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ABSTRACT

This paper analyzes the effects of changes in government expenditures on both a domestically produced and an imported good in an open economy based on intertemporal optimizing behavior. The dynamic adjustment is characterized in detail and the critical role played by the accumulating capital stock is highlighted. The evolution of the current account is seen to mirror that of capital. The welfare of such policies is also assessed in terms of the intertemporal utility of the representative agent. Both permanent and temporary policy changes are considered, with the latter being shown to have a permanent effect on the economy.

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1. INTRODUCTION

The emergence of large fiscal and current account deficits in the United States during the 1980's stimulated interest in the theoretical analysis of fiscal policies in open economies. The recent literature addressing this problem does so using various versions of the utility maximizing representative agent framework. Two types of issues have received attention; the effects of changes in government expenditure policy, on the one hand, and issues pertaining to debt and tax-financing policies, on the other. Of necessity, formal analyses of this type are restrictive, being required to invoke abstractions which permit them to focus on the specific issues at hand in the most lucid way. But in almost all cases, the role of capital accumulation, a central component of the adjustment process, is either ignored, or incorporated in restrictive ways.

Particularly significant contributions to the literature are contained in a series of papers by Frenkel and Razin, most of which are brought together in Frenkel and Razin (1987). This book provides a comprehensive treatment of the impact of deficits in a two-country world economy. For a large part, their analysis abstracts from investment and output effects, and when these are included, they are restricted to a two-period analysis. Buiter (1987) introduces capital accumulation into a true dynamic setting, although like Frenkel and Razin, he assumes that employment remains fixed. The main question he addresses is the choice between borrowing and tax-financing of a given level of government expenditure. In order to obtain a nontrivial analysis of this issue, Ricardian equivalence must be broken and he achieves this by adopting the finite life consumer model of Blanchard (1985). More recently, Obstfeld (1989) also introduces capital accumulation, while maintaining the assumption of a fixed employment of labor. Furthermore, in his model the capital stock can be augmented instantaneously by imports from abroad, in which case the rate of investment is infinite.

The present paper analyzes the effects of government expenditure policy using a two good model of a (semi-small) open economy.[1] Since we wish to focus on expenditure, rather than the details of government finance, we assume the existence of infinitely-lived private agents, when, with competitive markets, the conditions of Ricardian equivalence are known to prevail. A key aspect of the model is that both employment and the rate of capital accumulation are endogenously determined through the intertemporal optimizing behavior of a representative agent. Investment behavior is generated by a Tobin q-theoretic function, first developed within an intertemporal optimizing framework by Hayashi (1982), Abel (1982), and others.[2] The convex installation costs ensure that the rate of investment remains finite at all times. As will become evident in due course, the endogeneity of employment is crucial to the dynamics of the entire system. Without it, the dynamics would degenerate; a fiscal expansion would give rise to an instantaneous adjustment in the relative price. with no change in the capital stock.

Using this framework, we analyze the effects of changes in government expenditure on both a domestically produced good and an imported good on a number of key macroeconomic variables. These include, the rate of capital accumulation, employment, output, the current account deficit, the real interest rate, and the real exchange rate. In doing this, our analysis differs from the existing literature in several important respects. First, the model is sufficiently tractable to enable us to characterize in detail the dynamic adjustment of the economy and to highlight the critical role played by the accumulating capital stock in this process. In particular, the evolution of the current account is seen to mirror that of capital. Secondly, as well as describing the responses of the variables enumerated above, our approach provides a natural framework for assessing the effects of such policy shocks on the welfare of the representative agent, as measured by his utility along the entire adjustment path.[3]

Thirdly, for both forms of government expenditure, two types of changes are analyzed: namely an unanticipated permanent, and an unanticipated temporary expansion. A striking feature of the latter is that a temporary fiscal shock has a permanent effect on the economy. The reason for this is that, as we shall demonstrate below, the steady state corresponding to some sustained policy depends upon the initial conditions of the economy prevailing at the time this policy is introduced. The adjustment which occurs during some temporary policy change will have an important bearing on the initial conditions in existence at the time the temporary policy is permanently revoked.

The fact that the steady state may depend upon initial conditions in models with infinitely lived maximizing agents, having a constant rate of time discount and facing perfect capital markets (assumptions to be made below), has been previously discussed by Giavazzi and Wyplosz (1984). However, its significance for the implications of temporary shocks has received little consideration.[4] Yet this is a crucial issue, especially in the light of the recent interest pertaining to hysteresis and the random walk behavior of real variables such as output and employment. The present framework provides a plausible framework for generating this type of behavior.

In characterizing the dynamic adjustment paths generated by these fiscal disturbances, the analysis identifies several channels through which they are transmitted to the rest of the economy. First, there is the usual direct effect. This is simply the channel whereby a fiscal expansion on the domestic good impinges directly on the domestic output market, while a fiscal expansion on the imported good impacts directly on the trade balance. Secondly, a fiscal expansion induces a short-run change in the price of capital (the Tobin q), which in turn determines the transitional adjustment in the capital stock over time. The model is forward looking and as a consequence of this, the short-run adjustment depends upon the long-run response of the capital stock. As we will show below, this in turn depends upon the form of fiscal expansion. While government expenditure on the domestic good is unambiguously expansionary, government expenditure on the imported good is not. Thirdly, a fiscal expansion generates a wealth effect, which with perfect capital markets, remains constant over time. Moreover, because the economy changes its stock of wealth while a temporary policy is in effect, thereby determining the initial conditions in existence when the policy ceases, this wealth effect provides the channel whereby the temporary policy has a permanent effect. It is the essential source of the hysteresis generated by the model.

2. THE MACROECONOMIC FRAMEWORK

For present purposes the household and production sectors of the economy may be consolidated. The economy we consider is inhabited by a single infinitely-lived representative agent who accumulates capital (k) for rental at its competitively determined rental rate and supplies labor (ℓ) at its competitive wage. The agent is specialized in the production of a single commodity, using the stock of capital and labor by means of a neoclassical production function $F(k, \ell)$. Expenditure on any given increase in the capital stock is an increasing function of the rate of capital accumulation.[5] That is, there are increasing costs associated with investment (I) which we represent by the convex function $C(I) : C' > 0,\ C'' > 0$.[6] By choice of units, we assume

$$C(0) = 0, \quad C'(0) = 1$$

so that the total cost of zero investment is zero and the marginal cost of the initial installation is unity.[7]

Domestic output is used in part for investment, in part as a domestic consumption good (x), with the rest being exported. In addition to consuming part of this output, the agent also consumes another good (y), which is imported from abroad. While the price of this latter good is taken as given, the economy is large enough in the production of the domestic good to affect its relative price and therefore the terms of trade.

The agent can also accumulate net foreign bonds (b) that pay an exogenously given world interest rate (i^*). Equation (1a) describes the agent's instantaneous budget constraint

$$\dot{b} = \frac{1}{\sigma}[F(k,\ell) - C(I) - x - \sigma y + \sigma i^* b - T] \tag{1a}$$

where

σ = relative price of the foreign good in terms of the domestic good,

T = lump–sum taxes.

In addition, the rate of capital accumulation and investment are related by the constraint

$$\dot{k} = I \tag{1b}$$

where for simplicity we abstract from depreciation.

