Finite-Time Tracking Control with Prescribed Accuracy for Unknown Nonlinear Systems by Event-Triggered Input

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ABSTRACT This paper is dealt with the tracking control problem for strict-feedback systems with unknown nonlinearities and unmatched disturbances as well as event-triggered input. A robust control scheme together with an event-triggered update protocol is proposed to solve the problem, which outperforms the existing solutions in the following aspects. First, it guarantees that the tracking error converges to a given bound in a prescribed finite time, unlike the results on exponential convergence with unknown accuracy. Second, for control implementation, only a sequence of binary command signals needs to be sent to the actuator. This further economizes the communication cost in comparison with the conventional event-triggered control protocols. Third, our controller exhibits a simplicity attribute, due to the avoidance of using approximating structures and estimating algorithms and to calculating or filtering certain signal derivatives. The above theoretical findings are illustrated via a comparative simulation study.

INDEX TERMS Tracking control, finite-time convergence, predefined accuracy, nonlinear systems, event-triggered communication.

I. INTRODUCTION

THE past decades have witnessed considerable research effort in the area of robust control of nonlinear systems. As a result, various classical robust nonlinear control methodologies came out, which include but are not limited to adaptive control [1], neural or fuzzy control [2], [3], iterative learning control [4], and sliding mode control [5]. These are dedicated to different scenarios, introduced briefly as follows. Adaptive control laws are usually adopted to deal with constant parametric uncertainties. Iterative learning control schemes are capable of handling unknown time-varying parameters. The neural or fuzzy control approach is always applied to the system with unknown nonlinear functions. Sliding mode controllers are often employed to suppress external disturbances. The above methods were combined with the backstepping technique, forming diverse solutions to the control design for strict-feedback systems [6]–[11], a common type of nonlinear systems. Nonetheless, these solutions may encounter the issues on control complexity and system performance. The use of adaptive mechanisms, neural/fuzzy systems, or iterative learning algorithms requires real-time estimations of unknown parameters, and the application of backstepping needs recursive calculation of virtual control signal derivatives. Hence the conventional robust backstepping controllers consumes a large number of computation resources, even inducing the explosion of complexity issue. A typical control task is tracking control. The above methods mostly guarantee uniform ultimate boundedness (UUB)
of the tracking error, in the case of unmatched uncertainties or unknown nonlinearities. That is the tracking error converges, in infinite time, to a residual set whose size cannot be determined beforehand due to the dependence on some unknowns, e.g., the disturbance bound. This means that fast and accurate tracking control is unfulfillable, despite its practical significance in for instance satellites docking, automatic berthing of ships, and aerial refueling for aircraft.

Motivated by the above observation, some low-complexity robust prescribed performance control (PPC) schemes [12]–[14] for a variety of nonlinear systems were developed. The simplicity control attribute is reflected in the absence of approximating or estimating mechanisms and of the process for derivative calculation. By prescribed performance, it is meant that the convergence rate, overshoot and tracking accuracy can be freely predefined by the designer. Unfortunately, the exponential convergence rate leads to infinite settling time. To overcome this difficulty, several finite-time PPC approaches [15]–[18] were proposed with yet using adaptive algorithms, neural networks, disturbance observers, and the assumption on parametric uncertainties, i.e., the system nonlinear functions are known. At no expensive of robustness and simplicity, finite-time convergence was achieved by modifying the error signal via a tuning function [19] or by imposing a finite-time convergent bound on the tracking error [20]. Even so, it is notable that all the aforesaid robust nonlinear control methods, whether classical or recent, require the control signal to be applied to the plant continuously. This on the one hand aggravates wear and tear of the actuator and increases the energy consumption. On the other hand, for networked control systems (NCSs) where the controller communicates with the plant via networks, it results in a waste of communication resources. This is due to the fact that there is no need to send and apply the control signal to the plant, when the system performance is satisfactory.

Given this fact, some event-triggered control (ETC) protocols [21]–[30] were proposed in recent years. Different from continuous control, the control signal is implemented at certain discrete time instants determined by an event-triggered scheduling scheme. For NCSs, this greatly reduces the number of communication over the controller-to-actuator channel. Nevertheless, the control objective is still realized, and the robustness of the controller is preserved. To further alleviate the communication burden, two event-triggered binary communication policies [31], [32] were put forward, where only an 1-bit command signal (either 0 or 1) instead of the control policies [31], [32] were put forward, where only an 1-bit decision burden, two event-triggered binary communication protocols [21]–[30] were proposed in recent years. Different performance is satisfactory.

This on the one hand aggravates wear and tear of the actuator and increases the energy consumption. On the other hand, the control implementation does not use the initial value of the control signal, thus eliminating its transmission [31] or modification [32].

