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Abstract

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Reference

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Fitting oscillating string gas cosmology to supernova data

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arXiv:0812.4182v2 [hep-th] 2 Apr 2009

Contents

1. Introduction

String gas cosmology. String gas cosmology is a cosmological scenario motivated by string theory [1–3] (see [4,5] for reviews and [6] for another perspective). In the original formulation of string gas cosmology, all spatial dimensions are treated on an equal footing: they are all toroidal and start out at the string size. The aim is that dynamical processes in the early universe will allow only three dimensions to expand to macroscopic size, while the extra dimensions are stabilised at the string size by a gas of strings. Assuming that the dilaton is stabilised by some other mechanism, the string gas can stabilise the extra dimensions during the radiation-dominated era [7,8] (see also [1,9–16]). However, when the universe becomes matter-dominated, the matter will push the extra dimensions to open up [1, 7, 8, 17]. It was shown in [18] that the gas of strings can still prevent the extra dimensions from growing too large, but they cannot be completely stabilised. There is a competition between the push of matter and the pull of strings. If the number density of the strings is too small, the extra dimensions will grow to macroscopic size. If the strings win, the size of the extra dimensions will undergo damped oscillations around the self-dual radius. The oscillations between expansion and contraction of the extra dimensions induce oscillations in the expansion rate of the large dimensions, which can involve alternating periods of acceleration and deceleration [18]. (This kind of a mechanism has also been studied in [19].)

Since the oscillations can start only after the universe becomes matter-dominated, they provide an in-built mechanism for late-time acceleration in string gas cosmology, one that alleviates the coincidence problem in a manner similar to scaling and tracker fields [20, 21]. The mechanism is based on ingredients already present in string gas cosmology and does not require adding new degrees of freedom or turning on new

interactions. However, the oscillating expansion history is quite different from the ΛCDM model which is known to be a good fit to the observations. (For comparison of some oscillating models to observations, see [22, 23].)

We compare the model studied in [18] to observations of type Ia supernovae (SNe Ia), using the Big Bang Nucleosynthesis (BBN) constraint on new radiation degrees of freedom. In section 2 we describe the string gas model, and in section 3 we fit the model to the Union and ESSENCE sets of SNIa data. The distances predicted by the model provide an acceptable fit to the Union data, but the fit to the ESSENCE data is poor. In section 4 we summarise our results and discuss how to make the model more realistic.

2. The string gas model

The metric and the equation of motion. We consider the string gas model discussed in [18]. The spacetime is ten-dimensional, with the metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} dx^i dx^i + b(t)^2 \sum_{j=1}^{6} dx^j dx^j \, , \tag{2.1}$$

where $i = 1 \ldots 3$ labels the visible dimensions and $j = 1 \ldots 6$ labels the extra dimensions. All spatial dimensions are taken to be toroidal. We take the value $b = 1$ to correspond to extra dimensions at the self-dual radius given by the string length $l_s \equiv \sqrt{\alpha'}$.

We assume that the dilaton has been stabilised [10–14] in a way that leaves the equation of motion of the metric unconstrained, so that it reduces to the Einstein equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} \, , \tag{2.2}$$

where $G_{\mu\nu}$ is the Einstein tensor, κ^2 is the 10-dimensional gravitational coupling and $T_{\mu\nu}$ is the energy-momentum tensor. (We take the cosmological constant to be zero.)

Given the symmetries of the metric (2.1), the energy-momentum tensor has the form

$$T^\mu_\nu = \mathrm{diag}(-\rho(t), p(t), p(t), p(t), P(t), P(t), P(t), P(t), P(t), P(t)) \, , \tag{2.3}$$

where p and P are the pressure in the visible dimensions and the extra dimensions, respectively. With (2.1) and (2.3), the Einstein equation (2.2) reads

$$\kappa^2 \rho = 3H_a^2 + 18H_a H_b + 15H_b^2 \tag{2.4}$$

$$\dot{H}_a + H_a^2 = -\frac{1}{6}\kappa^2(\rho + 3p) - \frac{3}{8}\kappa^2(\rho - 3p + 2P) + 6H_a H_b + 10H_b^2 \tag{2.5}$$

$$\kappa^2(\rho - 3p + 2P) = 8\dot{H}_b + 24H_a H_b + 48H_b^2 \, , \tag{2.6}$$

where $H_a \equiv \dot{a}/a$ is the expansion rate of the visible dimensions and $H_b \equiv \dot{b}/b$ is the expansion rate of the extra dimensions.

The distance. In Friedmann-Robertson-Walker models, the luminosity distance D_L is determined by the Hubble rate as a function of redshift and the spatial curvature at one time,

$$D_L = (1+z)\frac{1}{\Omega_{K0}H_{a0}}\sinh\left(\Omega_{K0}H_{a0}\int_0^z\frac{\mathrm{d}z'}{H_a(z')}\right) \ , \tag{2.7}$$

where Ω_K is the spatial curvature density parameter and the subscript 0 refers to the present day (see e.g. [24, 25]).