The agent's decisions are to choose consumption levels x, y, labor supply ℓ, the rate of investment I, and the rates of asset accumulation $\dot{b}, \dot{k}$ to

$$\begin{gathered}\text{Maximize} \quad \int_0^\infty [U(x,y) + V(\ell) + W(g_x, g_y)]e^{-\delta t}dt \\ U_x > 0, \quad U_y > 0, \quad V' > 0, \quad W_{g_x} > 0, \quad W_{g_y} > 0\end{gathered} \tag{1c}$$

where g_x, g_y real domestic government expenditure on the domestic good and the import good, respectively. The optimization is subject to the constraints (1a), (1b), and the given initial conditions $K(0) = K_0,\ b(0) = b_0$. For simplicity, the instantaneous utility function is assumed to be additively separable in the private consumption goods, x and y, labor, ℓ, and the public expenditures g_x and g_y. We also assume that the utility function is increasing in the consumption of both private and public goods, but decreasing in labor, and that it is strictly concave. Finally, the two private goods are taken to be Edgeworth complementary, so that $U_{xy} > 0$.

The current–value Hamiltonian for this optimization is expressed by

$$H \equiv U(x,y) + V(\ell) + W(g_x, g_y) + \frac{\lambda}{\sigma}[F(k,\ell) - C(I) - x - \sigma y + \sigma i^* b - T] + q^* I \tag{2}$$

where λ is the shadow value (marginal utility) of wealth in the form of internationally traded bonds, q^* is the shadow value of the agent's capital stock. Exposition of the model is simplified by using the shadow value of wealth as numeraire. Consequently, $q \equiv \sigma q^*/\lambda$ is defined to be the market price of capital in terms of the (unitary) price of foreign bonds.

The first order optimality conditions with respect to x, y, ℓ, and I are respectively

$$U_x(x,y) = \frac{\lambda}{\sigma} \tag{3a}$$

$$U_y(x,y) = \lambda \tag{3b}$$

$$V'(\ell) = -\frac{\lambda}{\sigma} F_\ell(k,\ell) \tag{3c}$$

$$C'(I) = q. \tag{3d}$$

Pairwise the first three equations describe the usual marginal rate of substitution conditions for consumers. Equation (3d) equates the marginal cost of investment to the market value of capital, which is essentially a Tobin q theory of investment.[8]

In addition, the shadow value of wealth and the market value of capital evolve in accordance with

$$\dot{\lambda} = \lambda(\delta - i^*) \tag{3e}$$

$$\dot{q} = \left(i^* + \frac{\dot{\sigma}}{\sigma}\right) q - F_k(k,\ell). \tag{3f}$$

Since δ and i^* are both fixed, the ultimate attainment of a steady state is possible if and only if $\delta = i^*$ and henceforth we assume this to be the case. This implies $\dot{\lambda} = 0$

everywhere, so that λ is always at its steady-state value $\overline{\lambda}$ (to be determined below). Given the assumption of interest rate parity, the domestic interest rate $i(t)$ is related to the world interest rate by

$$i(t) = i^* + \frac{\dot{\sigma}}{\sigma}. \tag{4}$$

Equation (3f) is therefore an arbitrage condition equating the rate of return on capital, $(F_k + \dot{q})/q$, to the domestic interest rate, $i(t)$.

Finally, in order to ensure that the private agent satisfies his intertemporal budget constraint, we need to impose the transversality conditions

$$\lim_{t\to\infty} \lambda b e^{-i^* t} = \lim_{t\to\infty} q k e^{-i^* b} = 0. \tag{3g}$$

Turning to the domestic government, its budget constraint, expressed in terms of the foreign good, is described by the equation

$$\dot{a} = \frac{1}{\sigma}[g_x + \sigma g_y + \sigma i^* a - T] \tag{5}$$

where a is the stock of (traded) bonds issued by the domestic government. This equation is perfectly straightforward and requires no further comment.

Subtracting (5) from (1b), yields the national budget constraint

$$\dot{n} = \frac{1}{\sigma}[F(k,\ell) - (x + g_x) - \sigma(y + g_y) - C(I) + \sigma i^* n] \tag{6}$$

where $n \equiv b - a$ = stock of net credit of the domestic economy. That is, the rate of change of net credit of the domestic economy equals the balance of payments on current account, which in turn equals the balance of trade plus the net interest earned on the traded bonds. To rule out the possibility that the country can run up infinite debt or credit with the rest of the world, we impose the following intertemporal budget constraint

$$\lim_{t\to\infty} ne^{-i^*t} = 0. \tag{7a}$$

This relationship, together with the transversality condition (3g), imposes a corresponding constraint on the domestic government, namely

$$\lim_{t\to\infty} ae^{-i^*t} = 0. \tag{7b}$$

The complete macroeconomic equilibrium can now be described as follows. First, there are the static optimality conditions (3a) - (3d), with $\lambda = \bar{\lambda}$, together with the domestic output market clearing condition,

$$F(k,\ell) = x + Z(\sigma) + C(I) + g_x \tag{8}$$

where $Z(\cdot)$ is the amount of the domestic good exported, with $Z'(\cdot) > 0$. Secondly, there are the dynamic equations (1b), (3f), (5), (6), together with the transversality conditions (3g), (7a), (7b).

The five static equations may be solved for x, y, ℓ, I, and σ, in terms of $\bar{\lambda}, k, q$, and g_x, as follows:

$$x = x(\bar{\lambda}, k, q, g_x) \quad \bar{x}_{\bar{\lambda}} < 0, x_k > 0, x_q < 0, x_{g_x} < 0 \tag{9a}$$

$$y = y(\bar{\lambda}, k, q, g_x) \quad \bar{y}_{\bar{\lambda}} < 0, y_k > 0, y_q < 0, y_{g_x} < 0 \tag{9b}$$

$$\ell = \ell(\bar{\lambda}, k, q, g_x) \quad \ell_{\bar{\lambda}} \gtrless 0, \ell_k \gtrless 0, \ell_q > 0, \ell_{g_x} > 0 \tag{9c}$$

$$\sigma = \sigma(\bar{\lambda}, k, q, g_x) \quad \sigma_{\bar{\lambda}} > 0, \sigma_k > 0, \sigma_q < 0, \sigma_{g_x} < 0 \tag{9d}$$

$$I = I(q) \qquad I' > 0 \tag{9e}$$

The following explanation of these partial derivatives may be given. An increase in the marginal utility of wealth leads to a reduction in the domestic consumption of both goods.

The reduction in the demand for the domestic good causes its relative price to fall, i.e., σ rises, thereby stimulating exports. The overall effect on the demand for domestic output depends upon whether or not this exceeds the reduction in x. If so, domestic output and employment both rise, if not, both fall. An increase in the stock of capital raises output and the real wage. The higher domestic income stimulates the consumption of x, though by a lesser amount, and the relative price σ rises. With the two private goods being complementary in utility ($U_{xy} > 0$), the increase in the consumption of the domestic good increases the demand for the import good as well. While the rise in the real wage rate tends to decrease V', thereby stimulating employment, the rise in σ has the opposite effect; the net effect on employment depends upon which influence dominates. An increase in q stimulates investment. This increases the demand for the domestic good and its relative price rises; i.e., σ falls. This in turn raises the marginal utility of the domestic good, implying that the consumption of x must fall, and with $U_{xy} > 0$, y falls as well. On balance, the increase in investment exceeds the fall in demand stemming from the reduction in x and lower exports, so that domestic output and employment rises.