3) The controller exhibits not only strong robustness but also significant simplicity. It does not involve information on the expressions of system nonlinear functions or the bounds of disturbances, without yet approximation [6], [8], [33], adaptation [7], [21], [22], [34], learning [9], [11], observation [16], hard calculation [21], [22], [31], filtering [35]–[37], etc.

Notations: The notations used in this paper are standard that are summarized as follows: $n$ denotes the system order; $\mathbb{R}$ denotes the $i$-dimensional Euclidean space with $\mathbb{R}^n$; $\mathbb{R}^i$ denotes the lumped vector of $x_1, \cdots, x_i$, i.e., $\mathbb{R}^i = \{ x_1, \cdots, x_i \}^T$; $\text{sgn}(\cdot)$ denotes the sign function; $L^\infty$ denotes the Banach space of all Lebesgue measurable.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. SYSTEM DESCRIPTION

Consider a class of NCSs, where the controller communicates with the plant through networks. In view of the fact that a diversity of engineering plants can be modeled in the strict-feedback form, e.g., single-link flexible robots, jet engine compressors, active suspension systems, aircraft wing rocks, biochemical processes [32], the plant is supposed to be within the following strict-feedback form:

$$\begin{align*}
\dot{x}_i &= f_i(x) + g_i(x)u + d_i(t), \\
\dot{x}_n &= f_n(x_n) + g_n(x_n)u + d_n(t), \\
y &= x_1, \quad i = 1, \cdots, n - 1, \\
\end{align*}$$

(1)
where $\xi_i \in \mathbb{R}^i$, $i = 1, \ldots, n$, are vectors of the state variables that are all available for measurement; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and system output, respectively; $f_i(\xi_i) \in \mathbb{R}$ and $g_i(\xi_i) \in \mathbb{R}$, $i = 1, \ldots, n$, are the continuous nonlinear functions; and $d_i(t) \in \mathbb{R}$, $i = 1, \ldots, n$, are the bounded and piecewise continuous disturbances.

To ensure the controllability of the system in (1), an assumption from [32] is introduced as follows.

**Assumption 1.** There exists a positive constant, $g_0$, such that

$$|g_i(\xi_i)| \geq g_0, \quad i = 1, \ldots, n, \quad \forall \xi_i \in \mathbb{R}^i.$$  

The attention of this study is focused on the robust control of the system in (1), and thus the nonlinear functions and the disturbance functions as well as their bounding functions are with unknown analytical expressions.

**B. TRACKING PERFORMANCE**

The control objective for the system in (1) is forcing its output, $y$, to track a reference, $y_r$, under the following condition [12]–[14].

**Assumption 2.** The reference, $y_r$, and its first derivative with respect to time, $\dot{y}_r$, are bounded.

**Remark 1.** For the strict-feedback system in (1), the backstepping design philosophy requires the first $n$ order derivatives of the reference to be available. The command filtered backstepping [35], [36] and the dynamic surface control method [37] are able to alleviate this requirement, whereas the second derivative of the reference still needs to be attainable. In this study we assume that only the reference is accessible. This is conducive to cost saving for communication and hardware in certain applications such as the formation control of multiple marine vessels.

Denote the tracking error as

$$e_1 = y - y_r.$$  

The desired transient and steady-state tracking performance is prescribed by

$$|e_1(t)| < \epsilon, \quad t \geq T,$$

where $T$ and $\epsilon$ are bounds of the settling time and the steady-state error, respectively. They reflect the speed of response and accuracy of output tracking. It is noted that all the signals involved in the feedback control system are the functions of time, e.g., $e_1 = e_1(t)$. The argument $t$ is dropped out in (2) for simplification of the notation, whereas it is emphasized in (3) to form a contrast that $e_1$ is time-varying but $\epsilon$ is fixed. The same principle is also adopted for notation of other variables below.

**Remark 2.** In the presence of unknown nonlinearities and unmatched disturbances, a majority of the exiting robust control schemes guarantee UUB of the tracking error, i.e., the convergence rate and accuracy fail to be determined a priori. While the recent PPC technique provides a way to predefine the performance specifications, the error convergence is completed in infinite time. Motivated by the above observation, this paper is concerned with the problem of tracking control with the preassigned accuracy and settling time in (3).

**C. PROBLEM FORMULATION**

Due to the finite-bandwidth constraint, the effective use of communication resources is imposed for most of the NCSs. A common way is to employ an event-triggered communication protocol, where the broadcast of the control signal is done at only triggering times [21]–[29]. This significantly lowers the frequency of data transmission over the controller-to-actuator network. On this basis, cutting down the bit of signal transmission is with greater significance. In this direction, two binary event-triggered communication protocols are proposed in [31], [32], in which only an 1-bit command signal is sent to the actuator for control implementation. In this way, both the number and the bit of data transmission between the controller and the actuator are shortened.

The problem treated in this paper reads as follows.

