Finite-time survival probability and credit default swaps pricing under geometric Lévy markets

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Abstract

We study the first-passage time over a fixed threshold for a pure-jump subordinator with negative drift. We obtain a closed-form formula for its survival function in terms of marginal density functions of the subordinator. We then use this formula to calculate finite-time survival probabilities in a structural model for credit risk, and thus obtain a closed-form pricing formula for a single-name credit default swap (CDS). This pricing formula is well calibrated on market CDS quotes. In particular, it explains why the par CDS credit spread is not negligible when the maturity becomes short.

Keywords: credit default swap; finite-time survival probability; first-passage time; Lévy process; structural model

JEL Classification: C10, G13

1 Introduction

Credit default swaps (CDSs) have become the most popular credit derivatives in the past two decades. Pricing CDSs is based on a reasonable model for credit risk. In the literature, there are basically two main classes of credit risk models: reduced-form models and structural models. In reduced-form models, the precise mechanism leading to default is left unspecified and the default time of a firm is modeled as a non-negative random variable, whose distribution typically depends on economic factors. In this paper we alternatively

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consider a structural model, in which a firm defaults at the first time its asset value breaches a low barrier. This barrier corresponds to the recovery value of the firm's debt.

Classic structural models use a stochastic process with continuous paths to model a firm's asset value process. For example, in the pioneering first-passage model developed by Black and Cox (1976), the asset value process was described by a geometric Brownian motion. For an asset value process with continuous paths, default always occurs when the asset value of the firm exactly hits the barrier. It is never the case that the firm's value would suddenly undershoot the default level. However, it is more reasonable and practical to include jumps in stochastic models of asset value and incorporate skewness in the return distribution, since default events are usually triggered by shocks, i.e. downward jumps in the asset value.

In the last decade more and more attention has been paid to using stochastic processes with jumps in structural models. For example, Zhou (2001) used a geometric jump-diffusion process in modeling the market value of a firm's asset and gave a pricing formula for the defaultable bond issued by the firm. It was originally discovered by Zhou (2001) that the credit spread of a defaultable bond does not go to zero as its maturity goes to zero. Hilberink and Rogers (2002) assumed that a firm's market value follows an exponential spectrally negative Lévy process and also studied credit spreads with short maturities. Kou and Wang (2004) used a double exponential jump-diffusion model and obtained analytically tractable pricing formulas for path-dependent options. Still using the double exponential jump-diffusion model, Chen and Kou (2009) studied credit spreads, optimal capital structure, and implied volatility of equity options. Lipton (2002) studied a geometric jump-diffusion model with log-exponential jumps and gave a numerical example in which the par credit default spread is not small with short maturity. More recently, Ruf and Scherer (2011) studied corporate bonds in a geometric jump-diffusion model having two-sided jumps and showed that the limiting credit spread as its maturity goes to zero is given by the product of the local default rate and the expected loss given default. All these papers focused on Poissonian jumps and did not address the important issue of infinite activity. In this paper we consider the asset value process that would possibly be of infinite activity but finite variation in a structural model proposed by Madan and Schoutens (2008).

Why processes with infinite activity are important in modeling asset values? First of all, pure-jump processes with infinite activity are able to capture both frequent small moves and rare large moves, which makes them reasonable alternatives for jump diffusions when describing asset returns. Also, dealing with pure-jump processes with good distributional properties sometimes allows one to calibrate models more quickly and describe dependence structures between assets in a more straightforward way. More importantly, according to Carr *et al.* (2002) who empirically investigated the fine structure of asset returns, there is evidence from market prices of equity and option indicating that both physical and risk-neutral processes for equity prices seem to be pure-jump processes of infinite activity and finite variation. Here we highlight some of their findings: (1) index returns tend to be pure-jump processes of infinite activity and finite variation, both physically and risk-neutrally. A diffusion component appears to be statistically insignificant, while it may be present in individual equity returns. (2) Jump components consistently account for significant skewness levels from both equity and option prices. (3) The shape of the mean corrected density for asset returns appears to be a long spike near zero conjoined with two convex curves describing large returns. It apparently departs from that of a normal distribution which is always concave within one standard deviation of the mean. In contrast, the densities of processes with infinite activity and finite variation are consistent with both equity and option prices.

Finite-time survival probability is an essential quantity in the calculation of a single-name CDS. In the literature there are many efficient and fast numerical algorithms to calculate finite-time survival probabilities. For example, Kou and Wang (2004) assumed a double exponential jump diffusion for asset value and found an analytic approximation of the first-passage time. Asmussen *et al.* (2004) obtained an explicit solution to the first-passage time problem in a jump diffusion with phase-type jumps in both directions. More recently, Asmussen *et al.* (2008) used a phase-type approximation to the CGMY Lévy process and obtained exact computation of finite-time survival probabilities. Under the same model as we use, Madan and Schoutens (2008) found a fast method to calculate survival probabilities, which exploits the remarkable Wiener-Hopf factorization in combination with results by Rogers (2000) and performs a very fast double Laplace transform inversion. In this paper, we adopt an alternative approach to deal with finite-time survival probabilities that originates from ruin theory. Motivated by Dickson and Waters (1993), who obtained a closed-form formula for finite-time survival probabilities for a gamma process with drift, we will show that a similar type of formula holds true for a general pure-jump subordinator with drift. Using this formula we successfully explain why the par credit spread of a CDS does not go to zero when the maturity goes to zero as discovered by Zhou (2001).

This paper is organized as follows. In Section 2 we describe the structural model and obtain a closed-form formula for finite-time survival probabilities. In Section 3 we investigate two special cases of subordinator for which the closed-form formula from Section 2 is completely known. Finally, calibration results and proofs are given in Section 4 and Section

5, respectively.

2 Structural approach for CDS pricing

2.1 Models and the CDS spread

On a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, where $\mathbb{Q}$ is a risk-neutral measure, suppose that a firm's asset value follows a stochastic process $V = \{V_t, t \geq 0\}$ such that

$$V_t = V_0 \,\mathrm{e}^{X_t},$$

where $X = \{X_t, t \geq 0\}$ is a Lévy process of the form

$$X_t = \mu t - S_t$$

with $\mu > 0$ and $S = \{S_t, t \geq 0\}$ a pure-jump *subordinator*. Recall that a pure-jump subordinator is a non-decreasing Lévy process whose Laplace exponent is given by

$$\varphi(s) := \log \mathbb{E}(\mathrm{e}^{-sS_1}) = \int_0^\infty (\mathrm{e}^{-sx} - 1)\,\Pi(\mathrm{d}x), \quad s \geq 0,$$

where $\Pi(\cdot)$ is the Lévy measure of S. Assume a constant risk-free interest rate r. Since $\mathbb{E}(V_t) = V_0\mathrm{e}^{rt}$ for $t \geq 0$ under $\mathbb{Q}$, we immediately have

$$\mu = r - \varphi(1).$$

Suppose a person owns a defaultable bond of the firm with face value N and maturity T years. For a predetermined barrier level $L(< V_0)$, the *default* of the bond is defined as the event that $V_t \leq L$ for some $t \in (0, T]$, or, equivalently,

$$X_t \leq \log(L/V_0) \quad \text{for some } t \in (0, T].$$

When default occurs, the firm only pays the bondholder RN, where $R \in (0, 1)$ is called the *recovery rate*. To protect himself from the default risk, the bondholder enters a CDS contract. The CDS has the same maturity as the bond. Under this CDS, the bondholder makes predetermined payments to the protection seller. The payments continue until the maturity date or until the bond defaults, whichever is earlier. In the case of default, the protection seller is required to pay to the bondholder $(1 - R)N$. The CDS spread (in basis points (bpts)), denoted by c, is the yearly rate paid by the bondholder to enter the CDS contract against the default of the bond.

Note that we assume for simplicity a fixed recovery rate throughout the lifetime of a CDS contract in this paper. However, the recovery rate at default in reality may depend on the firm's asset value at the time of default. The more the firm value process undershoots below the default threshold, the lower the recovery rate one should expect. Our model can be extended to cover this type of more realistic case since the closed-form formula for finite-time survival probabilities that we will derive allows us to take partial derivatives with respect to both time and threshold (see Theorem 2.1 below). We would like to investigate this extension in future research.