The metric (2.1) is not homogeneous and isotropic, so the relation (2.7) does not hold. The distance should instead be calculated from the general equation (the null geodesic shear has been neglected)

$$\partial_\lambda^2 D_A = -\frac{1}{2}G_{\mu\nu}k^\mu k^\nu D_A \ , \tag{2.8}$$

where $D_A = (1+z)^{-2}D_L$ is the angular diameter distance, ∂_λ is the derivative along the null geodesic and k^μ is the photon momentum (see e.g. [25]). We only consider light rays which propagate in the visible directions, and (2.8) reduces to

$$H_a\partial_z[(1+z)^2 H_a\partial_z D_A] = (\dot{H}_a + 3\dot{H}_b - 3H_aH_b + 3H_b^2)D_A \ . \tag{2.9}$$

Only if $H_b = 0$ can we integrate (2.9) to recover (the spatially flat case of) (2.7). The relation (2.7) was formulated as a consistency check for the FRW metric in [24]. String gas cosmology provides a concrete example of a model where the metric does not have the FRW form and the consistency condition is violated. In general, this is also the case in other models with dynamical extra dimensions. However, in models where the observers are confined to a brane, distances along the visible directions are calculated with the induced metric on the brane, and the evolution of the extra dimensions does not directly enter the light propagation equation (2.8).

The matter content. In addition to ordinary four-dimensional radiation (γ) and pressureless matter (m), we have a gas of massless strings (s) with winding and momentum modes in the extra dimensions and momentum modes in the visible dimensions. The contribution of radiation and matter to the energy-momentum tensor (2.3) is

$$\rho_\gamma = \rho_{\gamma,in}a^{-4}b^{-6} \ , \qquad p_\gamma = \frac{1}{3}\rho_\gamma \ , \qquad P_\gamma = 0 \tag{2.10}$$

$$\rho_m = \rho_{m,in}a^{-3}b^{-6} \ , \qquad p_m = 0 \ , \qquad P_m = 0 \ , \tag{2.11}$$

and for the string gas we have [18]

$$\rho_s = M^{-1}\rho_{s,in}a^{-3}b^{-6}\sqrt{M^2a^{-2} + b^{-2} + b^2 - 2} \tag{2.12}$$

$$p_s = \frac{1}{3}\frac{M^2a^{-2}}{M^2a^{-2} + b^{-2} + b^2 - 2}\rho_s \tag{2.13}$$

$$P_s = \frac{1}{6}\frac{b^{-2} - b^2}{M^2a^{-2} + b^{-2} + b^2 - 2}\rho_s \ , \tag{2.14}$$

where the subscript *in* refers to the initial values, and M is the initial momentum of a string in the visible directions in units of the string scale l_s^{-1}. Note that all strings are assumed to have the same momentum.

There are four parameters in the energy-momentum tensor: the scale M and the energy densities $\rho_{\gamma,in}, \rho_{m,in}$ and $\rho_{s,in}$. However, the parameter M only determines the absolute scale, and does not affect the dynamics, as we see by rescaling $a \to Ma$. The evolution of the system therefore depends on only two dimensionless combinations of the parameters, which we take to be the following:

$$r \equiv M^{-1}\frac{\rho_{\gamma,in}}{\rho_{m,in}}$$
$$f_s \equiv \frac{\rho_{s,in}}{\rho_{\gamma,in}} \ . \tag{2.15}$$

Now, with the rescaled a, the total energy density reads

$$\rho = \rho_{m,in}M^{-3}a^{-3}b^{-6}\left(1 + ra^{-1} + rf_s\sqrt{a^{-2} + b^{-2} + b^2 - 2}\right) \ , \tag{2.16}$$

and the pressures are written accordingly.

Deep in the radiation-dominated era (in particular, during BBN), the energy density of the string gas evolves like radiation, and contributes to the total energy density a fraction $\Omega_{s,in} = f_s/(1 + f_s)$, given that the contribution of matter is negligible and $b = 1$ in the radiation-dominated era. The string fraction f_s is related to the effective number of additional neutrino species ΔN_ν by $f_s = 7\Delta N_\nu/43$ [26]. From BBN we have, assuming negligible neutrino chemical potential, the constraint $\Delta N_\nu \leq 1.5$, giving $f_s \leq 0.24$, or $\Omega_{s,in} \leq 0.20$ [27]. Allowing for a large electron neutrino chemical potential, we have $\Delta N_\nu \leq 4.1$, which translates into $f_s \leq 0.7$, or $\Omega_{s,in} \leq 0.4$ [28]. The bound depends on the assumption that the gravitational coupling during BBN is the same as today, which is not necessarily true in the string gas model, since $G_N \propto b^{-6}$. If $b < 1$ today, the gravitational coupling at BBN is reduced relative to the present value, so there is more room for new degrees of freedom. However, generally b dips below unity only very slightly, and typically $b > 1$ today, so taking this into account would make the constraints tighter. It was observed in [18] that a requirement for the string gas being able to keep the extra dimensions small is $rf_s > 3/2$. There are no other constraints on r, since it depends on M, the initial momentum of the strings in the visible directions, on which there is no limit.