**Problem 1.** Consider the NCS, where the plant in (1) is with unknown nonlinearities and disturbances. Find a low-complexity robust controller and an event-triggered binary communication protocol such that the performance in (3) and the boundedness of the signals in the closed-loop system are both guaranteed.

**Remark 3.** An alternative way to reduce the resource utilization for NCSs is dwindling the transmission frequency of the state measurements over the sensor-to-controller channel. To the best of our knowledge, however, until now no solution to the problem of how to bring down the bit of measurement transmission is given in the literature.

**III. CONTROL DESIGN**

**A. ROBUST TRACKING CONTROL LAW**

For ease of exposition, let

$$s_i = \text{sgn}(g_i(\xi_i)), \quad i = 1, \ldots, n.$$  

The controller development is conducted recursively. It starts with the construction of the following bound for the tracking error:

$$b_1 = \lambda \xi(t) + \epsilon,$$

with

$$\xi(t) = \begin{cases} \frac{1}{2} \cos \left( \frac{n t}{T} \right) + \frac{1}{2}, & \text{if } t < T, \\ 0, & \text{if } t \geq T, \end{cases}$$

where

$$s_i = \text{sgn}(g_i(\xi_i)), \quad i = 1, \ldots, n.$$
where $\lambda$ is a positive constant such that

$$|e_1(0)| < b_1(0) = \lambda + \epsilon.$$  

(7)

To combat the tracking error, employ the following barrier function:

$$\eta_1 = \tan\left(\frac{\pi e_1}{2b_1}\right).$$

(8)

The intermediate control signal is thus designed as

$$\alpha_1 = c_1 s_1 \eta_1,$$

(9)

where $c_1 > 0$ is the constant control gain. Proceed with

$$e_i = x_i - \alpha_{i-1},$$

(10)

$$\eta_i = \tan\left(\frac{\pi e_i}{2b_i}\right),$$

(11)

$$\alpha_i = c_i s_i \eta_i,$$

(12)

for $i = 2, \cdots, n$, one by one, till $i = n$, which gives the actual control:

$$v = \alpha_n.$$  

(13)

Here, $b_i$ and $c_i$, $i = 2, \cdots, n$, are positive constants, with

$$|e_i(0)| < b_i, \quad i = 2, \cdots, n.$$  

(14)

Remark 4. Careful inspection of the above design process reveals the following. First, the controller does not refer to knowledge on the system nonlinearity and disturbances, in addition to the sign of the virtual control coefficients. Second, no approximating structures, iterative learning algorithms, adaptive mechanisms, or disturbance observers are adopted to acquire the above knowledge. Third, the recursive calculation of the intermediate control signal derivatives is circumvented. Finally, no auxiliary filters are employed to generate filtered estimates on these derivatives. Accordingly, the controller is both structurally simple and computationally inexpensive.

B. EVENT-TRIGGERED BINARY COMMUNICATION PROTOCOL

In the NCSs, the controller communicates with the actuator for control implementation. To scale down the frequency of signal transmission, the communication is triggered at only:

$$t_{k+1} = \{ t : |z(t)| = \delta \}, \quad k = 1, 2, 3, \cdots,$$

(15)

where the equation in the brace describes the event; $\delta > 0$ is a bounded threshold; and

$$z(t) = v(t) - v(t_k), \quad k = 0, 1, 2, \cdots,$$

(16)

with $t_0 = 0$ throughout in this paper. Let

$$o(t_k) = \begin{cases} 1, & \text{if } z(t_k) = \delta, \\ 0, & \text{if } z(t_k) = -\delta \end{cases}, \quad k = 1, 2, 3, \cdots,$$

(17)

which is a binary signal. We have $o(t_k)$, instead of the control signal $v(t_k)$, be sent to the actuator at $t_k$ in (15). Once it is received, the actuator output is updated by

$$u(t_k) = u(t_{k-1}) + \delta(2o(t_k) - 1), \quad k = 1, 2, 3, \cdots,$$

(18)

with $u(t_0)$ the bounded initial value of the actuator output, not $v(t_0)$. During every triggering interval, the control action is held constant, i.e.,

$$u(t) = u(t_k), \quad t_k \leq t < t_{k+1}, \quad k = 0, 1, 2, \cdots.$$  

(19)

Remark 5. The proposed communication protocol is with consideration of not only the finite-bandwidth constraint of digital channels but also the significant cost saving and network security. On the one hand, it reduces the number of communication between the controller and actuator, because the communication is built at only the update instants in (15), not serial. This can also be accomplished by the existing ETC policies [21]–[29], with yet the broadcast of the sampled control signal. In our protocol, only the 1-bit command signal in (17) needs to be transmitted over the controller-to-actuator channel, hence significantly curtailing the bit of data transmission. As a byproduct of such a communication protocol, some potential security issues can be evaded, since the control signal does not involve in the communication network. It is noteworthy that a prerequisite for performing some stealthy attacks on NCSs is that the control signal is available for the hacker [38]. Nonetheless, it is also seen from (10)–(12) that the serial communication between the sensor and controller is imposed, as the control algorithm requires real-time state feedback. This is a restriction of our approach, present also in [21]–[32].