If spread is paid continuously the value of the CDS can be expressed in terms of finite-time survival probabilities. The finite-time survival probability up to time t ($\leq T$), denoted by $\delta(t)$, is defined as the probability under $\mathbb{Q}$ that no default occurs by time t, i.e.

$$\begin{aligned}\delta(t) &= \mathbb{Q}\left(X_s > \log(L/V_0), \text{ for all } 0 \leq s \leq t\right) \\ &= \mathbb{Q}\left(\sup_{0\leq s\leq t}(S_s - \mu s) < \log(V_0/L)\right). \qquad (2.1)\end{aligned}$$

According to Schoutens and Cariboni (2009, Section 3.1.1), the value of the CDS is

$$(1-R)N\left(-\int_0^T \mathrm{e}^{-rt}\mathrm{d}\delta(t)\right) - cN\int_0^T \mathrm{e}^{-rt}\delta(t)\mathrm{d}t,$$

where the first term and the second term respectively correspond to the present value of the so-called loss leg and premium leg of the CDS contract. Pricing the CDS is to find the par spread $c = c(T)$ which makes the loss leg equal to the premium leg:

$$c(T) = \frac{(1-R)\left(-\int_0^T \mathrm{e}^{-rt}\mathrm{d}\delta(t)\right)}{\int_0^T \mathrm{e}^{-rt}\delta(t)\mathrm{d}t} = (1-R)\left(\frac{1-\mathrm{e}^{-rT}\delta(T)}{\int_0^T \mathrm{e}^{-rt}\delta(t)\mathrm{d}t} - r\right). \qquad (2.2)$$

2.2 Finite-time survival probability

As shown in formula (2.2), the finite-time survival probability $\delta(t)$ is the essential quantity for $c(T)$. Motivated by this, we derive a closed-form formula for the following finite-time survival probability in Theorem 2.1:

$$\delta(t,u) = \mathbb{Q}\left(\sup_{0\leq s\leq t}(S_s - \mu s) < u\right), \qquad t, u > 0. \qquad (2.3)$$

Note that $\delta(t)$, as defined in (2.1), becomes $\delta(t, \log(V_0/L))$ in (2.3). Let us also define the right-continuous version of the survival probability as

$$\delta^*(t,u) = \mathbb{Q}\left(\sup_{0\leq s\leq t}(S_s - \mu s) \leq u\right), \qquad t, u > 0. \qquad (2.4)$$

From (2.3) and (2.4) it is easy to see that if $\delta^*(t,u)$ is left continuous at u, then $\delta(t,u) = \delta^*(t,u)$. The survival probability $\delta^*(t,u)$ has been studied in ruin theory for a long time. If we consider the process $\{u + \mu t - S_t, t \geq 0\}$ as an insurance risk process, then $1 - \delta^*(t,u)$ is the finite-time ruin probability. Usually a positive safety loading condition, i.e. $\mu - \mathbb{E}(S_1) > 0$, is assumed in ruin theory to prevent ruin from happening for sure in the long run. But we do not require it in this paper since we focus within a finite-time horizon.

In the following, for $t \geq 0$ we denote by F_t the cumulative distribution function (cdf) of S_t and by f_t the probability density function (pdf) of F_t if it exists.

Theorem 2.1 *Let $\mu > 0$ and $S = \{S_t, t \geq 0\}$ be a Lévy subordinator without drift. Suppose that $F_t(\cdot) \in C^1(0,\infty)$. For $t, u \geq 0$, we have*

$$\delta(t,u) = \delta^*(t,u) \tag{2.5}$$

$$= F_t(u + \mu t) - \int_0^t \frac{1}{s} \left(\int_0^{\mu s} F_s(x)\,\mathrm{d}x \right) f_{t-s}(u + \mu(t-s))\,\mathrm{d}s \tag{2.6}$$

Remark 2.1 When S is a compound Poisson process formula (2.6) can be found in, for example, Asmussen and Albrecher (2010, Theorems V.2.1 and V.2.2). See also Dickson and Waters (1993) for S being a gamma process.

Remark 2.2 By partial integration, formula (2.6) can be rewritten as

$$\begin{aligned} \delta(t,u) = {} & F_t(u + \mu t) - \mu \int_0^t F_s(\mu s) f_{t-s}(u + \mu(t-s))\,\mathrm{d}s \\ & + \int_0^t \frac{1}{s} \left(\int_0^{\mu s} x f_s(x)\,\mathrm{d}x \right) f_{t-s}(u + \mu(t-s))\,\mathrm{d}s. \end{aligned} \tag{2.7}$$

We notice that formula (2.7) involves double integrals of f_t with variable upper limits, which inevitably cause various computational difficulties in general. However, $\delta(t,u)$ is explicitly expressed and feasible to calculate provided that F_t and f_t are both explicitly presented. This enables us to perform the integrations and compute $\delta(t,u)$ for some known distributions with valid probability densities (e.g. gamma and inverse Gaussian distributions that are to be discussed in Section 3) using readily available computation packages.

Remark 2.3 Actually, we can easily extend formula (2.6) to the case where S is a spectrally positive Lévy process with possibly unbounded variation; see e.g. Michna (2011) for α-stable cases. Since we focus on processes with infinite activity but finite variation in this paper, we will realize this extension in future research.

Remark 2.4 To apply Theorem 2.1 one needs to check the smoothness of F_t. Here we list several criteria:

(i) Let $\widehat{F}_1$ be the characteristic function of F_1. If

$$\int_{\mathbb{R}} |\widehat{F}_1(x)||x|^{n-1}\,\mathrm{d}x < \infty,$$

for some integer n, then $F_t(\cdot) \in C^n(\mathbb{R})$. This result is essentially due to the Fourier inversion theorem; see Sato (1999, Theorem 5.28.2).

(ii) F_t has a pdf in $C^\infty(0,\infty)$ if

$$\liminf_{t\downarrow 0} t^{\alpha-2} \int_0^t x^2 g(x)\,\mathrm{d}x > 0,$$

for some $\alpha \in (0,2)$, where g is the density of Π; see, e.g. Orey (1968).

(iii) $F_t(\cdot) \in C^1(0,\infty)$ if $k(x) := xg(x)$ is decreasing, which is equivalent to the self-decomposability of F_t, and $k(0+) < \infty$. Moreover, $F_t(\cdot) \in C^\infty(0,\infty)$ if $k(0+) = \infty$. See Sato (1999, Theorem 5.28.4) for these results.

As a direct application of Theorem 2.1, we are able to obtain a closed-form formula for the par credit spread $c(T)$ by plugging formula (2.6) with $u = \log(V_0/L)$ into formula (2.2).

2.3 Term structure of CDS with short maturity

One advantage of a closed-form formula for $\delta(t,u)$ is that it can explain why $c(T)$ does not go to zero as T goes to zero. Actually, if $\lim_{t\downarrow 0}\delta(t) = 1$ and $\delta(\cdot) \in C^1(0,\infty)$, then by L'Hôpital's rule we obtain from (2.2) that

$$\begin{aligned}\lim_{T\downarrow 0} c(T) &= (1-R)\left(\lim_{T\downarrow 0}\frac{r\mathrm{e}^{-rT}\delta(T) - \mathrm{e}^{-rT}\delta'(T)}{\mathrm{e}^{-rT}\delta(T)} - r\right)\\ &= -(1-R)\lim_{T\downarrow 0}\delta'(T). \end{aligned} \tag{2.8}$$

As Theorem 2.2 will show, downward jumps in S guarantee that $\lim_{T\downarrow 0}\delta'(T) < 0$ and thus a positive limiting credit spread. Note that a positive limiting credit spread can also be achieved in a continuous geometric diffusion model where the information set available is not complete. See Jarrow and Protter (2004) and references therein for details on this direction of research.

Theorem 2.2 *Given conditions in Theorem 2.1, we additionally assume that* $F_t(x)$ *and* $f_t(x)$ *are continuous in a neighborhood of* $(t = 0, x)$ *for all* $x > 0$*. Furthermore, assume that* $(\partial/\partial t)F_t(x)$*,* $(\partial/\partial t)f_t(x)$ *and* $(\partial/\partial x)f_t(x)$ *exist and are also continuous at* $(t = 0, x)$ *for all* $x > 0$*. If the Lévy measure* Π *admits a density* g*, then for fixed* $u > 0$ *we have*

$$\lim_{t\downarrow 0} \delta(t,u) = 1 \quad \textit{and} \quad \lim_{t\downarrow 0} \frac{\partial}{\partial t}\delta(t,u) = -\Pi([u,\infty)) = -\int_u^\infty g(x)\mathrm{d}x. \tag{2.9}$$

Remark 2.5 Relations (2.8) and (2.9) together give that

$$\lim_{T\downarrow 0} c(T) = (1-R)\int_{\log(V_0/L)}^\infty g(x)\mathrm{d}x. \tag{2.10}$$

So the credit spread does not go to zero when maturity becomes short and the limiting spread is determined by the Lévy measure of S and the recovery rate. Note that interest rate r does not show up on the right-hand side of relation (2.10), which is reasonable since interest rates should not affect the credit spread with very short maturity.

