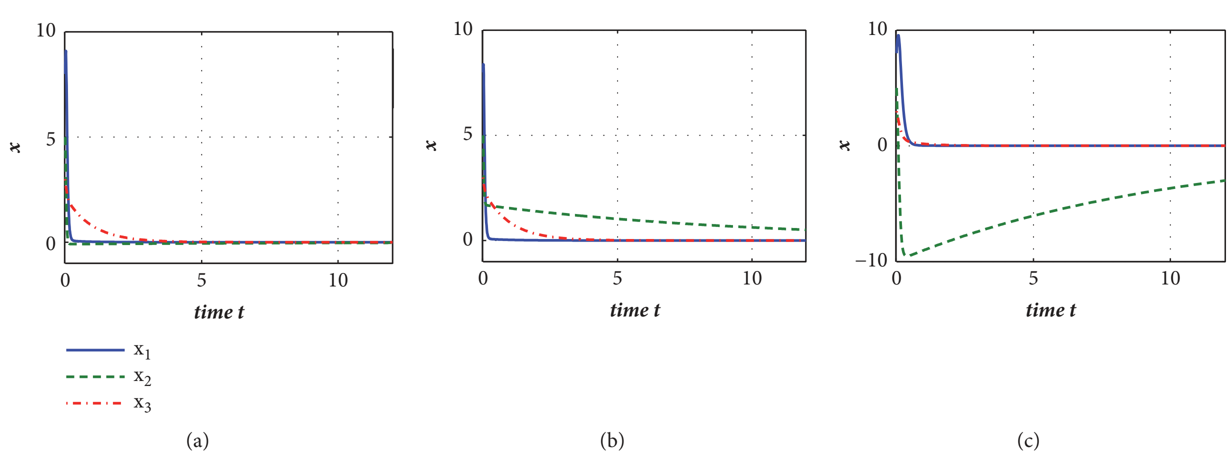


FIGURE 6: The control results of system (9) with different method: (a) finite time method, (b) adaptive method, and (c) linear feedback method.

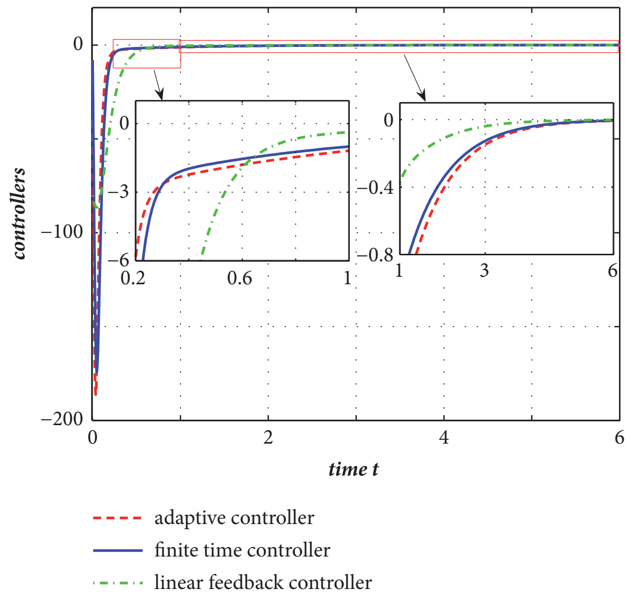


FIGURE 7: Controllers of the finite time method, adaptive method and linear feedback method of system (9).

