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First-order phase transition of a vacuum and the expansion of the Universe

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Summary. The progress of a first-order phase transition of a vacuum in the expanding Universe is investigated. The expansion of bubbles of a stable vacuum is calculated simultaneously with the cosmic expansion with the aid of the following two simplified nucleation rates of bubbles p: (i) $p = p_T T_c \delta(T - T_c)$ in the hot Universe models, (ii) $p = 0$ for $n > n_c$ and $p = p_Q$ for $n < n_c$ in the cold Universe models, where T is the cosmic temperature, T_c the critical temperature, n the cosmic number density of the fermions coupled to the order parameter of the vacuum, n_c the critical density, and p_T and p_Q are parameters.

The following results are obtained: (1) If the nucleation rates are small and the vacuum stays at the metastable state for a long time, the Universe begins to expand exponentially. As a result, the progress of the phase transition is delayed more and more by the rapid cosmic expansion. In particular, in model (i), if p_T is less than a critical value, the phase transition never finishes. (2) The lower limits of the nucleation parameter p_T and p_Q are obtained from observation of the number ratio of photons to baryons in the present Universe. (3) If the phase transition of the vacuum in SU(5) GUT is of first order or if there exists a hypothetical first-order phase transition of the vacuum in the very early stage in which baryon number is not conserved, the density and the velocity fluctuations created by the phase transition may account for the origin of galaxies.

1 Introduction

Currently, gauge theories with spontaneous symmetry breaking are of great interest. These gauge theories not only suggest the possibility of the unification of the basic four interactions between particles, but also imply a cosmological evolution of the interactions; namely, that there may have been solely a unitary interaction between particles when the Universe was created. This unitary interaction, however, may have shot branches by the successive cosmological phase transitions of vacua with the expansion of the Universe, and thus the present

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four basic interactions may have been evolved in a similar way to the evolution of creatures (see, e.g. Weinberg 1979).

It is, however, very natural to question whether these phase transitions of vacua necessarily affected the cosmic expansion and/or the thermal history of the Universe. If we take a simple ϕ^4 model for a model of phase transitions, the energy density of the vacuum $V(\phi)$ is given by

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \text{constant}, \tag{1.1}$$

where ϕ is a Higgs field or an order parameter of the vacuum. The energy density difference ρ_V between two vacua, $V(0)$ and $V(\mu/\sqrt{\lambda})$ is $\rho_V = \mu^4/4\lambda$. If we stand by the principle that the symmetric vacuum is the true vacuum and the energy density of the vacuum $V(0)$ should be zero, the present stable vacuum with $\phi = \mu/\sqrt{\lambda}$ has the negative energy density, $V(\mu/\sqrt{\lambda}) = -\rho_V$. The value of $V(\mu/\sqrt{\lambda})$, however, would be extremely small from the upper limit of the cosmological constant $|\Lambda| < 10^{-56}\,\text{cm}^{-2}$, because the cosmological constant is given as $\Lambda = 8\pi G V(\mu/\sqrt{\lambda})$ as discussed by Dreitlein (1974). In order that $8\pi G V(\mu/\sqrt{\lambda}) < 10^{-56}\,\text{cm}^{-2}$, the Higgs meson mass $m_\phi (\equiv \sqrt{2}\,\mu)$ should be smaller than $10^{-16}\,\text{eV}$ in the Weinberg–Salam model, because $\mu^2/\lambda = (\sqrt{2}\,G_F)^{-1}$ in this theory, where G_F is the Fermi coupling constant of the weak interaction. Such small masses are ruled out not only by some astrophysical observations (Sato & Sato 1975) but also by the radiative correction to the vacuum energy density (Linde 1976a; Weinberg 1976). It is reasonable to assume that the present vacuum energy density $V(\mu/\sqrt{\lambda}) = 0$ instead of $V(0) = 0$. Linde (1974) and Bludman & Ruderman (1977) investigated whether the energy density of vacuum $V(0) = \rho_V$ in the early stage of the Universe would affect the expansion of the Universe before phase transition. They showed that the energy density is always less than the radiation energy density T^4 before the phase transition ($T \geqslant T_c$), because the critical temperature T_c is an order of $\mu/\sqrt{\lambda}$. Furthermore, the ratio ρ_V/T_c^4 is an order of λ, which is smaller than unity in conventional gauge models. (See also the estimate of ρ_V/T_c^4 in the Higgs model given by Linde 1979; Kolb & Wolfram 1979.) This means that the second-order phase transition described by a simple ϕ^4 model would have practically no effect on cosmic expansion.

On the other hand, Linde (1976a) and Weinberg (1976) calculated the radiative correction to the classical potential of the vacuum energy density (equation 1.1) for the Higgs model and obtained the effective potential. They showed that the zero temperature effective potential has an additional local minimum at $\phi = 0$, if λ is smaller than a critical value. In this case, the vacuum stays at the symmetric state $\phi = 0$ for a while, even if the cosmic temperature decreases to lower than the critical temperature (see Fig. 1). Kirzhnits & Linde (1976) and Linde (1979) discussed this first-order phase transition in detail. This false vacuum, $\phi = 0$, decays into the stable vacuum ϕ_0 by quantum fluctuation (Coleman 1977) or by thermal fluctuation (Linde 1977). Obviously, if the time-scale of this first-order phase transition is much longer than that of cosmic expansion, the energy density of the vacuum ρ_V becomes dominant and changes the expansion law of the Universe, since the radiation energy density decreases with cosmic expansion, but ρ_V is a constant.