The string gas behaves like a scaling solution [20] in the radiation-dominated era and like a tracker solution [21] in the matter-dominated era [18]. The value $b = 1$ is an attractor point: as long as the initial value of b is not too large ($b < \sqrt{2}$ is a necessary condition), b will rapidly evolve to unity, and the extra dimensions are stable. Then the energy density of the string gas behaves exactly like radiation. When the universe becomes matter-dominated, the string gas starts tracking the matter as the extra dimensions expand. When the extra dimensions are pulled

back and contracted by the strings, the visible dimensions start oscillating between deceleration and acceleration. (If the string gas is too weak to prevent the extra dimensions from opening up, they will grow without bound, and there will be no acceleration in the visible dimensions. We are not interested in this possibility.)

3. Comparison with observations

The observations. We want to see how well the expansion history of the system of equations (2.4)–(2.6) with the energy density (2.16), and the distance given by (2.9), agrees with cosmological observations. The interpretation of many observations such as the cosmic microwave background (CMB) and the baryon acoustic oscillations requires perturbation theory. The effect of the string gas and the extra dimensions on the perturbation equations has not been completely worked out [29–31], so we will consider only observations which are independent of perturbations. Two important sets of observations which depend only on the background are luminosity distances of SNe Ia and the primordial abundance of light elements. The ages of passively evolving galaxies also provide a measurement of the Hubble parameter as a function of redshift independent of the distance scale [32]. In addition, there are local measurements of the Hubble parameter, the age of the universe and the matter density.

We will use the ESSENCE SNIa dataset [33] and the Union compilation [34] separately. The Union dataset is the newest and most comprehensive collection, but it has been analysed with the assumption that the ΛCDM model is correct. Therefore the results cannot, strictly speaking, be used to compare between cosmological models in an unbiased manner, especially in the case of models which are significantly different from ΛCDM, like the string gas model. Therefore, we also fit to the ESSENCE dataset, which has been analysed differently, for comparison. We find that, despite the bias in the Union analysis, the string gas model fits the Union dataset better than the ESSENCE data. This could be due to more conservative treatment of errors in the Union analysis.

We will also take into account the BBN constraint on new radiation degrees of freedom from the observed abundance of light elements [28]. We will not use the data on the ages of passively evolving galaxies, due to possible systematic effects related to stellar evolution. In addition to above data, a number of other general dark energy probes have also been suggested. In particular, the baryon acoustic oscillations [35] and the CMB shift parameter [36] have been considered as standard rulers. However, both of these probes suffer from model dependence and caution should be exercised when applying them to models other than ΛCDM [37, 38].

The supernova datafit. We use a grid method to scan the model parameter space (r, f_s), because the complicated confidence contour structure makes Monte Carlo Markov Chain methods ineffective. We refined the grid until, for a typical size
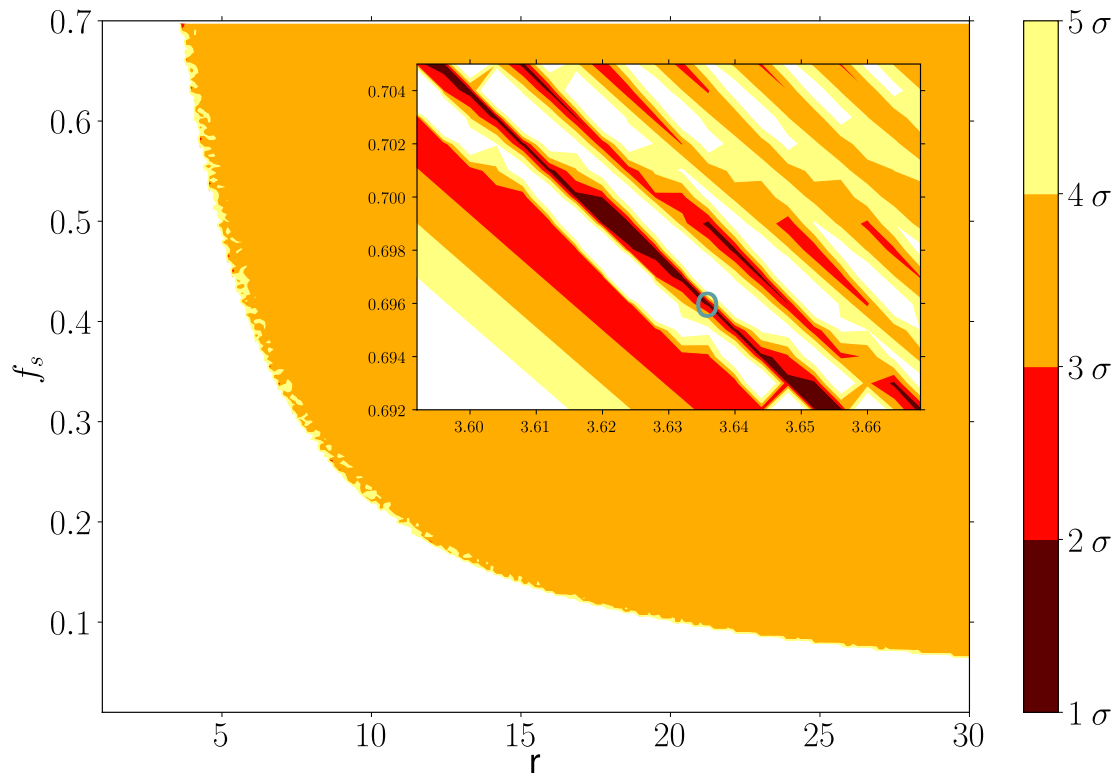
Figure 1: Confidence level contours in the (r, f_s)-plane for the Union dataset. The best-fit model is marked with a circle.