An increase in government expenditure on domestic output raises the demand for that good, thereby raising its relative price (lowering σ). Employment and domestic output are therefore stimulated. However, the increased output, together with the reduced exports stemming from the fall in σ, is smaller than the increase in demand generated by the additional government expenditure, so that x must fall in order for domestic goods market equilibrium to prevail. With $U_{xy} > 0$, the reduced demand for the domestic good spills over to the import good. All this describes only the partial effects of a short-run change in government expenditure on good x. In addition, such an expenditure generates jumps in $\bar{\lambda}$ and q, thereby inducing further responses. The complete short-run response consists of a combination of these effects and will be discussed in Section 5 below. Finally, we may note that given the separability of the utility function in private and public goods, the short-run equilibrium does not depend directly upon g_y. However, as we shall see in

Section 5, g_y has an indirect effect through its impacts on $\bar{\lambda}$ and q.

The evolution of the system is determined by substituting the short–run equilibrium into the dynamic equations and ensuring that the transversality conditions are met. It is readily apparent that in fact the dynamics can be determined sequentially. Equations (1b) and (3f) can be reduced to a pair of autonomous differential equations in q and k and these constitute the core of the dynamics. This is achieved by first differentiating (9d) with respect to t

$$\dot{\sigma} = \sigma_k \dot{k} + \sigma_q \dot{q} \tag{9d'}$$

and then substituting this equation, together with (9c) and (9d), into (1b) and (3f). Next (6) equates the accumulation of foreign assets by the economy to its current account surplus. Using the domestic goods market clearing condition (8), this may be expressed equivalently in terms of exports minus imports plus the interest service account

$$\dot{n} = \frac{1}{\sigma}[Z(\sigma) - \sigma(y + g_y) + \sigma i^* n] \tag{6'}$$

This equation may in turn be reduced to an autonomous differential equation in n, after substituting the solutions for q and k. The same applies to the government budget constraint (5).

3. EQUILIBRIUM DYNAMICS

Carrying out the procedure outlined above, (3f) and (1b) may be reduced to the following pair of linearized differential equations around the steady state:[9]

$$\begin{bmatrix} \dot{q} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} i^* & -\theta[F_{kk} + F_{k\ell}\ell_k] \\ \frac{1}{C''} & 0 \end{bmatrix} \begin{bmatrix} q - \tilde{q} \\ k - \tilde{k} \end{bmatrix} \tag{10}$$

where $\theta \equiv \frac{\sigma}{(\sigma - q\sigma_q)} > 0$, and $\tilde{}$ denotes steady-state values.

The determinant of the coefficient matrix in (10) can be shown to be negative and therefore the long-run equilibrium is a saddlepoint with eigenvalues $\mu_1 < 0, \mu_2 > 0$. It is clear that while the capital stock always evolves continuously, the shadow price of capital, q, may jump instantaneously in response to new information. Along the stable arm, therefore, k and q follow the paths

$$k = \tilde{k} + (k_0 - \tilde{k})e^{\mu_1 t} \tag{11a}$$

$$q = \tilde{q} + (\frac{\mu_1}{I'})(k - \tilde{k}) \tag{11b}$$

To determine the dynamics of the current account, we consider (6′) in the form

$$\dot{n} = \frac{Z[\sigma(\bar{\lambda}, k, q, g_x)]}{\sigma(\bar{\lambda}, k, q, g_x)} - y(\bar{\lambda}, k, q, g_x) + i^* n \tag{12}$$

Linearizing this equation around steady state yields

$$\dot{n} = \frac{1}{\tilde{\sigma}}[(\beta\sigma_k - \sigma y_k)(k - \tilde{k}) + (\beta\sigma_q - \sigma y_q)(q - \tilde{q})] + i^*(n - \tilde{n})$$

where $\beta \equiv Z' - Z/\sigma$. Following Sen and Turnovsky (1989), assume that the economy starts out with an initial stock of traded bonds $n(0) = n_0$. The solution for $n(t)$ consistent with the intertemporal budget constraint for the economy (7a) is

$$n(t) = \tilde{n} + \frac{\Omega}{\mu_1 - i^*}(k_0 - \tilde{k})e^{\mu_1 t} \tag{13}$$

where

$$\tilde{n} - n_0 = \frac{\Omega}{\mu_1 - i^*}(k_0 - \tilde{k}) \tag{14}$$

and

$$\Omega \equiv \frac{1}{\tilde{\sigma}}\left[\beta\left(\sigma_k + \sigma_q\frac{\mu_1}{I'}\right) - \sigma\left(y_k + y_q\frac{\mu_1}{I'}\right)\right] \tag{15}$$

Equation (13) describes the relationship between the accumulation of capital and the accumulation of traded bonds. Of particular significance is the sign of this relationship. The definition of Ω given in (15) emphasizes that capital exercises two channels of influence on the current account. First, an increase in k raises the relative price σ, both directly, but also through the accompanying fall in q, as seen in (11b). What this does to the trade balance depends upon β. From the above definition of β, $\beta > 0$ if and only if the relative price elasticity of the foreign demand for exports exceeds unity. At the same time, the increase in k increases imports both directly, and again through the fall in q, and this reduces the trade balance. While either case is possible, we shall assume that the relative price effect dominates, so that $\Omega > 0$.

Performing the same procedure for government debt, one can obtain an analogous set of equations to (13) - (15), namely

$$a(t) = \tilde{a} + \frac{\Phi}{i^* - \mu_1}(k_0 - \tilde{k})e^{\mu_1 t} \tag{16}$$

where

$$\tilde{a} - a_0 = \frac{\Phi}{i^* - \mu_1}\,(k_0 - \tilde{k}) \tag{17}$$

$$\Phi \equiv (\frac{g_y + i^* a}{\sigma})(\sigma_k + \frac{\sigma_q \mu_1}{I'}) > 0 \tag{18}$$

The steady state of the economy is obtained when $\dot{k} = \dot{q} = \dot{n} = \dot{a} = 0$ and is given by the following set of equations

$$U_x(\tilde{x}, \tilde{y}) = \frac{\bar{\lambda}}{\tilde{\sigma}} \tag{19a}$$

$$U_y(\tilde{x},\tilde{y}) = \bar{\lambda} \tag{19b}$$

$$V'(\tilde{\ell}) = -F_\ell(\tilde{k},\tilde{\ell})\frac{\bar{\lambda}}{\tilde{\sigma}} \tag{19c}$$

$$\tilde{q} = 1 \tag{19d}$$

$$F(\tilde{k},\tilde{\ell}) = \tilde{x} + Z(\tilde{\sigma}) + g_x \tag{19e}$$

$$F_k(\tilde{k},\tilde{\ell}) = i^* \tag{19f}$$

$$F(\tilde{k},\tilde{\ell}) = \tilde{x} + g_x + \tilde{\sigma}(\tilde{y} + g_y) - \tilde{\sigma} i^* \tilde{n} \tag{19g}$$

$$\tilde{n} - n_0 = \frac{-\Omega}{i^* - \mu_1}(\tilde{k} - k_0) \tag{19h}$$

$$\tilde{a} - a_0 = \frac{\Phi}{i^* - \mu_1}(\tilde{k} - k_0) \tag{19i}$$

$$T = g_x + \tilde{\sigma} g_y + \tilde{\sigma} i^* \tilde{a} \tag{19j}$$

These equations jointly determine the steady–state equilibrium values of $\tilde{x}, \tilde{y}, \tilde{k}, \tilde{\ell}, \tilde{\sigma}, \tilde{q}, \tilde{n}, \tilde{a}$, and T.

