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"Firm Turnover in Imperfectly Competitive Markets"

by

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Firm Turnover in Imperfectly Competitive Markets[1]

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Abstract

This paper is motivated by the empirical regularity that industries differ greatly in the level of firm turnover, and that entry and exit rates are positively correlated across industries. Our objective is to investigate the effect of sunk costs and, in particular, market size on entry and exit rates, and hence on the age distribution of firms.

We analyze a stochastic dynamic model of a monopolistically competitive industry. Each firm's efficiency is assumed to follow a Markov process. We show existence and uniqueness of a stationary equilibrium with simultaneous entry and exit: efficient firms survive while inefficient ones leave the market and are replaced by new entrants. We perform comparative dynamics with respect to the level of sunk costs: entry costs are negatively and fixed production costs positively related to entry and exit rates. A central empirical prediction of the model is that the level of firm turnover is increasing in market size. The intuition is as follows. In larger markets, price-cost margins are smaller since the number of active firms is larger. This implies that the marginal surviving firm has to be more efficient than in smaller markets. Hence, in larger markets, the expected life span of firms is shorter, and the age distribution of firms is first-order stochastically dominated by that in smaller markets. In an extension of the model with time-varying market size, we explore the comovements between market size and entry and exit rates. It is shown that both entry and exit rates tend to rise over time in growing markets.

In the empirical part, the prediction on market size and firm turnover is tested on industries where firms compete in well-defined geographical markets of different sizes. Using data on hair salons in Sweden, we show that an increase in market size or fixed costs shifts the age distribution of firms towards younger firms, as predicted by the model.

JEL Classification: L11, L13, D43

Keywords: firm turnover, monopolistic competition, industry dynamics, entry, exit, market size, firm age distribution.

1 Introduction

There is much simultaneous firm entry and exit going on at the industry level, and there is considerable variation in firm turnover across industries.[1] To explain these cross-industry differences in firm turnover is one of the important research agendas in the field of industrial economics. For example, Dunne, Roberts, and Samuelson (1988) conclude their study as follows.

> The high correlation between entry and exit across industries indicates that industries differ substantially in their degree of firm turnover. One area for further study is then to identify the characteristics of industry technology and demand that give rise to across industry differences in turnover.

It is probably fair to say that the cross-industry differences in firm turnover are not yet very well understood. Although there are some notable exceptions (discussed below), there appear to be only a few theories which make empirically testable predictions regarding the determinants of firm turnover. This paper considers observable industry characteristics – sunk costs and, in particular, market size – and explores their effects on entry and exit rates in an imperfectly competitive industry. The same factors should also be expected to cause cross-industry variations in gross job reallocation.[2]

We analyze a stochastic dynamic model of an imperfectly competitive industry. Upon entry, a new firm gets an initial draw of its productive efficiency or the perceived quality of its product. Over time, a firm's efficiency is likely to change, as it is hit by idiosyncratic shocks. To illustrate, the perceived quality of a restaurant is partly given by the chef's skills. If the chef quits, the restaurant needs to hire a new chef, who may turn out to be better or worse than the previous one. In stationary equilibrium, firms follow a threshold exit policy: firms that are hit by a sufficiently bad shock optimally decide to leave the market and are replaced by new entrants. The focus of this paper is to relate the equilibrium rate of firm turnover to entry costs, fixed production (or opportunity) costs, and market size. Since the equilibrium distribution of firm efficiencies depends on industry characteristics, our model also provides testable predictions regarding the well-documented differences in productivity between and within industries.[3]

The central new prediction of the model is that the rate of firm turnover is increasing in market size. The intuition is the following. In a free entry equilibrium, a rise in market size causes the population of active firms to increase. This, in turn, leads to lower price-cost margins. Hence, there tend to be two opposing effects on firms' profits: lower price-cost margins and larger sales. In equilibrium, the overall effect is positive for more efficient firms, and negative for less efficient firms, and so the marginal surviving firm has to be more efficient in larger markets. Hence, in larger markets, the expected life span of firms is shorter, and the rate of firm turnover larger. It follows that the age distribution of firms in larger markets is shifted toward younger firms. A direct implication of the stronger "selection effect" in larger markets is that firms are on average more efficient.

Our result on market size and firm turnover is closely related to the *price competition effect* in standard oligopoly models, according to which equilibrium prices (and hence price-cost margins) fall with an increase in the number of firms. It is commonly believed that the price competition effect implies that, in a free entry equilibrium, the number of active firms rises less than proportionally with market size (see, e.g., Sutton (1997a)). This is, in fact, what Bresnahan and Reiss (1991) find in their seminal empirical study on local service firms in isolated US towns. More recent papers by Asplund and Sandin (1999) and Campbell and Hopenhayn (2002) provide further empirical support. In our model, however, firms produce differentiated goods and differ in their efficiency level, and so the price competition effect does *not* necessarily imply that the number of firms rises less than proportionally with market size.

[1] Caves (1998) provides a recent survey of the empirical literature on turnover and mobility of firms. See also Sutton (1997b) and Cabral (1997).

[2] See Davis and Haltiwanger (1999) for a survey of the literature on gross job creation and destruction.

[3] See Bartelsman and Doms (2000) for a review of the evidence.

Another prediction of our model is that firm turnover is lower in markets with higher entry costs. As entry costs increase, fewer firms find it profitable to enter the market. This reduces the competitive pressure and allows less efficient firms to survive. The effect of fixed costs is more subtle. An increase in fixed costs also makes entry less attractive, which corresponds to fewer firms in the market and less intense competition with higher equilibrium prices. There is, however, the opposing effect that each firm has to spend more on fixed costs which reduces net profits. In equilibrium, the overall effect is positive for more efficient firms as they gain more from higher prices, and negative for less efficient firms. Consequently, the marginal surviving firm has to be more efficient in markets with higher fixed costs, and the expected life span of firms is shorter.

How can our predictions be tested empirically? The magnitude of the underlying fluctuations in the pattern of demand or technology is likely to vary greatly across industries. As pointed out by Sutton (1997b), this factor may be of primary importance, but it is very difficult to measure it or to control for its impact empirically. This causes a serious problem for any empirical test of cross-industry predictions on firm turnover. Fortunately, an attractive feature of the explanatory variable "market size" is that this problem can be largely circumvented. The idea is to study turnover rates in geographically independent ("local") markets of different sizes but within the same industry. This should control for most of those factors that would differ across industries. This is the route taken in the empirical part of the paper, where we set out to test two predictions of our theory, using data on the age distribution of hair salons in Sweden: firms tend to be younger in (i) larger markets, and (ii) markets with higher fixed costs. Using non-parametric tests of first-order stochastic dominance (Anderson (1996); Davidson and Duclos (2000)), we find empirical support for both predictions.

The starting point of the recent literature on stochastic dynamic industry equilibria with heterogenous firms is the seminal paper by Jovanovic (1982).[4] Jovanovic considers a perfectly competitive industry where firms have different but time-invariant efficiency levels. Firms only gradually learn their types over time by observing their "noisy" cost realizations. Firms which learn that they are efficient grow and survive, while firms that obtain consistently negative information decline and eventually leave the market. The model produces a rich array of empirical predictions on the relationship between firm growth and survival on the one hand and firm age and size on the other. However, all firms eventually learn their efficiency level, and so there is no firm turnover in the long run. Lambson (1991) considers another model with atomistic price takers. In his paper, there are no idiosyncratic shocks but instead common shocks to input price (and demand). In equilibrium, firms may choose different technologies and hence be affected differently by the common shocks. The model predicts that the variability of firm values is negatively related to the level of sunk costs. Some empirical evidence for this prediction is given in Lambson and Jensen (1998) and Gschwandtner and Lambson (2002). Ericson and Pakes (1995) analyze a stochastic dynamic oligopoly model. There are two sources of uncertainty in their model: the outcomes of firms' investments in "quality" are stochastic and firms are subject to (negative) aggregate shocks. The equilibrium distribution of qualities at any time is itself stochastic. Few analytic restrictions can be placed on equilibrium outcomes. Instead, the authors have developed a simulation package; see Pakes and McGuire (1994).[5]

Hopenhayn (1992) is closely related to our model. The key difference between his model and ours is the assumed form of competition: Hopenhayn considers a perfectly competitive industry. The main prediction of his model is that firm turnover is negatively related to entry costs. Due to the absence of the price competition effect, however, market size has no effect on entry and exit rates. There are a few

[4]Note that we study the properties of a stationary industry equilibrium; we do not analyze the life-cycle of an industry as in Klepper (1996). For more evidence on how entry and exit rates are related to the evolution of an industry, see also Carroll and Hannan (2000).

[5]The passive learning model by Jovanovic (1982) differs from a number of other models (such as Ericson and Pakes (1995), Hopenhayn (1992), and our model) in that the stochastic process generating the size of a firm is non-ergodic. This is used by Pakes and Ericson (1998) to empirically distinguish between the two classes of models.

other papers that build on Hopenhayn's framework. Hopenhayn and Rogerson (1993) apply a general equilibrium version of the model to study the effect of changes in firing costs on total employment and welfare. Bergin and Bernhardt (1999) consider business cycle effects in a model of perfect competition. Das and Das (1997) and Melitz (1999) both introduce monopolistic competition but assume a particular demand structure. Das and Das analyze the effect of entry adjustment costs on the convergence path to the stationary state. Melitz considers the impact of trade costs in a two-country version of Hopenhayn's model. In his model, the efficiency of incumbents does not vary over time, and the death rate of incumbents is exogenously given. There are no market size effects on firm turnover.

The plan of the paper is as follows. In Section 2, we present the basic model. Then, in Section 3, we characterize (stationary) equilibrium, and show existence and uniqueness. In Section 4, we investigate the comparative dynamics properties of the stationary equilibrium, which lie at the heart of the paper. We extend the model in Section 5 by analyzing the case of growing and declining markets, and providing results on the co-movements of entry and exit rates and market size. The robustness of the main results is discussed in Section 6. In the empirical part of the paper, Section 7, we investigate our prediction on market size and firm turnover, using Swedish data on hair salons. Finally, we conclude in Section 8.

2 The Model

We consider a stochastic dynamic model of an imperfectly competitive industry. There are a continuum of consumers and a continuum of (potential) firms. Each firm produces a unique differentiated product, and hence faces a downward-sloping demand curve. In a stochastic dynamic model, it is more convenient to work with a continuum of monopolistically competitive firms rather than with a finite number of oligopolistic players. First, if firms are atomistic, we do not have to worry about integer constraints. In a free entry equilibrium, the value of a new entrant is exactly equal to its outside option. Second, with a continuum of firms, idiosyncratic uncertainty washes out at the aggregate level. Hence, if uncertainty enters at the individual level only, all aggregate variables are deterministic. Third, the assumption of monopolistic competition greatly reduces the set of equilibria.

Firms differ in their "efficiency levels", or types, which are subject to idiosyncratic shocks. Under our leading interpretation, the shocks directly affect firms' marginal costs. In this case, the marginal cost of each new entrant is independently drawn from the continuous cumulative distribution function $G(\cdot)$ with support $[0, 1]$. An incumbent's marginal cost in period t, c_t, is given by

$$\begin{aligned} c_t &= c_{t-1} \text{ with probability } \alpha, \\ c_t &\sim G(\cdot) \text{ otherwise,} \end{aligned} \tag{1}$$

where $\alpha \in [0, 1)$, measures the persistence of an incumbent's costs. The "shocks" to incumbents' efficiencies are assumed to be firm specific. There is, however, an alternative interpretation where firms differ in the perceived quality of their product; see Example 2 below. In this case, a firm's perceived quality (relative to marginal cost) is negatively related to firm type c. For brevity, we will refer to c as a firm's marginal cost in the remainder of the paper. The simple stochastic process (1) allows us to obtain closed-form solutions. Moreover, this particular Markov process can be motivated by a simple economic model: the efficiency of a firm depends on the quality of its match with a manager. In each period, the manager leaves the firm with (exogenous) probability $1-\alpha$. If the manager stays, the quality of the firm-manager match (and thus the firm's efficiency) remains unchanged. If the manager leaves, however, the firm has to get a new manager from the pool of (potential) managers. The quality of the firm-manager match is distributed according to $G(\cdot)$. Importantly, in Section 6 we show that the main predictions of the paper remain unchanged if we replace (1) by a more general class of Markov processes with the property that a currently efficient firm is more likely to be efficient next period than a less efficient firm.

Regarding sunk costs, we assume that a firm has to pay an irrecoverable entry fee $\epsilon > 0$ when it enters the market. Additionally, a firm faces a fixed production (or opportunity) cost of $\phi > 0$ per period.[6]

Time is discrete and indexed by t. Firms have an infinite horizon and maximize the discounted sum of profits. The common discount factor is denoted by $\delta \in [0, 1)$. In each period, the timing is as follows.

1. *Entry stage.* The potential entrants decide whether to enter the market or, instead, take up the outside option (the value of which is normalized to zero).

2. *Learning stage.* The entrants and the incumbents observe the realization of their current costs, c_t.

3. *Exit stage.* The new entrants and incumbents decide whether to leave the market forever (and take up the outside option) or not.

4. *Output stage.* The active firms play some "market game", pay a fixed production cost ϕ, and receive profits.

Let us make two remarks on the sequence of moves. First, potential entrants decide whether or not to enter the market before knowing their current efficiency. This assumption is common to most dynamic industry models. It allows us to avoid assumptions about the size of the pool of potential entrants (other than that it is sufficiently large to ensure there is always a positive mass of firms which do not enter in equilibrium). This is of particular importance in our model as we are interested in the effects of market size. It seems plausible that the number of potential entrants is not independent of market size. Fortunately, with the assumed sequence of moves, we can remain agnostic about this relationship.[7] Second, new entrants are treated as incumbents at the exit stage, which takes place after firms have learnt their current types. This sequence of moves gives rise to a convenient and novel mathematical structure. In Section 6, we show that the assumption is not essential for the results.

Formally, the model can be described as an *anonymous sequential game*; see Jovanovic and Rosenthal (1988). Let $\mathcal{M}$ denote the set of Borel measures on $[0, 1]$, and $\mu \in \mathcal{M}$, the measure of firms' cost levels at the output stage. That is, for any Borel set (or interval) $A \subset [0, 1]$, $\mu(A)$ gives the mass of active firms with costs in A. The payoff relevant state of the industry in period t, stage 4, can then be summarized by μ_t, the measure of active firms' types at the output stage. We confine attention to Markov strategies.[8] Appealing to the law of large numbers (for a continuum of random variables), we assume that all idiosyncratic uncertainty washes out at the aggregate level.[9] Hence, the evolution of the industry (from an arbitrary initial state μ_0) is deterministic.

To allow for a large class of models describing competition at the output stage, we do not model the "market game" explicitly. In particular, we do not specify firms' strategic variables (prices or

[6] We could easily introduce a scrap value for exiting firms. However, any such scrap value would affect equilibrium in the same way as the fixed cost ϕ.

[7] In Nocke (2000), a different route is taken which also avoids making arbitrary assumptions about the relationship between the size of the pool of potential entrants and market size. There, it is assumed that potential entrants, knowing their current type, self-select into markets of different size. Reassuringly, it is found that the central prediction of the present paper is insensitive to the assumptions on the entry process.

[8] To focus on pure entry strategies, each potential entrant in period t is assigned a unique (and payoff irrelevant) label $l_t \in \mathbb{R}$. Entry decisions only depend on the firm's label l_t and last period's industry state μ_{t-1}. An entry strategy is thus a mapping $\eta : \mathbb{R} \times \mathcal{M} \rightarrow \{0, 1\}$. An incumbents' exit decision only depends on its own current costs c_t, last period's state μ_{t-1}, and the mass of new entrants, M_t. An exit strategy is thus a mapping $\chi : [0, 1] \times \mathcal{M} \times \mathbb{R}_+ \rightarrow \{0, 1\}$. Since there is no aggregate uncertainty and each firm is atomistic, the requirements on firms' information could be weakened substantially. For instance, incumbents may or may not condition their exit decisions on the current mass of entrants.

[9] See Feldman and Gilles (1985) and Uhlig (1996) for a precise statement of the conditions under which a law of large numbers can be justified for a continuum of random variables.

quantities) nor the details of the demand system. Instead, each firm's equilibrium profit is summarized by a reduced-form profit function. A firm's equilibrium profit depends on its own type, on market size, and on the endogenous distribution of active firms. The equilibrium profit (gross of fixed costs) of a type-c firm when the measure of firms is μ is denoted by

$$S\pi(c;\mu) \geq 0, \tag{2}$$

where S is a measure of market size (e.g., the mass of consumers in the market). By writing a firm's profit as in (2), we make a number of implicit assumptions. First, firms differ only in their types; they are symmetric in all other respects. Hence, if c denotes marginal costs, then the aggregate demand system is symmetric and competition non-localized. Second, an increase in market size means a replication of the population of consumers, leaving unchanged the distribution of preferences and income. Moreover, firms' marginal production costs are independent of output levels. This implies that market size enters (2) in a multiplicative way.[10] Of course, in equilibrium, the measure μ will depend on market size, but it is taken as given at the output stage.

To be more specific, let us consider the case where c denotes marginal costs. Denote by $SD(\cdot;\mu)$ and $P(\cdot/S;\mu)$ the demand and inverse demand functions, respectively, faced by an individual firm in equilibrium when the measure of active firms is given by μ. In this case, equilibrium gross profit can be written as

$$\begin{aligned} S\pi(c;\mu) &\equiv [p(c;\mu)-c]\, SD\left(p(c;\mu);\mu\right) \\ &= [P(q(c;\mu,S)/S;\mu)-c]\, q(c;\mu,S), \end{aligned} \tag{3}$$

where $p(c;\mu)$ and $q(c;\mu,S)$ are equilibrium price and output.