Remark 6. The concept of binary communication was also adopted by other scholars [31], [32], with yet noting the following. In Ref. [31], the initial control signal still needs to be sent to the actuator for implementation. This implies an assumption that the initial values of the control signal and actuator output are same, i.e., $u(0) = v(0)$. Otherwise, a process of off-line initialization or online modification [32] is needed. Besides, only parametric uncertainties are taken into account, which means that the knowledge of system nonlinearities should be available for the control design. Moreover, the transient and steady-state tracking performance is unknown a priori, rendering PPC infeasible. The above issues were partially addressed in Ref. [32] in a way, mainly with a remainder that the error convergence is finished in infinite time. In practical applications, fast tracking control is preferable or even necessary sometimes, e.g., missile interception. The finite-time control method [39]–[41] provides a recipe for such a task. Following this way, this paper is further dedicated to freely prescribed finite-time control, in the sense that the settling time can be a priori prescribed arbitrarily small. To this end, a novel bounding function in (5) with
(6) is constructed and imposed to the tracking error by the barrier function in (8).

The following lemma shows that the control signal can be applied to the plant, with a bounded deviation.

**Lemma 1.** With (15)–(19), there holds
\[ u(t) = v(t) + \Delta(t), \quad (20) \]
where
\[ \Delta(t) = u(t_0) - v(t_0) - z(t), \quad \Delta(t) \in L^\infty. \quad (21) \]

**Proof.** From (15) and (16), there is
\[ v(t_k) = \begin{cases} v(t_{k-1}) + \delta, & \text{if } z(t_k) = \delta, \\ v(t_{k-1}) - \delta, & \text{if } z(t_k) = -\delta, \end{cases} \]
for \( i = 1, 2, 3, \ldots \). By (17), one further has
\[ v(t_k) = v(t_{k-1}) + \delta(2\alpha(t_k) - 1) = v(t_0) + \delta \sum_{j=1}^{k} (2\alpha(t_j) - 1), \quad (22) \]
for \( i = 1, 2, 3, \ldots \). Similarly, rewrite (18) as
\[ u(t_k) = u(t_0) + \delta \sum_{j=1}^{k} (2\alpha(t_j) - 1), \quad (23) \]
for \( i = 1, 2, 3, \ldots \). Making a difference between (22) and (23) yields
\[ u(t_k) = v(t_k) + u(t_0) - v(t_0), \quad k = 1, 2, 3, \ldots. \quad (24) \]
Rewrite (16) as
\[ v(t_k) = v(t) - z(t). \quad (25) \]
Putting (25) into (24) gives (20). It follows from (15) and (16) that \( |z(t)| < \delta \). Owing to (14), by (11)–(13) one has (21). This completes the proof. \( \square \)

**IV. THEORETICAL ANALYSIS**

For performance analysis, a sufficient condition on the boundedness of the intermediate control signal derivatives is needed.

**Lemma 2.** For \( i \in \{1, \cdots, n\} \), \( \dot{\alpha}_i \) is bounded, if \( |e_i| < b_i \), \( |\eta_i| < \infty \), and \( |\dot{\epsilon}_i| < \infty \).

**Proof.** Differentiating (5) with (6) yields
\[ |\ddot{b}_i| \leq \frac{\lambda \pi}{2T}, \quad t \geq 0. \quad (26) \]
Suppose \( |e_i| < b_i \), \( i = 1, \cdots, n \). Then (8) and (11) are differentiable with
\[ \dot{\eta}_1 = \frac{\pi}{2b_i \phi_i^2} \left( \ddot{e}_1 - \frac{e_1 \dot{b}_1}{b_1} \right), \quad (27) \]
\[ \dot{\eta}_i = \frac{\pi \dot{e}_i}{2b_i \phi_i^2}, \quad i = 2, \cdots, n, \quad (28) \]
where
\[ \phi_i = \cos \left( \frac{\pi e_i}{2b_i} \right), \quad i = 1, \cdots, n. \]

Assumption 1 indicates \( s_i, i = 1, \cdots, n \), in (4) keep fixed. Hence the time derivatives of (9) and (12) exist that are given by
\[ \dot{\alpha}_i = c_i s_i \dot{\eta}_i, \quad i = 1, \cdots, n. \]
Suppose \( |\eta_i| < \infty \), which implies \( \sum_{i=1}^{n} |\ddot{\alpha}_i| < \infty \), \( i = 1, \cdots, n \). The above facts substantiate the proposal in Lemma 2.