Several aspects of this steady state merit comment. First, the steady state value of q is unity, consistent with the Tobin q theory of investment. Secondly, the steady–state marginal physical product of capital is equated to the exogenously given foreign interest rate, thereby determining the capital–labor ratio. Thirdly, equation (19g) implies that in steady–state equilibrium, the current account balance must be zero. Combined with (19e), we see that this requires the trade balance to offset the interest earnings on traded bonds. Equation (19h) describes the equilibrium relationship between the change in the stock of capital and the change in the net credit of the economy. Equation (19i) describes an analogous relationship between the stock of capital and the level of government debt, while (19j) determines the required lump sum tax, which will ensure that the steady–state government budget remains in balance.[10] Finally, the steady state depends upon the initial stocks of assets k_0, n_0, and a_0. It is this dependence upon initial conditions which is the source of the *temporary* fiscal (or other) shocks having *permanent* effects.

4. LONG–RUN EFFECTS OF FISCAL EXPANSIONS

The long–run effects of fiscal expansions, taking the forms of increases in government expenditure on domestic goods and on import goods respectively, are reported in Parts A and B of the Appendix and shall be discussed in turn.

A. Increase in Government Expenditure on Domestic Good

First, since the world interest rate i^* remains fixed, the marginal product condition (19f) implies that the long–run capital–labor ratio is a constant, independent of either g_x or g_y. Capital and labor therefore change in the same proportions, so that the marginal physical product of labor and hence the real wage rate also remains constant. An increase in g_x stimulates domestic output, thereby increasing employment and capital. The stimulus in demand through government expenditure may, or may not, exceed the addition to output and the relative price of the import good σ may either rise or fall. At the same time, the increase in the steady–state stock of capital leads to a decline in the steady–state stock of traded bonds held by the domestic economy. The increase in taxes necessary to finance the additional government expenditure, coupled with the reduction in net interest earnings by the economy, means a reduction in real disposable income. As a consequence, the private consumption of the two goods, $\tilde{x}$ and $\tilde{y}$ both decline. While the reduction in $\tilde{y}$ tends to raise the marginal utility $\bar{\lambda}$, the reduction in $\tilde{x}$ tends to lower it (since $U_{xy} > 0$). In general, the overall effect is unclear, although $\bar{\lambda}$ will certainly be increased if the utility function is additively separable in the two goods. Finally, we see from (19i) that the long–run increase in $\bar{k}$ must lead to a steady– state increase in the stock of government debt.

Substituting (19e) into (19g), the equilibrium trade balance is given by

$$Z(\tilde{\sigma}) - \tilde{\sigma}(\tilde{y} + g_y) = -\tilde{\sigma} i^* \tilde{n}$$

The fiscal expansion on the domestic good raises the equilibrium trade balance, when measured in terms of the foreign good $(-i^*\tilde{n})$. It will do the same, even more strongly, in terms of the domestic good, as long as the domestic economy is a net creditor nation ($\tilde{n} > 0$). However, for a debtor country, the trade balance in terms of domestic goods may fall, if the relative price effect is sufficiently strong.

B. Increase in Government Expenditure on Import Good

The long-run effects on domestic activity, as measured by employment, capital, and output, may all be either expansionary or contractionary. What is going on is the following. The increase in government expenditure on the import good raises its relative price, thereby stimulating the demand for the domestic good, and this is expansionary. But at the same time, the increase in lump sum taxes necessary to finance the additional expenditure reduces disposable income, reducing private expenditure on the domestic good (without any corresponding increase in public expenditure on that good), and this is contractionary. The net impact on domestic activity depends upon which effect dominates. In addition, the reduction in disposable income lowers the private consumption of the import good as well. The response of the marginal utility $\bar{\lambda}$ is an unambiguous increase. This is because the government expenditure impacts more on the private consumption of the import good $\tilde{y}$ than on the domestic good $\tilde{x}$, with the result that the increase in the marginal utility resulting from the fall in the former outweighs the decrease stemming from the fall in the latter.

The response of the equilibrium level of national credit $\tilde{n}$ and the equilibrium level of government debt $\tilde{a}$, both depend upon whether the long-run effect of this form of fiscal expansion is expansionary or contractionary. In the former case, $\tilde{n}$ will fall and $\tilde{a}$ will rise; in the latter case the reverse occurs. What happens to the trade balance as measured in terms of the foreign good, depends upon what happens to $\tilde{n}$. In the expansionary case it will rise, while in the contractionary case it will fall. In terms of the domestic good, the relative price effect also needs to be taken into account.

5. TRANSITIONAL DYNAMICS

As discussed in Section 3, the dynamics of k and q are described by a saddlepoint in $k-q$ space. The stable arm XX (in Fig. 1.A) is given by

$$q = 1 + \frac{\mu_1}{I'}(k - \tilde{k}) \tag{20a}$$

and is negatively sloped; the unstable arm (not illustrated), is described by

$$q = 1 + \frac{\mu_2}{I'}(k - \tilde{k}) \tag{20b}$$

and is positively sloped. The effects of the two types of fiscal expenditure shall be considered in turn.

A. Government Expenditure on Domestic Good

As long as no future change is anticipated, the economy must lie on the stable locus XX. The initial jump in $q(0)$, following an unanticipated permanent increase in g_x is

$$\frac{dq(0)}{dg_x} = \frac{-\mu_1}{I'}\frac{d\tilde{k}}{dg_x} > 0 \tag{21}$$

The long–run increase in the capital stock thus gives rise to a short–run increase in the shadow price $q(0)$.

The dynamics following an unanticipated permanent increase in g_x is illustrated in Fig. 1.A and 1.B. Part A describes the adjustment in q and k, while Part B describes the evolution of net credit. Suppose that the economy starts in steady–state equilibrium at the point P on the stable arm XX and that there is a permanent increase in g_x. The new steady state is at the point Q, with the higher equilibrium capital stock $\tilde{k}$, and an unchanged shadow price of capital $\tilde{q} = 1$. In the short run, q jumps from P to A on the new stable locus $X'X'$. From (3d) it is seen that the increase in q has an immediate expansionary effect on investment and capital begins to accumulate.

The initial responses of other key variables are

$$\frac{d\ell(0)}{dg_x} = \frac{\partial \ell}{\partial g_x} + \frac{\partial \ell}{\partial \bar{\lambda}}\frac{\partial \bar{\lambda}}{\partial g_x} + \frac{\partial \ell}{\partial q}\frac{dq(0)}{dg_x} > 0 \tag{22a}$$

$$\frac{d\sigma(0)}{dg_x} = \frac{\partial \sigma}{\partial g_x} + \frac{\partial \sigma}{\partial \bar{\lambda}}\frac{\partial \bar{\lambda}}{\partial g_x} + \frac{\partial \sigma}{\partial q}\frac{dq(0)}{dg_x} < 0 \tag{22b}$$

$$\frac{dx(0)}{dg_x} = \frac{\partial x}{\partial g_x} + \frac{\partial x}{\partial \bar{\lambda}}\frac{\partial \bar{\lambda}}{\partial g_x} + \frac{\partial x}{\partial q}\frac{dq(0)}{dg_x} < 0 \tag{22c}$$

$$\frac{dy(0)}{dg_x} = \frac{\partial y}{\partial g_x} + \frac{\partial y}{\partial \bar{\lambda}}\frac{\partial \bar{\lambda}}{\partial g_x} + \frac{\partial y}{\partial q}\frac{dq(0)}{dg_x} < 0 \tag{22d}$$

which consist of two channels of influence. First, there are the direct effects, consisting of the partial derivatives such as $\frac{\partial \ell}{\partial g_x}$ and discussed in Section 3. Secondly, there are the indirect effects, which operate through induced jumps in $\bar{\lambda}$ and q.