Throughout the paper, we make the following assumptions on the reduced-form profit function.

(MON) *The reduced-form profit function* $\pi(\cdot;\mu)$ *is strictly decreasing in* c *on* $[0,\overline{c}(\mu))$, *and* $\pi(c;\mu) = 0$ *for all* $c \in [\overline{c}(\mu), 1]$, *where* $\overline{c}(\mu) \in [0,1]$.

That is, firms with lower marginal costs have higher profits. We allow for the possibility that some inefficient firms (the types above $\overline{c}(\mu)$) may not sell their products even when offered at marginal cost and hence make zero gross profit.

Since the distribution of active firms is endogenous, we have to compare different distributions. For this purpose, we define a partial ordering, denoted by $\succeq$, on the set $\mathcal{M}$. Formally, let

$$\mu' \succsim \mu \Leftrightarrow \ \left\{\forall c \in [0,1],\ \pi(c;\mu') \leq \pi(c;\mu)\right\},$$

and

$$\mu' \succ \mu \Leftrightarrow \ \left\{\mu' \succsim \mu \text{ and } \forall c \in [0,\overline{c}(\mu)),\ \pi(c;\mu') < \pi(c;\mu)\right\}.$$

Note that the ordering implies $\overline{c}(\mu') \leq \overline{c}(\mu)$ for $\mu' \succsim \mu$. Measure μ' is said to be (weakly) *larger* than μ if $\mu' \succeq \mu$. Measures μ' and μ are said to be equivalent if $\mu' \sim \mu$. We define the *equivalence class* of measure μ as the set of Borel measures μ' in $\mathcal{M}$ such that $\mu' \sim \mu$.

(DOM) *If* $\mu'([0,z]) \geq \mu([0,z])$ *for all* $z \in (0,1]$, *then* $\mu' \succsim \mu$. *If, in addition, the inequality is strict for some* $\widetilde{z} \in (0,\overline{c}(\mu))$, *then* $\mu' \succ \mu$.

In words, the measure of active firms is larger if the mass of active firms is larger and the population of firms more efficient. We can remain completely agnostic about the effect on profits of increasing "average efficiency" of firms while reducing the mass of active firms. Assumption (DOM) implies that a rise in the mass of firms reduces the profit of any active firm and thus the value of an entrant.

(ORD) *The set* $(\mathcal{M},\succeq)$ *is completely ordered.*

[10] The assumption that market size enters multiplicatively into the gross profit function can easily be relaxed; see Nocke (2000).

Complete ordering of $(\mathcal{M}, \succeq)$ is a common property of models of symmetric and non-localized competition. The assumption rules out that some firm makes larger profits when the distribution of active firms is given by μ rather than μ', while some other firm is better off under μ'. If firms differ only in their types and are symmetric otherwise, then all types should have the same ranking of distributions.[11]

(CON) *The reduced-form profit function $\pi(c;\mu)$ is continuous.*[12]

For two of our main comparative dynamics results, we have to impose further structure on the reduced-form profit function. The following two conditions summarize properties of a large class of oligopoly models with heterogeneous firms.

A.1 *For $\mu' \succ \mu$, the profit difference $|\pi(c;\mu) - \pi(c;\mu')|$ is strictly decreasing in c on $[0, \overline{c}(\mu))$.*

A.2 *For $\mu' \succ \mu$, the profit ratio $\pi(c;\mu')/\pi(c;\mu)$ is strictly decreasing in c on $[0, \overline{c}(\mu'))$.*

Consider an increase in the distribution of active firms. By definition, this causes the gross profit of any firm to decrease, provided the firm makes a positive profit in the first place. Assumption A.1 says that efficient firms suffer more than inefficient firms in terms of the absolute decrease in profit. Assumption A.2 is the condition for our central result on the relationship between market size and market turbulence. It says that the percentage decrease in gross profit is larger for inefficient firms than for efficient ones. While both general and intuitive, these properties appear to have remained widely unnoticed in the literature.[13]

To improve our understanding of assumptions A.1 and A.2, the following proposition shows that they are equivalent to properties of equilibrium prices and quantities.

Proposition 1 *Suppose firm type c denotes marginal costs, and firms compete either in prices or in quantities. That is, equilibrium profit is given by (3). Assume also that the demand and inverse demand functions faced by an individual firm in equilibrium, $SD(\cdot;\mu)$ and $P(\cdot/S;\mu)$, are differentiable.*

1. *Assumption A.1 holds if and only if equilibrium output $q(c;\mu) = SD(p(c;\mu);\mu)$ is decreasing in μ; that is, if and only if $q(c;\mu') < q(c;\mu)\ \forall c \in [0, \overline{c}(\mu)), \mu' \succ \mu$.*

2. *Assumption A.2 holds if and only if the equilibrium price $p(c;\mu) = P(q(c;\mu)/S;\mu)$ is decreasing in μ; that is, if and only if $p(c;\mu') < p(c;\mu)\ \forall c \in [0, \overline{c}(\mu')), \mu' \succ \mu$.*

Proof. The assertions can be shown by taking the derivative of the profit difference and profit ratio with respect to c and applying the envelope theorem. ■

By definition, an increase in the distribution of active firms causes the profits of firms to fall. For this to hold, the equilibrium price or quantity of any given firm must fall. If firm type c denotes marginal costs, assumptions A.1 and A.2 say that both quantities and prices must fall in equilibrium. Proposition 1 is of independent interest. Since its proof does not make use of the fact that μ summarizes the distribution of active firms, it can be applied to any shift in consumers' tastes or incomes which reduces firms' profits.

Assumptions A.1 and A.2 (as well as the other conditions we impose on the reduced form profit function) are satisfied by a wide class of oligopoly models. Examples include the standard Cournot model with homogenous products, and the linear demand model with a finite number of differentiated products and either price or quantity competition; in Appendix A this is shown for the Cournot model. As to models of monopolistic competition with a continuum of firms, the widely used Dixit-Stiglitz model

[11] Alternatively, we could assume the following. There exist functions $h : \mathcal{M} \to \mathbb{R}$ and $\widehat{\pi} : [0,1] \times \mathbb{R} \to \mathbb{R}_+$ such that $\pi(c;\mu) \equiv \widehat{\pi}(c;h(\mu))$ for all $c \in [0,1]$ and $\mu \in \mathcal{M}$, where $\widehat{\pi}$ is strictly decreasing in its second argument. Hence, by definition, $h(\mu') \geq h(\mu)$ if and only if $\mu' \succeq \mu$.

[12] Formally, we endow $\mathcal{M}$ with the topology of weak* convergence.

[13] However, Boone (2000) – in independent work – provides a few parametric examples to show that an increase in the toughness of competition results in an increase in the profit ratio between a more and a less efficient firm. This corresponds to A.2.

satisfies most of our assumptions, including A.1, but not A.2. In fact, in the Dixit-Stiglitz model, firms use a simple markup pricing rule in which the markup is a function of some substitutability parameter in the utility function, but not of the mass of active firms. In the following, we give two examples of models of monopolistic competition that satisfy our assumptions.

Example 1 (The linear demand model with a continuum of firms.) *There is a continuum of S identical consumers whose utility U is defined over a continuum of substitute goods and a Hicksian composite commodity. Specifically,*

$$U(\mathbf{x}; H) = \int_0^n \left(x(i) - x^2(i) - 2\sigma \int_0^n x(j)x(i)dj \right) di + H,$$

where $x(i)$ is the consumption of variety i, and H the consumption of the Hicksian composite commodity.[14] *The parameter $\sigma \in (0,1)$, measures the substitutability between different varieties. Suppose each active firm in the industry produces a unique variety. Denote by $p(i)$ and $c(i)$ the price and marginal cost of variety i, respectively, and by Y consumer income. Normalizing the price of the Hicksian composite commodity to 1, $H = Y - \int_0^n p(i)x(i)di$. Re-label firms in increasing order of marginal costs, i.e., $c(j) > c(i) \Rightarrow j > i$. Let $m \in (0, n]$ denote the least efficient producer with positive sales in equilibrium. Then, if firms compete in prices (or quantities), equilibrium profit of firm $i \in [0, m]$, is given by*

$$\frac{S}{8} \left(\frac{2 + \sigma \int_0^m c(j)dj}{2 + m\sigma} - c(i) \right)^2 .$$

That is, profit is of the form $S\pi(c; \mu) = Sa(\overline{c}(\mu) - c)^2$ if $c < \overline{c}(\mu)$, and $S\pi(c; \mu) = 0$ otherwise. It is straightforward to check that the equilibrium profit function satisfies our assumptions.$\Box$

Example 2 (The linear demand model with perceived qualities.) *This example is similar to the preceding one. All firms now have the same constant marginal cost, normalized to zero. The utility of the representative consumer is given by*

$$U(\mathbf{x}; \mathbf{u}; H) = \int_0^n \left(x(i) - \frac{x^2(i)}{u^2(i)} - 2\sigma \int_0^n \frac{x(j)}{u(j)} \frac{x(i)}{u(i)} dj \right) di + H,$$

where $u(i) \geq 1$, is the perceived quality of variety i.[15] *Utility is strictly increasing in quality $u(i)$, provided that $x(i) > 0$. Re-label firms in decreasing order of quality, i.e., $u(i) > u(j) \Rightarrow i < j$. Let m again denote the marginal producer. Firm i's equilibrium profit under price (or quantity) competition can then be written as*

$$\frac{S}{8} \left(u(i) - \frac{\sigma}{2 + m\sigma} \int_0^m u(j)dj \right)^2 ,$$

which is again of the form $S\pi(c; \mu) = Sa(\overline{c}(\mu) - c)^2$, using, for example, the transformation $u = 2 - c$.$\Box$

3 Stationary Equilibrium

We now turn to the equilibrium analysis, where we confine attention to a free entry stationary equilibrium, in which firms' strategies and the distribution of firms are stationary. We characterize the

[14] This is the continuum version of the quadratic utility function which goes back to Bowley (1924). The associated demand system is widely used in oligopoly models; see Vives (1999).

[15] This is the continuum version of the utility function in Sutton (1997c).

stationary equilibrium, and show existence and uniqueness. We defer the analysis of the comparative dynamics properties, which lie at the heart of this paper, to the next section.

In a stationary equilibrium, the value of a type-c incumbent at the start of the exit stage, $V(c)$, can be written as

$$V(c) = \max\left\{0, \overline{V}(c)\right\}, \tag{4}$$

where

$$\overline{V}(c) = [S\pi(c;\mu) - \phi] + \delta\left[\alpha V(c) + (1-\alpha)\int_0^1 V(z)G(dz)\right]$$

is the value conditional on staying in the market for one period and behaving optimally thereafter. (In other words, $\overline{V}(c)$ is the value of a type-c firm at the output stage.) The expression for the conditional value consists of two terms: the first term is the firm's current net profit $S\pi(c;\mu) - \phi$, the second captures the expected continuation value – with probability α the firm will be of the same efficiency in the next period, while with the remaining probability it obtains a new draw from the distribution function $G(\cdot)$.

Let c^* be defined by

$$c^* \equiv \begin{cases} \sup\{c \in [0,1] \mid V(c) > 0\} & \text{if } V(1) = 0, \\ 1 & \text{if } V(1) > 0. \end{cases}$$

Our assumptions ensure that the value function $V(c)$ is continuous on $[0,1]$, and constant on $[\overline{c}(\mu), 1]$. If $\pi(0;\mu) > 0$, then standard arguments from dynamic programming imply that $V(c)$ is strictly decreasing on $[0, \min\{c^*, \overline{c}(\mu)\}]$. Hence, a firm's equilibrium exit strategy takes the form of a simple threshold rule, c^*, according to which the firm exits if and only if $c > c^*$. Let M denote the mass of entering firms in each period. In a stationary equilibrium,

$$\begin{aligned} V(c^*) &\geq 0, \\ \text{with } V(c^*) &= 0 \text{ if } M > 0. \end{aligned} \tag{5}$$

The value of an entrant at stage 1, V^e, is given by

$$V^e = \int_0^1 V(c)G(dc) - \epsilon. \tag{6}$$

Free entry implies that

$$\begin{aligned} V^e &\leq 0, \\ \text{with } V^e &= 0 \text{ if } M > 0. \end{aligned} \tag{7}$$

Suppose the stationary equilibrium exhibits simultaneous entry and exit, i.e., $M > 0$ and $c^* < 1$. In this case, $V^e = 0$, and so $\int_0^1 V(c)G(dc) = \varepsilon$. This allows us to compute the conditional value function in closed form as

$$\overline{V}(c) = \begin{cases} \frac{S\pi(c;\mu) - \phi + \delta(1-\alpha)\epsilon}{1-\alpha\delta} & \text{if } c \leq c^*, \\ S\pi(c;\mu) - \phi + \delta(1-\alpha)\epsilon & \text{if } c \geq c^*. \end{cases} \tag{8}$$

In stationary equilibrium with simultaneous entry and exit, the free entry condition $V^e = 0$ becomes

$$\int_0^{c^*} \left[S\pi(c;\mu) - \phi + \delta(1-\alpha)\epsilon\right] G(dc) - (1-\alpha\delta)\epsilon = 0. \tag{E}$$

Similarly, the condition for optimal exit, $V(c^*) = 0$, becomes

$$S\pi(c^*;\mu) - \phi + \delta(1-\alpha)\epsilon = 0. \tag{X}$$

From exit condition (X), the current net profit of the marginal incumbent firm, $S\pi(c^*;\mu)-\phi$, is negative. This firm stays in the market only because there is a probability $1-\alpha$ that it will get a new draw from $G(\cdot)$, and the value of this option is $\delta(1-\alpha)\epsilon$. In a stationary free entry equilibrium with simultaneous entry and exit, the value of a new draw must be equal to the entry cost ϵ. To simplify the exposition, we henceforth assume that $\phi \neq \delta(1-\alpha)\epsilon$. Then, a necessary condition for a stationary equilibrium with entry and exit is $\phi > \delta(1-\alpha)\epsilon$, which implies that the current gross profit of the marginal incumbent firm must be strictly positive, i.e., $c^* < \overline{c}(\mu)$.

Let $\overline{V}^e(x;\mu)$ denote the value of a new entrant who uses exit policy x in the period of entry and behaves optimally thereafter,

$$\overline{V}^e(x;\mu) = \int_0^x \overline{V}(c)G(dc) - \epsilon. \tag{9}$$

In stationary equilibrium with simultaneous entry and exit, the free entry condition (E) can then be rewritten as

$$\overline{V}^e(c^*;\mu) = 0, \tag{E'}$$

and the condition for optimal exit as

$$\frac{\partial}{\partial x}\overline{V}^e(c^*;\mu) = 0. \tag{X'}$$

The entry condition (E') says that the value of an entrant who uses the equilibrium policy c^* in the first period, and behaves optimally thereafter, is equal to zero. The exit condition (X') says that the derivative of this value function of an entrant with respect to the exit policy (and evaluated at the equilibrium exit policy c^*) is equal to zero. This close link between the entry and exit conditions is a consequence of the assumed sequence of moves. Since incumbents and new entrants face the same decision problem at the exit stage, the equilibrium exit policy of an incumbent, c^*, must maximize the value of an entrant. Hence, at the equilibrium exit policy, both the value of a new entrant and the derivative of the value function with respect to the exit policy have to be zero.

These properties of the value function $\overline{V}^e(x;\mu)$ are straightforward to illustrate in Figure 1. Note first that $\overline{V}^e(\cdot;\mu)$ is (for $\phi \neq \delta(1-\alpha)\epsilon$) single-peaked on $[0,1]$.[16] Furthermore, $\overline{V}^e$ is continuous, and continuously differentiable with respect to its first argument. Observe also that $\overline{V}^e(0;\mu) = -\epsilon < 0$. Hence, if μ gives the cost distribution in the stationary equilibrium, and c^* the equilibrium exit policy, then $\overline{V}^e(\cdot;\mu)$ takes its unique maximum at $\widehat{x}(\mu) = c^*$. This implies that there can be at most one stationary equilibrium with simultaneous entry and exit. Note finally that, for all $x > 0$, $\overline{V}^e(x;\cdot)$ is decreasing in μ (provided $\overline{c}(\mu) > 0$).

The distribution of active firms is determined by firms' entry and exit decisions. Let $\mu[c^*, M]$ denote the invariant measure of firms' efficiencies at stage 4 if all firms follow exit policy $c^* \in (0,1)$, and the mass of entrants in each period is M. This measure is uniquely defined by

$$\mu[c^*, M]\left([0,z]\right) = \frac{M}{(1-\alpha)\left(1-G(c^*)\right)} G(\min\{z, c^*\}),\ \forall z \in [0,1]. \tag{D}$$

The stationary distribution has thus the shape of $G(\cdot)$, is truncated at c^*, and scaled by the factor $M/\left[(1-\alpha)\left(1-G(c^*)\right)\right]$. The stationary equilibrium with simultaneous entry and exit can now be defined as the triplet (μ, M, c^*) satisfying equations (E), (X), and (D).