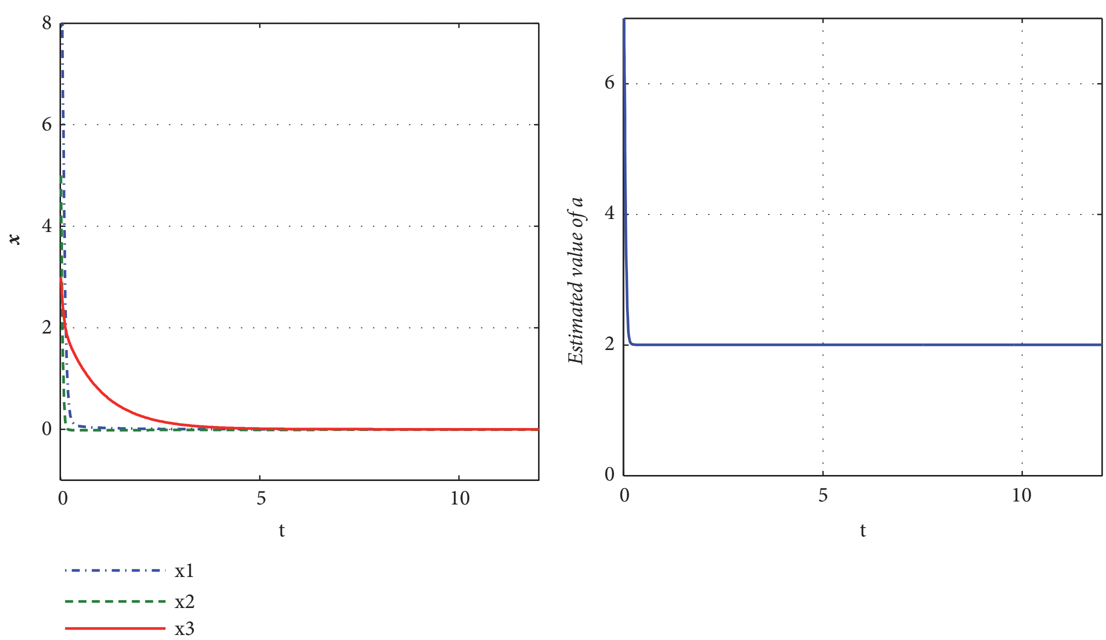


FIGURE 8: The control results of 3D finance system (9) with unknown parameter a, and the estimated value of a.

$0.0029x_1 + 3.0292x_2 - 4.782x_3 = 7.2381$, the chaotic attractor and projects of poincare section of system (35) on coordinate planes are provided in Figure 10.

The simulation results in Figures 9 and 10 show that the dynamics of the finance system (35) is unstable when the parameter c varies in the range between 1.4 and 1.3. The existence of period doubling bifurcation cascade means that the economic volatility increases exponentially with the parameter changing and finally results in economic crisis. For any country, this unstable phenomenon, especially the large economic fluctuations, is undesirable. So, it is essential to control these unstable behaviors in the finance system. Under the following choices of initial conditions $y_1(0) = 1.5$, $y_2(0) = 2$, $y_3(0) = 0.3$, $y_4(0) = 0.1$, $k(0) = -1$, and the control parameters $\zeta = 0.00001$, $\alpha = 4/5$, $\xi' = 15$, the dynamics behavior of the controlled system (39) designed according to Theorem 5 is exhibited in Figure 11(a). The control signals u_1 and u_4 varying with time are shown in Figure 12, which means