Model	Dataset	χ^2_{bf}	$p(\%)$	r	f_s	Remarks
ΛCDM	Union	308.3	44	-	-	$\Omega_\Lambda = 0.68$
ΛCDM	Essence	196.0	37	-	-	$\Omega_\Lambda = 0.75$
String gas	Union	329.8	15	3.636	0.696	$f_s \leq 0.7$
String gas	Union	317.6	28	0.893	3.246	$f_s \leq 9$
String gas	Essence	262.2	0.03	3.636	0.696	$f_s \leq 0.7$
String gas	Essence	234.5	1	0.833	3.497	$f_s \leq 9$

Table 1: Goodness-of-fit and best-fit parameters for ΛCDM and the string gas model for different datasets, with and without the BBN constraint.

of 400×400, the fit no longer improved significantly. In order to determine the best fit values, we further zoomed into regions with high values of χ^2. When we do not apply the BBN constraint $f_s < 0.7$, we restrict the scan to $f_s < 9$, corresponding to $\Omega_{s,in} < 0.9$.

In Figure 1 we plot the goodness-of-fit in the (r, f_s)-plane for the Union dataset (the behaviour is similar for the ESSENCE data). The inset shows the region around

the best-fit model, marked with a circle. The χ^2 contours have a striking structure. The lines of equal χ^2 are disjoint, and nearby points can have radically different values of goodness-of-fit. This is not an artifact of the analysis. (A complicated confidence level contour structure for an oscillating model was also found in [23].) In order to have enough acceleration in the visible dimensions at sufficiently late times, the present day has to be in a specific location, just after the rise of one of the first few oscillations. The details of the oscillations, in turn, depend on r and f_s in a complex manner. (Note that the late-time evolution depends on the parameter r only via the initial conditions, as the radiation term in the energy density (2.16) is negligible at late times.) Also, in order to have strong acceleration, the extra dimensions have to expand almost to the point of not turning back, and then contract rapidly. If the extra dimensions were to expand slightly more, they would not turn around, and there would be no acceleration. Therefore, the best fits are obtained on the border of very poor fits, as seen in Figure 1.

In Table 1 we give the parameter values as well as the χ^2 (and the corresponding probability p of obtaining the data given the model) for the best-fit string gas model with and without the BBN constraint for both datasets; values for the best-fit ΛCDM model are shown for comparison. Because of the complex dependence of the goodness-of-fit on r and f_s, we cannot definitely rule out the possibility that there would be a better fitting model somewhere in the regions that we did not completely study. In the patches that we did cover in detail, the quality of fit is already saturated at the scale visible in Figure 1 and does not improve when zooming to smaller regions.

For the Union dataset, the χ^2 for the best-fit string gas model without the BBN constraint is 9.3 points worse than for the ΛCDM model, and 21.5 points worse when the BBN constraint is taken into account. For the ESSENCE data, we have $\Delta\chi^2 = 40.2$ and 66.2, respectively. In Figure 2 we show the distance-redshift relation for the best-fit model to the Union dataset with and without the BBN constraint. The string gas behaviour is clearly different from the ΛCDM model, and provides a worse fit to the data, though for the Union data, the quality of the fit is still good. The string gas model would be further disfavoured if we took into account that it has one extra parameter compared to ΛCDM.