In a stationary equilibrium *without* entry and exit, we have $M = 0$ and $c^* = 1$, and the stationary distribution is given by

$$\mu_\lambda([0,z]) = \lambda G(z),\ \forall z \in [0,1], \tag{10}$$

[16] Either $\overline{V}^e(\cdot;\mu)$ is increasing in x on $[0,1]$, decreasing on $[0,1]$, or there exists a unique $\widehat{x}(\mu)$ such that $\overline{V}^e(\cdot;\mu)$ is increasing in x on $[0,\widehat{x}(\mu)]$ and decreasing on $[\widehat{x}(\mu),1]$. In the non-generic case $\phi = \delta(1-\alpha)\epsilon$, $\overline{V}^e(\cdot;\mu)$ is increasing in c on $[0,\overline{c}(\mu)]$ and constant on $[\overline{c}(\mu),1]$.

where $\lambda > 0$ is a scaling parameter. The stationary equilibrium without entry and exit is summarized by the triplet $(\mu_\lambda, 0, 1)$, satisfying (5), (7), and (10). Note that any stationary equilibrium distribution must be an element of $\mathcal{M}^* \equiv \{\mu \in \mathcal{M} \mid \forall z \in [0,1], \mu([0,z]) = kG(\min\{z, c^*\}),\ k > 0, c^* \in (0,1]\}$.

Before we turn to the issues of existence and uniqueness of equilibrium, let us consider some features of the equilibrium which are consistent with stylized facts on industry dynamics. The probability of immediate exit of a new entrant is $1 - G(c^*)$, whereas an incumbent leaves the market with a smaller probability, namely $(1-\alpha)(1 - G(c^*))$. Empirical studies have indeed shown that new firms are the ones most likely to exit the market. Moreover, the model implies that new entrants are on average more efficient than exiting firms, but less efficient than surviving incumbents. This is again consistent with the empirical evidence. Finally, if firm size (e.g., measured by output) decreases with marginal cost c, the simple stochastic process given by (1) implies that firm growth is negatively related to firm size, as found by Evans (1987) and others. In Section 6, we re-consider the model and allow for a broader class of Markov processes. This permits the model to be consistent with a number of other empirical findings.

To simplify the proof of existence and uniqueness, we impose a technical condition on the reduced-form profit function π.

(FREE) *Fix any positive measure $\mu \in \mathcal{M}^*$, let $k > 0$ a scaling parameter, and denote by μ^0 the null measure, i.e., $\mu^0(A) \equiv 0$ for any Borel set A. Then,*

(i) $\lim_{k\to\infty} \pi(c; k\mu) = 0$ *for all $c > 0$, and*

(ii) $\int_0^1 S\pi(c; \mu^0)G(dc) > \phi + (1-\delta)\epsilon$.

Part (i) of the free entry assumption (FREE) ensures that unlimited entry drives profits down to zero, while part (ii) implies that any stationary equilibrium distribution must be positive. We are now in a position to state our existence result.

Proposition 2 *There always exists a stationary equilibrium. Moreover, if a stationary equilibrium with simultaneous entry and exit exists, it is unique.*

Proof. See Appendix. ■

There are two kinds of stationary equilibria: (i) with simultaneous entry and exit, and (ii) without simultaneous entry and exit. If the first kind exists, it is the unique equilibrium. If it does not, there exists an equilibrium in which a (positive) mass of firms is active, and no entry and exit take place. Since in this case, the entry and exit conditions become inequalities ($V^e \leq 0$ and $V(1) \geq 0$, respectively), the mass of active firms in this equilibrium is not uniquely determined. In our proof of existence, we proceed in two steps. First, we neglect condition (D) and find the equilibrium exit policy c^* by varying the distribution of firms in $\mathcal{M}^*$ until (E) and (X) are satisfied. (If such a c^* does not exist in $(0,1)$, then the stationary equilibrium does not exhibit entry and exit.) Since $\overline{V}^e(c; \cdot)$ is strictly decreasing in μ for $c \in (0,1]$, conditions (E) and (X) also pin down the equivalence class of the stationary distribution. Only then do we consider the stationary distribution generated by c^* and M, as given by (D). From assumptions (DOM) and (CON), and condition (D), it then follows that there exists a unique mass M of entrants such that the stationary distribution generated by M and the exit policy c^* is equivalent to the distribution determined in the first part of the proof. Note that our (novel) method of proof is applicable also to models of perfect competition where firms are price-takers.

It is straightforward to find conditions under which the unique stationary equilibrium involves simultaneous entry and exit.

Proposition 3 *If the entry cost ϵ is sufficiently small, there exists a unique stationary equilibrium with simultaneous entry and exit.*

Proof. See Appendix. ■

For a stationary equilibrium to exhibit no entry and exit, the value of the least efficient incumbent (of type $c = 1$), conditional on staying in the market for another period, has to be larger than or equal to the value of an entrant: $\overline{V}(1) \geq 0 \geq V^e$. An incumbent of type $c = 1$ is less efficient than a potential entrant (whose efficiency is drawn from $G(\cdot)$) – but (in contrast to the potential entrant) has already sunk the entry cost ε. Clearly, for small entry costs, the efficiency difference will outweigh the incumbent's sunk cost advantage, and we must have $\overline{V}(1) < V^e$.

In the remainder of the paper, we will focus on the case of sufficiently small entry costs so that the stationary equilibrium exhibits firm turnover.

4 The Effects of Market Size and Sunk Costs on Firm Turnover

In this section, we analyze the comparative dynamics properties of the stationary equilibrium. We begin by defining a measure of firm turnover and showing the close connection between firm turnover and the age distribution of firms. Then, we analyze the effect of changes in the level of the entry cost ϵ and the fixed cost ϕ on the equilibrium level of firm turnover. Next, we turn to the main concern of the paper, namely the relationship between market size S on the one hand, and firm turnover and the age distribution of firms on the other. We also consider the relationship between market size and the number of active firms.

Measuring firm turnover. A natural measure of (relative) firm turnover is the ratio between the mass of new entrants and the total mass of active firms in each period. This suggests defining the turnover rate θ as

$$\theta \equiv \frac{M}{\mu\left([0,1]\right)},$$

where the denominator is the mass of active firms at stage 4, and the numerator is the mass of firms that have entered at stage 1 of the same period.

Using (D), the turnover rate in the stationary equilibrium can be written as

$$\theta = (1-\alpha)\frac{1-G(c^*)}{G(c^*)}. \tag{11}$$

That is, given persistence α, there is a monotonically decreasing relationship between the equilibrium exit policy c^* and the turnover rate θ. The monotonicity of the relationship between c^* and θ not only holds for the simple Markov process (1) considered here, but for a large class of Markov processes, as we will show in Section 6.

Age distribution of firms. In a stationary equilibrium, firms' exit policy shapes the age distribution of firms in a market. Let $\widehat{\theta}$ denote the (average) probability of exit of incumbents. In stationary equilibrium, $\widehat{\theta} = (1-\alpha)[1-G(c^*)] = G(c^*)\theta$.[17] We introduce the convention that the age of a newly entered firm (which is still active at the output stage) is one. Conditional on not leaving the market in the period of entry, the probability that a firm will survive until it reaches age a is then equal to $(1-\widehat{\theta})^{a-1}$. Consequently, the share of active firms whose age is less than or equal to a is given by

$$A(a|c^*) \equiv \frac{\sum_{t=0}^{a-1}(1-\widehat{\theta})^t}{\sum_{t=0}^{\infty}(1-\widehat{\theta})^t} = 1-(1-\widehat{\theta})^a,$$

[17] The exit probability $\widehat{\theta}$ is another natural measure of firm turnover. The main results of the paper are not sensitive to the particular choice of turnover measure.

which is strictly decreasing in c^*. Hence, a decrease in exit policy c^* shifts the age distribution towards younger firms in the sense of first-order stochastic dominance. In Section 6, we show that this stochastic dominance result holds much more generally for a large class of Markov processes.

Entry costs and firm turnover. Having defined a measure of firm turnover, we can now analyze the effects of changes in the parameters of the model on the turnover rate. Let us begin by looking at the effect of an increase in the level of entry costs.

Proposition 4 *An increase in the entry cost ϵ leads to an increase in exit policy c^*, and hence to a lower turnover rate θ and a shift in the age distribution of firms towards older firms. Furthermore, it causes the distribution of active firms, μ, and the mass of entrants per period, M, to decrease.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in the entry cost from ϵ_0 to $\epsilon_1 > \epsilon_0$. Let us assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e., $\theta_1 > 0$. (The proposition holds trivially if $\theta_1 = 0$.) From (8) and (9), it is easy to see that

$$\overline{V}^e(x; \mu_0; \epsilon_1) < \overline{V}^e(x; \mu_0; \epsilon_0) \text{ for all } x \in [0,1],$$

abusing notation by inserting argument ϵ into the value function $\overline{V}^e$, as defined by (9). Hence, for condition (E) to hold in the new stationary equilibrium, we must have $\mu_1 \prec \mu_0$. Since $\mu_1 \prec \mu_0$ and $\epsilon_1 > \epsilon_0$, we must have $c_1^* > c_0^*$ for condition (X) to hold:

$$\begin{aligned} S\pi(c_0^*; \mu_1) - \phi + \delta(1-\alpha)\epsilon_1 &> S\pi(c_0^*; \mu_0) - \phi + \delta(1-\alpha)\epsilon_0 \\ &= 0 \\ &= S\pi(c_1^*; \mu_1) - \phi + \delta(1-\alpha)\epsilon_1. \end{aligned}$$

From (11), we thus obtain $\theta_1 < \theta_0$. To see that $M_1 < M_0$, notice that $\mu_1 \prec \mu_0$ and $c_1^* > c_0^*$, in conjunction with (DOM), imply that $\mu_1([0, c_0^*]) < \mu_0([0, c_0^*])$. Using (D), the result follows. ■

The assertion of the proposition may be roughly explained as follows. Both the marginal incumbent (with cost level c^*) and the new entrant have a value of zero in equilibrium. However, since incumbents have already sunk the entry cost, the average entrant has to be more efficient than the marginal incumbent. Clearly, this wedge in efficiency is increasing in the level of entry costs. That is, exit policy c^* increases with ϵ. This, in turn, implies, that the hazard rate of incumbents is negatively related to the level of entry costs. A more formal explanation is the following. For any exit policy c^* and distribution μ, an increase in entry cost ϵ reduces the value of an entrant; that is, the curve $\overline{V}^e$ in Figure 1 shifts downwards. For the entry condition (E) to hold in the new equilibrium, the distribution of active firms must be smaller. This implies that the net profit $S\pi(c; \mu) - \phi$ of any type c goes up. This effect is reinforced by the fact that the option value of staying in the market rises with the level of entry cost. Hence, the value of the marginal incumbent in the initial equilibrium is now positive. Consequently, the marginal incumbent is less efficient in the new equilibrium. Hopenhayn (1992) derived the same prediction in a model with price-taking firms. A corollary of our result is that, in markets with higher entry costs, firms are on average less efficient, while the induced intensity of price competition is lower (in that the induced measure μ is smaller in the sense of our ordering on measures).

Fixed costs and firm turnover. Next, we analyze the effect of a change in ϕ, which may be interpreted as a fixed production cost or as an opportunity cost. The following proposition summarizes our results.

Proposition 5 *Suppose assumption A.1 holds. Then, an increase in the fixed (or opportunity) cost ϕ leads to a decrease in exit policy c^*, and hence to a higher turnover rate θ and a shift in the age distribution towards younger firms. Furthermore, it causes the distribution of active firms, μ, and the total mass of active firms, $\mu([0,1])$, to decrease.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in ϕ from ϕ_0 to ϕ_1, $\phi_1 > \phi_0$. Assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e. $\theta_1 > 0$. From entry condition (E), it follows that

$$\overline{V}^e(c^*; \mu_0; \phi_1) < \overline{V}^e(c^*; \mu_0; \phi_0) \text{ for all } c^* \in (0, 1],$$

and $\overline{V}^e(0; \mu_0; \phi_1) = \overline{V}^e(0; \mu_0; \phi_0)$. This implies that we must have $\mu_1 \prec \mu_0$ for (E) to hold again in the new stationary equilibrium. We now claim that $c_1^* < c_0^*$. To see this, suppose otherwise that $c_1^* \geq c_0^*$. According to condition (X),

$$S\pi(c_1^*; \mu_1) - \phi_1 + \delta(1-\alpha)\epsilon = 0 = S\pi(c_0^*; \mu_0) - \phi_0 + \delta(1-\alpha)\epsilon,$$

which implies that $S\pi(c_0^*; \mu_1) - \phi_1 \geq S\pi(c_0^*; \mu_0) - \phi_0$. Assumption A.1 then ensures that

$$S\pi(c; \mu_1) - \phi_1 > S\pi(c; \mu_0) - \phi_0 \text{ for all } c \in [0, c_0^*). \tag{12}$$

Thus, we obtain

$$\begin{aligned} \overline{V}^e(c_1^*; \mu_1; \phi_1) &\geq \overline{V}^e(c_0^*; \mu_1; \phi_1) \\ &> \overline{V}^e(c_0^*; \mu_0; \phi_0) \\ &= 0, \end{aligned}$$

where the first inequality follows from the fact that $\overline{V}^e(\cdot; \mu_1; \phi_1)$ assumes a maximum at c_1^*, and the second inequality from (12). Now, $\overline{V}^e(c_1^*; \mu_1; \phi_1) > 0$ cannot hold as it is in contradiction with (E). That is, we must have $c_1^* < c_0^*$. Finally, notice that, from (11), the turnover rate decreases monotonically with c^*, holding α fixed. Let us now show that we must indeed have $\theta_1 > 0$ (as assumed above), given that $\theta_0 > 0$. Define μ_0' and μ_1' by $\overline{V}^e(1; \mu_0'; \phi_0) = 0$ and $\overline{V}^e(1; \mu_1'; \phi_1) = 0$, respectively. (In the proof of Proposition 2, we have already shown that such measures exist.) It is easy to see that $\phi_1 > \phi_0$ implies $\mu_1' \prec \mu_0'$. Since $\theta_0 > 0$ by assumption, we have $\overline{V}^e(c^*; \mu_0'; \phi_0) = 0$ for some $c^* \in (0, 1)$. From assumption A.1, $\mu_1' \prec \mu_0'$, and $\phi_1 > \phi_0$, we get $\overline{V}^e(c^*; \mu_1'; \phi_1) > \overline{V}^e(c^*; \mu_0'; \phi_0)$ for all $c^* \in (0, 1)$. This concludes the proof of the assertion on turnover. The result on the total mass of active firms follows immediately from $\mu_1 \prec \mu_0$ and $c_1^* < c_0^*$ (and (DOM)). ■

Holding fixed the distribution of active firms, an increase in the fixed cost ϕ shifts the curve $\overline{V}^e$ downwards (see Figure 1): for any exit policy, the value of an entrant decreases. Since the equilibrium value of an entrant is zero, this implies that, in the new equilibrium, the measure of active firms μ is smaller (in terms of our ordering on measures). Hence, there are two opposing effects on an incumbent's net profit $S\pi(c; \mu) - \phi$. On the one hand, the increase in fixed cost ϕ reduces the net profit of all types by the same amount. On the other hand, the endogenous decrease in μ reduces the intensity of price competition. All firms benefit from the higher equilibrium prices, but (A.1 ensures that) less efficient firms gain less in absolute terms (as they produce a smaller quantity). For the entry condition (E) to continue to hold, the overall effect on profit must be positive for the most efficient firms, and negative for the least efficient active firms. In particular, the increase in fixed cost must decrease the profit of the marginal incumbent in the initial equilibrium. Hence, the exit policy c^* decreases with ϕ, which implies the predicted negative relationship between the fixed cost ϕ and turnover rate θ. Note that, in markets with higher fixed costs, the distribution of firm types is shifted towards more efficient firms, while the induced intensity of price competition is lower (in that μ is smaller in the sense of our ordering on measures). That is, if one were to correlate observable price-cost margins with the average efficiency level, one would find a *positive* cross-sectional correlation (assuming markets only differed in their level of fixed costs).

Market size and firm turnover. We now turn to our major concern, namely the relationship between market size and firm turnover. The central prediction of this paper is summarized in the following proposition.