Recently Lasher (1979) discussed the progress of the quark-nucleon phase transition in a cold Universe assuming it to be of first order. He calculated the expansions of bubbles of nucleon phase (a stable vacuum) created in quark phase matter (a false vacuum), but assumed that the cosmic expansion law was not changed.

The purpose of the present paper is to investigate generally, without using special gauge models, how the first order phase transitions proceed in the expanding Universe. In Section 2, basic equations which describe the progress of the phase transition in the expanding

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Universe are shown. In Section 3, nucleations of bubbles of a stable vacuum are discussed and it is demonstrated with the aid of simplified nucleation rate models that the progress of the phase transition and the expansion of the Universe are closely coupled. In Section 4, applications to some special gauge models are discussed.

2 Expansion of the Universe with a first order phase transition of a vacuum

2.1 ASSUMPTIONS AND BASIC EQUATIONS

First, we assume the primordial Universe before the phase transition is filled with radiation (in the hot Universe model) or with relativistic degenerate fermions (in the cold Universe model), and we define ρ_r as the energy density. Second, we assume the energy density of the vacuum before the phase transition $V(0) = \rho_v$ and the energy density of the present stable vacuum $V(\phi_0) = 0$, following Linde (1974) and Bludman & Ruderman (1977) (see Fig. 1).

When the temperature of the Universe or the number density of fermions coupled to the Higgs field ϕ becomes less than the critical temperature T_c or density n_c by the expansion of the Universe, the symmetric vacuum $\phi = 0$ becomes unstable and nucleations of bubbles of a stable vacuum begin. This first-order phase transition finishes when all the space is covered by bubbles. Let r be the volume ratio of the stable vacuum $\phi = \phi_0$ to the metastable vacuum $\phi = 0$ and ϵ be the energy density difference between two vacua at the cosmic time t. Then time evolution of a cosmic scale factor of a Universe with a homogeneous Minkowski space R is described by the following equation

$$\dot{R}^2/R^2 = 8\pi G(\rho_r + \rho_v - \epsilon r)/3. \tag{2.1}$$

The speed of expansion of the bubbles becomes almost that of the velocity of light after the radius of the bubble becomes longer than the characteristic thickness of the bubble wall, provided that the energy released by phase transition ϵ goes to the kinetic velocity of the bubble wall as discussed by Coleman (1977). We can include this kinetic energy of bubble walls among the relativistic particle energy ρ_r, because this energy also decreases with the expansion of the Universe as R^{-1}.

In practice, however, a part of the energy released by the phase transition is thermalized, because bubble walls collide with particles which fill the Universe. The degree of thermalization depends on the strength of interaction between particles and the Higgs field. Lasher (1979) assumed that all the released energy is thermalized instantaneously and he applied the theory of a detonation wave in order to estimate the expansion speed of bubbles. The expansion velocity of a bubble in this model is also about light velocity when the radiation

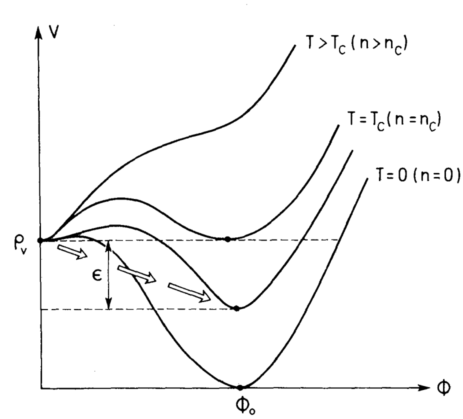

Figure 1. The schematic diagram of the effective potential of the vacuum energy density.

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energy density is greater than that of non-relativistic particles in the bubble. The energy conservation law and the volume fraction of bubbles r are, therefore, independent of the degree of thermalization and are given as

$$R^3 \epsilon\, dr/dt = (\rho_r/3)\, dR^3/dt + d(\rho_r R^3)/dt \tag{2.2}$$

and

$$r(t) = \int_0^t p(t')\,[R(t')/R(t)]^3\, V(t',t)\, dt', \tag{2.3}$$

where $p(t')$ is the nucleation rate of bubbles per unit time and unit volume of the false vacuum ($\phi = 0$) at the cosmic time t'. The volume of a bubble at the cosmic time t, which has been created at t' is

$$V(t',t) = 4\pi R^3(t) \int_{t'}^{t} [1 - r(t'')]\, \chi^2(t',t'') \frac{d\chi(t',t'')}{dt''}\, dt'' \tag{2.4}$$

where

$$\chi(t',t) = \int_{t'}^{t} dt/R(t). \tag{2.5}$$

Here we have assumed that a bubble expands spherically at the velocity of light and the increase in the volume of a bubble is depressed by the factor $[1 - r(t'')]$ due to the overlapping of bubbles.