Now we are in a position to state our main result as follows.

**Theorem 1.** Under Assumptions 1 and 2, the controller in (4)–(13) together with the protocol in (15)–(19) solves Problem 1.

**Proof.** For ease of exposition, the time or state dependence of some functions may be omitted in the sequel. Let us begin with claiming
\[ |e_i| < b_i, \quad i = 1, \cdots, n, \quad t \geq 0, \quad (29) \]
which is shown by contradiction. Due to (7) and (14), (29) is met at \( t = 0 \). Under Assumptions 1 and 2, \( e_1 \) in (2) and \( s_i, i = 1, \cdots, n \), in (4) are continuous in time. If \( |e_1| < b_1 \), then \( \eta_1 \) in (8) is continuous as well. This leads to the continuity of \( \alpha_1 \) in (9), further ensuring the continuity of \( \epsilon_2 \) in (10). Following the same line, it can be obtained from (10)–(12) in a recursive manner that \( e_i \) is continuous as long as \( |e_{i-1}| < b_{i-1}, i = 3, \cdots, n \). The continuity of the error signals implies that if (29) is violated, there exists a time instant \( t^* > 0 \) such that
\[ |e_i| < b_i, \quad i = 1, \cdots, n, \quad t < t^*, \quad (30) \]
and
\[ \lim_{t \to t^*} |e_j| = b_j, \quad j \in \{1, \cdots, n\}. \quad (31) \]
We suppose (31) with (30), and enumerate \( e_1, \cdots, e_n \) trying to validate (31). By (1), the differential equations for (2) and (10) are obtained as
\[ \dot{e}_1 = \dot{y}_r - f_1 - g_1 x_2 - d_1, \]
\[ \dot{e}_i = \dot{\alpha}_{i-1} - f_i - g_i x_{i+1} - d_i, \quad i = 2, \cdots, n - 1, \quad (32) \]
\[ \dot{e}_n = \dot{\alpha}_{n-1} - f_n - g_n u - d_n. \]
Rearrange (10) with (9) and (12) as
\[ x_{i+1} = e_{i+1} + c_i s_i \dot{\eta}_i, \quad i = 1, \cdots, n - 1. \quad (33) \]
Substituting (12) for \( i = n \) into (13) gives
\[ v = c_n s_n \dot{\eta}_n. \]
Inserting it into (20) leads to
\[ u = c_n s_n \dot{\eta}_n + \Delta. \quad (34) \]
With (4), putting (33) and (34) into (32) yields
\[ \dot{e}_i = h_i - c_i g_i |\eta|, \quad i = 1, \ldots, n, \]
where
\[ h_i = \dot{y}_r - f_1 - g_1 e_2 - d_1, \]
\[ h_i = \dot{\alpha}_i - f_i - g_i e_{i+1} - d_i, \quad i = 2, \ldots, n - 1, \]
\[ h_n = \dot{\alpha}_n - f_n - g_n \Delta - d_n. \]

Construct the following barrier Lyapunov functions, similarly to [9], [12], [13]:
\[ V_i = \frac{1}{\pi} \eta_i^2, \quad i = 1, \ldots, n. \]    (38)

Differentiating \( V_i \) by using (27) and (28), we have
\[ \dot{V}_i = \frac{m_i}{b_i \theta_i^2} \left( h_i - \frac{e_i b_1}{b_i} - c_i g_i |\eta| \right), \]    (39)
\[ \dot{V}_i = \frac{m_i}{b_i \theta_i^2} (h_i - c_i g_i |\eta|), \quad i = 2, \ldots, n. \] (40)

Substituting (35) into (39) and (40), we obtain
\[ \dot{V}_1 = \frac{m_1}{b_1 \theta_1^2} \left( h_1 - \frac{e_1 b_1}{b_1} - c_1 g_1 |\eta| \right), \] (41)
\[ \dot{V}_i = \frac{m_i}{b_i \theta_i^2} (h_i - c_i g_i |\eta|), \quad i = 2, \ldots, n. \] (42)

Next we analyze \( \dot{V}_1, \ldots, \dot{V}_n \), one by one. Discuss first the boundedness of the term, \( h_1 - \frac{e_1 b_1}{b_1} \), in (41) with (36). It follows from (26) and (29) that \( b_1 \in L^\infty, |e_2| < b_2 \) and \( \frac{e_1 b_1}{b_1} < 1 \) for \( t < t^* \). Under Assumption 2 and (2), one has \( \dot{y}_r \in L^\infty \) and \( |y| < \infty \) on \([0, t^*)\). With \( y = x_1 \), due to the continuity of the nonlinear functions, there hold \( |f_1| < \infty \) and \( |g_1| < \infty \) for \( t < t^* \). Together with the boundedness of disturbances, the above facts imply
\[ |h_1 - \frac{e_1 b_1}{b_1}| < \infty, t < t^*. \]