Despite the fact that the various effects may, or may not, work in the same direction (and in fact the effects through $\bar{\lambda}$ are ambiguous), we are able to establish that overall a permanent increase in g_x will have the same qualitative effects on employment and consumption in the short run, as it does in the steady state. Namely it will raise employment, while reducing the consumptions of the two goods. In addition it will lower the relative price σ. How the magnitudes of these short-run responses compare with the long run depends upon whether the short-run effects resulting from the rise in the shadow price of investment $q(0)$ dominates the long-run effects stemming from the eventual increase in the capital stock.

From equation (9d') and the fact that upon reaching the point A in Fig. 1.A on the new stable locus, $\dot{q} < 0, \dot{k} > 0$, we can infer that in the short run and during the subsequent transition, $\dot{\sigma} > 0$, i.e. that the relative price of the import good must be increasing. This means that the short-run fall in the relative price overshoots its long-run response. At the same time, the fact that $\dot{\sigma} > 0$ means that the fiscal expansion initially raises the domestic interest rate above the fixed real world rate.[11]

Differentiating (9a), (9b) analogously with respect to t, one can show using a similar argument that during the transition $\dot{x} > 0, \dot{y} > 0$, so that these consumptions also overreact in the short run. In both cases, the shadow price of investment effect dominates. In the case of employment, however, we are unable to determine the relative sizes of the short–run and long–run adjustments.

Critical to these adjustments is the endogeneity of employment. To see this, consider the steady–state relationships (19) and assume instead that employment is fixed, so that the optimality condition (19c) is no longer applicable. The marginal productivity condition (19f) now implies that the equilibrium stock of capital (rather than the capital–labor ratio) is determined exogenously by i^* and is independent of g_x. It therefore follows from (11a), (11b) that the capital stock and the shadow price of investment remain constant at all points of time. Output is therefore unchanged. There are no dynamics and all that happens is that the fiscal expansion leads to a once–and–for–all decline in the relative price σ and the private consumptions x, and y.[12]

Part B of Fig. 1 illustrates the relationship between n and k, which combining (11a) and (13) is

$$n(t) - \tilde{n} = \frac{-\Omega}{i^* - \mu_1}(k(t) - \tilde{k})$$

This is a negatively sloped line, denoted by ZZ. Since $\frac{d\tilde{n}}{dg_x} = -[\frac{\Omega}{(i^* - \mu_1)}]\frac{d\tilde{k}}{dg_x}$ this line remains fixed. The movement along A to Q in Fig. 1.A corresponds to a movement along LM in Fig. 1.B. From this figure we see that an increase in government expenditure on the domestic good leads to an immediate decumulation of foreign bonds. This is brought about by the fact that the increase in g_x leads to an immediate reduction in the relative price σ, which with $\Omega > 0$ creates an immediate current account deficit. With the stock of traded bonds being predetermined, the trade balance, measured in terms of the foreign good also falls, and with the fall in σ, the trade balance in terms of domestic goods falls even more. Over time, the initial decumulation of foreign bonds is reversed. This occurs through the rising relative price σ, which causes the trade deficit to decline over time.

Consider now a temporary increase in g_x. Specifically, suppose that at time 0 the government increases its expenditure on the domestic good, but is expected to restore its expenditure to its original level at time T. The transitional adjustment is now as follows.[13] As soon as the increase in g_x occurs, the stable arm XX will shift up instantaneously (and temporarily) to X'X', while the shadow price q increases to the point B, which lies below X'X', at which point the initial rate of capital accumulation is moderated. The same is true of employment. As is the case for a permanent expansion, the initial increase in (iq) is less than the increase in the marginal physical product of capital resulting from the additional employment, so that q begins to fall; see (3f). Moreover, the accumulation of capital is accompanied by a decumulation of traded bonds. Hence immediately following the initial jump, q and k follow the path BC in Fig. 1.A, while k and n follow the corresponding path LH in Fig. 1.B. At time T, when the level of government expenditure is restored to its original level, the stock of capital and traded bonds will have reached a point such as H in Fig. 1.B. The accumulated stocks of these assets, denoted by k_T and n_T respectively, will now serve as initial conditions for the dynamics beyond time T when g_x reverts permanently to its original level. As noted in Section 3, they will therefore in part determine the new steady state equilibrium. With no new information being received at time T (since the temporary nature of the fiscal expansion was announced at the outset), and no further jumps, the stable locus relevant for subsequent adjustments in q and k beyond time T is the locus X''X'', parallel to XX which passes through the point $k = k_T$. Likewise, the relevant locus linking the accumulation of capital and traded bonds is now Z'Z'.

After time T, q and k follow the stable locus CR in Fig. 1.A to the new steady state equilibrium at R, while correspondingly k and n follow the locus HN in Fig. 1.B to the new equilibirum point N. One can establish formally that X''X'' lies above the original stable locus XX, while Z′Z′ lies below ZZ, as these curves have been drawn. In the new steady state, the shadow price q reverts to 1, but with a higher stock of capital and a lower stock of traded bonds than originally. The striking feature of the adjustment is that the temporary increase in government expenditure leads to a permanent increase in the stock of capital, accompanied by a lower stock of traded bonds. This is because during the transitional adjustment period, during which the fiscal expansion is in effect, the accumulation of capital and bonds will influence subsequent initial conditions, which in turn will affect the subsequent steady state.[14]

B. <u>Government Expenditure on Import Good</u>

The initial response of $q(0)$ to a fiscal expansion taking the form of an increase in government expenditure on the import good g_y is given by

$$\frac{dq(0)}{dg_y} = -\frac{\mu_1}{I'}\frac{d\tilde{k}}{dg_y} \gtrless 0 \tag{23}$$

and depends upon whether the long-run effect on the capital stock is expansionary or contractionary. In the former case, the dynamics are essentially as illustrated in Fig. 1. There is an initial stimulus to investment, leading to a long-run accumulation of capital, and decumulation of traded bonds. In the latter case, the adjustment paths are as depicted in Fig. 2. The fiscal expansion now generates an initial drop in the shadow price $q(0)$, leading to a long- run decumulation of capital, accompanied by an accumulation of traded bonds.

The initial responses of ℓ, σ, x, and y are given by

$$\frac{d\theta(0)}{dg_y} = \frac{\partial\theta}{\partial\bar{\lambda}} + \frac{\partial\theta}{\partial q}\frac{dq(0)}{dg_y} \qquad \theta \equiv \ell, \sigma, x, y \tag{24}$$

In contrast to g_x, there is no direct effect; government expenditure on the import good operates entirely through $\bar{\lambda}$ and q. This is a consequence of our assumption of additive separability of utility in private and public consumption.

By a parallel argument to that given above, we can show the short–run response of the relative price σ, overadjusts in the direction of the long–run response. In the case that the long–run effect of the fiscal expansion is expansionary, the initial fall in $\sigma(0)$ exceeds the corresponding long–run reduction in $\bar{\sigma}$. On the other hand, if the long–run effect of g_y is contractionary, σ overincreases in the short run. In this case, the fact that $\dot{\sigma} < 0$ over time as the capital stock falls and the shadow price of investment increases, means that the domestic interest rate falls below the world rate during the transition. The overresponse of x and y is also true when g_y is expansionary. But in the contractionary case this is not necessarily so. In this case, x and y may actually increase on impact, though they will thereafter fall continuously to their lower equilibrium levels. Finally, the short–run employment effect is unclear. This is in part due to the fact that one effect of the higher g_y is to raise the marginal utility $\bar{\lambda}$, the effects of which on employment are ambiguous.