Proposition 6 *Suppose assumption A.2 holds. Then, an increase in market size S leads to a decrease in exit policy c^*, and hence to a rise in the turnover rate θ and a shift in the age distribution of firms towards younger firms. Furthermore, an increase in market size causes the distribution of active firms, μ, and the mass of entrants per period, M, to rise.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in the size of the market from S_0 to $S_1 > S_0$. Let us assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e., $\theta_1 > 0$. (It is straightforward to show that turnover must indeed be positive in the new equilibrium, given that $\theta_0 > 0$. The argument is similar to that in the proof of Proposition 5, replacing assumption A.1 by A.2.) The proof proceeds in several steps. First, notice that

$$\overline{V}^e(x; \mu_0; S_1) > \overline{V}^e(x; \mu_0; S_0) \text{ for all } x \in (0, 1],$$

and $\overline{V}^e(0; \mu_0; S_1) = \overline{V}^e(0; \mu_0; S_0)$. For entry condition (E) to hold in the new equilibrium, we thus need $\mu_1 \succ \mu_0$. Second, suppose there exists a $y \in (0, \overline{c}(\mu_1))$ such that $S_1\pi(y; \mu_1) = S_0\pi(y; \mu_0)$. Assumption A.2 then implies that $S_1\pi(c; \mu_1) > S_0\pi(c; \mu_0)$ for all $c \in [0, y)$, and the reverse inequality for all $c \in (y, \overline{c}(\mu_1))$. Third, assume the assertion of the proposition does not hold, and so $c_1^* \geq c_0^*$. Then,

$$\begin{aligned} S_1\pi(c_1^*; \mu_1) &= S_0\pi(c_0^*; \mu_0) \\ &\geq S_0\pi(c_1^*; \mu_0), \end{aligned}$$

where the equality follows from condition (X). From A.2 we then obtain

$$S_1\pi(c; \mu_1) > S_0\pi(c; \mu_0) \text{ for all } c \in [0, c_1^*) \tag{13}$$

Consequently,

$$\begin{aligned} \overline{V}^e(c_1^*; \mu_1; S_1) &\geq \overline{V}^e(c_0^*; \mu_1; S_1) \\ &> \overline{V}^e(c_0^*; \mu_0; S_0) \\ &= 0, \end{aligned}$$

where the first inequality follows from the fact that $\overline{V}^e(\cdot; \mu_1; S_1)$ is maximized at c_1^*, and the second inequality from (13). Thus, entry condition (E) cannot hold in the new equilibrium: a contradiction. Consequently, we must have $c_1^* < c_0^*$, and hence $\theta_1 > \theta_0$. Observe that y exists and is in $(0, c_0^*)$; otherwise (E) would be violated. Finally, let us consider the effect of the increase in market size on the mass of firms that enter each period. Since $\mu_1 \succ \mu_0$ and $c_1^* < c_0^*$, we obtain (using (DOM)) $\mu_1([0, c_1^*]) > \mu_0([0, c_1^*])$, and hence, using (D), $M_1 > M_0$. ■

The result may be explained as follows. In a free entry equilibrium, the measure of active firms, μ, is positively related to market size. Holding the distribution of active firms fixed, an increase in market size raises the value of firms for any exit policy. Graphically, this means that the curve $\overline{V}^e$ in Figure 1 shifts upwards. Free entry then implies that the measure of firms has to increase with market size. This shifts the curve $\overline{V}^e$ downwards. Hence, there are two opposing effects on a firm's gross profit $S\pi(c; \mu)$. On the one hand, the rise in market size S increases the profits of all firms proportionally. This can be thought of as an increase in output levels, holding prices fixed. On the other hand, as the distribution of firms increases, prices (and, hence, price-cost margins) fall. A.2 implies that the percentage decrease in profit from an increase in μ is greater the less efficient is the firm. Hence, if there is some type for which its value remains unchanged, then the value of all better types increases, and that of all worse types decreases. Since the equilibrium value of an entrant is zero, independently of market size, there must be some types in $[0, c^*]$, which are worse off in the larger market. In particular, the marginal incumbent in the smaller market would have a negative value in the larger market if it were forced to use the same exit policy as in the smaller market. This implies that the marginal surviving firm has to be more efficient in larger markets. That is, exit policy c^* is decreasing with market size. The hazard rate of the average incumbent firm is therefore higher in larger markets: firm turnover and market size are positively correlated. An immediate consequence is that the age distribution of firms in a smaller market first-order stochastically dominates that in a larger market. Moreover, in larger markets, the distribution of firm types is shifted towards more efficient firms, while the induced intensity of price competition is higher (in that μ is greater in the sense of our ordering on measures). That is, if one were to correlate observable price-cost margins with the average efficiency level of active firms, one would find a *negative* cross-sectional correlation (assuming markets only differed in their size).

It is important to point out that our prediction on market size and firm turnover (Proposition 6) would not obtain in a model of perfect competition (as in Hopenhayn (1992)) or in a Dixit-Stiglitz type model of monopolistic competition (as in Melitz (1999)). The reason is that, in these models, the price competition effect is absent, which implies that equilibrium price-cost margins are independent of market size.

Let us illustrate this in a model of a homogenous goods industry with price-taking firms. In such a model, the gross profit of a type-c firm may be written as $\pi(c;p)$, where p is the equilibrium price. Market size enters the profit function only indirectly through p. The value of a type-c incumbent at stage 4 may then be denoted by $V(c;p)$. Under weak assumptions, $V(c;p)$ is strictly increasing in p for all $c \in [0, c^*]$, where c^* is the optimal exit policy, given price p. Furthermore, $V(c;p) > 0$ for all $c \in [0, c^*)$, and $V(c;p) = 0$ for all $c \in [c^*, 1]$. The entry condition for a stationary equilibrium is given by

$$\int_0^1 V(c;p)G(dc) - \epsilon = 0.$$

Hence, the entry condition uniquely determines the equilibrium price p, which is independent of market size. Given p, the exit threshold is uniquely determined by the exit condition $V(c^*;p) = 0$. Consequently, in a model of perfect competition, exit policy and turnover rate do not vary with market size.

The reasoning in the proof of Proposition 6 shows that efficient firms make higher profits in larger markets, and hence are more valuable. In contrast, less efficient firms are better off in smaller markets. That is, the distribution of profits and firm values is more "dispersed" in larger markets, while the distribution of efficiency levels is less "dispersed". The effect on profits is illustrated graphically in Figure 2.

Corollary 1 *The range of profits and firm values across active firms in the same market is increasing with market size. Formally, $\Delta\pi(S) \equiv S\pi(0;\mu) - S\pi(c^*;\mu)$ and $\Delta V(S) \equiv V(0) - V(c^*)$ are increasing with S.*

Proof. Exit condition (X) implies that $S\pi(c^*;\mu) = \phi - \delta(1-\alpha)\epsilon$, and hence $\Delta\pi(S_1) - \Delta\pi(S_0) = S_1\pi(0;\mu_1) - S_0\pi(0;\mu_0)$, using the same notation as in the proof of Proposition 6. For $S_1 > S_0$ (and, therefore, $\mu_1 \succ \mu_0$), the last expression is strictly positive since $y > 0$. Similarly, $V(c^*) = 0$, and hence $\Delta V(S) = V(0)$. Using (8) and $y > 0$, one obtains the result. ■

Market size and the number of active firms: the price competition effect. Our result on market size and firm turnover is closely related to the *price competition effect* (Sutton (1997a)) in standard models of oligopolistic competition: equilibrium prices fall with an increase in the number of firms (as formally stated in our assumption A.2). In the Cournot model with homogenous goods and identical firms, the price competition effect implies that, in a free entry equilibrium, the number of active firms rises less than proportionally with market size (see, e.g., Campbell and Hopenhayn (1999)). This is, in fact, what Bresnahan and Reiss (1991) find in their seminal paper. Assuming that firms within a market are identical, Bresnahan and Reiss (1991) use the relation between the number of firms and market size to back out parameters in a reduced form profit function. Examining the extent to which the market size per firm increases in the number of firms, they are able to infer how competition changes with the number of firms. Later studies (e.g., Asplund and Sandin (1999) and Campbell and Hopenhayn (2002)) have confirmed Bresnahan and Reiss' initial finding that an increase in market size leads to a less-than-proportionate increase in the number of active firms.

In our model, however, firms are heterogeneous, and the endogenous distribution of efficiencies depends on characteristics such as market size. Moreover, firms produce differentiated goods. Does the price competition effect still imply that the number of firms rises less-than-proportionately with market size? The answer is that it does not necessarily. This is for two reasons.

1. *Differentiated products.* If firms produce differentiated goods and consumers have a preference for variety, then a *market expansion effect* may counteract the price competition effect. In larger markets, consumers may spend a larger fraction of their income on the products in the differentiated goods industry as free entry of firms ensures that more variety is offered than in smaller markets. In fact, in the linear demand model (Example 1), the market expansion effect outweighs the price competition effect in small markets: even if all firms have the same marginal cost, the ratio between the mass of firms and market size, $\mu([0,1])/S$, rises (decreases) with market size, provided market size is sufficiently small (large).

2. *Efficiency differences.* If firms differ in their marginal costs, then the fractional decrease in gross profits from an endogenouse rise in the intensity of competition in larger markets is smaller for more efficient firms (as stated in A.2). Consider now the thought experiment of scaling up market size and the population of active firms by the same factor. This will decrease the profit of sufficiently inefficient firms but *increase* the profit of sufficiently efficient firms. Moreover, even if such a replication of the population of consumers and firms reduces the profit of the average entrant, the expected profit of an entrant may still rise since the profit of a firm is strictly convex in its marginal cost.[18]

In numerical simulations using the linear demand model, we found indeed that the ratio between the number of firms and market size, $\mu([0,1])/S$, is first *increasing* and then decreasing with market size.

Cost persistence and firm turnover. What is the effect of changes in the persistence of costs, as measured by parameter α, on the turnover rate? In the first discussion paper version of this paper (Asplund and Nocke (2000)), we show that an increase in α has two opposing effects on the turnover rate θ: the first term in (11), $(1-\alpha)$, clearly decreases with α, but the second term, $[1-G(c^*)]/G(c^*)$, is positively correlated with α. (The intuition for the latter observation is the following. The marginal incumbent makes a negative net profit, $S\pi(c^*;\mu)-\phi<0$, and is only in the market because of the prospect of lower costs in the future ("option value"). An increase in the persistence of costs decreases the value of this option, and so the marginal incumbent has to be more efficient.) The overall effect on θ is ambiguous.[19]

For empirical work, this has important implications. In a cross-industry study of firm turnover, it is likely to be difficult to control for differences in the stochastic process governing the evolution of firms' efficiencies or consumers' tastes (such as the persistence parameter α). To minimize measurement problems in this dimension, in Section 7, we analyze turnover rates across independent local markets within the same industry.

[18] For a given output level, the profit of a firm is linear in its (constant) marginal cost. However, since firms with lower marginal costs will find it profitable to produce at a greater scale, a firm's profit is strictly convex in its marginal cost.

[19] However, an increase in α has an unambigously negative effect on the exit probability $\widehat{\theta}$ (which does not keep track of those entrants that leave immediately after learning their current type).

5 Growing and Declining Markets

Intuitively, one may expect entry rates to be high in periods of market growth and low in periods of decline. Conversely, exit rates would be low when markets are growing and high when markets are shrinking. This intuition is not correct. The aim of this section is to analyze entry and exit rates when market size changes over time. In nonstationary markets, entry and exit rates will, in general, be different. Hence, our previous analysis of stationary markets does not carry over. We now show that, even in nonstationary markets, both entry and exit rates tend to be positively correlated with the *level* of market size. Holding (current) market size fixed, however, entry rates will tend to be higher in growing than in declining markets.

The exogenous evolution of market size is summarized by the deterministic sequence $\{S_t\}$, which is common knowledge to all firms. All other exogenous variables remain constant over time. To make the analysis more tractable, we assume throughout in this section that changes in market size and entry costs are sufficiently small so that there is simultaneous entry and exit in each period along the equilibrium path. Moreover, we simplify the analysis further by assuming $\alpha = 0$; that is, in each period, all incumbents get a new draw from distribution $G(\cdot)$.

The value function of an incumbent at the period-t exit stage is given by

$$V_t(c) = \max\left\{0, S_t\pi(c;\mu_t) - \phi + \delta\int_0^{c_{t+1}^*} V_{t+1}(z)G(dz)\right\},$$

where μ_t is the measure of active firms at the period-t output stage, and c_{t+1}^* the exit policy in $t+1$. The value of an entrant at the beginning of period t can be written as

$$V_t^e = \int_0^1 V_t(c)G(dc) - \epsilon.$$

Since we assume entry costs and the changes in market size to be small, there is simultaneous entry and exit in each period, i.e., $c_t^* < 1$ and $M_t > 0$ for all t. Consequently, the value of an entrant and the value of the marginal incumbent are always zero: $V_t^e = 0$ and $V_t(c_t^*) = 0$ for all t. The entry and exit conditions can then be expressed as

$$\int_0^{c_t^*} \left[S_t\pi(c;\mu_t) - \phi + \delta\epsilon\right] G(dc) - \epsilon = 0, \tag{E$_t$}$$

and

$$S_t\pi(c_t^*;\mu_t) - \phi + \delta\epsilon = 0, \tag{X$_t$}$$

respectively. Given that all firms follow exit policy $\{c_t^*\}$ and the mass of new entrants is given by $\{M_t\}$, we obtain the following relationship between the distribution of active firms in period t and the total mass of active firms in $t-1$:

$$\mu_t([0,z]) = [M_t + \mu_{t-1}([0,1])]\, G(\min\{z, c_t^*\}),\ \forall z \in [0,1]. \tag{D$_t$}$$

The term in brackets gives the total mass of firms at the beginning of the period-t exit stage: the sum of the mass of new entrants, M_t, and the mass of incumbents, $\mu_{t-1}([0,1])$. Each of these firms gets a draw from distribution function $G(\cdot)$, and thus stays in the market with probability $G(c_t^*)$. As before, the equilibrium measure μ_t is an element of $\mathcal{M}^*$. Given the mass of initially active firms, equilibrium is described by the sequence $\{\mu_t, M_t, c_t^*\}$ satisfying equations (E$_t$), (X$_t$), and (D$_t$) for all t.

Note that the future enters conditions (E$_t$) and (X$_t$) only through the option value of staying in the market, which is constant over time and given by $\delta\epsilon$, provided (as assumed) that there is positive gross entry in all future periods. In contrast, the future does not directly enter condition (D_t). The only way the past enters the equilibrium conditions is in (D_t) through the total mass of firms active in the previous period. As a result, the dynamic entry and exit conditions, (E_t) and (X_t), are equal to the static ones, (E) and (X), except for the fact that $\alpha = 0$, and S and μ are indexed by t. Now, from our previous analysis (proof of Proposition 2), we know that the entry and exit conditions uniquely determine the equilibrium exit policy and the "equivalence class" of the stationary distribution. That is, c_t^* and the equivalence class of μ_t are uniquely defined by (E_t) and (X_t). In particular, they are independent of future and past values of market size. Applying Proposition 6, we obtain that the exit policy c_t^* is decreasing in current market size S_t. According to (D_t), the stationary distribution in period t is given by the distribution function $G(\cdot)$, truncated at c_t^*, and scaled by $M_t + \mu_{t-1}([0,1])$. Consequently, conditions (E_t) and (X_t) also pin down the scaling factor, and hence the total mass of active firms, $\mu_t([0,1])$. Thus, condition (D_t) uniquely determines the mass of new entrants, M_t, as a function of $\mu_{t-1}([0,1])$ and S_t. More specifically, since (E_t) and (X_t) pin down $M_t + \mu_{t-1}([0,1])$, any change in the mass of last period's active firms will be exactly offset by a change in this period's mass of entrants (again, provided there is some entry and exit in all periods).

We define the entry rate to be "forward looking", and the exit rate to be "backward looking". The period-t entry rate, η_t, is the ratio between period-t entrants and the mass of active firms at the period-t output stage. Formally,[20]

$$\eta_t \equiv \frac{M_t}{\mu_t([0,1])}.$$

The period-t exit rate, χ_t, is the share of firms active in period $t-1$, which leave the market in period t. That is,

$$\begin{aligned} \chi_t &\equiv \frac{[1 - G(c_t^*)]\,\mu_{t-1}([0,1])}{\mu_{t-1}([0,1])} \\ &= 1 - G(c_t^*). \end{aligned}$$

We are now in the position to state our two main results on firm turnover when market size changes over time. First, we consider a given sequence $\{S_t\}$, and analyze the effect of changing market size in a single period. Then, we analyze the co-movements between $\{S_t\}$, $\{\eta_t\}$, and $\{\chi_t\}$.

Proposition 7 *Consider two sequences of market size, $\{S_t\}$ and $\{S_t'\}$, where $S_r < S_r'$, and $S_t = S_t'$ for all $t \neq r$. Then, the resulting equilibrium sequences of entry rates are such that $\eta_t = \eta_t'$ for all $t < r$ and $t > r+1$, $\eta_r < \eta_r'$, and $\eta_{r+1} > \eta_{r+1}'$ if and only if $\mu_r([0,1]) < \mu_r'([0,1])$. Equilibrium exit rates are characterized by $\chi_t = \chi_t'$ for all $t \neq r$, and $\chi_r < \chi_r'$.*

Proof. See Appendix. ■

Corollary 2 *Consider an arbitrary sequence of market size, $\{S_t\}$. In equilibrium, exit rates are positively correlated with current market size. More precisely, χ_t is increasing in S_t, and independent of S_r, $r \neq t$. Equilibrium entry rates are positively related to current market size, too, but only if controlling for last period's market size: holding S_{t-1} fixed, entry rate η_t is increasing in S_t, and independent of S_r, $r \neq t, t-1$. Moreover, holding S_t fixed, η_t is negatively correlated with the mass of active firms in $t-1$.*

From the dynamic entry and exit conditions, (E_t) and (X_t), the exit policy c_t^* and the mass of active firms, $\mu_t([0,1])$, is a function of current market size S_t, and independent of past and future market size.

[20]Again, we could use $\widehat{\eta}_t \equiv G(c^*)\eta_t$ as an alternative measure, which does not keep track of those entrants that exit immediately after learning their current efficiency.

From our analysis of stationary markets, we then know that c_t^* is decreasing in S_t. Consequently, the exit rate x_t is increasing in S_t. From our previous analysis, we also know that the measure of currently active types μ_t is increasing in S_t (in terms of our ordering on measures). Given that the mass of incumbents, $\mu_{t-1}([0,1])$, is independent of S_t, it follows that the mass of new entrants, M_t, must be positively related to S_t. Furthermore, since firms use a tougher exit policy in larger markets, an increase in current market size must cause M_t to rise by a larger fraction than the total mass of active firms, $\mu_t([0,1]) = [M_t + \mu_{t-1}([0,1])]\, G(c_t^*)$. That is, the entry rate η_t must increase with S_t, holding S_{t-1} (and hence $\mu_{t-1}([0,1])$) fixed.