In the following discussion, we use, for convenience, a characteristic scale factor of this cosmic model,

$$l \equiv (8\pi G \rho_V/3)^{-1/2} \tag{2.6}$$

to define units of time and length.

In the era before the phase transition ($r = 0$), the equation of cosmic expansion, equation (2.1), reduces to a simple equation,

$$\dot{R}^2 = R^2 + bR^{-2} \tag{2.7}$$

where

$$b \equiv (\rho_r R^4)/\rho_V. \tag{2.8}$$

The value of the parameter b is a constant from the energy conservation law equation (2.2). The solution of this equation is $R(t) = b^{1/4} \sinh^{1/2}(2t)$. Since the absolute value of the scale factor R has no meaning in the model of the Universe with a Minkowski space, we set $b = 1$ for simplicity

$$R(t) = \sinh^{1/2}(2t) = \begin{cases} (2t)^{1/2} & \text{for } 2t \ll 1 \\ e^t/\sqrt{2} & \text{for } 2t \gg 1. \end{cases} \tag{2.9}$$

3 Nucleation models and their results

Nucleations of bubbles of a stable vacuum are induced by two factors, one is the thermal fluctuation and the other is the quantum fluctuation. These factors give very different types of nucleation rates, and we therefore, in the following, discuss them separately in order to

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make clear the characteristic properties of the progress of the phase transition induced by each of these factors.

3.1 NUCLEATIONS BY THERMAL FLUCTUATION AND THE PROGRESS OF PHASE TRANSITION – TF MODEL

When the cosmic temperature becomes less than T_c, nucleations begin. The nucleation rate p by thermal fluctuations is essentially the same as the probability of bubble formations in a boiling liquid, which is given as $p \propto \exp[-S_3(T)/T]$. The factor $S_3(T)$ is the action of an $O(3)$ symmetric bubble (Linde 1977), which is given as $S_3(T) = 16\pi a^3/3\,\epsilon^2$ if we take a thin wall approximation, i.e. the radius of a bubble is much greater than the thickness of the wall, where a is the surface energy per unit area. The nucleation rate $p(T)$ has a sharp peak at T_c' which is less than T_c, but is very close to T_c, i.e. $T_c - T_c' \ll T_c$. In order to represent the strength of this peak, we introduce a parameter p_T as

$$p_T\, T_c \equiv \int_0^{T_c} p(T)\, dT. \tag{3.1}$$

In the present work, we approximate p by a δ-function and the energy-density difference ϵ by a step function simply as,

$$p = p_T\, T_c\, \delta\,(T - T_c) \tag{3.2}$$

and

$$\epsilon = \begin{cases} 0 & \text{for } T \geq T_c \\ \rho_v & \text{for } T < T_c \end{cases}, \tag{3.3}$$

where we neglect the difference between T_c and T_c'. The number of bubbles created at the phase transition per unit volume is calculated by the integration of equation (3.2),

$$N_b \equiv \int p_T\, T_c\, \delta\,(T - T_c)\, dt = p_T\, T_c\, (dt/dT)_{T=T_c} = p_T\, R_c\, (dt/dR)_{R=R_c} \tag{3.4}$$

where R_c is the scale factor when $T = T_c$. With the aid of the expansion law equation (2.9), we obtain

$$N_b = p_T \tanh 2t_c = \begin{cases} 2t_c\, p_T & \text{for } 2t_c \ll 1 \\ p_T & \text{for } 2t_c \gg 1, \end{cases} \tag{3.5}$$

where t_c is the cosmic time when the phase transition begins ($T = T_c$),

$$t_c = \tfrac{1}{2} \operatorname{arcsinh}\, [\rho_v/(S\pi^2 T_c^4/15)]^{1/2}. \tag{3.6}$$

The statistical weight of particles in unit of photon energy density S is an order of 100.

The equation of the volume fraction of bubbles (2.3) is reduced to a simple equation as

$$r(t) = 1 - \exp\left[-\frac{4\pi}{3} N_b\, R^3(t_c)\, \chi^3(t_c, t)\right]. \tag{3.7}$$

In the limit $\chi(t_c, t) \to \infty$, the fraction $r(t)$ becomes unity and the phase transition finishes. The δ-function approximation of the nucleation rate equation (3.2) is justified provided that the duration time of the phase transition is much longer than the duration time of the nucleation. Therefore, because the duration time of the nucleation seems to be much shorter than the cooling time scale $T_c\,(dt/dT)_{T=T_c}$, this condition is sufficiently satisfied if the duration

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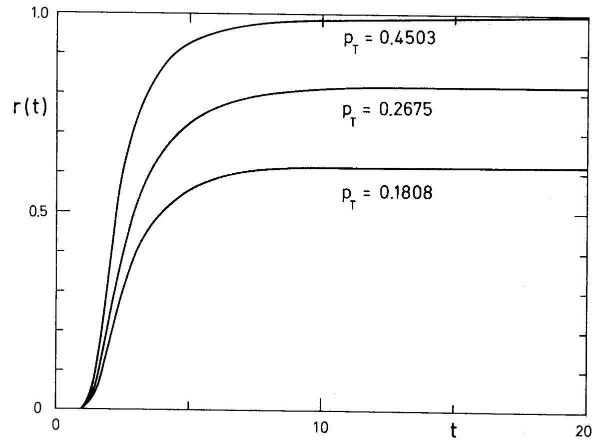

Figure 2. The time evolutions of the volume fraction of bubbles $r(t)$. If the parameter $p_T < 0.4503$, bubbles cannot cover all the space forever. It is assumed that the phase transition begins at the cosmic time $t = 1$.

time of the phase transition is longer than tanh $2t_c$. This will be checked in the following calculations.