In Figure 3, we plot some quantities for the best-fit model to the Union dataset (with the BBN constraint included). In Figure 3 a), we show the density parameters of radiation, matter and the string gas. The energy density of the string gas is completely subdominant at late times, $\Omega_{s0} = 0.02$. However, the string gas can still have a large impact on the dynamics, because its energy-momentum tensor (2.12)–(2.14) violates the null energy condition. When the expansion is faster than in the Einstein-de Sitter case, the matter density parameter $\Omega_m \equiv \kappa^2 \rho_m/(3H_a^2)$ is smaller than unity, and in principle it could be in the observationally allowed range $\Omega_{m0} \approx$ 0.2–0.3 today. However, for the best-fit model we have $\Omega_{m0} = 0.73$, far too large.
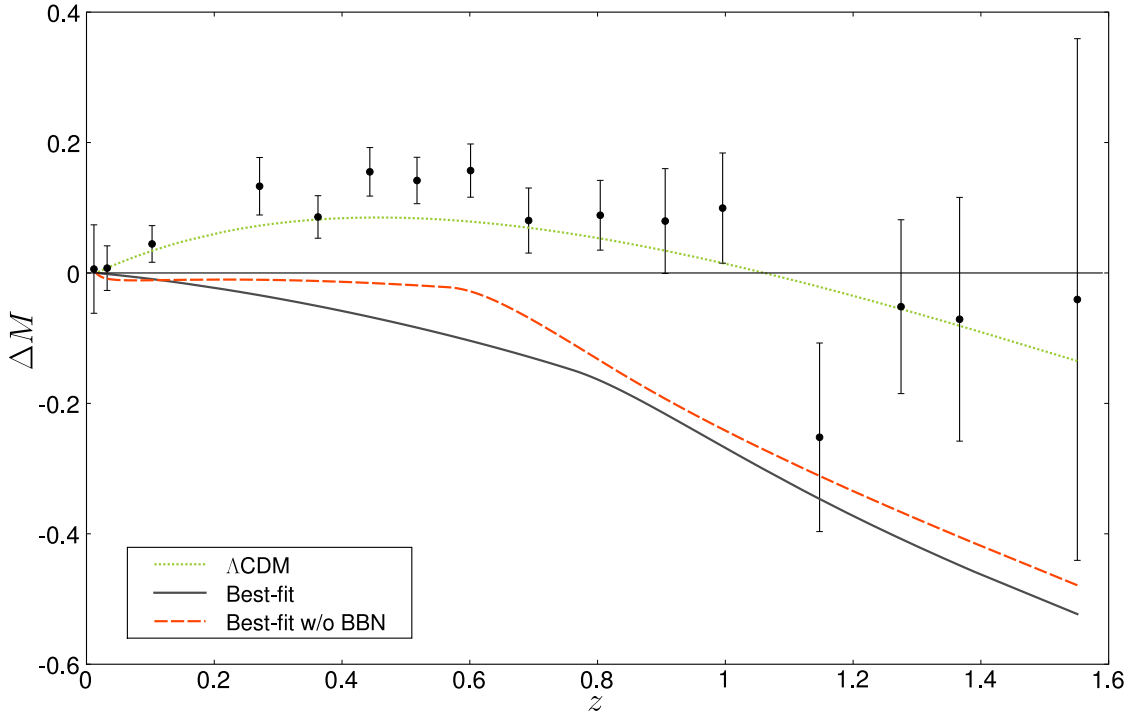
Figure 2: Magnitude as a function of redshift compared to an empty universe for the Union data, for the string gas model with the BBN constraint and the ΛCDM model.

In Figure 3 b) we show the scale factor of the extra dimensions b and the four-dimensional gravitational coupling $G_{\rm N} \propto b^{-6}$. The difference between b at BBN and today is small, and well within the observational limits discussed in [18]. However, b deviates noticeably from unity at last scattering at $z = 1100$: $b_{LS} = 1.14$, $G_{{\rm N},LS}/G_{{\rm N},in} = 0.45$. This is a generic feature of the string gas model, because last scattering is soon after the matter-radiation equality, when the extra dimensions start opening up. This prediction could provide a stringent constraint. However, quoted limits on the variation of $G_{\rm N}$ (or on new radiation degrees of freedom) from the CMB and other non-BBN probes are model-dependent [38, 39], and rely on perturbation theory. (Note that the string gas does not behave like radiation at last scattering.)

In Figure 3 c) we show the expansion rate of the visible dimensions H_a relative to what it would be without the extra dimensions and the string gas, denoted by $H_{4{\rm D}}$. (In the matter-dominated era, $H_{4{\rm D}} = 2/(3t)$.) Comparing to the plot of H_b/H_a in Figure 3 d), we see how acceleration in the visible dimensions correlates with contraction of the extra dimensions. The Hubble parameter today in the model is somewhat low, which is related to the large value of Ω_{m0}. At late times $3H_{4{\rm D}}^2 = \kappa^2 \rho_{m,in} a^{-3}$, so we have $\Omega_m = (H_a/H_{4{\rm D}})^{-2} b^{-6}$. In order to get enough acceleration in the recent past, it seems that the extra dimensions must have recently collapsed, so
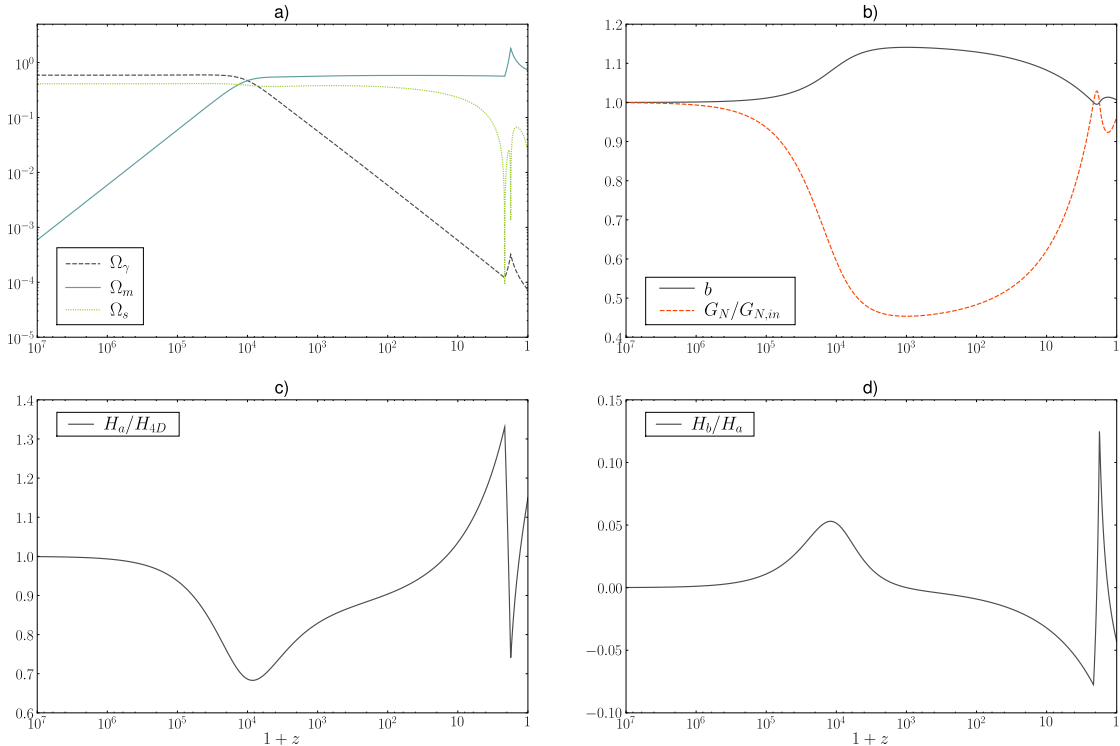
Figure 3: a) Density parameters $\Omega_i \equiv \kappa^2 \rho_i / (3 H_a^2)$, b) size of the extra dimensions and Newton's constant, c) expansion rate of the large dimensions (H_{4D} is the Hubble parameter in the usual four-dimensional case) and d) expansion rate of the extra dimensions, for the best-fit model to the Union data, with the BBN constraint.