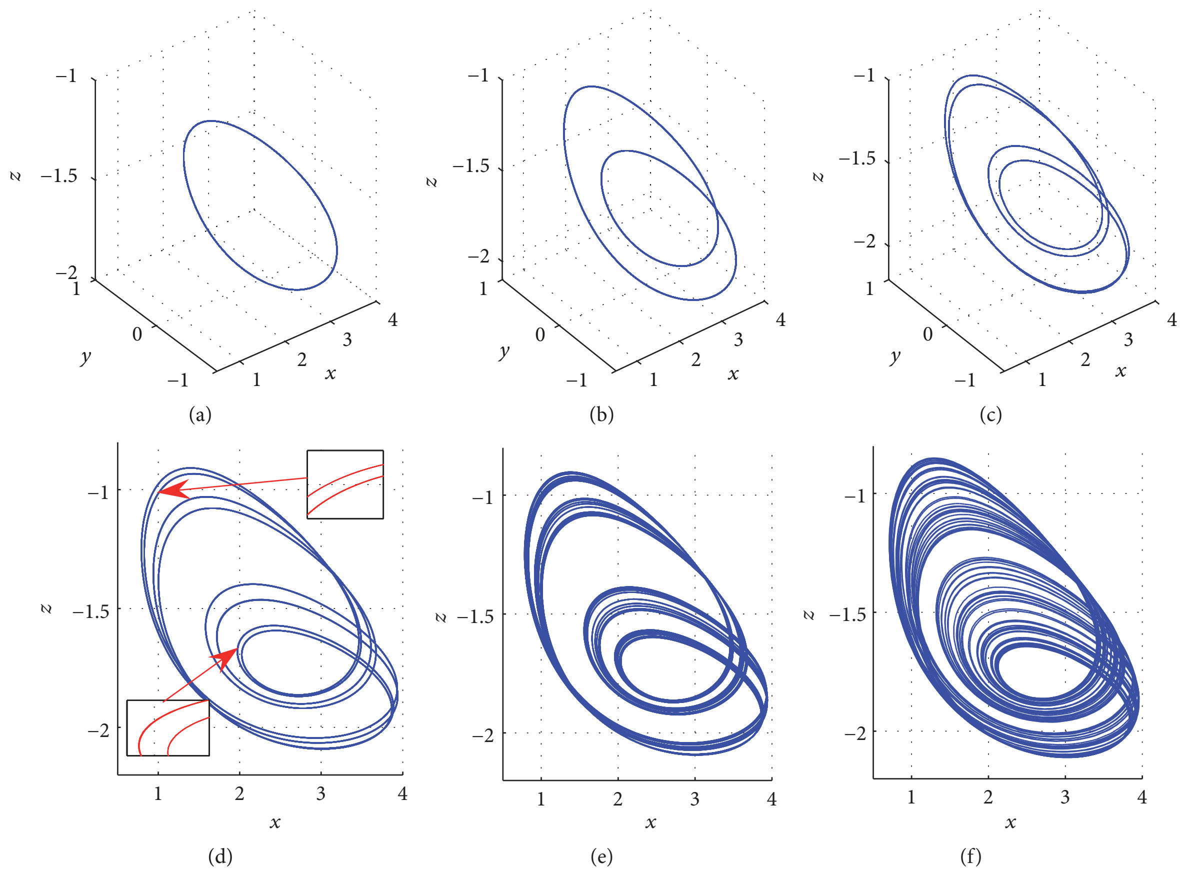


Figure 9: The period doubling bifurcation route to chaos of the 4D finance system (35) while the parameter c changes.