In Figs 2–5, the results of numerical integrations of equations (2.1)–(2.3) are shown. As shown in Fig. 2, the phase transition never finishes provided that the value of the nucleation rate parameter p_T is less than 0.4503 in the models with the beginning time $t_c = 1$. The reason for this is that the event horizon $\chi(t_c, \infty)$ is finite in the de Sitter Universe model (Weinberg 1972): If we assume the expansion law as $R \sim \exp(t)$ (see equation 2.9) simply, the event horizon $\chi(t_c, \infty)$ is $\exp(-t_c)$. In order for the value $r(\infty)$ to be unity, $\chi(t_c, \infty)$ must be infinite from the equation (3.7). This condition is satisfied when the energy density of the radiation becomes dominant due to the rapid phase transition. In Fig. 3, this critical value p_T is shown as a function of the beginning time of the phase transition t_c (curve A). In the limit $t_c \to 0$, the critical value is given as $p_T = 10^{-1.21} t_c^{\,2.14}$ approximately from the numerical computations. In order that the phase transition finishes, the region below curve (A) must be forbidden. In these models, however, we have neglected nucleations by quantum

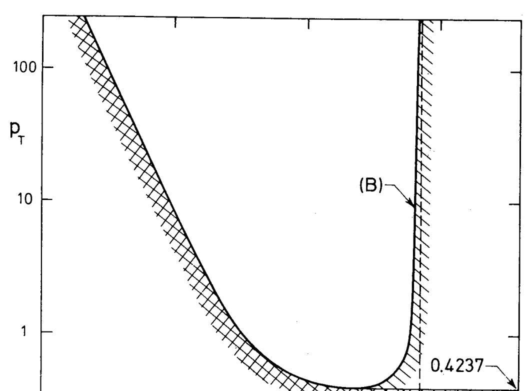

Figure 3. Constraints on the parameters p_T and t_c. If the values of p_T are less than the curve (A), the phase transition never finishes. If the values of p_T are less than the curve (B), the ratio $(n_\gamma/n_b)_f$ becomes greater than 10^9 for any value of the primordial ratio of photons to baryons before the phase transition. The regions lower than these curves can be ruled out from the observation of the ratio $(n_\gamma/n_b)_0 = 10^9$ in the present Universe.

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fluctuations. After the Universe becomes cold, nucleations by quantum fluctuations become dominant. This problem is discussed in the next subsection.

We can also obtain a more severe constraint on the nucleation rate parameter p_T and the beginning time t_c from the number ratio of photons to nucleons in the present Universe $(n_\gamma/n_b)_0 \approx 10^9$ (Weinberg 1972), provided that the temperature during the phase transition is much lower than the characteristic temperature of the SU(5) GUT (grand unified theory) $\sim 10^{15}$ GeV (Georgi & Glashow 1974).

Let the number ratio of photons to baryons before the phase transition be an order of unity, for simplicity, then the ratio at the cosmic time t is estimated as

$$(n_\gamma/n_b)_t = \{\rho_r(t) R^4(t)/\rho_v\}^{3/4}, \tag{3.8}$$

where for simplicity we have neglected the presence of particles except photons and baryons because most of their entropy is eventually reduced to that of photons in the late stage of cosmic expansion. Equation (3.8) can be rewritten as

$$(n_\gamma/n_b)_t = \left\{1 + \int_0^{r(t)} R^4(t)\, dr(t)\right\}^{3/4}, \tag{3.9}$$

with the aid of equations (2.2) and (3.3).

In Fig. 4, time evolutions of the ratio $(n_\gamma/n_b)_t$ of the models which give the final ratio $(n_\gamma/n_b)_f = 10^9$ are shown. In the models with the beginning time $t_c = 0.01$, 1, 3, the rise times for growth of the volume fraction $r(t)$ are much longer than $\tanh 2t_c$ and the photon productions near the beginning time t_c are negligible compared with the productions near the finishing time of the phase transition. These results justify the δ-function approximation of the nucleation rate and the step function approximation of the energy density difference (equations 3.2 and 3.3). As seen in Fig. 4, the photon productions in the very late stage of the phase transition, where $r(t)$ is almost unity, are very important. Since the speed of the phase transition is extremely decelerated by the factor $[1 - r(t)]$ in equations (2.3) and (2.4), the duration time of the phase transition is stretched very much. As a result, the scale factor R becomes very large and photon production continues even in the very late stage $(r(t) \ll 1)$ as described by equation (3.9).