$b \approx 1$ today. The value $\Omega_m = 0.3$, for example, then requires $H_a/H_{4D} = 1.8$. The maximum value of H_a/H_{4D} in the best-fit model is only 1.3, and the value today is 1.2. Without the BBN constraint, the situation would be better, with higher values of H_a/H_{4D}.

The quantity H_a/H_{4D} also gives the relation between the age of the universe and the present value of the Hubble parameter, since $H_a/H_{4D} = 3H_a t/2$ at late times. A model-independent observational constraint on the age of the universe is given by the ages of globular clusters [40], which lead to the lower limit $t_0 \geq 11.2$ Gyr at 95% C.L. and a best-fit age of $t_0 = 13.4$ Gyr. The best model-independent measure of the current value of the Hubble parameter comes from the Hubble Key Project [41]. The result is sensitive to the treatment of Cepheids, and two different analyses yield $H_{a0} = 0.73 \pm 0.06$ km/s/Mpc and $H_{a0} = 0.62 \pm 0.05$ km/s/Mpc (1σ limits). Taking the best-fit value for t_0 and the mean values for H_{a0} gives $H_a/H_{4D} = 1.5$ and $H_a/H_{4D} = 1.27$, respectively. The value in the best-fit model is too low, but not drastically so, taking into account the uncertainties in t_0 and H_{a0}.
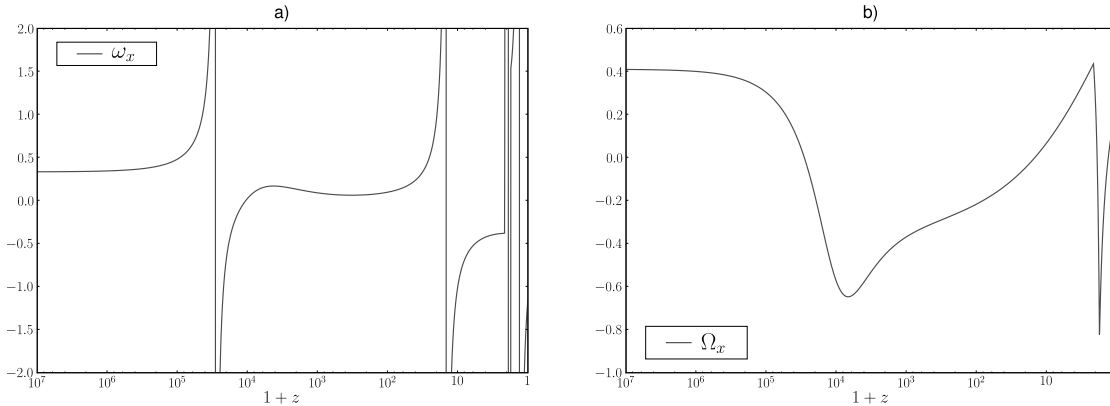
Figure 4: a) Effective equation of state and b) effective density parameter, for the best-fit model to the Union data, with the BBN constraint.

The effective equation of state. One reason for the poor fit is the extra-dimensional modification of the relationship between the expansion rate and the distance in (2.9). Rapid oscillations of the Hubble parameter do not by themselves rule out the expansion history. If we take the expansion rate H_a for the string gas model and calculate the distance using the FRW relation (2.7), the χ^2 of the best fit without the BBN constraint improves by 4.2 points for the Union data and 30.7 points for the ESSENCE data, and the fits correspond to a probability of 29% and 20%, respectively.