Remark 2.6 Relation (2.10) has also been obtained by Ruf and Scherer (2011, Theorems 3.1 and 3.2) for the case where S is a jump-diffusion process having two-sided jumps. Their results show that if S contains only Poissonian jumps then the limiting credit spread as maturity goes to zero is given by the product of the local default rate and the expected loss given default, which is exactly as our relation (2.10) shows. So our Theorem 2.2 complements their results for the case where S has infinite downward jumps.

3 Examples

As stated in Remark 2.2, to evaluate $\delta(t,u)$ by using formula (2.7) one needs to know f_s for $0 < s \le t$. In this section, we consider two special cases of subordinator: gamma processes and inverse Gaussian processes, for which the marginal pdfs are explicitly known.

3.1 Gamma process

Recall that the gamma(a, b) distribution with parameters $a, b > 0$ has the pdf

$$f(x;a,b) = \frac{b^a}{\Gamma(a)}x^{a-1}\mathrm{e}^{-bx}, \quad x > 0.$$

A Lévy process $S = \{S_t, t \ge 0\}$ is called a gamma(a, b) process if S_t follows the gamma(at, b) distribution for every $t > 0$. The Lévy density of a gamma(a, b) process is given by $g(x) = ax^{-1}\mathrm{e}^{-bx}$, $x \in (0,\infty)$.

The following is a corollary of Theorems 2.1 and 2.2:

Corollary 3.1 *Let $\mu > 0$ and S be a gamma(a, b) process with parameters $a, b > 0$.*

(i) For $t, u > 0$,

$$\delta(t,u) = F_t(u+\mu t) - \mu \int_0^t F_s(\mu s)\, f_{t-s}\,(u+\mu(t-s))\,\mathrm{d}s + \frac{a}{b}\int_0^t F_{s+1/a}(\mu s) f_{t-s}(u+\mu(t-s))\mathrm{d}s, \tag{3.1}$$

where f_t and F_t are the pdf and cdf of the gamma(at, b) distribution, respectively.

(ii) For fixed $u > 0$,

$$\lim_{t\downarrow 0} \delta(t,u) = 1 \quad \textit{and} \quad \lim_{t\downarrow 0} \frac{\partial}{\partial t}\delta(t,u) = -\int_u^\infty a x^{-1} \mathrm{e}^{-bx}\mathrm{d}x.$$

Again, formula (3.1) with $a = b = 1$ was first obtained by Dickson and Waters (1993).

3.2 Inverse Gaussian process

Recall that the inverse Gaussian distribution, IG(a, b), with parameters $a, b > 0$ has the pdf

$$f(x; a, b) = \frac{a}{\sqrt{2\pi}} \mathrm{e}^{ab} x^{-3/2} \exp\{-(a^2 x^{-1} + b^2 x)/2\}, \quad x > 0.$$

A Lévy process $S = \{S_t, t \geq 0\}$ is called an IG(a, b) process if S_t follows the IG(at, b) distribution for every $t > 0$. The Lévy density of an IG(a, b) process is given by $g(x) = a\mathrm{e}^{-b^2x/2}/\sqrt{2\pi x^3}$, $x \in (0, \infty)$.

The following is also a corollary of Theorems 2.1 and 2.2:

Corollary 3.2 *Let $\mu > 0$ and S be an IG(a, b) process with parameters $a, b > 0$.*

(i) For $t, u > 0$,

$$\delta(t,u) = F_t(u+\mu t) - \mu \int_0^t F_s(\mu s) f_{t-s}(u+\mu(t-s))\,\mathrm{d}s + \int_0^t \frac{1}{s}\left(\int_0^{\mu s} x f_s(x)\,\mathrm{d}x\right) f_{t-s}(u+\mu(t-s))\,\mathrm{d}s,$$

where f_t and F_t are the pdf and cdf of the IG(at, b) distribution, respectively.

(ii) For fixed $u > 0$,

$$\lim_{t\downarrow 0} \delta(t,u) = 1 \quad \textit{and} \quad \lim_{t\downarrow 0} \frac{\partial}{\partial t}\delta(t,u) = -\int_u^\infty \frac{a\mathrm{e}^{-b^2x/2}}{\sqrt{2\pi x^3}}\mathrm{d}x.$$

4 Calibration

We calibrate the gamma model and the IG model from Section 3 to two families of CDS quotes: CDX.NA.IG index (125 North American companies) and iTraxx.EUR index (125 European companies). The quotes were collected on December 9, 2011. For each company, we have its market CDS par spreads with seven maturities (1 year, 2 years, 3 years, 4 years, 5 years, 7 years, and 10 years). Altogether we have 250 real market CDS term structures.

In the calibration, models will be calibrated in order to match the real market spreads as accurately as possible. Specifically, for each reference entity we minimize the mean absolute error (MAE) given by

$$\text{MAE} = \frac{\sum_{\text{CDS par spreads}} |\text{Market CDS par spread} - \text{Model CDS par spread}|}{\text{Number of CDS par spreads}},$$

where the sum over CDS par spreads refers to all the CDS quotes from the company's real market CDS term structure. We carry out the calibration using R packages on a Linux computer with an Intel Core i5 CPU (2.27 GHz). Aided by formula (2.7), the calculations for $\delta(t, u)$ and $c(T)$ are very efficient. Under the gamma(IG) model the computer takes about 0.004(0.056) seconds to calculate 10,000 CDS spreads.

Table 1 and Figure 1 are here.

Table 1 shows part of the calibration results for the CDX index. According to Moody's long-term credit ranking on December 9, 2011, we select two companies from each risk category from Aa2 to Baa3 for demonstration. We apply the gamma model and the IG model on each company and always assume that $r = 1\%$ and $R = L/V_0 = 40\%$. Here the 40% recovery rate is used for all companies. In reality, recovery rates vary across industries as shown in Table 3 of Altman and Kishore (1996), which also shows an average recovery rate around 40% of 696 defaulted bonds from different industries during the time period from 1971 to 1995. For simplicity we use this average recovery rate in the calibration. Our target is to find out the optimal pair (a, b) with the minimum MAE for each company. It can be seen from Table 1 that both gamma and IG models well fit CDS term structures in the CDX index. In particular, Figure 1 illustrates the results for McDonald's CDS term structure.

Table 2 and Figure 2 are here.

Table 2 shows the similar calibration results for the iTraxx index. Note that in iTraxx we are able to find companies such as Nestle and Royal Dutch Shell from a higher category

Aa1. It can be seen from Table 2 that both gamma and IG models well fit CDS term structures in the iTraxx index. In particular, Figure 2 illustrates the results for Siemens' CDS term structure.

Figures 3 and 4 are here.

It is worthwhile to point out that almost all the term structures in the 250 companies are upward sloping, namely, CDS spreads are increasing over years. Both gamma and IG models overall fit this kind of term structure very well. Among the 125 North American companies in the CDX index, 79(80) companies have MAE of less than 5 bpts using the gamma(IG) model. Among the 125 European companies in the iTraxx index, 102(99) companies have MAE of less than 5 bpts using the gamma(IG) model. Moreover, the models can also be used to cope with the situations in which a decreasing or a humped term structure is present. See Figures 3 and 4 for artificial spread term structures of these two kinds from both models. The other side of the coin is that the models do not fit well for the term structures with large CDS spread values. For instance, the MAE is as high as 26 bpts when calibrating the gamma model on Computer Sciences' CDS term structure (210, 263, 293, 334, 366, 397, 423). This is definitely a big concern for using our model. We are still trying to understand the underlying problem causing this limitation.

Tables 3 and 4 are here.