The later the phase transition begins, the sooner the phase transition must finish in order for the final ratio $(n_\gamma/n_b)_f$ to be 10^9, because the entropy production per baryon becomes

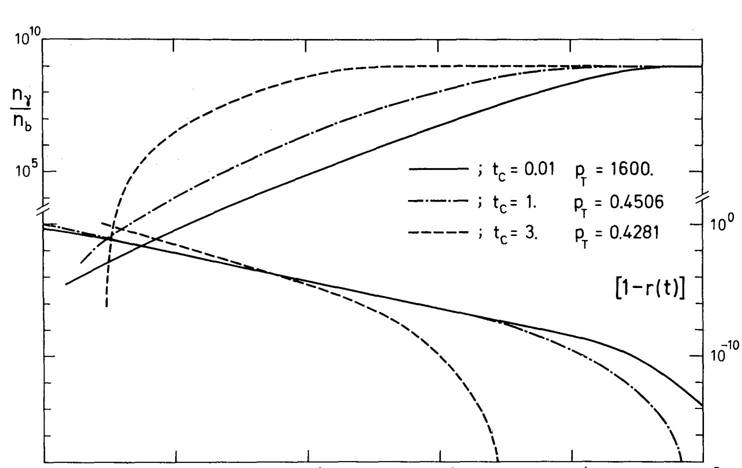

Figure 4. The time evolution of the ratio (n_γ/n_b) and the volume fraction of the metastable vacuum $[1 - r(t)]$ in the models which give the final ratio of photons to baryons $(n_\gamma/n_b)_f = 10^9$.

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larger and larger if the phase transition begins at a later time. The final ratio $(n_\gamma/n_b)_f$ is given as

$$(n_\gamma/n_b)_f = \left\{1 + \int_0^1 R^4(t)\, dr(t)\right\}^{3/4}$$

from equation (3.9). If the phase transition finishes very quickly, this ratio is reduced to

$$(n_\gamma/n_b)_f = (1 + \sinh^2 2t_c)^{3/4} \approx (\sinh 2t_c)^{3/2}. \tag{3.10}$$

In order for this ratio to be 10^9, t_c should be 7.254. From this result, the phase transition must begin at least before $t_c \leqslant 7.254$. The δ-function approximation equation (3.2), however, is not adequate when the phase transition begins near this time. But it should be noticed that the beginning time t_c is earlier than the cosmic time unity in the plausible gauge models, because the radiation energy density at the critical temperature is comparable or greater than the vacuum energy density in the usual Higgs models (see, e.g. Linde 1979). It seems rather difficult to make models in which the beginning time t_c is far later than the cosmic time unity.

In Fig. 3, the value of p_T which gives the final ratio $(n_\gamma/n_b)_f = 10^9$ is shown as a function of t_c (curve B). This curve is almost unchanged even if the value of the primordial ratio of photons to baryons before the phase transition is far less than unity. But if this value is greater than unity, the curve will shift upwards. Even though this curve depends on the primordial ratio, we can conclude that the region lower than the curve (B) must be forbidden, because in this region the final ratio $(n_\gamma/n_b)_f$ is always greater than 10^9 for any value of the primordial ratio.

3.2 NUCLEATIONS BY QUANTUM FLUCTUATIONS AND THE PROGRESS OF PHASE TRANSITION – QF MODEL

The broken symmetry of a vacuum is restored not only by the increase of the temperature but also by the increase of the number density of fermions which are coupled to a Higgs field★ as shown in the σ-model (Lee & Wick 1974). Even though the early Universe was cold, the state of the vacuum in the early Universe might have been at the symmetric state (Fig. 1).

When the fermion number density becomes less than a critical value n_c, the phase transition begins, and bubbles of a stable vacuum are created solely by quantum fluctuation. The nucleation rate p, by quantum fluctuations, is given as $p \propto \exp(-S_4)$, where S_4 is the action of an $O(4)$ symmetric bubble (Coleman 1977). If we take a thin wall approximation, the factor S_4 is given as $S_4 = 27\pi^2 a^4/2\epsilon^3$. The surface energy a and the energy density difference ϵ change with the decrease of the fermion density n (see Fig. 1). In the present work, we approximate the nucleation rate and the energy density difference by a step function simply,

$$p = \begin{cases} 0 & \text{for } n > n_c \\ p_Q & \text{for } n < n_c \end{cases} \tag{3.11}$$

and

$$\epsilon = \begin{cases} 0 & \text{for } n > n_c \\ p_v & \text{for } n < n_c. \end{cases} \tag{3.12}$$

★ In the Weinberg–Salam model, however, the degree of the asymmetry increases with the increase of the weak charge density, for example, the neutrino number density instead of the restoration (Sato & Nakamura 1976; Linde 1976b).