The string gas cosmology context aside, this provides an interesting demonstration of how a model with an expansion history radically different from ΛCDM is consistent with the SNIa data. In Figure 4 a), we plot the effective equation of state ω_x of the best-fit model, defined by treating the string gas and the extra-dimensional geometrical contributions to the Friedmann equation as one effective component, so that (2.4) reads $3H_a^2 = 8\pi G_N(\rho_{\gamma,in}a^{-4} + \rho_{m,in}a^{-3} + \rho_x)$, with p_x defined correspondingly for (2.5), and $w_x \equiv p_x/\rho_x$. The variation in the effective equation of state w_x is extreme: in fact, the equation of state diverges, because ρ_x passes through zero (in the plot, we cut w_x off at ± 2). The equation of state is far from constant, and far from slowly varying, unlike assumed in most parametrisations. (For the importance of the assumed parametrisation of the equation of state for analysing the data, see [42].) Because w_x diverges, the evolution of the effective energy density is better displayed via the effective density parameter $\Omega_x \equiv \kappa^2 \rho_x/(3H_a^2)$ shown in Figure 4 b). The effective energy density is negative for a significant part of the evolution, as could be expected on the basis of the strong deceleration seen in Figure 3 c).

4. Discussion

Conclusions. We have studied the late-time acceleration due to the oscillations of extra dimensions in string gas cosmology in the simple model discussed in [18]. We have fitted the expansion history to the Union and ESSENCE sets of type Ia supernovae. The string gas model does not fit the SNIa data as well as the ΛCDM model. With the Union SNIa data, the difference in the goodness-of-fit is small, but the fit to the ESSENCE data is poor. Also, the best-fitting string gas models have a significant fraction of the energy density during BBN in the string gas, and taking into account the BBN constraint on new radiation degrees of freedom makes the fit worse. Further, we have considered a rather conservative BBN limit, allowing for neutrino chemical potential. Since the best-fit model is at the boundary of the region allowed by BBN, we would expect the fit to become worse as the BBN limit becomes more stringent. In the model, the matter density is also too high and the Hubble parameter today somewhat low, so taking further observational constraints into account would be likely to degrade the fit further. In any case, the model can still provide a stabilisation mechanism for the extra dimensions during the matter dominated era, for which it was originally introduced.

Leaving aside the physical origin of the oscillations and the constraint from BBN, the model demonstrates how an expansion history which is very different from the ΛCDM model, with strong oscillations of the Hubble parameter, can still provide a good fit to the supernova data. (In this context, it may be interesting that the Hubble parameter inferred from observations of the ages of passively evolving galaxies shows oscillations [32], though it is premature to draw strong conclusions from the data.) The fit only becomes poor when the change in the expansion rate-distance relationship due to the extra dimensions is taken into account. This in turn is a concrete example of a model where this FRW consistency condition, discussed in [24], is strongly violated.

Improving the model. As discussed in [18], the energy-momentum tensor for the string gas is expected to be more complex than (2.12)–(2.14). The energy density (2.12) corresponds to a gas of strings which all have the same momentum Ml_s/a in the visible dimensions, while a realistic gas would have a distribution of strings with different momenta. The evolution of terms with different values of M is qualitatively the same: they scale like radiation in the radiation-dominated era and start tracking the matter during the matter-dominated era until the onset of oscillations. However, the different terms will lead to quantitatively slightly different oscillations, and as we have seen, the evolution is very sensitive to the parameters of the string gas. In order to explore this possibility, we would have to know the distribution of string momenta, which depends on how the string gas was created in the early universe and whether it has thermalised.

Acknowledgments

References

[1] J. Kripfganz and H. Perlt, *Cosmological impact of winding strings, Class. Quant. Grav.* **5** (1988) 453.

[2] R.H. Brandenberger and C. Vafa, *Superstrings in the early universe, Nucl. Phys.* **B316** (1989) 391.

[3] A.A. Tseytlin and C. Vafa, *Elements of string cosmology, Nucl. Phys.* **B372** (1992) 443 [arXiv:hep-th/9109048].

[4] T. Battefeld and D. Watson, *String Gas Cosmology, Rev. Mod. Phys.* **78** (2006) 435 [arXiv:hep-th/0510022].

[5] R.H. Brandenberger, *String Gas Cosmology* [arXiv:0808.0746 [hep-th]].

[6] S.D. Mathur, *What is the state of the Early Universe?, J. Phys. Conf. Ser.* **140** (2008) 012009 [arXiv:0803.3727 [hep-th]].

[7] S.P. Patil and R. Brandenberger, *Radion stabilization by stringy effects in general relativity, Phys. Rev.* **D71** (2005) 103522 [arXiv:hep-th/0401037].

[8] S.P. Patil and R.H. Brandenberger, *The cosmology of massless string modes*, JCAP01(2006)005 [arXiv:hep-th/0502069].

[9] S. Watson and R.H. Brandenberger, *Stabilization of extra dimensions at tree level*, JCAP11(2003)008 [arXiv:hep-th/0307044].

[10] A.J. Berndsen and J.M. Cline, *Dilaton stabilization in brane gas cosmology, Int. J. Mod. Phys.* **A19** (2004) 5311 [arXiv:hep-th/0408185].

[11] R. Brandenberger, Y.-K.E. Cheung and S. Watson, *Moduli stabilization with string gases and fluxes*, JHEP05(2006)025 [arXiv:hep-th/0501032].