Next in Tables 3 and 4 we check that the limiting par credit spread when the maturity goes to zero is indeed determined by formula (2.10). We first calculate the limiting par spread directly by plugging formula (2.7) into (2.2) and letting T go to zero. Then we compare the numerical results with the theoretical results obtained from (2.8). Here we still assume $R = L/V_0 = 0.4$. For a variety of values for (a, b, r), it is clearly seen that the numerical results and the theoretical results are very close and that interest rates do not affect the limiting credit spread.

5 Proofs

5.1 Proof of Theorem 2.1

Although the corresponding result is essentially obtained by Dickson and Waters (1993); see also Dufresne *et al.* (1991) in the context of a gamma subordinator, we will give an alternative proof for a general subordinator using a weak convergence result in $\mathbb{D}$-space, which is a space of all the cádlág functions on $[0,t]$ embedded in the *Skorokhod topology*.

Since $\{S_t - \mu t, t \geq 0\}$ is a finite variation process with downward drift, the point 0 is irregular for $(0,\infty)$; see, e.g. Kyprianou (2006, Theorem 6.5). Then by the same argument as in the proof of Theorem V.2.2 of Asmussen and Albrecher (2010) we have that

$$\delta^*(t,u) = F_t(u+\mu t) - \mu \int_0^t \delta^*(t-s,0) f_s(u+\mu s)\,\mathrm{d}s. \tag{5.1}$$

We can also show (5.1) by the convergence in (5.2) below. Note that $f_t(x)$ is continuous in $x>0$ under the assumption. Then, relation (5.1) and the dominated convergence theorem imply that the function $\delta^*(t,u)$ for every $t>0$ is continuous in $u \in (0,\infty)$. Therefore, relation (2.5) holds.

We derive a closed-form formula for $\delta(t,0)$ first. Consider a sequence of drifted compound Poisson processes

$$S_t^{(n)} - \mu_n t = \int_0^t \int_{z \geq 1/n} z\, N(\mathrm{d}s, \mathrm{d}z) - \mu_n t, \quad n = 1,2\ldots,$$

where $\mu_n \to \mu$ as $n \to \infty$, and N is a jump-counting measure of S. Then we see that $\{S_s^{(n)} - \mu_n s, 0 \leq s \leq t\}$ converges in law to $\{S_s - \mu s, 0 \leq s \leq t\}$ in $\mathbb{D}[0,t]$-space; see, e.g., Jacod and Shiryayev (2003, Corollary VII.3.6) or Asmussen *et al.* (2004, Proposition 1). Moreover, it is easy to see that a supremum $f[\cdot] := \sup_{0\leq s\leq t}(\cdot)$ is a continuous functional on $\mathbb{D}[0,t]$; see Jacod and Shiryayev (2003, Chapter VI) or Embrechts *et al.* (1997, Section A.2.5). Hence, it follows from the continuous mapping theorem that

$$\sup_{0\leq s\leq t} \left(S_s^{(n)} - \mu_n s\right) \xrightarrow{\mathcal{D}} \sup_{0\leq s\leq t} \left(S_s - \mu s\right), \quad n \to \infty,$$

which yields that

$$\delta_n^*(t,u) := \mathbb{Q}\left(\sup_{0\leq s\leq t} \left(S_s^{(n)} - \mu s\right) \leq u\right) \to \delta^*(t,u), \quad n \to \infty, \tag{5.2}$$

for every $t,u>0$. Since $\delta_n^*(t,u)$ is non-decreasing, uniformly bounded, and continuous in $u \in (0,\infty)$, we see that the convergence in (5.2) is uniform in $u \in (0,\infty)$. So, for any $\epsilon > 0$, $\sup_{u>0} |\delta_n^*(t,u) - \delta^*(t,0)| < \epsilon/3$ for n large enough. Moreover, noticing (5.1) and the similar equality for δ_n^*, we see that $\delta(t,u)$ and $\delta_n^*(t,u)$ are right-continuous at $u=0$. Hence it follows that for any $\epsilon>0$ there exists a sequence $\{u_n, n=1,2,\ldots\}$ with $|\delta^*(t,u_n) - \delta^*(t,0)| < \epsilon/3$ and $|\delta_n^*(t,u_n) - \delta_n^*(t,0)| < \epsilon/3$ such that

$$\begin{aligned}|\delta_n^*(t,0) - \delta^*(t,0)| &\leq |\delta_n^*(t,0) - \delta_n^*(t,u_n)| + |\delta_n^*(t,u_n) - \delta^*(t,u_n)| + |\delta^*(t,u_n) - \delta^*(t,0)| \\ &\leq \epsilon/3 + \sup_{u>0} |\delta_n^*(t,u) - \delta^*(t,0)| + \epsilon/3 < \epsilon,\end{aligned}$$

which yields that

$$\delta_n^*(t,0) \to \delta^*(t,0), \quad n \to \infty. \tag{5.3}$$

On the other hand, denoted by $F_t^{(n)}$ the cdf of $S_t^{(n)}$, we see from Theorem V.2.1 of Asmussen and Albrecher (2010) that

$$\delta_n^*(t,0) = \frac{1}{\mu t}\int_0^{\mu t} F_t^{(n)}(x)\,\mathrm{d}x.$$

Furthermore, it follows that for every $t, x > 0$

$$F_t^{(n)}(x) \to F_t(x), \quad n \to \infty, \tag{5.4}$$

since any marginal distributions of $S^{(n)}$ converges to those of S when $S^{(n)} \xrightarrow{\mathcal{D}} S$. Hence, it follows from (5.3), (5.4) and the dominated convergence theorem that

$$\delta^*(t,0) = \frac{1}{\mu t}\int_0^{\mu t} F_t(x)\,\mathrm{d}x.$$

The above equality and relation (5.1) together yield (2.6).

5.2 Proof of Theorem 2.2

First, we prove the first relation of (2.9). Recall formula (2.7):

$$\begin{aligned}\delta(t,u) &= F_t(u+\mu t) - \mu\int_0^t F_s(\mu s) f_{t-s}(u+\mu(t-s))\,\mathrm{d}s \\ &\quad + \int_0^t \frac{1}{s}\left(\int_0^{\mu s} x f_s(x)\,\mathrm{d}x\right) f_{t-s}(u+\mu(t-s))\,\mathrm{d}s \\ &= I_1(t) - I_2(t) + I_3(t).\end{aligned}$$

By Corollary 3 of Rüschendorf and Woerner (2002), for each fixed $x > 0$, we have

$$\lim_{t\downarrow 0}\frac{1}{t}(1-F_t(x)) = \Pi([x,\infty)) \tag{5.5}$$

and

$$\lim_{t\downarrow 0}\frac{1}{t} f_t(x) = g(x). \tag{5.6}$$

Relation (5.5) and the continuity of $F_t(x)$ at $(t=0, x=u)$ immediately imply that $\lim_{t\downarrow 0} I_1(t) = 1$. For $I_2(t)$, we derive as follows:

$$\begin{aligned}0 \le I_2(t) &\le \mu\int_0^t f_{t-s}(u+\mu(t-s))\,\mathrm{d}s \\ &= \mu\int_0^t f_s(u+\mu s)\,\mathrm{d}s \\ &\to 0, \quad \text{as } t \to 0,\end{aligned}$$

where in the last step we use relation (5.6) and the continuity of $f_t(x)$ at $(t=0, x=u)$. As to $I_3(t)$, since for each $s>0$,

$$\frac{1}{s}\int_0^{\mu s} x f_s(x)\,\mathrm{d}x \leq \mu F_s(\mu s) \leq \mu,$$

we similarly obtain that $\lim_{t\downarrow 0} I_3(t) = 0$. Hence,

$$\lim_{t\downarrow 0} \delta(t,u) = 1.$$

Next, we prove the second relation of (2.9). Note that

$$\begin{aligned}\frac{\partial}{\partial t}\delta(t,u) = \mu f_t(u+\mu t) + \frac{\partial}{\partial t}F_t(x)\Big|_{x=u+\mu t} \\ &- \mu\int_0^t F_s(\mu s)\frac{\partial}{\partial t}f_{t-s}(u+\mu(t-s))\,\mathrm{d}s \\ &+ \int_0^t \frac{1}{s}\left(\int_0^{\mu s} x f_s(x)\,\mathrm{d}x\right)\frac{\partial}{\partial t}f_{t-s}(u+\mu(t-s))\,\mathrm{d}s \\ &= J_1(t) + J_2(t) - J_3(t) + J_4(t).\end{aligned}$$