Note the asymmetry between entry rates in growing and declining markets. If market size and the mass of active firms (i.e., S_t and $\mu_t([0,1])$) are positively correlated, as one would intuitively expect, then entry rates will be higher in growing than in declining markets, holding fixed the level of market size: given S_t, η_t is then negatively correlated with S_{t-1}.

Let us point out that the predictions of Proposition 7 and Corollary 2 would remain unchanged if we assumed that market size followed some stochastic process, and the realization of current market size became common knowledge at the start of each period (prior to entry decisions). The results of this section provide us with a useful benchmark. The caveat to keep in mind, however, is that they have been obtained under the rather strong assumption that incumbents' cost draws are i.i.d., i.e., $\alpha = 0$. A more general treatment of growing and declining markets is left for future research.

6 Robustness of Results

In this section, we explore the robustness of our results along two dimensions, namely the sequence of moves and the evolution of firms' efficiencies. So far, we have assumed that exit decisions take place *after* incumbents and entrants learn their current type. This gave rise to a simple mathematical structure. Moreover, it captured the observation that, in many markets, a subset of new entrants make initial investments but never reach the production stage and, hence, exit again. The obvious question is whether our results are sensitive to the sequence of moves. The timing we want to analyze here is as in Hopenhayn's (1992) model. At the first stage, entry and exit decisions take place. At the second stage, the new entrants and surviving incumbents learn their current type. Finally, the active firms play a market game and receive profits.

In the main part of this paper, we assumed that the evolution of an incumbent's efficiency is governed by equation (1). This stochastic process allowed us to obtain simple closed-form solutions for firms' value functions and the stationary distribution, expressed only in terms of the exit policy c^* and the mass of entrants M. We now want to show that the main results of this paper hold more generally for a larger class of Markov processes. This is of particular interest since the simple stochastic process considered so far cannot account for some of the stylized facts regarding the relationship between firm size and firm age on the one hand, and firm growth and survival on the other. For instance, assuming that a firm's output $q(c;\mu,S)$ is decreasing in marginal cost c, equation (1) implies that the probability of exit, $(1-\alpha)(1-G(c^*))$, is independent of an incumbent's size. This is inconsistent with the empirical finding that firm size and failure are negatively correlated (e.g., Evans (1987), Hall (1987), and Dunne, Roberts and Samuelson (1989)). With the class of Markov processes considered here, this and other stylized facts are implied by or consistent with our model.

For brevity of exposition, we change both the timing and the stochastic process in our model, rather than discussing each modification in turn. Incorporating the new Markov process into the model of Section 2 proceeds in a very similar fashion.

Consider an incumbent of type c. The probability that, in the next period, his marginal cost is less than or equal to c' is given by $F(c'|c)$. For any $c' \in (0,1)$, we assume that $F(c'|c)$ is strictly decreasing in c. This means that a currently efficient firm is more likely to be efficient tomorrow than a currently less efficient firm; this is formally expressed in terms of first-order stochastic dominance. Suppose that firm size is measured by output $q(c;\mu,S)$, which is decreasing in c, and suppose that sufficiently inefficient firms exit the market, $c^* < 1$. Then, the probability of survival is increasing with firm size, which is consistent with the empirical evidence. For convenience, we assume that $F(\cdot|c)$ is strictly increasing on $[0,1]$, and $F(c'|c)$ is continuous in c' and c. Moreover, we posit that the distribution of entrants' efficiencies is given by the distribution of a particular type of incumbent. That is, there exists a cost level $\widehat{c} \in (0,1)$ such that $G(\cdot) \equiv F(\cdot|\widehat{c})$. For some results, we impose a further technical condition, which will be discussed below.

At the entry and exit stage (the new stage 1), the value of an incumbent with previous cost level c can be written as

$$V(c;\mu,S) \equiv \max\left\{0, \overline{V}(c;\mu,S)\right\},$$

where $\overline{V}(c;\mu,S)$ is the value of the incumbent conditional on staying in the market in the current period and behaving optimally thereafter. This conditional value is given by

$$\overline{V}(c;\mu,S) \equiv \int_0^1 W(c';\mu,S)F(dc'|c)$$

where $W(c';\mu,S) \equiv S\pi(c';\mu) - \phi + \delta V(c';\mu,S)$ is the value of a firm *after* learning that its new type is c'. Similarly, prior to learning his current type, the value of a new entrant is given by

$$\begin{aligned} V^e(\mu,S) &\equiv \int_0^1 W(c;\mu,S)G(dc) - \epsilon \\ &= \overline{V}(\widehat{c};\mu,S) - \epsilon. \end{aligned}$$

Standard arguments in dynamic programming imply that the conditional value $\overline{V}(c;\mu,S)$ is strictly decreasing in marginal cost c, provided the conditional value of the most efficient firm is positive, $\overline{V}(0;\mu,S) > 0$. Furthermore, under the same condition, $\overline{V}(c;\mu,S)$ is strictly decreasing in the measure μ and strictly increasing in market size S.

In a stationary equilibrium with simultaneous entry and exit, where $c^* < 1$,

$$V^e(\mu,S) = 0 \tag{E_R}$$

and

$$\overline{V}(c^*;\mu,S) = 0. \tag{X_R}$$

Consequently, if $c^* < 1$, the "average entrant" $\widehat{c}$ is more efficient than the marginal incumbent c^*: $\widehat{c} < c^*$.

Exit policy c^* and the mass of entrants per period, M, induce a unique stationary distribution. The mass of active firms with costs in $[0,z]$ is given by

$$\mu\left([0,z]\right) = MG\left(z\right) + \int_0^{c^*} F(z|c)\mu(dc),\ \forall z \in [0,1]. \tag{D_R}$$

A stationary equilibrium is a triplet $\{\mu, M, c^*\}$ satisfying equations (E_R), (X_R), and (D_R). Proving existence and uniqueness of a stationary equilibrium is beyond the scope of this section. Instead, we focus on the main comparative dynamics results.

The turnover rate is defined by $\theta \equiv M/\mu([0,1])$. In a stationary equilibrium, we can interpret the total mass of firms active at the output stage in period t as being the survivors amongst the firms that entered in periods t, $t-1$, $t-2$, $t-3$, and so on. Decomposing the mass of active firms into different cohorts of surviving firms, we obtain

$$\mu\left([0,1]\right) = M\left[1 + \sum_{s=1}^{\infty}\sigma_s(c^*)\right],$$

where $\sigma_s(c^*)$ is the probability of surviving s periods. For $s \geq 1$, this probability is given by

$$\sigma_s(c^*) \equiv \int_0^{c^*} \int_0^{c^*} ... \int_0^{c^*} \int_0^{c^*} F(dc_s|c_{s-1})F(dc_{s-1}|c_{s-2})...F(dc_2|c_1)G(dc_1)$$

Since $d\sigma_s(c^*)/dc^* > 0$, the equilibrium turnover rate $\theta = 1/\left[1 + \sum_{s=1}^{\infty} \sigma_s(c^*)\right]$ is strictly decreasing with exit policy c^*. Very generally, there is a close relationship between the exit policy c^* (which acts as an absorbing barrier for the Markov process governing a firm's efficiency) and the probability distribution of a firm's age at the time of exit: the probability that a firm exits less than or equal to s periods after entry is given by $1 - \sigma_s(c^*)$. Hence, an increase in c^* shifts the probability distribution of a firm's age at exit towards later exit (in terms of first-order stochastic dominance) and raises the expected life time of a new entrant, $1/\theta = 1 + \sum_{s=1}^{\infty} \sigma_s(c^*)$.

What is the effect of a change in the exit policy c^* on the (stationary) age distribution of firms at a given point in time? Let $A(\cdot|c^*)$ denote the cumulative age distribution function, given that all firms use exit policy c^*. The share of active firms whose age is less than or equal to a is given by

$$A(a|c^*) = \frac{1 + \sum_{s=1}^{a-1} \sigma_s(c^*)}{1 + \sum_{s=1}^{\infty} \sigma_s(c^*)}.$$

For $A(a|c^*)$ to be decreasing in c^* for all a, we need to impose a regularity condition on the Markov process.

Proposition 8 *Suppose F and G are such that the probability that a firm exits $t \geq 1$ periods after entry, conditional on having survived for $t-1$ periods, is decreasing in the exit policy c^*. That is, let the hazard rate $\left(\sigma_{t-1}(c^*) - \sigma_t(c^*)\right)/\sigma_{t-1}(c^*)$ be decreasing in c^*. Then, an increase in exit policy c^* shifts the (stationary) age distribution of firms towards older firms in terms of first-order stochastic dominance. Formally, $A(a|c^*)$ is decreasing in c^* for all $a \geq 1$.*

Proof. See Appendix. ■

We are now in the position to analyze the effects of sunk costs and market size on market turbulence. Throughout, we assume that there is simultaneous entry and exit in the initial stationary equilibrium. First, we consider an increase in entry costs from ϵ_0 to ϵ_1. Holding the distribution of active firms fixed at μ_0, this reduces the value of a new entrant:

$$\overline{V}(\widehat{c}; \mu_0, S) - \epsilon_1 < \overline{V}(\widehat{c}; \mu_0, S) - \epsilon_0 = 0.$$

For the entry condition (E_R) to hold in the new equilibrium, the distribution of active firms must decrease, i.e., $\mu_1 \prec \mu_0$. This raises the conditional value of any firm. In particular, the conditional value of the marginal incumbent in the initial equilibrium is now positive:

$$\overline{V}(c_0^*; \mu_1, S) > \overline{V}(c_0^*; \mu_0, S) = 0.$$

The exit condition (X_R) then implies that the marginal incumbent is less efficient in the new equilibrium: $c_1^* > c_0^*$. Hence, the rise in entry costs causes the equilibrium turnover rate θ to decrease, as predicted by Proposition 4.

Next, we consider the effect of an increase in market size from S_0 to $S_1 > S_0$. As before, we assume A.2. In addition, we impose a technical condition on the Markov process. Specifically, the conditional (cumulative) distribution function $F(c'|c)$ can be decomposed as a weighted average of two distribution functions:

$$F(c'|c) = a(c)\underline{F}(c') + [1 - a(c)]\,\overline{F}(c'), \tag{14}$$

where the weight $a(c) \in (0,1)$, is continuous and strictly decreasing in c. The (continuous) distribution functions $\underline{F}(\cdot)$ and $\overline{F}(\cdot)$ have support $[0,\varphi]$ and $[\varphi,1]$, respectively, where $\varphi \in (0,1)$. That is, $\underline{F}(\cdot)$ gives the distribution of "good types", and $\overline{F}(\cdot)$ the distribution of "bad types". The more efficient the firm is currently, the more "likely" it is to get a draw from the good distribution in the future: $F(c'|c)$ is decreasing in c. We assume that φ is not too large so that $\varphi < \overline{c}(\mu_0)$.

For a given distribution of active firms, the increase in market size raises the value of a new entrant:

$$V^e(\mu_0, S_1) > V^e(\mu_0, S_0) = 0.$$

Hence, the distribution of active firms is increasing with market size; that is, $\mu_1 \succ \mu_0$. Assumption A.2 implies that there exists a type $y \in (0, c_0^*)$ such that all better types have higher current profits in the larger market, while all worse types have lower profits (provided they make profits at all in the smaller market). Moreover, our assumption on the stochastic process ensure that

$$\int_0^{\varphi} S_1 \pi(c;\mu_1)\underline{F}(dc) > \int_0^{\varphi} S_0 \pi(c;\mu_0)\underline{F}(dc),$$

and

$$\int_{\varphi}^{1} S_1 \pi(c;\mu_1)\overline{F}(dc) < \int_{\varphi}^{1} S_0 \pi(c;\mu_0)\overline{F}(dc).$$

It should be clear that the two inequalities cannot point the same way: if they did, the conditional value of *all* types would rise (fall) with market size, which is inconsistent with the entry condition (E_R). Since the value of an entrant can be written as

$$V^e(\mu, S) = a(\widehat{c}) \int_0^{\varphi} W(c;\mu,S)\underline{F}(dc) + [1 - a(\widehat{c})] \int_{\varphi}^{1} W(c;\mu,S)\overline{F}(dc),$$

we obtain

$$\int_0^{\varphi} W(c;\mu_1,S_1)\underline{F}(dc) > \int_0^{\varphi} W(c;\mu_0,S_0)\underline{F}(dc)$$

and

$$\int_{\varphi}^{1} W(c;\mu_1,S_1)\overline{F}(dc) < \int_{\varphi}^{1} W(c;\mu_0,S_0)\overline{F}(dc).$$

That is, conditional on obtaining a draw from the good distribution, the conditional value of a firm is larger in the larger market, while the opposite relationship holds conditional on getting a draw from the bad distribution. For an entrant, these two effects cancel each other out in expectation. Now, the marginal incumbent firm in the smaller market, c_0^*, is less efficient than the average entrant $\widehat{c}$, and hence obtains a bad draw with a larger probability. This implies that the conditional value of type c_0^* is negative in the larger market, i.e., $\overline{V}(c_0^*;\mu_1,S_1) < 0$. It follows that $c_1^* < c_0^*$. The efficiency of the marginal incumbent rises with market size. This implies the positive relationship between market size and firm turnover predicted by Proposition 6.

It is important to point out that we can dispense with condition (14) and still obtain our result on market size and firm turnover (and the age distribution), provided we assume that the discount factor is not too large. If δ is small, then single-crossing of the profit function implies single-crossing of the value function. Hence, in a larger and endogenously more competitive market, the value function will cross the value function in the smaller market only once, and firms will use a tougher exit policy.

The analysis of the effect of a change in the fixed cost ϕ proceeds analogously to that of a change in market size. It is omitted for brevity. The results of this section should be reassuring. The main predictions of this paper are quite robust to changes in the sequence of moves and the stochastic process governing the evolution of incumbents' efficiencies.

7 Empirical Application: Hair Salons in Sweden

We now turn to the empirical application of our theory. As stressed in the Introduction, we have focused on the comparative dynamics properties of our model with respect to "observables" such as entry costs, fixed costs, and market size. However, one important variable in our model, namely the magnitude of the underlying idiosyncratic shocks (as captured by the persistence parameter α in our benchmark formulation) is hard to measure or to control for, and is likely to vary across industries. In addition, there are likely to be other factors that contribute to cross-industry differences in entry and exit patterns (e.g., financial constraints, regulations, industry life-cycles) and that are left out of our model. Our empirical application is therefore based on data from a single sector where competition takes place in many geographical markets: hair salons in Sweden. Thus, we use variations in local market conditions, in particular market size and fixed costs, to explain differences in entry and exit rates. An identifying assumption of this approach, discussed further below, is that other factors remain constant across geographical markets. In the context of studying differences in turnover rates, we believe this assumption to be much more reasonable within an industry rather than across industries.

Competition among hair salons corresponds closely to the assumptions of our model. Even in small towns, there is typically a large number of hair salons to choose from.[21] Products are clearly differentiated in terms of location and quality of service (which is closely tied to the skills and personalities of employees). The assumption of monopolistic competition seems therefore to hold good in this industry. Moreover, casual observation suggests that there is a great deal of entry and exit of hair salons. An important source of idiosyncratic shocks is likely to be the turnover of employees.

The central prediction of our theory is that an increase in market size causes a rise in the turnover rate of firms, and hence a shift in the age distribution towards younger firms. We test this prediction as follows: we split our sample into small and large markets and then test if the age distribution of firms in the subsample of small markets first-order stochastically dominates that of firms in the subsample of large markets. Furthermore, we use information on land values to test our prediction that higher fixed costs result in higher rates of firm turnover, as evidenced by a shift in the age distribution towards younger firms. We begin by describing the data, and then turn to non-parametric tests of first-order stochastic dominance (FOSD).

7.1 Data

The 2001 edition of the Swedish Yellow Pages lists 7,243 hair salons, out of which we contacted 1,100 by phone.[22] The sample contains information from interviews with 1,030 of these; the remaining were either unwilling to participate or not possible to reach. The majority of hair salons in Sweden are small, single establishment operations. Chains play only a minor role in the sector (our impression from studying the Yellow Pages and discussions with a number of hairdressers is that less than five percent of the salons are part of a chain) and we identify each establishment with a firm.

The measure of firm age is the number of years the salon has been established at the current location, AGE. The median age in the sample is nine years (see Table 1) but the data show, as expected, a wide

[21] In Asplund and Nocke (2000), we tested our predictions by relating hazard rates of Swedish driving schools to market size. This data set gave some support for the theory, but had two weaknesses. First, very few markets had more than five firms such that the assumption of monopolisticc competition is untenable. Second, a demographic shift in the population aged 16 to 24 years lead to a distinct decrease in market size.

[22] The selection was conducted as follows. Each hair salon was assigned a number, and in a first step we randomly selected 1,000 of these. With such a non-stratified sample, most observations are from medium to large towns. To obtain greater representation from small markets, we randomly selected an additional 100 hair salons from the subsample of markets (postal areas) with less than 10 hair salons. Some hair salons may have chosen not to pay to appear in the Yellow Pages, but we believe that given the small cost (<SEK900≈ USD100) these make up only a tiny fraction of all hair salons.

range of ages – with the 10th and 90th percentile at 2 and 25 years, respectively.[23]

Our theory is concerned with the effects of market size on firm turnover. Finding an appropriate measure of market size requires careful consideration. Since haircuts are not very costly and purchased frequently, and most consumers have many nearby salons to choose from, consumers are unlikely to travel significant distances to purchase the service. This suggests to measure market size by the population living in an area, and to assume that the population is solely served by the salons located within the same area.[24] The smallest areas that the Yellow Pages allow us to identify, and for which population figures are available, are the 8,977 five-digit postal codes. At this level of market definition, however, we are unlikely to measure market size correctly since many consumers frequent salons outside the postal code where they reside. On the other hand, defining markets very broadly, e.g., by municipalities (of which there are 289 in Sweden), runs into the opposite problem: a municipality is likely to contain several submarkets with little or no overlap. We have therefore decided to use an intermediate level of aggregation, namely postal areas, to define market boundaries. Our measure of market size, $MSIZE$, is then the population living within a postal area. In Sweden, there are 1,534 postal areas, ranging in size from small villages with less than 200 inhabitants up to the three largest postal areas Stockholm, Gothenburg, and Malmo with more than 200,000 inhabitants each. In our sample, 368 postal areas are represented, with a median population of 7,096 inhabitants. To measure the number of firms in a market, we use the number of hair salons listed in the Yellow Pages for a given postal area, $FIRMS$. The median is 11 hair salons in a postal area.