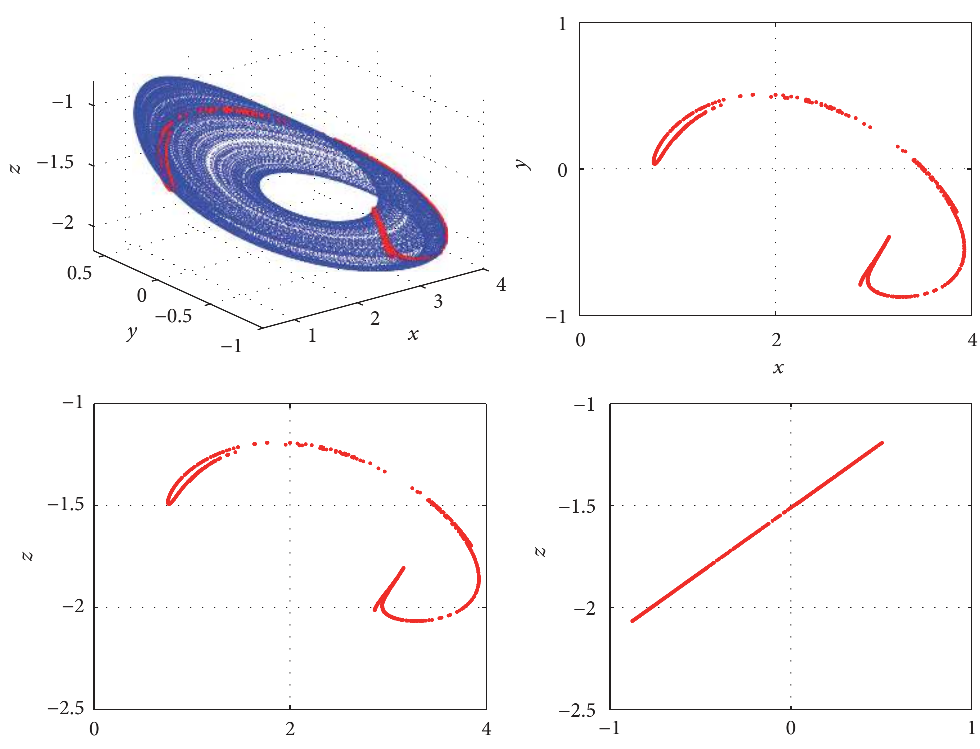
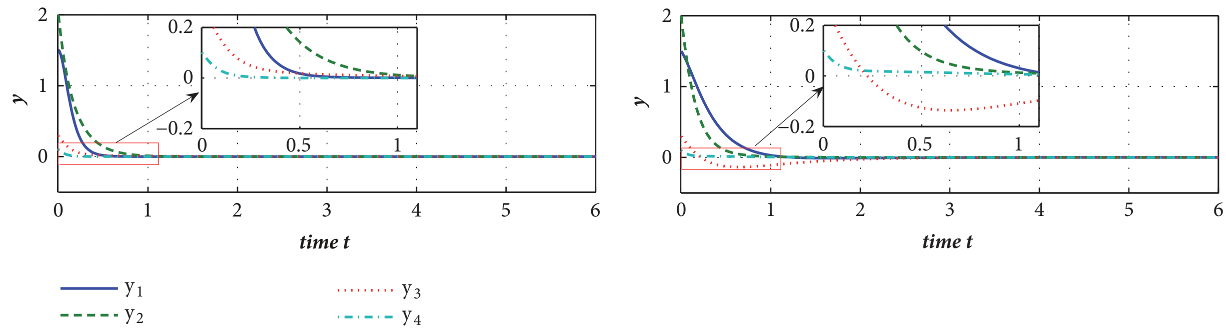


Figure 10: The chaotic attractor and projects of poincare section of the system (35).



(a) (b)

Figure 11: The control results of 4D finance system (35) with different method: (a) finite time method; (b) linear feedback method.

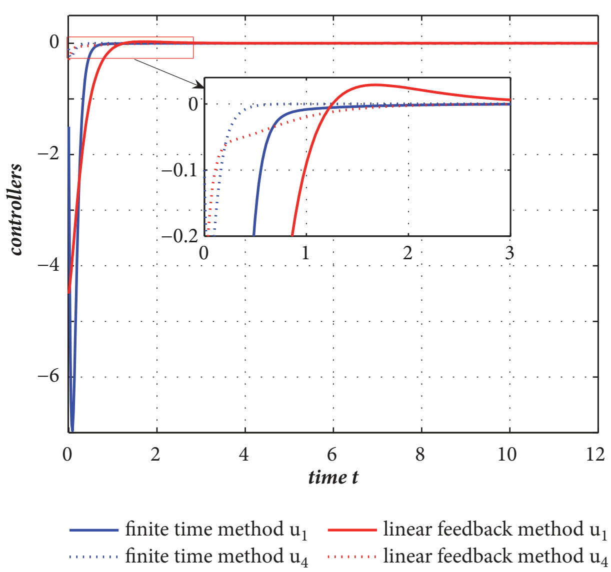


Figure 12: Controllers of the finite time method and linear feedback method of 4D finance system (35).

that the use of the interest rate tool and the rebuilding of the market confidence can put the economy on track when the economic chaos arises.

Remark 9. Linear feedback control designed as $\boldsymbol{f}(\boldsymbol{y}) = (f_1(\boldsymbol{y}) - \epsilon y_1,\ f_2(\boldsymbol{y}), f_3(\boldsymbol{y}),\ f_4(\boldsymbol{y}) - \epsilon y_4)^{\mathrm{T}}$ is employed to compare with the presented method. When $[y_1(0), y_2(0), y_3(0), y_4(0)] = [1.5, 2, 0.3, 0.1]$, $\epsilon = 2$, the control results and the controllers varying with time are shown in Figures 11(b) and 12, where we can see that the states and controllers of the controlled system designed by finite time method have a better speed of convergence. The results show that the economic control measures designed according to the finite time method can stabilize the economic system in a shorter time than that of linear feedback control method.