The relationship between the current account and the accumulation of traded bonds is analogous to that already given and requires no further discussion. The same applies with respect to the analysis of temporary increases. This is illustrated in Fig. 2 and should be self–explanatory.

6. WELFARE EFFECTS

Thus far, we have been describing the adjustment of the economy to the various fiscal expansions. Of particular relevance is the impact of such shocks on the welfare of the representative agent in the economy. This can be conveniently analyzed by considering the effect on the intertemporal utility function (with $\delta = i^*$):

$$\Phi \equiv \int_0^\infty [U(x,y) + V(\ell) + W(g_x, g_y)]e^{-i^* t}dt. \tag{25}$$

In this section we briefly consider the welfare effects of permanent fiscal expansions, when consumptions x, y and employment ℓ follow the solutions given by (9a) - (9c), where k and q evolve in accordance with the dynamic paths (11a), (11b).[15]

The evaluation is based on a linear approximation to Ω. To obtain this we first linearize the instantaneous utility function $U(\cdot)+V(\cdot)$ about steady state

$$U(x,y)+V(\ell)\cong U(\tilde{x},\tilde{y})+V(\tilde{\ell})+U_x(x-\tilde{x})+U_y(y-\tilde{y})+V'(\ell-\tilde{\ell})$$

where the marginal utilities U_x, U_x, V' are all evaluated at steady state. The transitional paths followed by consumption and employment may be linearly approximated by

$$x(t)\cong\tilde{x}+(x(0)-\tilde{x})e^{\mu_1 t} \tag{26a}$$

$$y(t)\cong\tilde{y}+(y(0)-\tilde{y})e^{\mu_1 t} \tag{26b}$$

$$\ell(t)\cong\tilde{\ell}+(\ell(0)-\tilde{\ell})e^{\mu_1 t}. \tag{26c}$$

With g_x, g_y constant, one can then readily show that welfare Φ may be linearly approximated by

$$\begin{aligned}\Phi\cong\frac{1}{i^*}[U(\tilde{x},\tilde{y})+V(\tilde{\ell})+W(g_x,g_y)]+\frac{U_x[x(0)-\tilde{x}]}{i^*-\mu_1}\\+\frac{U_y[y(0)-\tilde{y}]}{i^*-\mu_1}+V'\frac{[\ell(0)-\tilde{\ell}]}{i^*-\mu_1}.\end{aligned} \tag{27}$$

The first term in (27) represents the level of welfare, if the steady state were attained instantaneously. The remaining terms reflect adjustments to this, due to the fact that the steady state is reached only gradually along the transitional paths followed by consumption and employment. It is possible to express this linearized solution in terms of the time path of the capital stock, but for present purposes this is not necessary.

Using the equilibrium conditions (3a) - (3c), the effect of an increase in government expenditure on good $i(i=x,y)$ on total welfare is given by

$$\frac{d\Phi}{dg_i} = \frac{1}{i^*}\frac{\partial W}{\partial g_i} - \frac{\mu_1 U_x}{i^*(i^* - \mu_1)}\left[\frac{d\tilde{x}}{dg_i} + \sigma\frac{d\tilde{y}}{dg_i} - F_\ell\frac{d\tilde{\ell}}{dg_i}\right]$$
$$+ \frac{U_x}{i^* - \mu_1}\left[\frac{dx(0)}{dg_i} + \sigma\frac{dy(0)}{dg_i} - F_\ell\frac{d\ell(0)}{dg_i}\right] \qquad i = x, y \tag{28}$$

This is seen to consist of two types of effects. The first is the direct effect, $\frac{\partial W}{\partial g_i}$, which being permanent, is capitalized at the constant rate i^*. This effect is welfare improving. The second are indirect effects which operate through private consumption and employment. These in turn comprise the steady–state effects and those along the transitional path. Because the adjustment path can be characterized in terms of the initial points and steady state (see (26)), these effects can be expressed in terms of changes in the initial consumptions $x(0), y(0)$, and employment $\ell(0)$, and the corresponding steady–state quantities $\tilde{x}, \tilde{y}$, and $\tilde{\ell}$.[16]

Since an increase in government expenditure on the domestic good g_x lowers the consumption of both goods and raises employment (lowers leisure), both initially and in the steady state (and therefore along the entire adjustment path), we can see from (28) that what we have called the indirect effects are negative. Thus whether an increase in g_x is welfare improving or not depends upon whether these dominate the positive direct effects. The optimal level of government expenditure on the domestic good is obtained by equating the discounted direct marginal gains to the marginal losses resulting from the crowding out of private consumption and leisure.[17]

The welfare effects of an increase in g_y are generally similar. Even in the case where increased expenditure on the import good is contractionary, so that employment declines, the welfare improving increase in leisure is dominated by the welfare deteriorating reduction in private consumption. This net welfare loss must then be weighed up against the positive direct effect, as before.

7. CONCLUSIONS

This paper has considered the impacts of both permanent and temporary changes in government expenditures on various key macroeconomic variables. The main results of the analysis can be summarized as follows.

When the increase in government expenditure is directed towards the domestic good, the adjustment in the domestic economy is essentially expansionary. Employment and output are stimulated both in the short run and over time. Despite the transitional rise in the domestic real interest rate, investment is stimulated by the increase in q, leading to a higher equilibrium stock of capital. In addition, the increase in demand for the domestic good lowers the relative price of the imported good, leading to initial trade and current account deficits. Over time, the relative price of the imported good rises, the current account deficit falls, and in the long run, the initial trade deficit is transformed to a surplus of sufficient magnitude to finance the interest payments on the debt accumulated during the transition. Along the transitional adjustment path consumption and leisure are both below their respective initial starting levels leading to reductions in welfare. These losses need to be offset against any direct gains resulting from the additional government expenditure, in assessing the net benefits.

In the case where the government expenditure is directed towards the import good, the dynamic adjustment of the economy depends critically upon the long–run response of the capital stock. Here two effects are in operation. While the rise in the relative price of the import good is expansionary, the increase in lump sum taxes required to finance the additional government imports is contractionary. If the former effects dominates, the dynamic adjustments are much the same as when the expenditure is on the domestic good. However, if the latter effect prevails, the adjustments are reversed. There is a reduction in output and employment, capital decumulates, the current account is in surplus, and the interest rate falls below its steady–state equilibrium level.

But perhaps the most interesting conclusion of the analysis is that *temporary* fiscal expansions will give rise to *permanent* effects. These are qualitatively the same as the corresponding responses to permanent expansions, although their magnitudes are dampened appropriately. They stem from the fact that the steady state depends upon the initial stocks of assets. As a consequence, temporary fiscal expansions, by altering these initial conditions for some later date when the expansions cease, lead to permanent effects on the economy. Repeated shocks will therefore generate random walk behavior. This conclusion is very relevant for recent discussions of hysteresis. Recent studies have suggested that temporary real and monetary shocks have led to permanent effects on employment and unemployment in various European countries.[18] Existing models explaining these phenomena have done so in terms of union power over wages. The fact that they can emerge from the competitive intertemporal optimizing framework adopted in this paper provides them with an alternative and arguably a more rigorous theoretical underpinning. The dependence of the steady state upon the stocks of initial assets, which we have noted is the cause of this result, is in turn generated by the assumption that the domestic consumer's time discount rate equals the exogenously determined foreign real interest rate.[19] This assumption, necessary for a steady state equilibrium in this model to exist, does not seem an unreasonable one for small open economies such as the United Kingdom and France, for which the hysteresis phenomenon has been observed.