Are postal areas a reasonable market definition for hair salons? To a first approximation, the number of consumers needed to make a hair salon viable should be roughly the same across markets, which would translate into a strong correlation between population and the number of firms. The raw correlation (for 1,534 postal areas) between $MSIZE$ and $FIRMS$ is 0.92. In contrast, the corresponding correlation for postal codes is only 0.16. A simple example can explain this. Consider a typical medium sized town, which is a single postal area with 20,000 inhabitants and 20 postal codes. The center of the town has three postal codes, and the 17 other postal codes comprise residential suburbs. Although relatively few people live in the center, many of the hair salons are located there and, as a consequence, the majority of the other postal codes do not have a single hair salon. This explains the low correlation between between population and the number of firms in a postal code. Aggregating to the postal area level reflects our lack of information on exactly how market demand is geographically distributed. In larger towns (say, with a population above 100,000), there are often several centers, and so the postal area is too broad a market definition. To sum up, the population in a postal area is an imperfect measure of market size, but should be a sufficiently powerful measure to allow us to distinguish between small and large markets. Since measurement problems are likely to be most pronounced in large towns, we focus our analysis on smaller markets.

Our paper is also concerned with the effects of fixed costs on firm turnover. For hair salons, the cost of floor space is an important source of fixed costs that varies considerably between, and even within, towns. The closest proxy for which public information exists is the average assessed value per square meter for commercial properties, $RENT$ (collected to provide a basis for property taxation). These data are broken down by postal code. We base our tests of FOSD on this low level of aggregation so as to be able to measure as exactly as possible the fixed costs a particular firm faces. With reference to the example above, rents vary not only between residential suburbs and the center, but certainly also between the three postal codes in the center. For the 890 postal codes represented in the sample,

[23] We also have information on whether the present owner had been previously established in the same area but at a different adress; approximately 40 percent of the observations fall into this category. There is no significant correlation between having moved location on the one hand, and firm age, market size, and land values on the other.

[24] The number of inhabitants in a market may not be a perfect measure of market size. Unfortunately, official income statistics are only broken down to the far more aggregated municipality level. The same applies to demographic information on gender and age composition. Preferences, and thereby per capita demand, could potentially also vary across markets.

the median is SEK 2,720/m^2 and the 10th and 90th percentiles are 1,240 and 6,190, respectively. For comparison, we also calculate a measure of the average fixed costs in a postal area, $RENTMEAN$, by using the total value of commercial properties in the constituent postal codes as weights of $RENT$.

A potential problem of identification is that rents tend to be increasing with town size. In the sample, the raw correlation between $MSIZE$ and $RENT$ is 0.62. Also, the cost of floor space is usually higher in the center, partly since demand is greatest there. However, since rents are not only determined by the number of potential customers for hair salons, but by many other factors, it should in principle be possible to separate the effects of market size and fixed costs. To test this assertion we argue that, to a first approximation, the number of firms (i) rises less-than-proportionally with market size[25], and (ii) is decreasing in fixed costs (as predicted by Proposition 5). In Table 2, we report the results from a parsimonious Tobit specification with $FIRMS$ as the dependent variable.[26] In (2:1), where the largest markets are excluded, the coefficient of $MSIZE$ is positive and that of $MSIZE^2$ negative. The coefficient of the interaction term $MSIZE \times RENTMEAN$ is negative and significant, which implies that, conditional on market size, there are fewer firms where rents are high. These results are in line with our predictions. In (2:2), where we include all but the 11 largest markets, the coefficient of $MSIZE \times RENTMEAN$ remains negative, but the one of $MSIZE^2$ becomes insignificant. Using the full sample, as in (2:3), produces markedly different coefficients. As noted above, problems of measuring market size in the large towns, and using a market average of rents, are the most likely explanations as to why including the largest markets may distort the estimates. It is also conceivable that we underestimate the size of the largest markets by ignoring the possibility that per capita demand is higher there (e.g., people in big towns go more often to the hairdresser). Overall, the results suggests that it is reasonable to focus on the smaller markets so as to reduce measurement problems.

Finally, we use regressions to check the robustness of our results and examine some competing explanations. The control variables we use are discussed in conjunction with the results in Table 5.

7.2 Statistical Tests of FOSD

Let Y and Z be two random variables with cumulative distribution functions $F_Y(.)$ and $F_Z(.)$. Random variable Y first-order stochastically dominates Z, denoted $Y \succ_1 Z$, if

$$\begin{aligned} F_Y(a_i) &\leq F_Z(a_i) \text{ for all } a_i, \text{ and} \\ F_Y(a_i) &\neq F_Z(a_i) \text{ for some } a_i. \end{aligned}$$

Suppose the data consist of N_Y and N_Z independent observations from $F_Y(.)$ and $F_Z(.)$, which form the empirical distributions, $\widehat{F}_Y(.)$ and $\widehat{F}_Z(.)$.[27] In the present context, $\widehat{F}_Y(.)$ and $\widehat{F}_Z(.)$ correspond to the proportions of firms that are less than or equal to a_i years old, in two different subsets of markets (small

[25] While this is not guaranteed in our model with heterogeneous firms, numerical analysis (using the linear demand specification) suggests that this holds true, provided market size is not too small; see section 4.

[26] The specification follows from assuming that $FIRMS/MSIZE$ decreases linearly in both $MSIZE$ and $RENTMEAN$. The regression equation is obtained by multiplying both sides of $\frac{FIRMS}{MSIZE} = \beta_1 + \beta_2 MSIZE + \beta_3 RENTMEAN + \varepsilon$ with $MSIZE$ and adding a constant. A Tobit model is used since many (mostly small) markets lack a hair salon.

[27] There is a subtle issue whether the observations in our data are completely independently drawn. While we have randomly sampled about one in eight hair salons in Sweden, we drew each observation without returning it into the "urne". Hence, sampling an observation from one market reduces the probability of drawing again from the same market. Moreover, the fact that one has drawn a firm of a certain age may change the expected age distribution of the firms from the same market that remain in the urne. We believe that this does not significantly reduce the power of our tests. First, we have sampled only a small fraction of firms, and (due to the large number of markets) most are from different markets. Indeed, we have rarely sampled more than one hair salon from a small market. Second, our simple Markov process (1) implies that, within the same market, a firm's age is independent of its efficiency, and hence independent of a rival firm's age.

vs. large, and low rent vs. high rent markets). To test for first-order stochastic dominance (FOSD), we use the non-parametric procedure proposed by Davidson and Duclos (2000), which compares the two distributions at a finite number of grid points a_i, $i = 1, ..., K$. Anderson (1996) provides an alternative test.

Davidson and Duclos show that, under the null hypothesis of equal distributions, $\widehat{F}_Y(a_i) - \widehat{F}_Z(a_i)$ is asymptotically normally distributed, $N(0, \widehat{V}(a_i))$, with variance

$$\begin{aligned}\widehat{V}(a_i) &= \widehat{V}_Y(a_i) + \widehat{V}_Z(a_i) \\ &= \frac{1}{N_Y}\left(\widehat{F}_Y(a_i) - \left(\widehat{F}_Y(a_i)\right)^2\right) + \frac{1}{N_Z}\left(\widehat{F}_Z(a_i) - \left(\widehat{F}_Z(a_i)\right)^2\right).\end{aligned}$$

The standardized test statistic is then given by

$$T^{DD}(a_i) = \frac{\widehat{F}_Y(a_i) - \widehat{F}_Z(a_i)}{\sqrt[2]{\widehat{V}(a_i)}}.$$

As the test for FOSD involves multiple comparisons (i.e., one for each grid point a_i), the critical values from the standard Student's t-statistics are not applicable. Instead, let $m_{\alpha,K,\infty}$ denote the critical value at the α percent level of significance of the studentized maximum modulus test with K and infinite degrees of freedom, tabulated in Stoline and Ury (1979). We accept the hypothesis of $Y \succ_1 Z$ if

$$\begin{aligned}-T^{DD}(a_i) &> m_{\alpha,K,\infty} \text{ for some } i, \text{ and} \\ T^{DD}(a_i) &< m_{\alpha,K,\infty} \text{ for all } i.\end{aligned}$$

If all $\left|T^{DD}(.)\right|$ are less than the critical value, we accept the null hypothesis of equal distributions. For $K = 10$ and $\alpha = 1, 5, 10, 20$ the critical values are 3.29, 2.80, 2.56 and 2.29. In both Table 3 and Table 4 below, all $T^{DD}(.) < 2.29$, and so we refer to $\left|T^{DD}\right| \equiv \max_i \left|T^{DD}(a_i)\right|$. We also report results from the test proposed by Anderson (1996).

7.3 Results

Table 3 shows the results for the test statistics of Anderson (1996) and Davidson and Duclos (2000). We use 10 grid points so as to divide the age distribution for the full sample into eleven intervals with an approximately equal number of observations (see Tse and Zhang (2000)). In the first three columns of Table 3, we split the sample into two (equally large) subsamples of "small" and "large" markets, and test whether the age distribution in the subsample of smaller markets first-order stochastically dominates that in the subsample of larger markets. For this split, we report the Davidson and Duclos test statistic for each of the 10 grid points, as well as the maximum value of the Anderson test statistic. We also report the maximum value of the Davidson and Duclos test statistic when comparing the age distribution in the smallest third of markets with that in the largest third. This split of the sample should be less sensitive to errors in measuring market size. We refer to the Davidson and Duclos maximum test statistics under the two splits as $\left|T^{DD}_{1/2}\right|$ and $\left|T^{DD}_{1/3}\right|$, and that of Anderson as $\left|T^{A}_{1/2}\right|$. In the last three columns of the table, we conduct analogous splits for our variable $RENT$. In Table 3, large negative values support our predictions.

Before examining the statistical results it is useful to first look at the raw data. Figure 3 shows the cumulative frequencies of AGE in small and large markets, conditional on $MSIZE$ <75,000 (this corresponds to the second column in Table 3). Over virtually the entire range of firm ages, the cumulative frequency in the set of large markets is above that in the set of small markets. This is in line with our prediction that firms in small markets should tend to be older than those in larger markets. Figure 4 gives the corresponding picture for the distribution of AGE in markets with low and high $RENT$, again conditional on $MSIZE$ <75,000. Here, the youngest firms tend to be in the set of markets with high rents, which is what we expect from our theory. However, the gap between the two curves narrows with increasing age, and the two curves intersect to the left of AGE =20; at higher age, the gap between the curves remains small.

In (3:1), we confine attention to markets (postal areas) with a population below 25,000. We find that $\left|T^{DD}_{1/2}\right| = 2.52 > 2.29 = m_{20,10,\infty}$ and $\left|T^{A}_{1/2}\right| = 2.67 > 2.56 = m_{10,10,\infty}$. The FOSD-test becomes significant at the 1%-level when using the 1/3-split, $\left|T^{DD}_{1/3}\right| = 3.79 > m_{1,10,\infty}$. As noted above, the result from the 1/3-split should be more reliable as it is less sensitive to measurement problems in $MSIZE$. This suggests that firms in larger markets tend to be younger (in the sense of FOSD).

In (3:2), we include all markets with up to 75,000 inhabitants. The result are now statistically stronger: all test statistics show significance at the 1%-level. However, including the observations from the 11 largest towns in Sweden, as in (3:3), the 1/2-split is only border-line significant at the 20%-level although $\left|T^{DD}_{1/3}\right|$ remains highly significant. One explanation for this is that the very largest towns contain a number of small submarkets, where competition is less intense (and hence firms are older) than in the city center.

The last three columns of Table 3 are concerned with the effects of $RENT$ on the age distribution of firms. In (3:4), where we restrict the sample to markets with less than 25,000 inhabitants, $\left|T^{DD}_{1/2}\right| = 2.88 > m_{5,10,\infty}$. This indicates that firms tend to be younger (in the the sense of FOSD) in markets with high fixed costs, as predicted by our theory. In (3:5), the test statistics for the sample of markets with less than 75,000 inhabitants are all significant at the 1%-level.[28] In (3:6), the same is true for the 1/2-split even when including the very largest markets. The surprisingly significant test statistics in the full sample may be due to the fact that our variable $RENT$ captures differences in rents across postal codes within the same postal area; in contrast, we assign the same value of $MSIZE$ to all firms, even in large towns (which may explain the less significant results for $MSIZE$ in the full sample).

It is encouraging that the results in Table 3 conform with our predictions. The caveat is, however, that $MSIZE$ and $RENT$ are highly collinear, and so it may not be too surprising to find similar results in the comparisons. To address this issue, we need to separate the two effects so as to verify whether both are significant.

Using the medians of $MSIZE$ and $RENT$ in markets with less than 75,000 inhabitants (which correspond to values of 15,889 and 2,509, respectively), we split the sample into four subsamples, which we call Q1 to Q4. Our theory makes predictions for five bilateral comparisons of age distributions across these four subsamples. For instance, conditional on being in a small market, the age distribution of hair salons in a low rent area should first-order stochastically dominate that of salons in high rent areas. For each of the four subsamples, Table 4 shows the cumulative age distributions at the ten grid points. In the table, we also report the test statistics $\left|T^{DD}_{1/2}\right|$ and $\left|T^{A}_{1/2}\right|$ for the five bilateral comparisons.

Examining the test statistic for each of the five bilateral comparisons, it is striking that all are above the critical value at the 20%-level, and most are above that of the 5%-level. The most straightforward comparison regards Q1 vs. Q4 (small markets with low rents vs. large markets with high rents), where the test statistics show significance at the 1%-level. Interestingly, the test statistics for those bilateral comparisons where $MSIZE$ is varied and $RENT$ is kept constant (i.e., Q1 vs. Q3 and Q2 vs. Q4),

[28]As seen in Figure 4, the two curves intersect to the left of AGE =20. The grid points we use are AGE =20 (in 3:5, $T^{DD}_{1/2}$ =0.595) and AGE =35 (in 3:5, $T^{DD}_{1/2} = -0.607$), and so one may ask whether our results are affected by the choice of grid points. Using AGE =28 rather than AGE =20 gives the highest value of $T^{DD}_{1/2}$, but it is only 0.874<2.29= $m_{20,10,\infty}$. Thus, the conclusion is robust to the choice of grid points.

tend to be more significant than those where $RENT$ is varied and $MSIZE$ kept constant (i.e., Q1 vs. Q2 and Q3 vs. Q4).

In the first three columns in Table 5, we seek to identify the effects by least squares regressions with $\ln(AGE)$ as dependent variable (using the same cut-off levels for $MSIZE$ as before to define the subsamples). The coefficients on $\ln(MSIZE)$ and $\ln(RENT)$ are always negative, as expected, but only the former is ever individually significant. Again, this is caused by the collinearity between the two, as evidenced by P-values below 0.02 for the test of the joint restriction that both coefficients are zero. The low explanatory power (adjusted R-squared around 0.01) shows that little of the variation of $\ln(AGE)$ can be attributed to differences in market size and fixed costs, which is not surprising in markets with high entry rates.

So far, we have implicitly assumed that markets differ only in terms of size and fixed costs. We acknowledge that conditions may differ also in other dimensions that could influence the age distribution. First, there are differences in demand growth, measured here by the growth rate in municipal population and average per capita income over the period 1990-2000, $POPGROWTH$ and $INCGROWTH$.[29] Second, there is more mobility of population in some areas which might give rise to more frequent shocks (c.f. the α parameter) either due to greater propensity of employees to move or change employer, or that customer relations are less stable. Our control variable, $MIGRATION$, is the ratio of the sum of inward and outward migration over the period 1990-2000 to the municipal population in 1995. Further, the age composition of the population across markets may differ, something we capture with the fraction of the municipal population in 2000 that is aged 25 to 44, $YOUNGPOP$. One reason for including $YOUNGPOP$ is to test the alternative explanation that large markets tend to have relatively more young people and the age of a hair salon is correlated with the age of its owner (which in turn is correlated with the age of the population). In the last three columns of Table 3 we report regressions with these as control variables.

Most importantly, the coefficients of key variables $\ln(MSIZE)$ and $\ln(RENT)$ are only affected to a limited extent by the introduction of the control variables and their joint significance remains high. This addresses the serious concern that the market size effect on the age distribution is driven by large markets being systematically different in several dimensions. In other words, our results are not driven by large markets being those that have grown the fastest, have the most mobility, and have the youngest population.[30] The control variables are generally statistically insignificant ($INCGROWTH$ is significant at the 10%-level in column 4 but the coefficient changes considerably across subsamples). The joint restriction that all control variables are zero can *not* be rejected. Overall, the lack of explanatory power of the control variables in the regressions strengthens our claim that the age distribution is driven largely by market size and fixed costs, as predicted by our theory.