Remark 10. The controlled system designed with adaptive method is

$$\boldsymbol{f}(\boldsymbol{y}) = \left(f_1(\boldsymbol{y}) - \tilde{\epsilon} y_1, f_2(\boldsymbol{y}), f_3(\boldsymbol{y}), f_4(\boldsymbol{y}) - \tilde{\epsilon}' y_4\right)^{\mathrm{T}}, \quad (52)$$

where $\tilde{\epsilon} = \beta y_1^2, \tilde{\epsilon}' = \beta y_4^2$. When $\beta = 10$, The trajectory of the controlled system and the controllers are displayed in Figures 13(b) and 14. It is clear that the presented method has the obvious advantage in convergence speed. From the simulation results, one can see that the financial control measures designed according to the adaptive method need much longer span of time to stabilize the economic system than the finite time method presented in this paper. This often means much more damage to the economic and the social development. Thus it can be seen that the finite time control method has a strong advantage in the application of stabilizing economic disarray.

Remark 11. The speed feedback control method presented in [25] cannot be applied to control the 4D finance system (35) by using one or two controllers.

Remark 12. The occurrence of chaos in the finance system most likely drives the system to slide into unpredictable state, even economic crisis. According to the method presented in this paper, an effective control measure can be designed to enable the economic state of the regions and countries affected by the crisis to be recovered to its normal state in finite time.

6. Conclusions

In general, a government would like to restore financial stability in finite time by using less financial regulation policies. For this reason, this paper investigates the finite time control of finance system with or without market confidence. The theoretical analysis proves that the finance systems can be stabilized effectively in finite time with less control input. The simulation results indicate that the proposed method can provide a faster convergence speed of the controlled finance systems than speed feedback method, linear feedback method, and adaptive control method. We hope that the

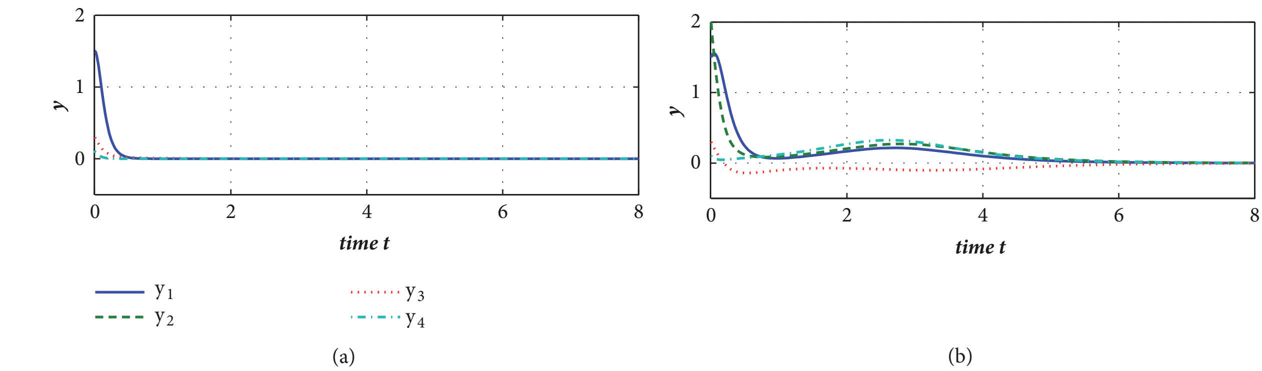


Figure 13: The control results of 4D finance system (35) with different method: (a) finite time method; (b) adaptive method.