For convenience we denote
\[ \sup_{t \in [0, t^*)} \left| h_1 - \frac{e_1 b_1}{b_1} \right| \leq \bar{h}_1. \] (43)

With Assumption 1 and (43), (41) is bounded by
\[ \dot{V}_1 \leq \frac{\eta_i}{b_i \theta_i^2} \left( \bar{h}_1 - c_1 g_1 |\eta| \right), \quad t < t^*. \]

This means \( \dot{V}_1 < 0 \) if \( |\eta| > \frac{\bar{h}_1}{c_1 g_1} \) for some \( t < t^* \). It in turn implies from (38) for \( i = 1 \) that
\[ |\eta| \leq \max \left\{ |\eta_0(0)|, \frac{\bar{h}_1}{c_1 g_0} \right\}, \quad t < t^*. \] (44)

This facilitates us to analyze \( \dot{V}_2 \) next.

As established above, \( h_1, g_1 \) and \( \eta_1 \) are bounded when \( t < t_1 \). It thus follows from (33) and (35) for \( i = 1 \) that
\[ |e_1| < \infty \text{ and } |x_2| < \infty \text{ as } t < t_1. \]

As shown in (29), \( e_i, i = 1, 2, 3 \), are bounded on \([0, t^*)\). By Lemma 2, one has \( |\alpha_i| < \infty, t < t^*. \) By virtue of the continuity of the nonlinear functions, \( f_2 \) and \( g_2 \) are bounded over \([0, t^*)\).

With \( d_2 \in L^\infty \) and the above facts, we can claim the boundedness of \( h_2 \) in (37). Denote for convenience
\[ \sup_{t \in [0, t^*)} |h_2| = \bar{h}_2. \] (45)

With Assumption 1 and (45), \( \dot{V}_2 \) in (42) is bounded by
\[ \dot{V}_2 \leq \frac{|\eta_2|}{b_2 \theta_2^2} \left( \bar{h}_2 - c_2 g_0 |\eta_2| \right), \quad t < t^*. \]

Apparently, \( \dot{V}_2 < 0 \) if \( |\eta_2| > \frac{\bar{h}_2}{c_2 g_0} \) for some \( t < t^* \). This shows from (38) for \( i = 2 \) that
\[ |\eta_2| \leq \max \left\{ |\eta_2(0)|, \frac{\bar{h}_2}{c_2 g_0} \right\}, \quad t < t^*. \] (46)

With trivial changes of the above arguments on \( \dot{V}_1 \) and \( \dot{V}_2 \), we can analyze \( \dot{V}_i \) and conclude
\[ |\eta_i| \leq \max \left\{ |\eta_i(0)|, \frac{\bar{h}_i}{c_i g_0} \right\}, \quad i = 3, \ldots, n, \quad t < t^*, \] (47)

where \( \bar{h}_i \) is a positive constant, \( i = 3, \ldots, n \). The inequalities in (44), (46) and (47) in turn imply from (8) and (11) that the error signal, \( e_i \), cannot approach \( b_i, i = 1, \ldots, n \). There hence holds
\[ \lim_{t \to t^*} |e_i| < b_i, \quad i = 1, \ldots, n, \]
which obviously contradicts (31). As a result, (31) is false and instead, each error signal never tend to the prescribed bound. There thus exist a set of positive constants, \( \bar{l}_i, i = 1, \ldots, n \), such that
\[ |e_i| \leq b_i - \bar{l}_i < b_i, \quad i = 1, \ldots, n, \quad t \geq 0. \] (48)

Accordingly, our claim in (29) is true. Note from (5) and (6) that \( b_1 = \epsilon, t \geq T \). With (29) for \( i = 1 \), we have (3). Therefore, the output tracking with the assigned settling time and accuracy is achieved by our controller.

Proceed with the proof of the boundedness of the signals in the closed-loop system. They are the intermediate control signals, \( \alpha_i, i = 1, \ldots, n - 1 \), the final control signal, \( v \), the system state variables, \( x_i, i = 2, \ldots, n \), and the control input \( u \). The inequalities in (48) guarantee the boundedness of \( \eta_i, i = 1, \ldots, n \), in (8) and (11). Thereby, \( \eta_i, i = 1, \ldots, n \), in (9) and (12) are bounded.

Due to (13), one thus has \( v \in L^\infty \). Under Lemma 1, one further has \( u \in L^\infty \). By (33), there hold \( x_i \in L^\infty, i = 2, \ldots, n \).