Tables 4 and 5 indicate that, while both market size and fixed costs shape the age distribution of firms as predicted by our theory, the effect of market size seems statistically stronger. One potential explanation for this is that our variable $RENT$ is not only correlated with fixed costs, but also with other factors that impact the age distribution. In particular, it seems plausible that $RENT$ is positively correlated with the level of sunk entry costs: upon entry, a salon is required to sign a lease contract for a minimum duration.[31] Furthermore, a new firm may need a certain time in the market to reach customers and to make an informed decision on exit. With the data at hand, we are not able to quantify the effects of $RENT$ on fixed costs and entry costs.[32]

[29] See section 5 for an examination of the effects of demand growth on the turnover rate.

[30] In our data, $MSIZE$ is postively correlated with $POPGROWTH$ ($\rho = 0.56$), $MIGRATION$ ($\rho = 0.29$), and $YOUNGPOP$ ($\rho = 0.67$). $MSIZE$ is virtually unrelated to $INCGROWTH$ ($\rho = 0.04$).

[31] Other costs for a hair salon do not appear to vary much across country. According to the interest organization Sveriges Frisörföretagare, wages show very little dispersion across regions. Equipment and material are typically bought from a few national distributors.

[32] Note that our theory predicts that both an increase in entry costs and an increase in fixed costs reduce the number

Summing up, our study of the age distribution of hair salons in different geographically defined markets supports the predictions of our theory: firms operating in larger markets, and those in locations where rents (being a proxy for fixed costs) are higher, tend to be younger (in the sense of FOSD).

Although our empirical focus has been on the determinants of firm turnover, we would like to point out that the model contains many testable predictions on the distribution of firm efficiencies. Indeed, Syverson (2002) explores variations in within-industry productivity for a sample of four-digit industries using a framework that builds on ours.[33]

8 Conclusion

Many empirical studies in industrial organization and labor economics have shown that industries differ substantially in the level of firm turnover and gross job reallocation. These differences are stable over time and similar across countries. This suggests that there are some systematic factors that determine the magnitude of reallocation of inputs and outputs within industries. This paper is concerned with an examination of the role played by industry characteristics such as entry costs, fixed production costs, and market size on market turbulence.

To this end, we have analyzed a stochastic dynamic model of an imperfectly competitive industry. Firms are heterogeneous and subject to idiosyncratic shocks to their "efficiencies". In our formulation, a firm's efficiency can be interpreted either as its productivity or as the perceived quality of its product.

Even in a stationary environment, the equilibrium exhibits simultaneous entry and exit: currently efficient firms survive while firms with sufficiently bad cost draws exit. Our analysis shows that the replacement of inefficient firms ("churning") is more rapid in markets with low entry costs and high fixed costs. The most important and novel prediction of our theory, however, is that the rate of firm turnover is positively related to the size of the market.

The paper provides a number of results in addition to those on market turbulence. Most importantly, if the idiosyncratic shocks affect firms' productivities, then the stationary equilibrium will exhibit a nondegenerate distribution of efficiency levels. That firms within an industry display considerable heterogeneity is empirically well-documented. Our model makes a number of testable predictions on the distribution of productivities. In particular, the model predicts that in larger markets firms tend to be more efficient. It has often been informally argued that "more intense product-market competition fosters efficiency". In our model, the *induced* intensity of price competition may differ across markets in that the endogenous distribution of firm types μ may vary with changes in entry costs, fixed costs, or market size. However, if one were to correlate, in a cross section of markets, price-cost margins with the average efficiency of active firms, one may find a negative or positive correlation: if markets differed mainly in their fixed costs, then one would find that firms are more efficient in markets with *higher* price-costs margins, while the reverse would hold if markets differed mainly in their size (or entry costs).

To see the broader implications of our result on market size and firm turnover, note that an increase in market size may be interpreted as the opening of industries to trade, e.g., as a move from two closed economies to a fully integrated economy. In a two-country model related to ours, Melitz (1999) analyzes the effect of trade costs on the distribution of efficiency levels. In his model, a move from infinite trade costs to zero trade costs has no effect on average efficiency and turnover levels. Our model, in contrast, predicts that average firm efficiency and turnover levels should rise as a result of economic integration.

of firms. Thus, the finding in Table 2 that the number of firms is negatively related to $RENTMEAN$ is consistent with both effects.

[33] Syverson (2000) conducts a detailed examination of the efficiency distribution of plants producing ready-made concrete in the U.S., and relates the differences to variations in local demand density.

In the empirical part of the paper, we test some of the model's predictions. The idea is to examine the age distribution of firms that compete in the same sector but in different geographical markets. This should avoid many of the measurement problems associated with cross-industry studies. To this end, we have collected data on hair salons in Sweden. We use the population in postal areas to capture differences in market size, and land values to proxy for differences in fixed costs (primarily rents). The empirical results are in line with the predictions. Hair salons tend to be younger in larger markets and in markets with higher fixed costs.

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Appendix A: Proofs

Properties of the Profit Function in a Cournot Model. We want to show that assumptions A.1 and A.2 hold in a homogenous goods Cournot model, where firms differ in their (constant) marginal costs. Let $P(Q/S)$ denote inverse demand when aggregate output is Q and market size is S. We assume that the demand function is downward-sloping, i.e. $P'(\cdot) < 0$. In equilibrium, aggregate output Q will be some (possibly complicated) function of the vector of firms' marginal costs, i.e. $Q = Sf(\mathbf{c})$, where $\mathbf{c}$ is the vector of firms' marginal costs. It is therefore convenient to consider changes in aggregate output which reflect changes in the underlying distribution of firms' efficiencies. Conditional on aggregate output Q, the equilibrium output of a firm with constant marginal cost c is denoted by $Sq(c; Q)$. (The function $Sq(c; \cdot)$ is sometimes called the backward-reaction function.) The first-order condition for profit maximization is given by

$$P\left(Q/S\right) - c + q(c;Q)P'(Q/S) = 0, \tag{15}$$

which implies

$$Sq(c;Q) = -S\frac{P(Q/S) - c}{P'(Q/S)}.$$

The associated second-order condition is given by

$$2P'(Q/S) + q(c;Q)P''(Q/S) < 0. \tag{16}$$

It is straightforward to show that (15) and (16) imply that a firm's equilibrium equilibrium profit $S\pi(c; Q)$ is strictly decreasing in industry output Q. Any change in the underlying distribution of efficiencies which reduces firms' profits must induce an increase in industry output Q. Hence, an increase in Q is equivalent to an increase in the distribution of firms as defined in the main text. Moreover, the assumption of complete ordering of distributions (ORD) is satisfied. Conditional on industry output Q, the equilibrium profit of a type-c firm can be written as

$$S\pi(c;Q) = -S\frac{\left(P(Q/S) - c\right)^2}{P'(Q/S)}.$$

We now consider an increase in the distribution of active firms: suppose aggregate output increases from Q to Q', $Q' > Q$. Assumption A.2 says that the profit ratio $\pi(c; Q')/\pi(c; Q)$ is decreasing in c for all c such that $q(c; Q) > 0$. It is immediate to see that this condition holds if and only if $P(Q'/S) < P(Q/S)$, which is clearly satisfied since the inverse demand function is downward-sloping. Assumption A.1 requires that the profit difference $\pi(c; Q') - \pi(c; Q)$ is increasing in c for all c such that $q(c; Q) > 0$. Taking the derivative with respect to c, we obtain that A.1 holds if and only if $q(c; Q') < q(c; Q)$, i.e. a firm's equilibrium output is decreasing in the distribution of active firms. This inequality is satisfied if

$$P'(Q/S) + q(c;Q)P''(Q/S) < 0,$$

which is equivalent to the assumption that quantities are strategic substitutes (firms' reaction curves are downward-sloping). It is a rather weak (and standard) assumption in Cournot models. Hence, in a Cournot model with homogenous products and constant marginal costs, A.1 and A.2 hold under fairly general conditions on demand.[34]

Proof of Proposition 2. The proof proceeds in several steps.

Step one. Consider any positive measure μ' in $\mathcal{M}^*$ such that $\overline{V}^e(1;\mu') = 0$. Conditions (CON) and (FREE) ensure that μ' exists. Since $\overline{V}^e(\cdot;\mu')$ is single-peaked, there are two possibilities.

[34] It is easy to check that assumptions (MON), (DOM), (ORD), and (CON) are satisfied as well. Part (i) of (FREE) holds if $\lim_{Q\to\infty} \left[P(Q)\right]^2/P'(Q) = 0$, while part (ii) simply says that fixed costs and entry costs are sufficiently small so as to make entry of at least one firm profitable.

(i) $\overline{V}^e(c;\mu') < 0$ for all $c \in [0,1)$,

(ii) $\overline{V}^e(c;\mu') > 0$ for some $c \in (0,1)$.

Step two. In case (i), there does not exist a stationary equilibrium with simultaneous entry and exit. To see this, suppose otherwise that there exists a stationary equilibrium with simultaneous entry and exit. Denote the associated stationary distribution by μ'', and the exit policy by c''. Since $\overline{V}^e(c;\mu)$ is decreasing in μ, condition (E) implies that $\mu'' \prec \mu'$. It follows that $\overline{V}^e(1;\mu'') > \overline{V}^e(c'';\mu'') = 0$. Since $\overline{V}^e(\cdot;\mu'')$ is single-peaked, we thus have $\partial\overline{V}^e(c'';\mu'')/\partial c > 0$, which contradicts condition (X). Although there does not exist a stationary equilibrium with simultaneous entry and exit, there does exist at least one without entry and exit. From (DOM) and (CON), it follows that there exists a positive number λ' such that $\mu_{\lambda'} \sim \mu'$, where $\mu_{\lambda'}$ is the stationary distribution defined by (10). (Indeed, (DOM) implies that for λ sufficiently large, $\mu_\lambda \succ \mu'$, and for λ sufficiently small, $\mu_\lambda \prec \mu'$. (CON) then implies that there exists a λ' such that $\mu_{\lambda'} \sim \mu'$.) It is easy to check that $(\mu_{\lambda'}, 0, 1)$ satisfies conditions (5), (7), and (10). Note that there may exist a multiplicity of stationary equilibria. More precisely, there exists a nonempty interval of λ-values, $[\underline{\lambda}, \overline{\lambda}]$, with $\overline{\lambda} \geq \underline{\lambda}$, such that $(\mu_\lambda, 0, 1)$, $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, forms a stationary equilibrium.

Step three. Consider case (ii), which can only arise if $\phi > \delta(1-\alpha)\epsilon$. We claim that, in this case, there exists a unique stationary equilibrium, which involves simultaneous entry and exit. Existence and uniqueness can be shown as follows. Starting from μ', we increase the measure of active firms: in Figure 1, this shifts the curve $\overline{V}^e$ downwards. (CON), (FREE), and single-peakedness of $\overline{V}^e(\cdot;\mu)$ imply that there exists a measure $\mu'' \succ \mu'$ such that $\overline{V}^e(\cdot;\mu'')$ assumes a unique maximum at some $c^* < 1$ and $\overline{V}^e(c^*;\mu'') = 0$. (It is easy to see that the exit policy c^* is unique.) From assumptions (DOM) and (CON), and condition (D), it follows that there exists a unique M such that $\mu[c^*, M] \sim \mu''$. The unique equilibrium distribution is then given by $\mu \equiv \mu[c^*, M]$.[35] ■

Proof of Proposition 3. Abusing our previous notation, let us denote by $\overline{V}^e(x;\mu;\varepsilon)$ the value of an entrant who uses exit policy x in the period of entry and behaves optimally thereafter, when the value of entry costs is given by ε. Note that $\overline{V}^e(x;\mu;\varepsilon)$ is continuous (and decreasing) in ε. Assumptions (CON) and (FREE) ensure that there exists a positive measure $\widehat{\mu}$ in $\mathcal{M}^*$ such that $\overline{V}^e(1;\widehat{\mu};0) = 0$. (MON) then implies that $S\pi(1;\widehat{\mu}) < \phi < S\pi(0;\widehat{\mu})$. It follows that there exists an exit policy $\widehat{c} < 1$, such that

$$\overline{V}^e(\widehat{c};\widehat{\mu};0) > 0 = \overline{V}^e(1;\widehat{\mu};0).$$

Hence, for ϵ sufficiently small, we have $\overline{V}^e(\widehat{c};\widehat{\mu};\varepsilon) > 0 > \overline{V}^e(1;\widehat{\mu};\varepsilon)$. Then, there exists a measure $\mu' \prec \widehat{\mu}$ such that

$$\overline{V}^e(1;\mu';\varepsilon) = 0 < \overline{V}^e(\widehat{c};\mu';\varepsilon).$$

That is, we are in case (ii) of the proof of Proposition 2. As we have already shown there, this implies that there exists a unique stationary equilibrium which involves simultaneous entry and exit. ■

Proof of Proposition 7. As pointed out in the main text, the future enters the equilibrium conditions only through the constant option value $\delta\epsilon$ in (E_t) and (X_t). Consequently, the change in market size in period r has no effect on endogenous variables in periods before r. Moreover, again from the discussion in the main text, c_t^* and the equivalence class of μ_t are independent of future and past values of market size. The same applies to $M_t + \mu_{t-1}([0,1])$. Using our comparative dynamics result on market size, Proposition 6, we then obtain the following characterization of endogenous variables:

[35] If $\phi = \delta(1-\alpha)\epsilon$ (which occurs with zero probability if the parameters are drawn from some continuous distribution), two cases can arise: either $\overline{c}(\mu') = 1$ or $\overline{c}(\mu') < 1$. The former case is given by (i), i.e., there exists a stationary equilibrium, but it does not exhibit entry and exit. In the latter case, $\overline{V}^e(c;\mu') < 0$ for all $c \in [0, \overline{c}(\mu'))$ and $\overline{V}^e(c;\mu') = 0$ for all $c \in [\overline{c}(\mu'), 1]$. In this case, there exists both a stationary equilibrium without entry and exit ($c^* = 1$) as well as a continuum of equilibria with simultaneous entry and exit, where $c^* \in [\overline{c}(\mu'), 1)$ and $\mu[c^*, M] \sim \mu'$.

$c_t^* = c_t^{*\prime}$ for all $t \neq r$, and $c_r^* > c_r^{*\prime}$, $\mu_t = \mu_t'$ for all $t \neq r$, and $\mu_r \prec \mu_r'$, $\eta_t = \eta_t'$ for all $t \leq r-1$ and $t \geq r+2$, $M_r < M_r'$, and $M_{r+1} > M_{r+1}'$ if and only if $\mu_r([0,1]) < \mu_r'([0,1])$. The prediction on the evolution of exit rates follows immediately. Let us now show that $\eta_r < \eta_r'$. Condition (D_t) and $\mu_{r-1} = \mu_{r-1}'$ imply

$$\frac{[M_r' + \mu_{r-1}([0,1])]\, G(c_r^{*\prime})}{\mu_r'([0,1])} = 1 = \frac{[M_r + \mu_{r-1}([0,1])]\, G(c_r^*)}{\mu_r([0,1])}.$$

Since $M_r < M_r'$, we then obtain

$$\frac{M_r' G(c_r^{*\prime})}{\mu_r'([0,1])} > \frac{M_r G(c_r^*)}{\mu_r([0,1])},$$

and hence

$$\frac{M_r'}{\mu_r'([0,1])} > \frac{M_r}{\mu_r([0,1])},$$

which is the desired result.[36] The remaining characterization of the evolution of entry rates follows from the discussion in the main text. ■

Proof of Proposition 8. Consider two exit policies, c' and c'', where $c'' > c'$. The regularity condition on the Markov process can then be written as

$$\frac{\Delta\sigma_t(c')}{\sigma_{t-1}(c')} > \frac{\Delta\sigma_t(c'')}{\sigma_{t-1}(c'')}, \tag{17}$$

where $\Delta\sigma_t(c^*) \equiv \sigma_{t-1}(c^*) - \sigma_t(c^*)$, and $\sigma_0(c^*) \equiv 1$. The first step in the proof consists in showing that (17) implies that $\sigma_t(c')/\sigma_t(c'')$ is decreasing in t. Using the identity

$$\frac{\sigma_t(c')}{\sigma_t(c'')} \equiv \frac{\sigma_{t-1}(c') - \Delta\sigma_t(c')}{\sigma_{t-1}(c'') - \Delta\sigma_t(c'')},$$

it is straightforward to see that

$$\frac{\sigma_t(c')}{\sigma_t(c'')} < \frac{\sigma_{t-1}(c')}{\sigma_{t-1}(c'')}$$

whenever (17) holds. Let

$$\psi_t(c^*) \equiv \frac{\sigma_t(c^*)}{\sum_{s=0}^{\infty} \sigma_s(c^*)},$$

and note that $\sum_{t=0}^{\infty} \psi_t(c^*) = 1$. The second step in the proof consists in showing that there exists a $\widehat{t} \geq 1$ such that $\psi_t(c') > \psi_t(c'')$ for all $t < \widehat{t}$, and $\psi_t(c') < \psi_t(c'')$ for all $t > \widehat{t}$. To see this, note first that $\psi_0(c') > \psi_0(c'')$ since $\sigma_0(c^*) \equiv 1$ and $d\sigma_s(c^*)/dc^* > 0$ for all $s \geq 1$, and hence

$$\frac{\sigma_0(c')}{\sigma_0(c'')} = 1 > \frac{\sum_{s=0}^{\infty} \sigma_s(c')}{\sum_{s=0}^{\infty} \sigma_s(c'')}.$$

Next, since $\sum_{t=0}^{\infty} \psi_t(c') = 1 = \sum_{t=0}^{\infty} \psi_t(c'')$, there exists some s such that $\psi_s(c') < \psi_s(c'')$. Finally, since $\sigma_t(c')/\sigma_t(c'')$ is decreasing in t (as shown in the first step), there exists a $\widehat{t} \geq 1$ such that

$$\frac{\sigma_t(c')}{\sigma_t(c'')} > (<) \frac{\sum_{s=0}^{\infty} \sigma_s(c')}{\sum_{s=0}^{\infty} \sigma_s(c'')} \text{ if } t < (>) \widehat{t}.$$

[36] Observe that the first inequality implies that the result also holds when using the alternative entry measure $\widehat{\eta}_t = G(c^*)\eta_t$.