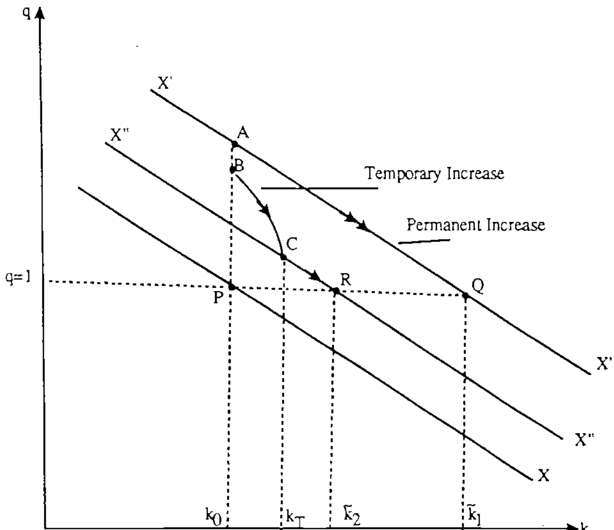

FIGURE 1.A

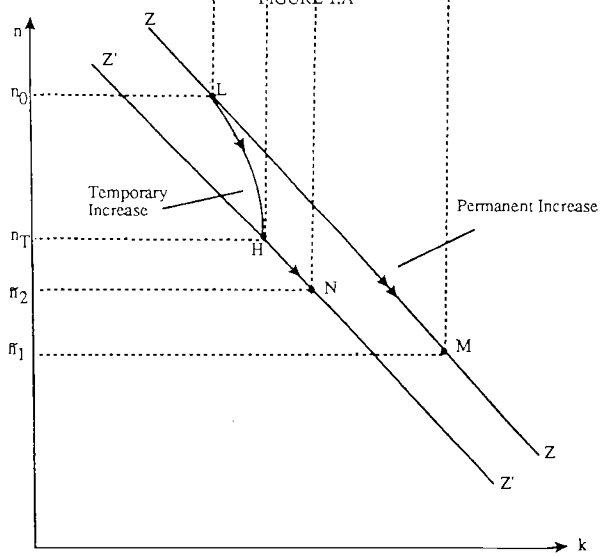

FIGURE 1.B

INCREASE IN GOVERNMENT EXPENDITURE ON EXPORT GOOD

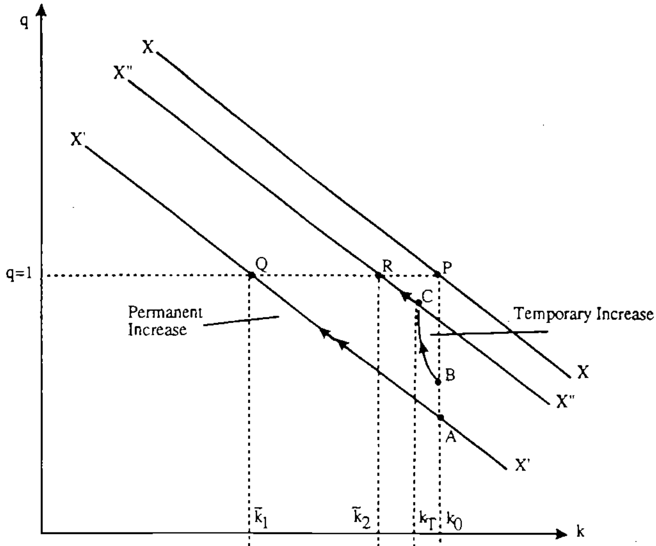

FIGURE 2.A

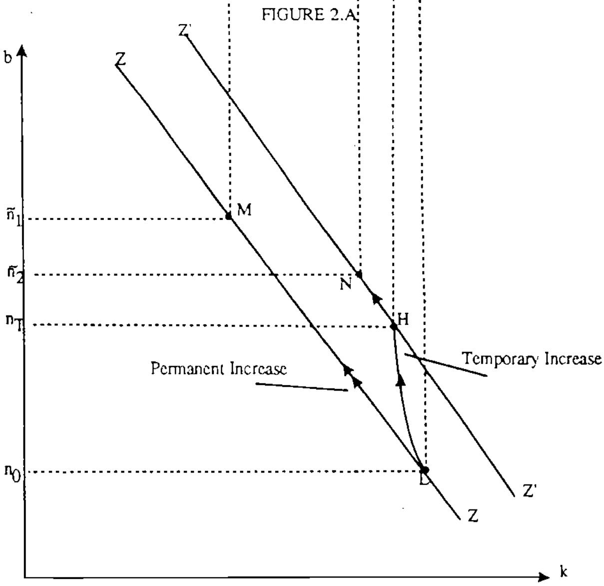

FIGURE 2.B

INCREASE IN GOVERNMENT EXPENDITURE ON IMPORT GOOD:
CONTRACTIONARY CASE

APPENDIX

LONG RUN EFFECTS OF INCREASE IN GOVERNMENT EXPENDITURES

A. Domestic Good g_x

1. Capital–Labor Ratio:

$$\frac{d(\tilde{k/\ell})}{dg_x} = 0$$

2. Capital, Employment and Output:

$$\frac{1}{k}\frac{d\tilde{k}}{dg_x} = \frac{1}{\ell}\frac{d\tilde{\ell}}{dg_x} = \frac{1}{z}\frac{d\tilde{z}}{dg_x} = \frac{F_\ell}{\sigma \ell D}[\beta\Delta - \lambda U_{xx}] > 0.$$

3. Relative Price:

$$\frac{d\tilde{\sigma}}{dg_x} = \frac{1}{D}[V''[U_{xy} - \sigma U_{xx}] - \frac{\Delta\psi F_\ell}{\sigma}] \gtrless 0.$$

4. Consumption of Domestic Good:

$$\frac{d\tilde{x}}{dg_x} = \frac{1}{D}[\frac{V''\beta}{\sigma}[\sigma U_{xy} - U_{yy}] + \frac{\lambda}{\sigma}[V'' + U_{xy}\frac{F_\ell\psi}{\sigma}]] < 0.$$

5. Consumption of Imported Good:

$$\frac{d\tilde{y}}{dg_x} = \frac{1}{D}[\frac{V''\beta}{\sigma}[U_{xy} - \sigma U_{xx}] - \frac{\lambda}{\sigma^2}U_{xx}F_\ell\psi] < 0.$$

6. Marginal Utility:

$$\frac{d\tilde{\lambda}}{dg_x} = \frac{1}{D}[U_{xy}V''\frac{\tilde{\lambda}}{\sigma} - \Delta(\beta V'' + \psi F_\ell \frac{\tilde{\lambda}}{\sigma^2})] \gtrless 0.$$

7. Net Credit:

$$\frac{d\tilde{n}}{dg_x} = \frac{-\Omega}{i^* - \mu_1}(\frac{d\tilde{k}}{dg_x}) < 0.$$

8. Net Government Debt:

$$\frac{d\tilde{a}}{dg_x} = \frac{\Phi}{i^* - \mu_1}(\frac{d\tilde{k}}{dg_x}) > 0.$$

where

$$\psi \equiv \frac{-\sigma i^* \Omega}{i^* - \mu_1}(\frac{\tilde{k}}{\tilde{\ell}}) < 0; \quad \beta \equiv Z' - \frac{Z}{\sigma} > 0; \quad \Delta \equiv U_{xx}U_{yy} - U_{xy}^2 > 0.$$

$$D \equiv -V''[U_{xy}Z' + \frac{\bar{\lambda}}{\sigma} - Z'U_{xx}\sigma] - V''\beta[U_{xy} - \frac{1}{\sigma}U_{yy}]$$

$$-F_\ell \psi U_{xy}\frac{\bar{\lambda}}{\sigma^2} - F_\ell \frac{F}{\ell}U_{xx}\frac{\bar{\lambda}}{\sigma} + \Delta\frac{F_\ell}{\sigma}(\frac{F\beta}{\ell} + \psi Z') > 0.$$