Obviously, $\lim_{t\downarrow 0} J_1(t) = 0$ by (5.6). Since $(\partial/\partial t)F_t(x)$ is continuous at $(t=0, x=u)$, by (5.5) we obtain

$$\lim_{t\downarrow 0} J_2(t) = -\Pi([u,\infty)).$$

Therefore, the proof is complete if we can show that

$$\lim_{t\downarrow 0} J_3(t) = 0 \quad \text{and} \quad \lim_{t\downarrow 0} J_4(t) = 0.$$

Actually,

$$\begin{aligned}|J_3(t)| &\leq \mu\int_0^t \left|\frac{\partial}{\partial t}f_{t-s}(u+\mu(t-s))\right|\mathrm{d}s = \mu\int_0^t\left|\frac{\partial}{\partial s}f_s(u+\mu s)\right|\mathrm{d}s \\ &= t\left|\frac{\partial}{\partial s}f_s(u+\mu s)\right|_{s=s(t)} \\ &\leq t\left|\frac{\partial}{\partial s}f_s(x)\right|_{s=s(t),x=u+\mu s(t)} + \mu t\left|\frac{\partial}{\partial x}f_s(x)\right|_{s=s(t),x=u+\mu s(t)}\end{aligned} \tag{5.7}$$

for some $s(t)\in[0,t]$. Since $\lim_{t\downarrow 0} s(t) = 0$, by (5.6) and the continuity of $(\partial/\partial t)f_t(x)$ at $(t=0, x=u)$ we have

$$\lim_{t\downarrow 0}\frac{\partial}{\partial s}f_s(x)\Big|_{s=s(t),x=u+\mu s(t)} = g(u). \tag{5.8}$$

Moreover, by (5.5) and the continuity of $(\partial/\partial x)f_t(x)$ at $(t=0, x=u)$ we have

$$\lim_{t\downarrow 0} \frac{\partial}{\partial x} f_s(x)\Big|_{s=s(t),x=u+\mu s(t)} = 0. \tag{5.9}$$

Plugging (5.8) and (5.9) into (5.7) we obtain

$$\lim_{t\downarrow 0} J_3(t) = 0.$$

Since

$$\begin{aligned}
|J_4(t)| &\le \int_0^t \frac{1}{s}\left(\int_0^{\mu s} x f_s(x)\,\mathrm{d}x\right)\left|\frac{\partial}{\partial t} f_{t-s}(u+\mu(t-s))\right| \mathrm{d}s \\
&\le \mu \int_0^t F(\mu s)\left|\frac{\partial}{\partial t} f_{t-s}(u+\mu(t-s))\right| \mathrm{d}s \\
&\le \mu \int_0^t \left|\frac{\partial}{\partial t} f_{t-s}(u+\mu(t-s))\right| \mathrm{d}s,
\end{aligned}$$

we similarly have

$$\lim_{t\downarrow 0} J_4(t) = 0.$$

This completes the proof.

5.3 Proof of Corollary 3.1

For part (i), since

$$\int_0^{\mu s} x F_s(\mathrm{d}x) = \frac{\Gamma(as+1)}{\Gamma(as)} \int_0^{\mu s} \frac{b^{as}}{\Gamma(as+1)} x^{as} \mathrm{e}^{-bx} \mathrm{d}x = \frac{as}{b} F_{s+1/a}(\mu s),$$

formula (3.1) follows immediately from formula (2.7).

We are going to apply Theorem 2.2 to prove part (ii). For a gamma(a,b) process S, obviously $F_t(x)$, $f_t(x)$, and $(\partial/\partial x)f_t(x)$ are continuous at $(t=0,x)$ for all $x>0$. Also, by Theorem 1 and Corollary 3 of Rüschendorf and Woerner (2002), relations (5.5) and (5.6) hold. So, to prove the continuity of $(\partial/\partial t)f_t(x)$ and $(\partial/\partial t)F_t(x)$ at $(t=0,x)$ for $x>0$ is equivalent to prove that

$$\lim_{t\downarrow 0} \frac{\partial}{\partial t} f_t(x) = a x^{-1} \mathrm{e}^{-bx} \quad \text{and} \quad \lim_{t\downarrow 0} \frac{\partial}{\partial t} F_t(x) = -\int_x^\infty a y^{-1} \mathrm{e}^{-by} \mathrm{d}y. \tag{5.10}$$

It is known that

$$\frac{\partial}{\partial t} f_t(x) = a f_t(x)\left(\log b + \log x - \psi(at)\right),$$

where $\psi(\cdot) = \Gamma'(\cdot)/\Gamma(\cdot)$ is the so-called *digamma function*. Note that we have the following summation representation for the digamma function:

$$\begin{aligned}\psi(t) &= -\gamma - \frac{1}{t} + \sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+t}\right), \quad t \neq 0, \\ &= -\gamma - \frac{1}{t} + O(t),\end{aligned} \tag{5.11}$$

and the following *Laurent expansion* for the gamma function:

$$\begin{aligned}\Gamma(t) &= \sum_{k=0}^{\infty}\frac{\Gamma^{(k)}(1)}{k!}t^{k-1}, \quad |t| < 1, \\ &= \frac{1}{t} - \gamma + \frac{1}{6}\left(3\gamma^2 + \frac{\pi^2}{2}\right)t + O(t^2),\end{aligned} \tag{5.12}$$

where γ is the *Euler-Mascheroni* constant. Hence, using (5.11) and (5.12),

$$\begin{aligned}\frac{\partial}{\partial t}f_t(x) &= \frac{ab^{at}x^{at-1}\mathrm{e}^{-bx}}{\Gamma(at)}\left(\log b + \log x + \gamma + \frac{1}{at} + O(t)\right) \\ &= \frac{ab^{at}x^{at-1}\mathrm{e}^{-bx}}{\Gamma(at)at} + o(1) \\ &\to ax^{-1}\mathrm{e}^{-bx} \quad \text{as } t \to 0.\end{aligned}$$

Then we prove the second relation in (5.10). Using (5.5) and (5.11) we derive

$$\begin{aligned}\frac{\partial}{\partial t}F_t(x) &= \int_0^x \frac{\partial}{\partial t}f_t(y)\mathrm{d}y \\ &= a(\log b - \psi(at))\int_0^x f_t(y)\mathrm{d}y + a\int_0^x f_t(y)\log y\,\mathrm{d}y \\ &= a(\psi(at) - \log b)\int_x^\infty f_t(y)\mathrm{d}y - a\int_x^\infty f_t(y)\log y\,\mathrm{d}y \\ &= a\psi(at)(1 - F_t(x)) + o(1) \\ &\to -\int_x^\infty ay^{-1}\mathrm{e}^{-by}\mathrm{d}y, \quad \text{as } t \to 0,\end{aligned}$$

where we use, in the third step, the fact that if a random variable X has the gamma(a, b) distribution, then $\mathbb{E}\log X = \psi(a) - \log b$.

5.4 Proof of Corollary 3.2

Part (i) is a direct application of Theorem 2.1. Again we are going to apply Theorem 2.2 to prove part (ii). For an IG(a, b) process S, obviously $F_t(x)$, $f_t(x)$, and $(\partial/\partial x)f_t(x)$ are continuous at $(t=0,x)$ for all $x>0$. Also, by Theorem 1 and Corollary 3 of Rüschendorf and Woerner (2002), relations (5.5) and (5.6) hold. So, to prove the continuity of $(\partial/\partial t) f_t(x)$ and $(\partial/\partial t) F_t(x)$ at $(t=0,x)$ for $x>0$ is equivalent to prove that

$$\lim_{t\downarrow 0}\frac{\partial}{\partial t} f_t(x)=\frac{a\mathrm{e}^{-b^2x/2}}{\sqrt{2\pi x^3}}\quad\text{and}\quad \lim_{t\downarrow 0}\frac{\partial}{\partial t} F_t(x)=-\int_x^\infty \frac{a\mathrm{e}^{-b^2y/2}}{\sqrt{2\pi y^3}}\mathrm{d}y.$$

It is easy to obtain that

$$\frac{\partial}{\partial t} f_t(x)=f_t(x)\left(ab+t^{-1}-a^2tx^{-1}\right).$$

Therefore,

$$\frac{\partial}{\partial t} f_t(x)=\frac{f_t(x)}{t}+o(1)\to\frac{a\mathrm{e}^{-b^2x/2}}{\sqrt{2\pi x^3}},\quad \text{as } t\to 0.$$

And, using (5.5),

$$\begin{aligned}\frac{\partial}{\partial t} F_t(x) &= \int_0^x \frac{\partial}{\partial t} f_t(y)\mathrm{d}y\\ &= (ab+t^{-1})\int_0^x f_t(y)\mathrm{d}y - a^2t\int_0^x f_t(y)y^{-1}\mathrm{d}y\\ &= -(ab+t^{-1})\int_x^\infty f_t(y)\mathrm{d}y + a^2t\int_x^\infty f_t(y)y^{-1}\mathrm{d}y\\ &= -\frac{1}{t}(1-F_t(x))+o(1)\\ &\to -\int_x^\infty \frac{a\mathrm{e}^{-b^2y/2}}{\sqrt{2\pi y^3}}\mathrm{d}y,\end{aligned}$$

where we use, in the third step, the fact that if a random variable X has the $\mathrm{IG}(a,b)$ distribution, then $\mathbb{E}X^{-1}=ba^{-1}+a^{-2}$. See, for example, Tweedie (1957, Section 6).