This proves the assertion. The third and final step in the proof consists in showing that $A(a|c') > A(a|c'')$ for all $a \geq 1$. Note that $A(a|c^*) = \sum_{s=0}^{a-1} \psi_s(c^*)$ and $\lim_{a\to\infty} A(a|c^*) = 1$. From the second step, $A(1|c') > A(1|c'')$. Moreover, $A(a|c') - A(a|c'')$ is increasing in a for all $a < \widehat{t} + 1$, and decreasing for all $a > \widehat{t} + 1$; i.e., $A(a|c') - A(a|c'')$ is single-peaked in a. Since $\lim_{a\to\infty} \{A(a|c') - A(a|c'')\} = 0$, it follows that $A(a|c') - A(a|c'') > 0$ for all $x \geq 1$. ■

Appendix B: Figures and Tables

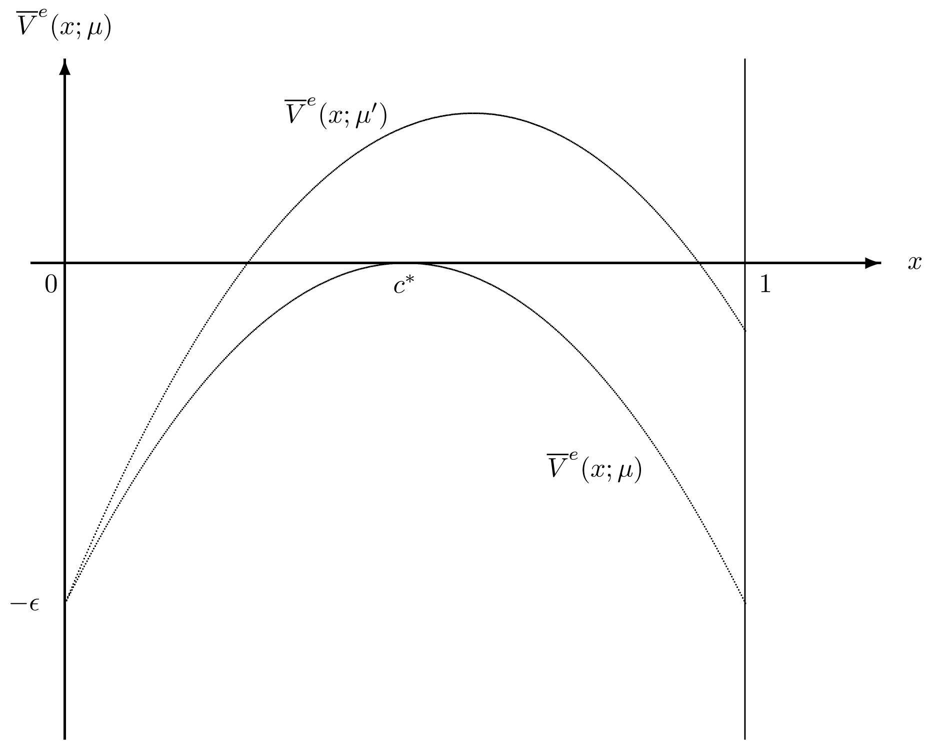

Figure 1: The effect of a decrease in the distribution of firms ($\mu' \prec \mu$) on the value of an entrant with exit policy x.

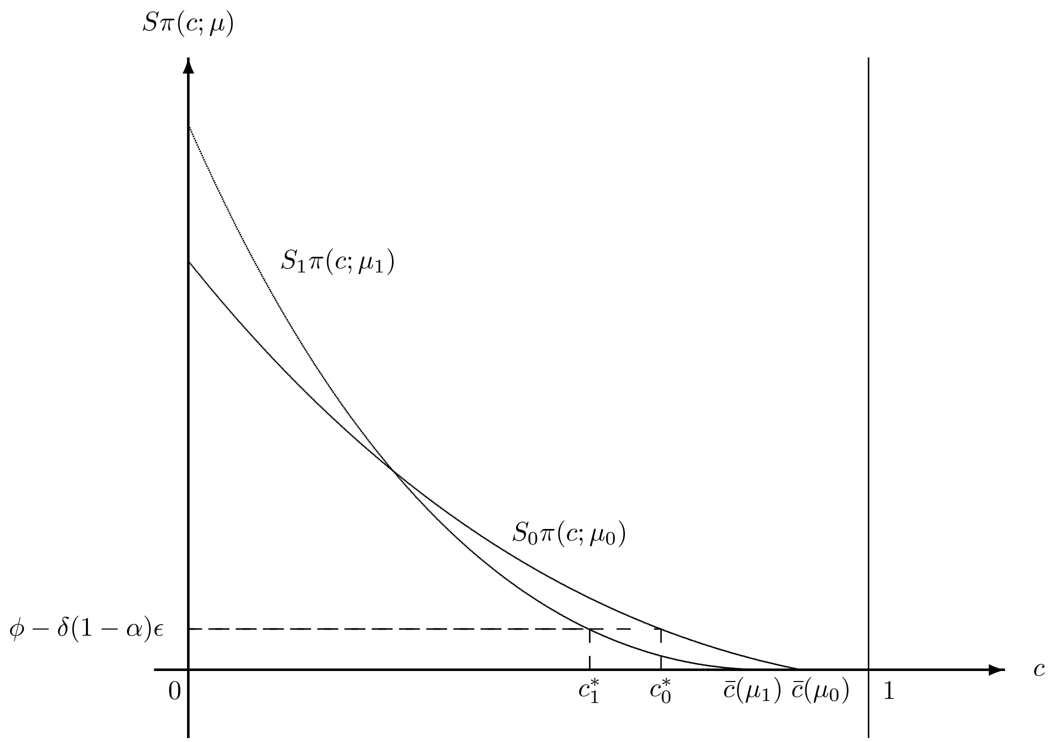

Figure 2: The effect of an increase of market size on gross profits: $S_1 > S_0$, and hence $\mu_1 \succ \mu_0$.

Table 1: Descriptive statistics.

	Mean	St.Dev	Min	10th	25th	50th	75th	90th	Max	Nobs
AGE	10.89	9.27	0.01	2	4	9	15	25	58	1030
$MSIZE$	72014	94541	355	2865	8271	27763	82788	277522	285981	1030
$FIRMS$	103	174	1	2	7	24	77	459	577	1030
$RENT$	3591	2649	209	1339	1942	2874	4370	6611	25580	1003
$RENTMEAN$	4338	3790	559	1547	2112	3217	4810	7608	16113	1030
$POPGROWTH$	0.031	0.082	-0.138	-0.068	-0.022	0.043	0.092	0.101	1.082	1030
$INCGROWTH$	0.336	0.041	0.255	0.288	0.311	0.328	0.357	0.393	0.555	1030
$MIGRATION$	0.927	0.262	0.287	0.648	0.716	0.926	1.069	1.175	2.116	1030
$YOUNGPOP$	0.277	0.039	0.198	0.232	0.244	0.269	0.301	0.349	0.380	1030

Table 2: Tobit regressions. The number of firms related to market size and fixed costs.

	$FIRMS$	$FIRMS$	$FIRMS$
$MSIZE$	1.2032***	1.1838***	0.5978***
	[0.0540]	[0.0380]	[0.0183]
$MSIZE * MSIZE$	-0.0107***	0.0001	0.0023***
	[0.0025]	[0.0007]	[0.0001]
$MSIZE * RENTMEAN$	-0.0274***	-0.0652***	0.0440***
	[0.0104]	[0.0063]	[0.0025]
Constant	-2.4489***	-3.0773***	-2.5400***
	[0.1463]	[0.1791]	[0.1998]
Sample	$MSIZE$ <25	$MSIZE$ <75	Full
N	1465	1523	1534
logL	2334.5	2890.3	3237.5
RlogL	1548.9	2067.5	3095.7
P-value	0.0000	0.0000	0.0000

Standard errors in brackets.
* significant at 10%; ** significant at 5%; *** significant at 1%.

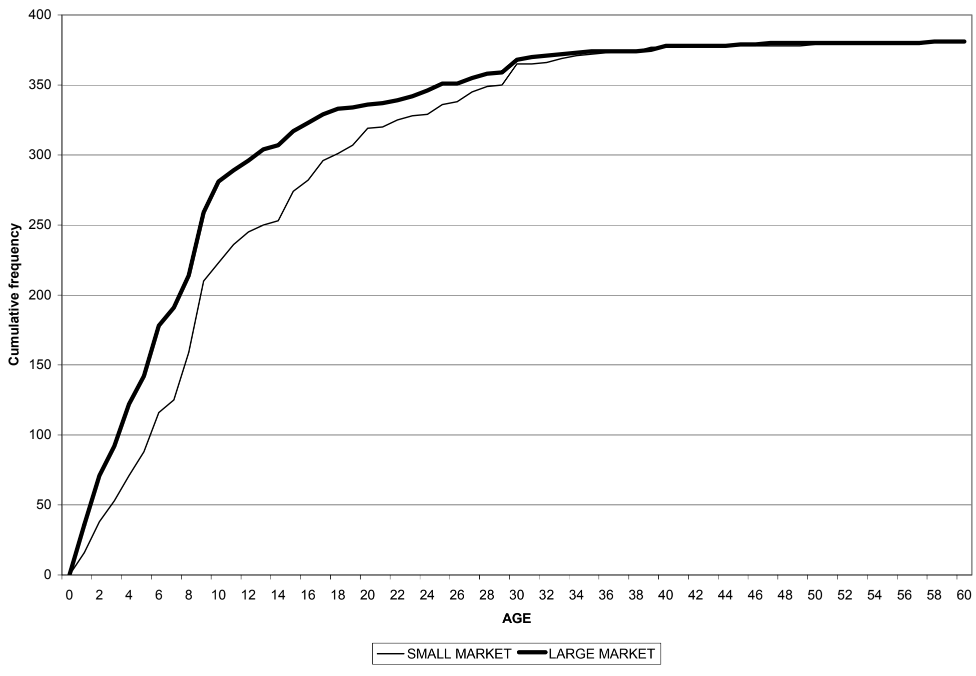

Figure 3: Cumulative age distributions in small and large markets.

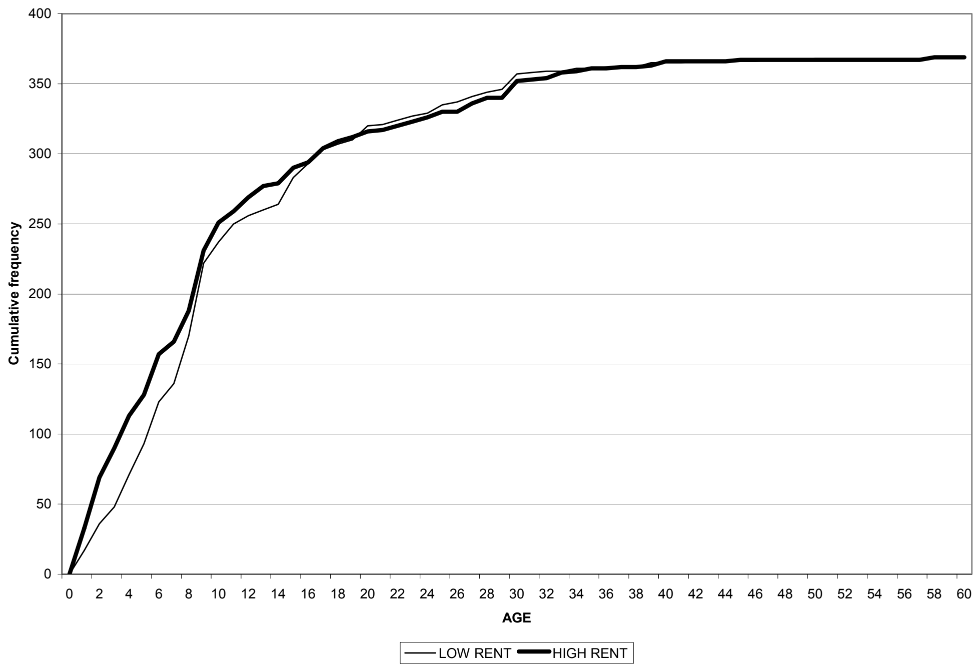

Figure 4: Cumulative age distributions in markets with low and high rents.

Table 3: Tests of first order stochastic dominance.

Compare	$MSIZE$	$MSIZE$	$MSIZE$	$RENT$	$RENT$	$RENT$
AGE	TDD	TDD	TDD	TDD	TDD	TDD
1	-2.105	-2.889	-0.246	-2.567	-2.352	-2.585
2	-2.410	-3.441	-0.349	-2.295	-3.506	**-3.327**
3	-2.500	-3.545	0.015	**-2.880**	**-4.008**	-2.975
5	-2.103	-4.237	-1.068	-1.983	-2.828	-3.309
7	-2.022	**-4.855**	-2.196	-1.516	-2.254	-2.485
9	**-2.524**	-3.681	-1.570	-1.048	-0.681	-1.131
12	-2.251	-4.117	-1.946	-0.977	-1.057	-0.924
15	-1.475	-3.768	**-2.275**	-0.186	-0.619	-1.157
20	-0.146	-1.776	-1.010	0.595	0.427	0.263
35	0.578	-0.505	-0.867	-0.607	0.000	-0.867
Max TA	-2.671	-4.828	-2.307	-2.868	-3.955	-3.317
Max TDD (1/3-split)	-3.769	-3.745	-3.269	-2.951	-3.483	-3.167
Sample	$MSIZE$ <25	$MSIZE$ <75	Full	$MSIZE$ <25	$MSIZE$ <75	Full
N	494	762	1030	475	738	1003

Critical values at 1, 5, 10, and 20 percent levels: 3.29, 2.80, 2.56, and 2.29.

Table 4: Tests of first order stochastic dominance

	(Q1)	(Q2)	(Q3)	(Q4)
MSIZE	SMALL	SMALL	LARGE	LARGE
RENT	LOW	HIGH	LOW	HIGH
AGE	Cum.Dens.	Cum.Dens.	Cum.Dens.	Cum.Dens.
1	0.02	0.09	0.10	0.09
2	0.08	0.15	0.13	0.20
3	0.12	0.19	0.15	0.27
5	0.23	0.27	0.31	0.38
7	0.33	0.34	0.46	0.50
9	0.57	0.50	0.67	0.68
12	0.66	0.61	0.77	0.78
15	0.74	0.67	0.82	0.84
20	0.85	0.81	0.90	0.88
35	0.97	0.97	0.99	0.98
N	257	113	112	256
FOSD test	Max TDD	Max TA	at *AGE*	
Q1 v Q2	-2.88	-2.30	1	
Q1 v Q3	-3.19	-2.52	1	
Q1 v Q4	-4.13	-4.23	3	
Q2 v Q4	-3.52	-3.28	15	
Q3 v Q4	-2.31	-2.60	3	

MSIZE SMALL: *MSIZE* < 15.889. *RENT* LOW: *RENT* < 2.509.
Critical values at 1, 5, 10, and 20 percent levels: 3.29, 2.80, 2.56, and 2.29.

Table 5: Least squares regressions. Age of firm related to market size and fixed costs.

	$lnAGE$	$lnAGE$	$lnAGE$	$lnAGE$	$lnAGE$	$lnAGE$
	(5:1)	(5:2)	(5:3)	(5:4)	(5:5)	(5:6)
$lnMSIZE$	-0.141**	-0.125***	-0.036	-0.144**	-0.137***	-0.073*
	[0.059]	[0.040]	[0.030]	[0.063]	[0.045]	[0.040]
$lnRENT$	-0.031	-0.072	-0.085	-0.149	-0.110	-0.088
	[0.116]	[0.093]	[0.078]	[0.130]	[0.105]	[0.090]
$POPGROWTH$				0.599	0.352	0.701
				[1.052]	[0.826]	[0.746]
$INCGROWTH$				3.079*	-0.533	-1.714
				[1.766]	[1.430]	[1.099]
$MIGRATION$				0.139	0.089	0.061
				[0.299]	[0.208]	[0.194]
$YOUNGPOP$				-0.108	0.835	1.105
				[2.956]	[2.335]	[2.128]
Constant	2.344***	2.342***	2.169***	1.305	2.281***	2.485***
	[0.105]	[0.088]	[0.072]	[0.940]	[0.669]	[0.492]
Sample	$MSIZE$ <25	$MSIZE$ <75	Full	$MSIZE$ <25	$MSIZE$ <75	Full
Nobs	476	738	1003	476	738	1003
Adj.R2	0.014	0.021	0.006	0.021	0.019	0.007
Test 1	0.018	0.000	0.008	0.008	0.000	0.016
Test 2				0.103	0.761	0.329

Robust standard errors in brackets

* significant at 10%; ** significant at 5%; *** significant at 1

Test 1 is the P-value of the restriction $lnMSIZE= lnRENT= 0$

Test 2 is the P-value of the restriction $POPGROWTH=INCGROWTH=MIGRATION=YOUNGPOP= 0$