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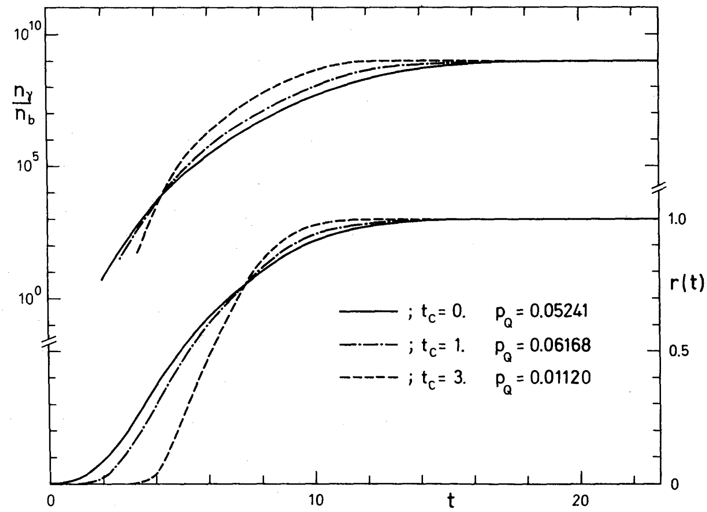

Figure 5. The time evolution of the ratio (n_γ/n_b) and the volume fraction of bubbles $r(t)$ in the models which give the final ratio $(n_\gamma/n_b)_f = 10^9$.

These approximations are valid if the rise time for growth of nucleation rate is much shorter than the duration time of the phase transition. This condition is sufficiently satisfied if the duration time of the phase transition is longer than tanh $2t_c$ as discussed in the previous subsection. In the cold model, the beginning time t_c instead of equation (3.6) is given as,

$$t_c = \tfrac{1}{2} \,\text{arcsinh}\, [\rho_v/\{S'(3\pi^2 n_c)^{4/3}/4\pi^2\}]^{1/2}, \tag{3.13}$$

if the number of species of degenerate fermions is S' and they have the same number density. The fraction of baryons (quarks) in these fermions is also a free parameter. If the fraction of baryons is comparable to that of leptons (this seems to be a reasonable assumption), the number ratio of photons to baryons at the cosmic time $t \gg t_c$ is estimated by equation (3.8) also.

In Fig. 5, time evolution of $r(t)$ and $(n_\gamma/n_b)_t$ in the models which give the final ratio $(n_\gamma/n_b)_f = 10^9$ are shown. The important difference between the QF (quantum fluctuation) model and the previous TF (thermal fluctuation) model is that the phase transition and the

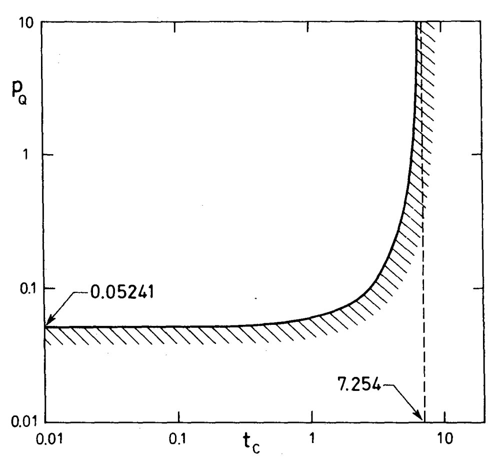

Figure 6. The constraint on the nucleation parameter p_Q and the beginning time of the phase transition t_c. If the values of p_Q are less than this curve, the ratio $(n_\gamma/n_b)_f$ becomes greater than 10^9 for any value of the primordial number ratio of baryons to leptons before the phase transition. The region lower than this curve can be ruled out from the present observation.

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photon production in the QF models finish earlier than in the TF models. In spite of the existence of the factor $[1 - r(t)]$ in equations (2.3) and (2.4), the duration time of the phase transition does not extend very long owing to the continuous nucleation of new bubbles. The size of bubbles in the QF model is also limited to the particle horizon the same as in the TF model, but the fraction of the volume of bubbles can increase by new bubbles being created continuously. The finishing time of phase transition t_f in the models giving the final ratio $(n_\gamma/n_b)_f = 10^9$ is always less than 20, as shown in Fig. 5.

The values of p_Q which give the final ratio $(n_\gamma/n_b)_f = 10^9$ are shown as a function of the beginning time t_c in Fig. 6. This curve, however, shifts upward if the primordial fraction of baryons is far less than unity. Even though this curve depends on the primordial fraction, we can conclude that the region lower than the curve shown in Fig. 5 must be forbidden, because in this region the final ratio $(n_\gamma/n_b)_f$ is always greater than 10^9 for any value of the primordial fraction.

It should be noticed that the phase transition in the QF model finishes eventually due to the continuous nucleation of new bubbles even if the value of the parameter p_Q is extremely small.

In the present subsection, we have assumed that the Universe before the phase transition is cold. Even if the Universe before the phase transition is hot, nucleations by quantum fluctuations become dominant when the cosmic temperature becomes less than S_3/S_4 $[= 32\,\epsilon(t)/81\,\pi a(t)]$. In particular, the present QF model may be more adequate than the TF model even in the hot Universe models, if the temperature T_q which is the solution of the equation, $T_q = 32\epsilon(T_q)/81\pi a(T_q)$, is comparable to the critical temperature T_c. Even if $T_q \ll T_c$, nucleations by quantum fluctuations are important in the models with small nucleation rates by thermal fluctuation (the region lower than the curve (A) in Fig. 3). In these models, the phase transition does not finish without the nucleations by quantum fluctuations.