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Table 1: Calibration on CDS term structures in the CDX index

Company	Moody's		1y	2y	3y	4y	5y	7y	10y	a	b	MAE
Wal-Mart	Aa2	market	17	26	34	46	54	63	75			
		gamma	16	27	38	46	54	65	75	1.0723	5.7763	0.9342
		IG	17	28	37	46	53	65	75	0.8581	2.9853	0.8779
GE	Aa2	market	129	170	193	208	235	230	230			
		gamma	129	169	193	209	219	228	230	0.9678	3.4109	3.1505
		IG model	129	168	193	208	218	228	230	0.9388	2.2315	3.1711
UPS	Aa3	market	20	28	37	48	54	63	73			
		gamma	19	30	39	47	54	64	73	0.8402	5.1503	0.8670
		IG	20	30	39	47	54	64	73	0.7165	2.8235	0.8149
IBM	Aa3	market	15	20	28	35	43	56	68			
		gamma	9	19	28	37	44	56	67	1.4571	6.9843	1.6264
		IG	10	19	28	37	44	56	67	1.0767	3.2980	1.4191
ACE	A1	market	33	45	57	71	80	88	94			
		gamma	33	48	60	70	77	88	97	0.7893	4.5058	1.7606
		IG	33	47	59	69	77	88	97	0.7236	2.6438	1.7869
Cisco	A1	market	33	53	70	88	106	130	149			
		gamma	26	53	76	95	108	126	138	2.3463	6.9338	5.3810
		IG	28	53	75	92	106	123	135	1.5774	3.1977	5.0485
McDonald's	A2	market	8	12	18	22	25	31	39			
		gamma	7	12	17	22	26	32	39	0.7844	6.0141	0.3810
		IG	8	12	17	21	25	32	39	0.6503	3.0983	0.4039
Honeywell	A2	market	15	22	31	41	47	55	65			
		gamma	15	24	33	40	46	56	65	0.8812	5.4827	0.8272
		IG	15	24	32	39	46	55	65	0.7569	2.9512	0.7943
Sherwin-Williams	A3	market	16	29	42	54	61	74	87			
		gamma	16	30	42	53	62	74	85	1.3622	6.2352	0.5173
		IG	16	29	42	53	62	76	87	1.1267	3.1649	0.4765
Baxter Intl	A3	market	13	19	25	32	36	45	55			
		gamma	12	19	26	32	37	45	53	0.8030	5.5796	0.7111
		IG	13	19	26	32	37	45	53	0.6590	2.9377	0.5291
Black & Decker	Baa1	market	14	21	30	38	46	57	66			
		gamma	12	21	31	39	46	56	66	1.1855	6.3079	0.4295
		IG	12	21	30	38	45	56	67	0.9254	3.1384	0.3700
Ingersoll-Rand	Baa1	market	11	22	28	36	40	48	56			
		gamma	14	22	29	35	40	48	56	0.7513	5.2958	0.6606
		IG	15	22	29	35	40	48	57	0.6422	2.8747	0.7934
McKesson	Baa2	market	10	16	22	29	36	45	52			
		gamma	9	16	23	29	35	44	53	1.0589	6.3852	0.6441
		IG	10	17	23	29	35	44	52	0.7700	3.0986	0.6379
Cox Comm	Baa2	market	21	32	44	57	66	79	90			
		gamma	18	33	46	57	66	79	90	1.3432	6.0853	0.9165
		IG	20	34	46	57	66	79	90	1.0077	3.0250	0.8539
Cardinal Health	Baa3	market	19	27	37	49	56	65	76			
		gamma	19	30	40	49	56	66	76	0.9365	5.3741	1.1603
		IG	19	29	39	48	55	66	76	0.8048	2.9138	1.1178
CSX	Baa3	market	24	31	41	52	63	76	89			
		gamma	17	31	43	54	63	76	87	1.3730	6.2221	1.9240
		IG	17	31	43	54	63	76	87	1.0758	3.1140	1.7282

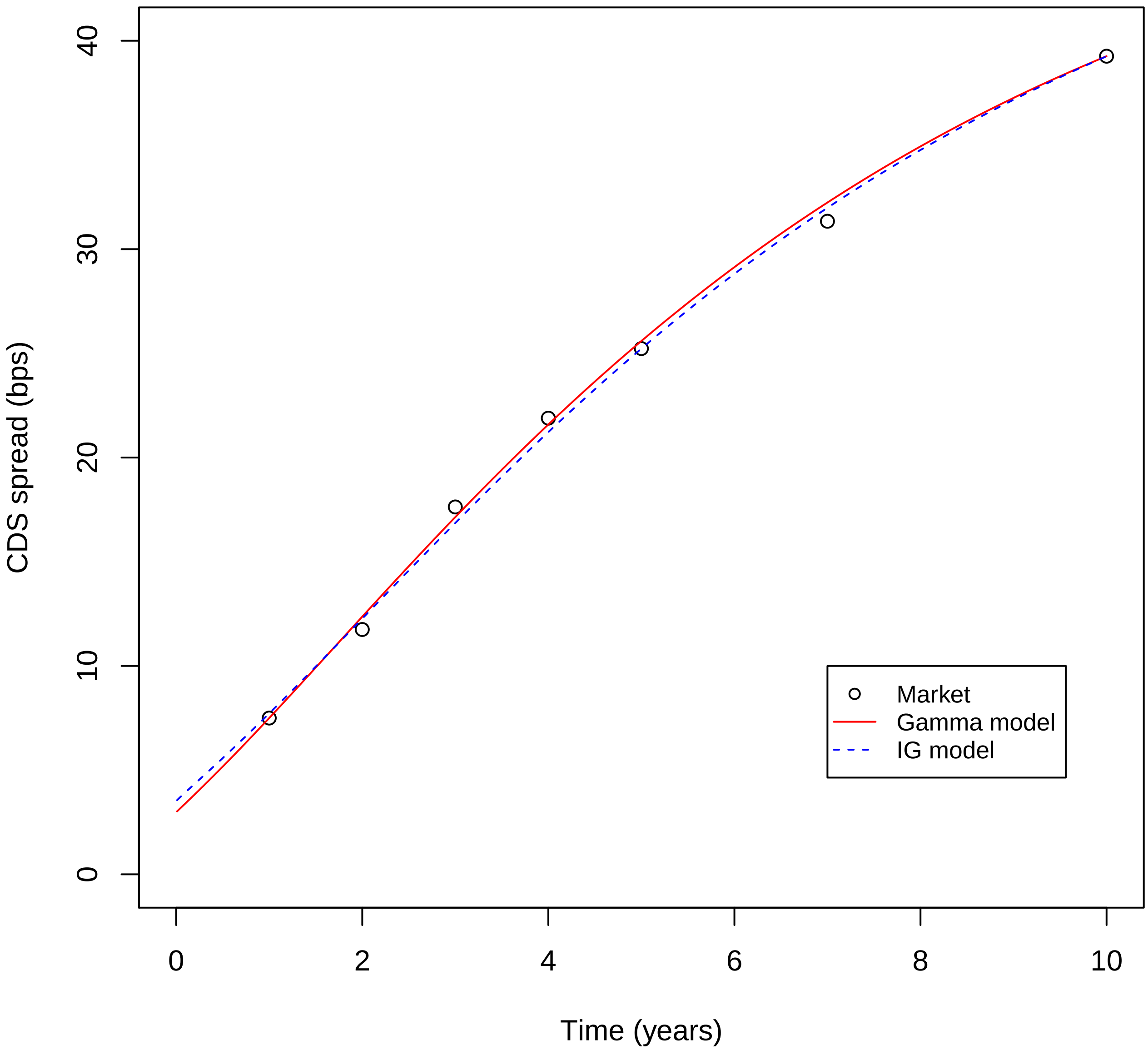

Figure 1: Calibration of gamma and IG models for McDonald's CDS term structure.