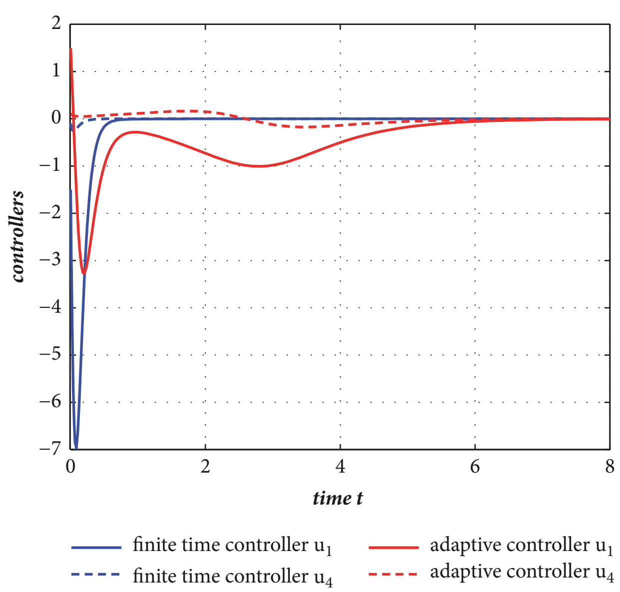


Figure 14: Controllers of the finite time method and adaptive method of 4D finance system (35).

results obtained in this paper would benefit the studies of healthy development of financial markets.

Appendix

The Jacobian matrix of system (9) is

$$J = \begin{pmatrix} \frac{1}{b} - a & 0 & -1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}, \quad \text{(A.1)}$$

and the characteristic equation of matrix (A.1) is

$$(\lambda + b)\left(\lambda^2 + \left(c - \left(\frac{1}{b} - a\right)\right)\lambda + 1 - c\left(\frac{1}{b} - a\right)\right) = 0. \quad \text{(A.2)}$$

Using Routh-Hurwitz criterion, the following proposition can be obtained.

Proposition 13. *System (9) is stable at the origin if the parameters satisfy the following conditions.*

$$\frac{1}{b} - a < c, \qquad \frac{1}{b} - a < \frac{1}{c}. \quad \text{(A.3)}$$

Case 1 ($-k\dot{x}_3, (k > 0)$ feedback on x_1). According to the speed feedback control method used in [25], the controlled 3D finance system is as follows:

$$\boldsymbol{f}(\boldsymbol{x}) = \left(f_1(\boldsymbol{x}) - k\dot{x}_3, f_2(\boldsymbol{x}), f_3(\boldsymbol{x})\right)^{\mathrm{T}}. \quad \text{(A.4)}$$

The Jacobian matrix and characteristic equation of the controlled system can be obtained as follows:

$$J = \begin{pmatrix} k + \left(\frac{1}{b} - a\right) & 0 & 1 + kc \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix}, \quad \text{(A.5)}$$

$$(\lambda + b) \cdot \left(\lambda^2 + \left(c - \left(k + \frac{1}{b} - a\right)\right)\lambda + 1 - c\left(\frac{1}{b} - a\right)\right) = 0, \quad \text{(A.6)}$$

where $a, b, c, k > 0$. Then, we can give the following proposition.

Proposition 14. *The controlled system (A.4) will converge to origin when*

$$k < c - \left(\frac{1}{b} - a\right), \qquad \frac{1}{b} - a < \frac{1}{c}. \quad \text{(A.7)}$$

Combining the above two propositions, the following theorem can be derived.

Theorem 15. *The relation between the control performance and the parameters of system is as follows:*

(1) $c = 1$: when the control is effective, system (9) is stable at the origin.

(2) $c > 1$: when the control is effective, system (9) is stable at the origin. When system (9) is unstable at the origin, effective control cannot be implemented.

(3) $c < 1$: when system (9) is unstable at the origin, effective control cannot be implemented.

Theorem 15 leads to the following conclusion. When system (9) is unstable at the origin, the controlled system (A.4) cannot converge to the origin no matter the value that k takes.

Case 2 ($-k\dot{x}_1, (k > 0)$ feedback on x_3). Similar to the analysis of Case 1, we conclude that the speed feedback method can realize the stabilization of the unstable system (9), when $c < 1$, $c < 1/b - a < 1/c$, and $k > (1/b - a) - c$.