What remains to be shown is the Zeno-free property of the data transmission over the controller-to-actuator network. With (12) for \( i = n \), differentiating (13) gives
\[ \dot{e}_n = c_n s_n \dot{\eta}_n, \]
where \( \dot{\eta}_n \) is given in (28) with (35) for \( i = n \). From the above proof process, \( \dot{e}_n \) is bounded, leading to
the boundedness of $\bar{\eta}_n$. Therefore, $v \in L^\infty$ holds. For convenience we denote
$$\sup_{t \in [0,t^*)} |v| = m.$$ 
Further, from (15) and (16), we have
$$t_{k+1} - t_k \geq \frac{\delta}{m}, \quad k = 0, 1, 2, \cdots, (49)$$
This indicates that the command signal, $o(t_k)$, is intermittently transmitted to the actuator for control implementation, with the minimal interexecution interval, $\frac{\delta}{m}$. Consequently, the possible Zero phenomenon is excluded. The proof is completed. \hfill \Box

Remark 7. The above proof process shows the robustness of our controller against the nonlinear functions with unknown yet fixed expressions and against the additive disturbances with unknown bounds in (1). This is attributed to the use of the barrier functions in the control design in (8) and (11). It is seen that such a function will tend to infinity, once the error approaches the prescribed bound. This enables the resulting controller to provide enough effort to suppress the effect of model uncertainties, such that the error cannot go outside the predefined bound. It is noted that the infinity property is just used for analyzing the potential of the controller, not really reflected in the control magnitude [12]-[14]. This is due to the fact that when the error evolves in a bounded region, the unknown terms in the system dynamics which are functions of the error or state are also bounded. Thereby, a finite control effort is sufficient to counteract their effects. This claim is substantiated by (48), which ensures the boundedness of the barrier functions in (8) and (11), and is also illustrated by the simulation result below.

Remark 8. The above proof process also implies the potential robustness of our controller against the changes in the system dynamics caused by, e.g., parameter variation and/or process faults. Such behavior is described by
$$\begin{aligned}
\dot{x}_i &= f_i(x_i, \zeta_i) + g_i(x_i, \zeta_i)x_{i+1} + d_i(t), \\
\dot{x}_n &= f_n(x_n, \zeta_n) + g_n(x_n, \zeta_n)u + d_n(t), \quad (50) \\
y &= x_1, \quad i = 1, \cdots, n - 1,
\end{aligned}$$
where $\zeta_i = \zeta_i(t), i = 1, \cdots, n$, are vectors of time-varying parameters [9] and/or fault functions [42]. Let
$$|f_i(x_i, \zeta_i)| \leq \psi_i(x_i), \quad g_i(x_i, \zeta_i) \leq \psi_i(x_i), \quad (51)$$
with $i = 1, \cdots, n$ and $g_0$ in Assumption 1. Here, $\psi_i(\cdot) > 0$ and $\psi_i(\cdot) > 0, i = 1, \cdots, n$, are uniformly continuous with respect to their arguments. This is a widely used assumption [9], [11], [14], [42] and is prone to be met in practice. In the above proof process, we first show the system state is bounded, and then the boundedness of the nonlinear functions is established by the continuity of $f_i(x_i)$ and $g_i(x_i), i = 1, \cdots, n$. This enables the subsequent analysis therein to proceed; see, e.g., the one between (42) and (43). Now, the same still holds for (50). Due to the continuity of $\psi_i(x_i)$ and $\psi_i(x_i)$ in (51), $f_i(x_i, \zeta_i)$ and $g_i(x_i, \zeta_i)$ are bounded, as long as the system state is bounded. Thereby, Theorem 1 holds for (50) as well. This means our controller is robust to the parameter variation and process faults, i.e., the control signal will be automatically tuned for compensation, if such changes occur in the system dynamics. The robustness of PPC against the changes in system dynamics was reported in [14] in detail.

V. SIMULATION STUDY

Now we illustrate the above theoretical result by a computer simulation. A comparative study on an inverted pendulum is performed to show the superiority of our approach.

A. INVERTED PENDULUM

Consider the problem of networked control of a second-order inverted pendulum. As pointed out in [32], inverted pendulums are a model abstracted from a variety of engineering systems, e.g., two-wheel self-balancing vehicles, rocket launchers, and biped robots. The dynamical model of the pendulum system is described by [32]
$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= f(x) + g(x)u + d(t),
\end{aligned}$$
with
$$f(x) = \frac{g \sin(x_1) - mlx_2^2 \sin(x_1) \cos(x_1)}{l \left(\frac{4}{3} - \frac{m \cos^2(x_1)}{m + m_c}\right)},$$
$$g(x) = \frac{\cos(x_1)}{l \left(\frac{4}{3} - \frac{m \cos^2(x_1)}{m + m_c}\right)},$$
$$d(t) = 0.1 \sin(0.2t),$$
where $u$ is the control torque; $x_1$ and $x_2$ are the angle and angular velocity of the pendulum, respectively; $m$ and $l$ are the mass and half length of the pole, respectively; $g$ is the acceleration of gravity; and $m_c$ is the mass of the cart. The pendulum dynamics is simulated with $m = 0.1kg, l = 0.5m, g = 9.81m/s^2$, and $m_c = 1kg$, under the initial condition of $x_1(0) = 0.3927\text{rad}, x_2(0) = -0.7092\text{rad}/s$, and $u(0) = 0\text{N-m}$.