Table 2: Calibration on CDS term structures in the iTraxx index

Company	Moody's		1y	2y	3y	4y	5y	7y	10y	a	b	MAE
Nestle	Aa1	market	22	27	36	43	50	57	65			
		gamma	19	28	36	43	48	57	65	0.7023	4.8862	0.8131
		IG	20	29	36	43	48	57	65	0.5958	2.7197	0.7525
Royal Dutch Shell	Aa1	market	46	59	72	83	90	99	105			
		gamma	46	61	73	82	90	99	107	0.6264	3.8343	0.8977
		IG	46	60	72	82	89	99	107	0.6043	2.4126	0.9185
Statoil ASA	Aa2	market	58	72	83	95	102	111	119			
		gamma	55	72	85	94	102	111	118	0.6008	3.5917	0.7814
		IG	57	73	85	94	102	111	119	0.5694	2.2919	0.6316
Credit Suisse	Aa2	market	111	125	138	153	163	164	165			
		gamma	111	130	143	151	157	164	167	0.4373	2.5501	2.7850
		IG	111	129	142	151	157	164	167	0.4652	1.8850	2.7879
Munich Reinsurance	Aa3	market	53	61	70	79	85	92	97			
		gamma	52	64	73	79	85	92	97	0.4003	3.1307	0.8949
		IG	53	64	73	79	85	92	98	0.4043	2.1323	0.8700
JTI UK Finance	Aa3	market	17	23	28	35	40	50	58			
		gamma	15	23	30	37	42	50	58	0.7229	5.1432	1.1124
		IG	16	23	30	36	42	50	58	0.6225	2.8289	0.9345
Siemens	A1	market	34	50	63	73	81	92	100			
		gamma	34	50	63	73	81	92	100	0.8195	4.5252	0.2489
		IG	35	50	62	72	80	92	101	0.7284	2.6239	0.5032
ENI SpA	A1	market	116	141	156	168	175	182	188			
		gamma	116	141	157	168	175	183	185	0.5637	2.7969	0.6538
		IG	116	140	157	168	175	183	186	0.5872	1.9954	0.6433
Sanofi-Aventis	A2	market	44	55	63	72	79	86	92			
		gamma	43	55	64	72	77	85	92	0.4816	3.5408	0.5602
		IG	44	55	64	72	77	86	93	0.4697	2.2911	0.5156
Vattenfall AB	A2	market	40	52	65	75	87	95	105			
		gamma	35	52	65	76	84	96	105	0.8712	4.5971	1.2847
		IG	36	52	65	76	84	96	105	0.7551	2.6293	1.1158
Groupe Danone	A3	market	41	56	68	80	86	100	107			
		gamma	39	56	69	79	87	98	107	0.7741	4.2896	0.7705
		IG	40	56	69	79	87	98	107	0.6915	2.5368	0.6274
TeliaSonera AB	A3	market	31	47	62	76	85	98	107			
		gamma	31	49	64	76	85	98	108	1.0541	5.0289	0.7387
		IG	31	49	63	75	85	98	108	0.9232	2.8034	0.7453
Pearson	Baa1	market	26	35	45	54	64	75	84			
		gamma	22	35	46	56	63	74	84	0.9505	5.2192	0.9299
		IG	23	35	46	55	63	74	84	0.7977	2.8343	0.8108
Wolters Kluwer	Baa1	market	30	46	60	72	79	89	97			
		gamma	30	46	59	70	78	89	99	0.9148	4.8261	0.9100
		IG	31	46	59	69	78	89	99	0.7936	2.7177	1.2009
Royal KPN NV	Baa2	market	55	76	93	108	116	128	136			
		gamma	55	78	95	107	116	128	136	0.8633	4.1123	0.6157
		IG	55	77	94	107	116	128	137	0.8048	2.5005	0.5750
British Telecom	Baa2	market	54	75	91	106	114	124	131			
		gamma	54	76	92	104	112	124	132	0.8117	4.0355	0.8200
		IG	54	75	91	103	113	125	133	0.7731	2.4842	1.0609
Royal Ahold	Baa3	market	54	80	101	121	132	145	154			
		gamma	54	83	104	120	131	145	154	1.2400	4.7376	1.1205
		IG	54	82	103	119	131	145	154	1.0840	2.6949	1.0450
Tate & Lyle	Baa3	market	25	42	57	75	86	98	107			
		gamma	25	44	60	73	84	98	109	1.4625	5.9228	1.6145
		IG	25	43	59	73	83	98	110	1.1952	3.0573	1.6257

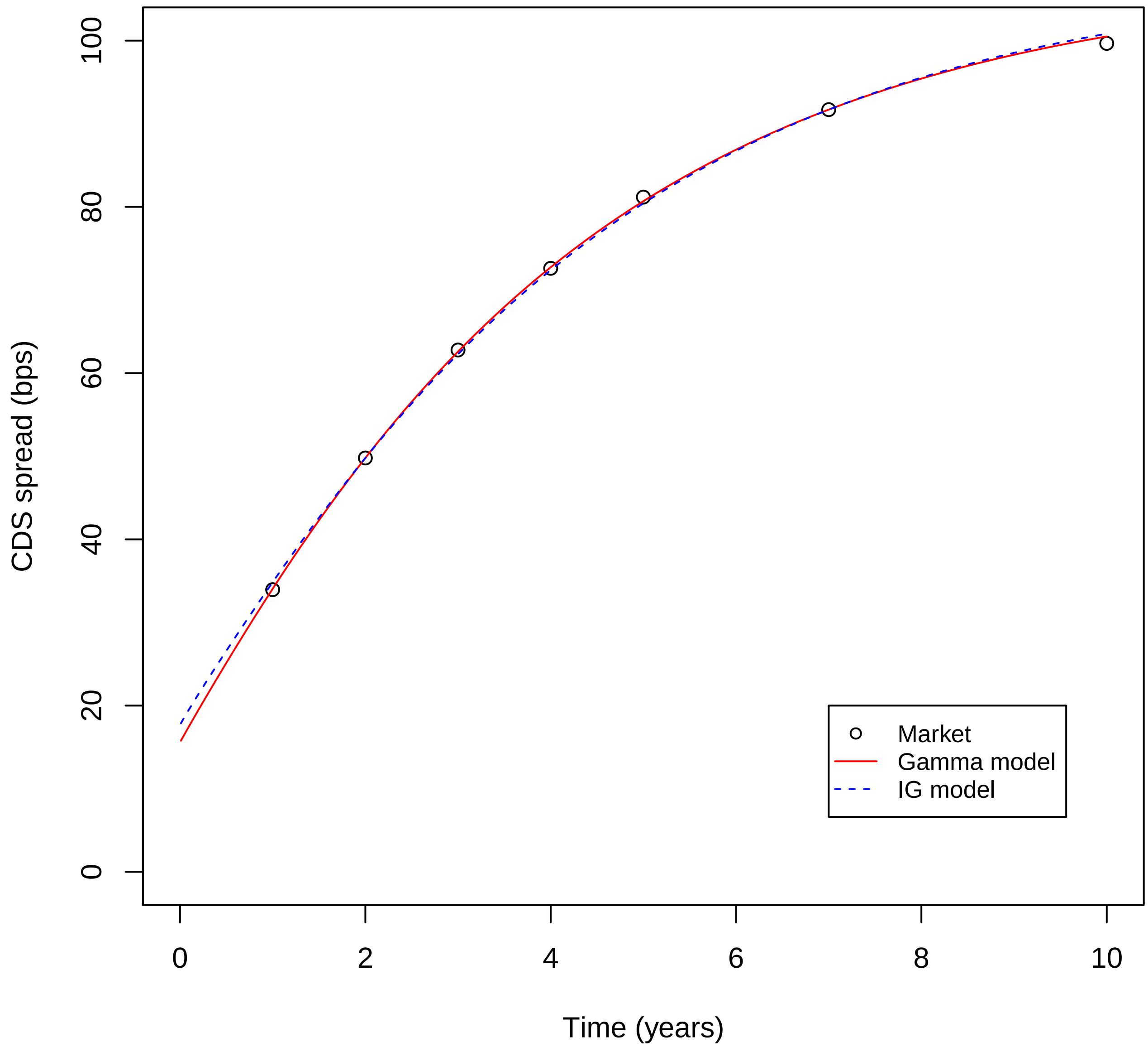

Figure 2: Calibration of gamma and IG models for Siemens CDS term structure.