Generally speaking, we can conclude that the phase transitions of the vacuum should satisfy one of the two constraints (Figs 3 and 4) from the present argument.

4 Discussions and conclusions

4.1 APPLICATION TO THE QUARK–NUCLEON PHASE TRANSITION

Lasher (1979) discussed the density fluctuations produced by the quark–nucleon phase transition in the cold Universe assuming that this phase transition was of first-order. In order to compare with his results, we estimate the total baryon mass in the biggest bubble in the QF model as follows. The mass in a bubble created at the cosmic time t_c is

$$M = m_N\,(4\pi R^3\,(t_c)\,n_b\,(t_c)/3)\,\chi^3\,(t_c, t_f),$$

if the coalescences between bubbles before the finishing time t_f are neglected, where m_N is the nucleon mass. The total baryon number in a sphere of the radius with the cosmic scale factor at the cosmic time t_c, $(4\pi R^3(t_c)\,n_b\,(t_c)/3)$ is an order of $4\pi\,l^3\,(\rho_v)^{3/4}/3$. (The unit of the length l (equation 2.6) is shown explicitly in order to avoid any misunderstanding.) In this phase transition, ρ_v is the bag constant B (Chodos *et al.* 1974) which is about $B \sim (100\ \mathrm{MeV})^4$. The beginning time t_c is an order of unity, because this phase transition begins when the quark kinetic energy ρ_r becomes equal to or less than the vacuum energy density ρ_v. The value of $\chi\,(t_c, t_f)$ depends on the nucleation rate p_Q. In particular, if p_Q is much greater than unity $\chi\,(t_c, t_f) \propto p_Q{}^{-1/4}$. But the value of p_Q must be much less than unity in order that the ratio $(n_\gamma/n_b)_f$ is 10^9, because the finishing time t_f must be much greater than unity as shown in Fig. 5. In this case, the value of $\chi\,(t_c, t_f)$ is about $\exp(-t_c)$ almost independent

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of the value t_f. This results in our estimate of the maximum mass of bubbles M being an order of $10(100\,\mathrm{MeV}/B^{1/4})^3 M_\odot$, which is about the baryon mass in the particle horizon at the cosmic time t_c. This estimate has been justified by numerical calculations.

On the other hand, Lasher (1979) derived $10^6 M_\odot$ for a characteristic mass, which is much greater than the present value. This discrepancy obviously comes from his unusual assumption regarding the quark phase energy density. He assumes that $B \sim n_b{}^{2/3}$ even after the baryon number density n_b becomes less than the critical density of the phase transition. This assumption leads to the expansion law $R \sim t$ instead of $R \sim \exp(t)$ as in the present model. In the Lasher model, the event horizon becomes infinite and the bubbles would also expand infinitely in principle. The assumption $B \sim n_b{}^{2/3}$ also means that the energy per unit volume released by the phase transition decreases as R^{-2}, and that a large volume per baryon would be necessary in order to create a sufficient number of photons per baryon, compared with the volume needed in our model. This also makes the size of the bubbles larger. Another important difference is the nucleation rate. Lasher assumes that nucleations occur instantaneously at the critical density and no new bubbles are formed at a later time. In our model, new bubbles are created during the phase transition and prevent the old ones from becoming large. We can conclude that the maximum mass of the fluctuations created by the phase transition is an order of $10 M_\odot$, at most, in the standard MIT bag model of hadrons (Chodos *et al.* 1974). This value is too small for the seeds of galaxies.

4.2 PHASE TRANSITION IN THE WEINBERG–SALAM MODEL

The phase transition in the Weinberg–Salam model has been studied by Kirzhnits & Linde (1976) and Linde (1979) in detail. According to Linde (1979), symmetry of the vacuum is restored in the early stage of the Universe ($\phi = 0$), provided the weak charge density of the Universe is roughly less than the photon number density. The phase transition from $\phi = 0$ to $\phi = \phi_0 (= \mu/\sqrt{\lambda})$ is of first order, provided that the value of a coupling constant λ is less than the fourth power of a gauge coupling constant $g^4 \equiv \pi^2 \alpha^2 (2\cos^2\theta + 1)/\sin^4\theta \cos^4\theta$, where α is the fine structure constant and θ is the Weinberg angle. Namely the effective potential V has a local minimum at $\phi = 0$, even though the temperature is less than the critical temperature. This local minimum, however, disappears with the decrease of the temperature if $3g^4/16\pi^2 < \lambda < g^4$, because the effective potential of the zero temperature has no local minimum if $3g^4/16\pi^2 < \lambda$. This means the value of the nucleation rate parameters p_T (equation 3.1) and p_Q (equation 3.11) diverge infinitely. The nucleation rates, however, are an extremely rapidly decreasing function of λ, if $\lambda < 3g^4/16\pi^2$. The constraints on p_T (Fig. 3) and p_q (Fig. 6) obtained from the ratio $(n_\gamma/n_b)_0 = 10^9$ in the present Universe predict, therefore, that the coupling constant λ should be equal or greater than $3g^4/16\pi^2$ roughly, i.e. $\lambda \gtrsim 3g^4/16\pi^2$. This limit is essentially the same as the one obtained from the constraint that the lifetime of the metastable vacuum should be shorter than the cosmic age (Linde 1977). A more stringent limit is obtained, in principle, if we calculate the ratio $(n_\gamma/n_b)_f$ with the aid of the precise nucleation rate based on the Weinberg–Salam model.