B. SIMULATION SETUP

The simulation is carried out in Matlab/Simulink software, with the start time, the stop time, and the fundamental sample time of 0s, 10s, and 0.001s, respectively. The control objective is as follows. Let the reference for the pendulum angle be zero, which means that the inverted pendulum should keep upright as a common
practice. Find a robust control law that does not use explicit information of the pendulum model but the full-state measurements. Choose a low-cost communication protocol, in which a sequence of 1-bit command signals rather than the control signals are sent to the actuator of the pendulum. The control action ought to guarantee the predefined specifications on the settling time and accuracy:

$$|x_1| < 0.02\text{rad}, \ t \geq 4s. \quad (53)$$

By Theorem 1, a robust event-triggered prescribed performance control scheme is given as follows. The control law is designed with $\lambda = 0.98$, $\epsilon = 0.02$, $T = 4$, $c_1 = 1$, and $c_2 = 4$. The threshold of the event-triggering rule is set to be $\delta = 0.1$.

Given robustness and simplicity of PID control, the following PID controller is employed for comparison:

$$u = -10z - 5 \int_0^t z(\tau) d\tau - 0.1 \dot{z}, \quad (54)$$

where $z = 2x_1 + x_2$ is a filtered signal with respect to the pendulum system. In this way, the recursive design for the controller is evaded. Lemma 2 in [43] points out that $x_1$ will converge to zero if so is $z$.

### C. SIMULATION RESULTS

We first perform simulation on the inverted pendulum model with application of the PID controller. The results in Figs. 1 and 2 show that the prescribed transient and steady-state performance in (53) is guaranteed by a bounded control action. Such performance is however established under the following situations. First, a trial and error process that is both arduous and time-consuming is conducted for determining the optimal control gains in (54). Moreover, such a process needs to be carried out again for performance guarantees, once the system dynamics changes. This is illustrated by Fig. 3, where we increase the frequency and amplitude of the disturbance in (52) as $d(t) = \sin(t)$, without changing the controller gains yet. Second, the control signal is applied to the system on a sampled-based time basis. With the sample time of 0.001s and the average bits of the control signal of $p$, e.g., $p = 16$, there is a total of $p \times 10^4$ bits of data transmission over the controller-to-actuator network in 10s.

Next perform simulation by using our control strategy. The results are displayed in Figs. 4–9 and discussed as follows. Figs. 4 and 5 indicate that the pre-specified performance in (53) is met, under a bounded control effort. It is remarkable that such performance has been explicitly specified before the control implementation, thereby irrespective to the choice of design parameters and robust against model uncertainties even with structural changes. For clarification, we scale down the controller gain from $c = 4$ to $c = 2$, while intensifying the disturbance from (52) to $d = \sin(t)$. The simulation
results exhibited in Fig. 6 verify our claim. As shown in Fig. 7, the amount of event triggering times is 223 that reflects the number of communication between the controller and the actuator. Fig. 8, plotting the triggering times locally, manifests the Zeno-free property of our communication protocol. At each triggering time, just an 1-bit command signal is broadcasted over the network; see Fig. 9. Thus, there is a total of only 223 bits of data transmission in ten seconds, which is much less than $p \times 10^4$ required by the PID controller. Therefore, the simulation results point out the superiority of our control strategy with respect to the well-established PID control methodology.

VI. CONCLUSION
A prescribed performance control law together with an event-triggered update protocol for a sort of networked nonlinear system is put forward in this paper. Fast and accurate output tracking is achieved in the sense that the tracking error enters into a preselected residual

![Figure 4: System output under our controller.](image1)

![Figure 6: System output under $c_2 = 2$ and $d = \sin(t)$.](image2)

![Figure 5: Control input under our controller.](image3)

![Figure 7: Number of event triggering times.](image4)

![Figure 8: Local event triggering times.](image5)
set in a given finite time. It is notable that this is established in the following scenarios. First, unknown nonlinearities and unmatched disturbances are involved in the system dynamics, whereas no approximating structures or estimating algorithms are employed in the controller. Second, only a sequence of 1-bit command signals need to be transmitted to the actuator for control implementation, as time goes on. Accordingly, a low-complexity, little-demanding, communication-aware control strategy is provided in this paper. Nevertheless, the current approach works under the assumption on serial communication between the controller and sensor. Therefore, our future investigation will be focused on how to relax such a requirement.

DISCLOSURE STATEMENT
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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