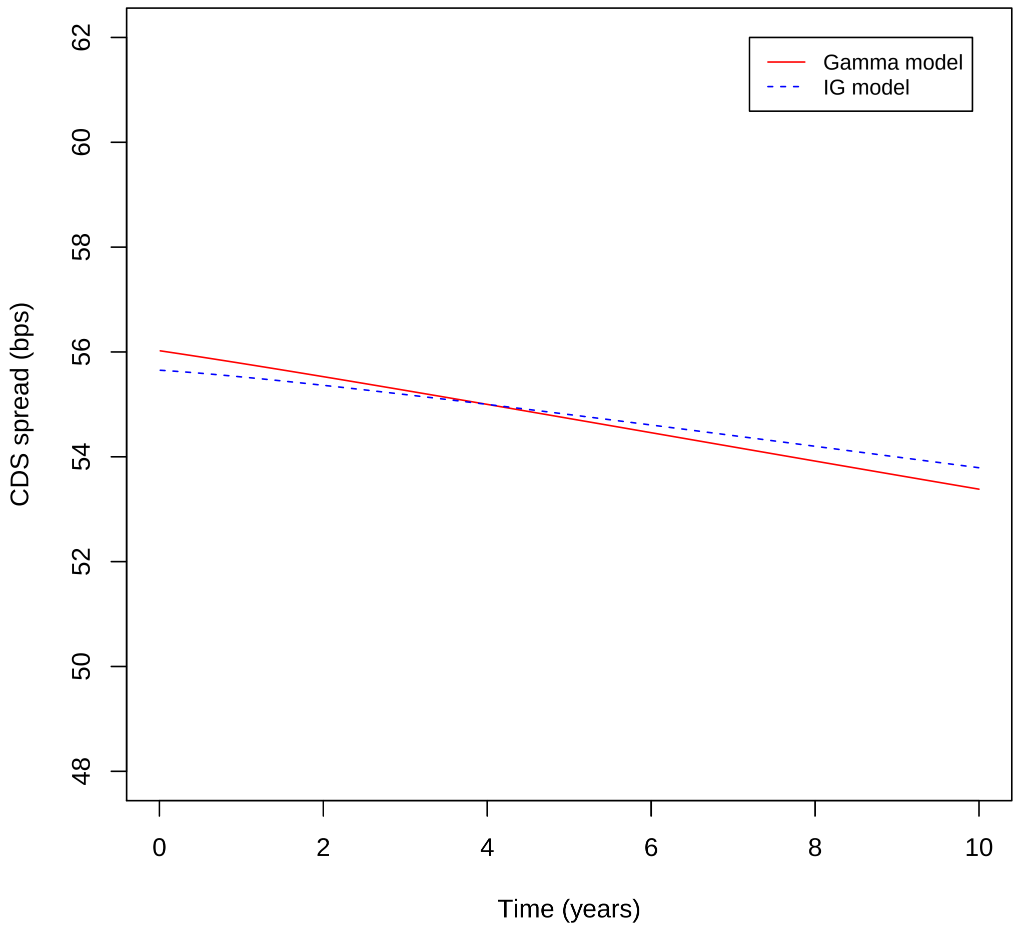

Figure 3: Artificial decreasing term structures for gamma($a = 0.0122, b = 0.3969$) and IG($a = 0.0165, b = 0.3085$) models.

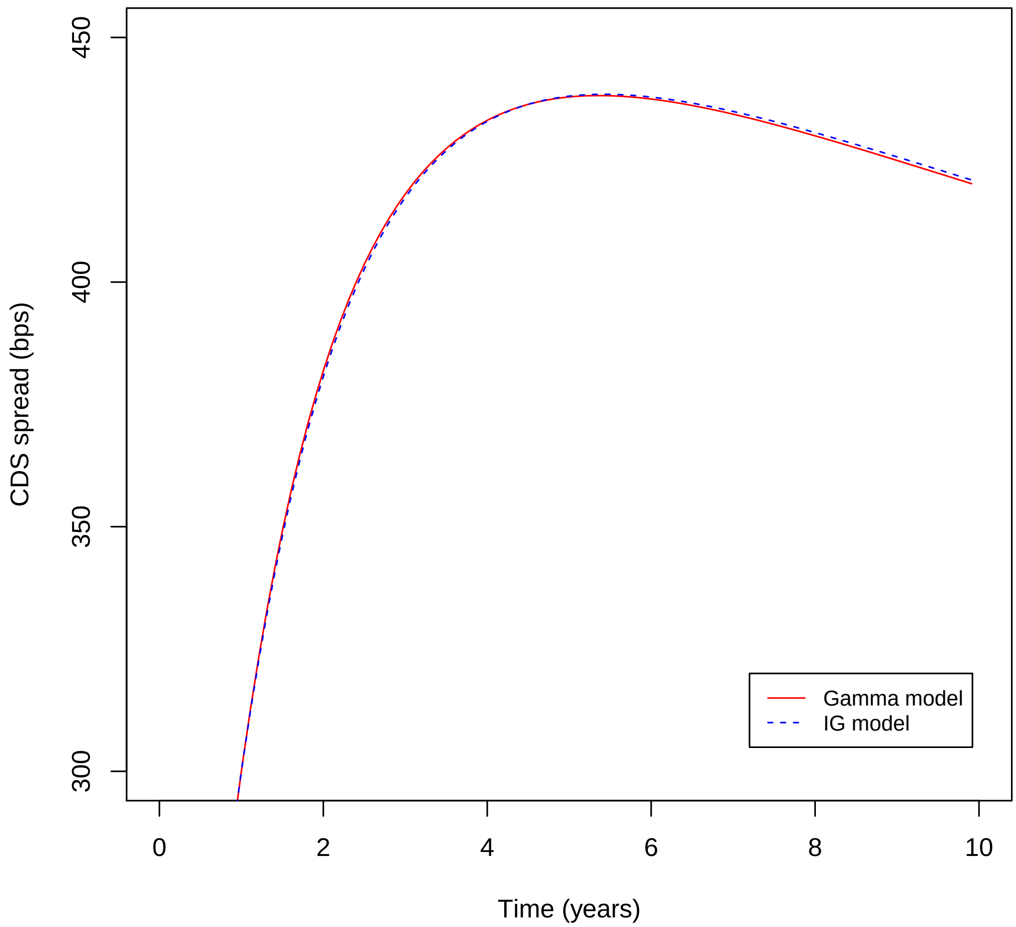

Figure 4: Artificial humped term structures for gamma($a = 1.7908, b = 3.4437$) and IG($a = 1.6758, b = 2.2185$) models.

Table 3: Limiting credit spread when maturity goes to zero: gamma model

a	b	r	Numerical	Theoretical
0.5	5	1%	5.645955	5.645982
0.5	5	5%	5.645975	5.645982
1.0	5	1%	11.29197	11.29196
1.0	5	5%	11.29199	11.29196
1.5	5	1%	16.93792	16.93795
1.5	5	5%	16.93794	16.93795
2.0	5	1%	22.58394	22.58393
2.0	5	5%	22.58396	22.58393

Table 4: Limiting credit spread when maturity goes to zero: IG model

a	b	r	Numerical	Theoretical
0.25	2.5	1%	8.705575	8.705537
0.25	2.5	5%	8.705595	8.705537
0.50	2.5	1%	17.41108	17.41107
0.50	2.5	5%	17.41110	17.41107
0.75	2.5	1%	26.11665	26.11661
0.75	2.5	5%	26.11667	26.11661
1.00	2.5	1%	34.82215	34.82215
1.00	2.5	5%	34.82224	34.82215