4.3 FORMATION OF FLUCTUATIONS WITH THE GALACTIC SCALE IN PHASE TRANSITION

As seen in Section 3, the duration time of the phase transition is strongly limited by the ratio $(n_\gamma/n_b)_0$ in the present Universe (Figs 3 and 6). We now look at the phase transition in the SU(5) GUT, or the phase transition which occurs at a temperature higher than that of the SU(5) GUT, and find that this limit becomes extremely relaxed or removed, because

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baryon number is no longer a conserved number. We will, therefore, discuss here such a phase transition and assume $t_f \gg t_c \approx 1$. The radius of the bubble at the finishing time of the phase transition t_f is about

$$L(t_f) = R(t_f)\,\chi(t_c, t_f) \sim \exp(t_f), \tag{4.1}$$

if the coalescences before the finishing time t_f are neglected. This is the scale of the particle horizon (Weinberg 1972). After the phase transition, the radiation energy density becomes dominant again. If we assume that the energy released by the phase transition is thermalized at this time t_f, for simplicity, then the temperature at time t_f is $T(t_f) = (15\,\rho_V/\pi^2 S)^{1/4}$, where statistical weights of particles S are an order of 100. In the course of the following expansion of the Universe, the reasonable baryon number is created as discussed by many authors (Yoshimura 1978; Ignatiev *et al.* 1978; Dimopoulous & Susskind 1978; Toussaint *et al.* 1979; Ellis, Gaillard & Nanopoulos 1979; Weinberg 1979; Barrow 1979). The characteristic length of the density and the velocity fluctuations at present is estimated by the ratio of the present cosmic radiation temperature T_0 to $T(t_f)$,

$$L_0 = L(t_f)(S^{1/3}\,T(t_f)/T_0), \tag{4.2}$$

where we have assumed the entropy of all the particles is reduced to that of photons in the course of cosmic expansion. We obtain the following inequality from the condition that this length L_0 should be greater than the mean separation length between galaxies, d, i.e. $L_0 > d$,

$$t_f > 52 + \ln\left[\left(\frac{T_0}{3\mathrm{K}}\right)\left(\frac{\rho_V{}^{1/4}}{10^{15}\,\mathrm{GeV}}\right)\left(\frac{d}{1\ \mathrm{Mpc}}\right)\left(\frac{100}{S}\right)^{1/12}\right]. \tag{4.3}$$

As discussed by Linde (1977), the transition time becomes longer than the cosmic age in the Weinberg–Salam model, when we take a small value for λ. It seems, there, that the condition $t_f > 52$ is not unreasonable, but is acceptable even in this phase transition.

It should also be noticed that fluctuations created by the first-order phase transition of a vacuum correspond to a growing mode in the linear perturbation theory (Lifshitz 1946). In the period of the phase transition $t_c < t < t_f$, the equation of state $p = p(\rho, T)$ changes spatially, i.e, $p = -\rho_V$ in the region of a metastable vacuum $\phi = 0$ and $p = S\pi^2 T^4/45$ in the region of a stable vacuum $\phi = \phi_0 \neq 0$. The change in the equation of state $p = p(\rho, T)$ changes the curvature from place to place and gives a growing mode (see, e.g. Press & Vishniac 1980, and papers cited therein).

Strictly speaking, the linear perturbation theory cannot be applied in the present first order phase transition, because perturbations in the equation of state are very big and non-linear effects cannot be neglected. It seems, however, reasonable to expect that fluctuations created by the first order phase transition do not decay but grow from the result of the linear perturbation theory.

4.4 CONCLUSION

In the present paper, we have discussed how the first order phase transition proceeds in the expanding Universe and have demonstrated that the progress of the phase transition and the cosmic expansion affect each other strongly. It has been shown that the duration time of the phase transition is stretched by this mutual interference. We have obtained the constraints on the nucleation rates of bubbles and the beginning time of the first-order phase transition from the observation of the ratio $(n_\gamma/n_b)_0 = 10^9$ in the present Universe (Figs 3 and 6). Gauge theories which predict values conflicting with these constraints can be ruled out from

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the present cosmological discussion. It is also pointed out that the density and the velocity fluctuations created by first-order phase transitions may account for the origin of galaxies, provided that baryon number is not conserved before the phase transitions.

At present, there is no direct evidence that the vacuum phase transitions in gauge theories are of first order. But it is very likely that the phase transitions are of first order, because the Weinberg–Salam theory, which is now looked upon as an almost established theory, predicts the first-order phase transition of the vacuum, provided that the four body self-coupling constant λ is less than a critical value.

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Note added in proof

Developments after the present paper was submitted are shown in M. Einhorn & K. Sato, Nordita preprint 80/37 and papers cited therein.

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