B. <u>Import Good g_y</u>

1. Capital–Labor Ratio:

$$\frac{d(\tilde{k/\ell})}{dg_x} = 0.$$

2. Capital, Employment, and Output

$$\frac{1}{k}\frac{d\tilde{k}}{dg_y} = \frac{1}{\ell}\frac{d\tilde{\ell}}{dg_y} = \frac{1}{z}\frac{d\tilde{z}}{dg_y} = \frac{F_\ell}{\sigma\ell D}[Z'\Delta - \bar{\lambda}U_{xy}] \gtrless 0.$$

3. Relative Price:

$$\frac{d\tilde{\sigma}}{dg_y} = \frac{1}{D}V''(U_{yy} - \sigma U_{xy}) + \Delta F\frac{F_\ell}{\ell}] > 0.$$

4. Consumption of Domestic Good

$$\frac{d\tilde{x}}{dg_y} = \frac{1}{D}[V''Z'[\sigma U_{xy} - U_{yy}] - \frac{\bar{\lambda}}{\sigma}U_{xy}F\frac{F_\ell}{\ell}] < 0.$$

5. Consumption of Imported Good

$$\frac{d\tilde{y}}{dg_y} = \frac{1}{D}[V''Z'[U_{xy} - \sigma U_{xx}] + \frac{\bar{\lambda}}{\sigma}[V'' + F\frac{F_\ell}{\ell}\frac{\bar{\lambda}}{\sigma}U_{xx}]] < 0.$$

6. Marginal Utility:

$$\frac{d\bar{\lambda}}{dg_y} = \frac{1}{D}[U_{yy}V''\frac{\lambda}{\sigma} + \Delta[\frac{F}{\ell}F_\ell\frac{\lambda}{\sigma} - V''Z'\sigma]] > 0.$$

7. Net Credit:

$$\frac{d\tilde{n}}{dg_y} = \frac{-\Omega}{i^* - \mu_1}\left(\frac{d\tilde{k}}{dg_y}\right) \gtrless 0.$$

8. Net Government Debt:

$$\frac{d\tilde{a}}{dg_y} = \frac{\Phi}{i^* - \mu_1}\left(\frac{d\tilde{k}}{dg_y}\right) \gtrless 0.$$

FOOTNOTES

*The constructive suggestions from the referees and the Managing Editor, Peter Sinclair, are gratefully acknowledged.

[1]By semi–small we mean that the economy is able to influence the price of its export goods.

[2]This approach to investment is also adopted by Buiter (1987). His analysis of the semi–small economy is based on numerical simulations; the model developed in this paper is investigated entirely analytically. For other recent applications of the cost of adjustment approach to investment to the analysis of alternative macro disturbances in open economies see e.g., Matsuyama (1987), Brock (1988), Sen and Turnovsky (1989).

[3]This is in contrast to a consideration of steady–state utility, which neglects welfare along the transitional adjustment path.

[4]For a discussion of this issue in the context of tariffs see Sen and Turnovsky (1989).

[5]Throughout the paper we shall adopt the following notation. Where appropriate, primes shall denote derivatives, subscripts shall denote partial derivatives, and a dot shall denote a derivative with respect to time.

[6]This formulation of the installation function follows the original specification of adjustment costs introduced by Lucas (1967) and Gould (1968). More recent work by Hayashi (1982) and others postulates an installation function which depends upon k as well as I. This modification makes little difference to our analysis and for simplicity we retain the simpler specification.

[7]Note that this specification implies that in the case where disinvestment may occur $I < 0, C(I) < 0$ for low rates of disinvestment. This may be interpreted as reflecting the revenue obtained as capital is sold off. The possibility that all changes in capital are costly can be incorporated by introducing sufficiently large fixed costs, so that $C(0) > 0$. This does not alter our analysis in any substantive way.

[8]In the case where the installation cost function is homogeneous of degree 1 in I and k, the investment function implied by (3d) is modified to

$$\frac{I}{k} = G(q).$$

[9]Note, the (1-1) element in the matrix (10) is obtained as follows. In general, following the procedure outlined it is equal to $\theta[i^* + \sigma k I_q/\sigma - F_{k\ell}\ell_q]$. Evaluating the partial derivatives σ_k, I_q, ℓ_q and noting the steady–state conditions given in (19), this expression evaluated at steady state reduces to i^*.

[10]We should note that the adjustment of some form of tax is necessary in order for the steady–state equilibrium to be sustainable. If instead of being lump sum, taxes were distortionary, then the appropriate

rate consistent with (19j) would need to be set. This chosen rate would of course impact on the decisions of the representative agent.

[11]These results imply that in spite of a higher (but declining) interest rate crowding in of investment occurs.

[12]Actually it is the assumption of endogeneity of employment in conjunction with infinitely-lived agents that is critical. This is because this gives rise to short-run dynamics which are driven by long-run changes in the capital stock alone; see equations (9), (11). This may be compared to the Buiter (1987) model, for example, where employment is fixed, but consumers have finite lives. In this case, the long-run capital stock is also independent of everything other than the foreign interest rate, and is therefore also independent of domestic fiscal policy. However, in contrast to the present analysis, temporary changes in the capital stock may still occur. This is due to the fact that the short-run dynamics are also driven by long-run changes in other forms of financial wealth which may be generated by changes in fiscal policy.

[13]The formal derivations of these adjustment paths are omitted, but are available from the authors on request.

[14]As the figures are drawn, C lies above R and H lies above N, respectively. The complete adjustment paths BCR and LHN are therefore monotonic. We are unable to rule out the possibility of C lying below R and H lying above N, in which case, the accumulation of capital and decumulation of bonds would be reversed at some point during the transition. In any event, the temporary increase in the relative price of domestic goods generates an initial current account deficit, which continues as long as capital is being accumulated.

[15]The welfare effects of temporary shocks can be analyzed similarly. They are a little more complicated due to the fact that part of the transition is along an unstable path of higher dimension.

[16]It is possible to express all the changes in consumption and employment appearing in (28) in terms of changes in the long-run equilibrium capital stock. However, this does not turn out to be particularly illuminating.

[17]In this regard a referee has raised the following point. If the government chooses an optimal level of expenditure, agents having perfect foresight will anticipate this level of expenditure and consequently no unanticipated changes in expenditure can occur. This point is one which is frequently made in conjunction with having unanticipated shocks in models of perfect foresight. The difficulty would appear to be largely a semantic one. Operationally, the assumption of perfect foresight, as we are using it, pertains to a consideration of the forward looking solution to the dynamic system describing the economy. It does not require that discrete unanticipated exogenous events be exactly predicted; indeed by their nature they typically will not

be.

[18]See Blanchard and Summers (1986), Lindbeck and Snower (1987).

[19]More fundamentally, the necessity of the requirement that $\delta = i^*$ for the existence of a steady-state equilibrium is a consequence of three assumptions: (i) an infinitely-lived representative agent; (ii) perfect capital markets; (iii) constant rate of time preference. If any of these assumptions is dropped, the equality between δ and i^* is no longer required at all times for a steady state to obtain. In this case the model will generally not give rise to hysteresis.

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