Other Cases. The speed feedback method cannot realize the stabilization of the unstable system (9) at origin.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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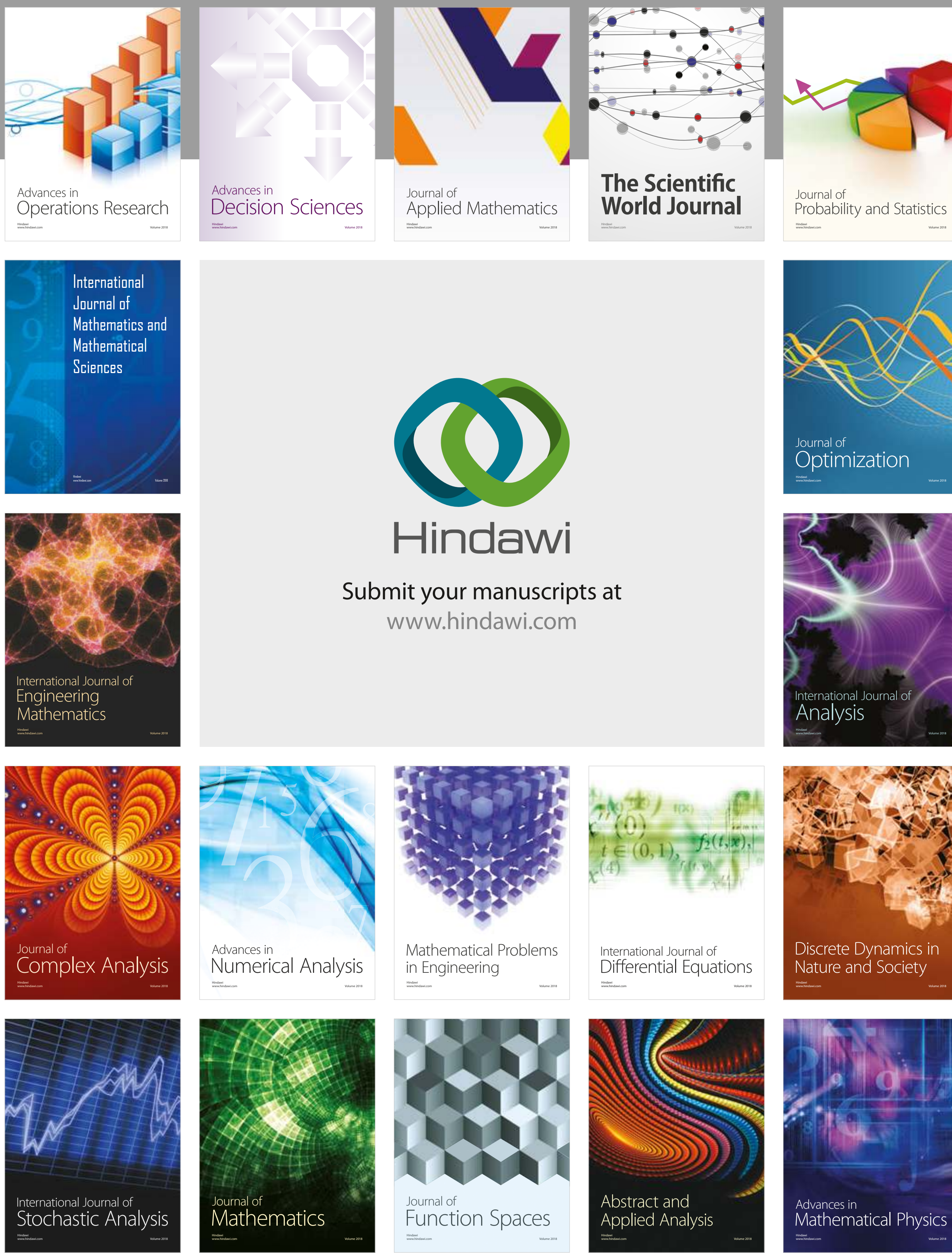
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