[12] S.P. Patil, *Moduli (dilaton, volume and shape) stabilization via massless F and D string modes* [arXiv:hep-th/0504145].

[13] A. Berndsen, T. Biswas and J.M. Cline, *Moduli stabilization in brane gas cosmology with superpotentials*, JCAP08(2005)012 [arXiv:hep-th/0505151].

[14] S. Kanno and J. Soda, *Moduli stabilization in string gas compactification, Phys. Rev.* **D72** (2005) 104023 [arXiv:hep-th/0509074].

[15] R.J. Danos, A.R. Frey and R.H. Brandenberger, *Stabilizing moduli with thermal matter and nonperturbative effects*, Phys. Rev. **D77** (2008) 126009 [arXiv:0802.1557 [hep-th]].

[16] M. Sano and H. Suzuki, *Moduli fixing and T-duality in Type II brane gas models*, Phys. Rev. **D78** (2008) 064045 [arXiv:0804.0176 [hep-th]].

[17] N. Weiss, *Superstring Cosmology: is it consistent with a matter-dominated universe?*, Phys. Lett. **B172** (1986) 180.

[18] F. Ferrer and S. Räsänen, *Dark energy and decompactification in string gas cosmology*, JHEP02(2006)016 [arXiv:hep-th/0509225].

[19] L. Perivolaropoulos and C. Sourdis, *Cosmological effects of radion oscillations*, Phys. Rev. **D66** (2002) 084018 [arXiv:hep-ph/0204155];

L. Perivolaropoulos, *Equation of state of oscillating Brans-Dicke scalar and extra dimensions*, Phys. Rev. **D67** (2003) 123516 [arXiv:hep-ph/0301237].

[20] C. Wetterich, *Cosmology and the Fate of Dilatation Symmetry*, Nucl. Phys. **B302** (1988) 668;

B. Ratra and P.J.E. Peebles, *Cosmological Consequences of a Rolling Homogeneous Scalar Field*, Phys. Rev. **D37** (1988) 3406;

A.R. Liddle and R.J. Scherrer, *A classification of scalar field potentials with cosmological scaling solutions*, Phys. Rev. **D59** (1999) 023509 [arXiv:astro-ph/9809272].

[21] I. Zlatev, L.-M. Wang and P.J. Steinhardt, *Quintessence, cosmic coincidence, and the cosmological constant*, Phys. Rev. Lett. **82** (1999) 896 [arXiv:astro-ph/9807002];

P.J. Steinhardt, L.-M. Wang and I. Zlatev, *Cosmological tracking solutions*, Phys. Rev. **D59** (1999) 123504 [arXiv:astro-ph/9812313].

[22] S. Nesseris and L. Perivolaropoulos, *A comparison of cosmological models using recent supernova data*, Phys. Rev. **D70** (2004) 043531 [arXiv:astro-ph/0401556];

J.-Q. Xia, B. Feng and X. Zhang, *Constraints on Oscillating Quintom from Supernova, Microwave Background and Galaxy Clustering*, Mod. Phys. Lett. **A20** (2005) 2409 [arXiv:astro-ph/0411501];

G. Barenboim, O. Mena Requejo and C. Quigg, *Undulant universe*, Phys. Rev. **D71** (2005) 063533 [arXiv:astro-ph/0412010];

R. Lazkoz, S. Nesseris and L. Perivolaropoulos, *Exploring Cosmological Expansion Parametrizations with the Gold SnIa Dataset*, JCAP10(2005)01 [arXiv:astro-ph/0503230];

G. Barenboim, O. Mena Requejo and C. Quigg, *Observational constraints on undulant cosmologies*, JCAP04(2006)008 [arXiv:astro-ph/0510178];

E.V. Linder, *On oscillating dark energy*, *Astrophys. J.* **25** (2006) 167 [arXiv:astro-ph/0511415];

J.-Q. Xia, G.-B. Zhao, H. Li, B. Feng and X. Zhang, *Features in Dark Energy Equation of State and Modulations in the Hubble Diagram*, *Phys. Rev.* **D74** (2006) 083521 [arXiv:astro-ph/0605366];

H. Wei and S.N. Zhang, *Observational $H(z)$ Data and Cosmological Models*, *Phys. Lett.* **B644** (2007) 7 [arXiv:astro-ph/0609597];

G.-B. Zhao *et al.*, *Probing for dynamics of dark energy and curvature of universe with latest cosmological observations*, *Phys. Lett.* **B648** (2007) 8 [arXiv:astro-ph/0612728];

H. Zhang and Z.-H. Zhu, *Natural Phantom Dark Energy, Wiggling Hubble Parameter $H(z)$ and Direct $H(z)$ Data*, JCAP03(2008)007 [arXiv:astro-ph/0703245];

A. Kurek, O. Hrycyna and M. Szydlowski, *Constraints on oscillating dark energy models*, *Phys. Lett.* **B659** (2008) 14 [arXiv:0707.0292 [astro-ph]];

D. Jain, A. Dev and J.S. Alcaniz, *Cosmological bounds on oscillating dark energy models*, *Phys. Lett.* **B656** (2007) 15 [arXiv:0709.4234 [astro-ph]];

A. Kurek, O. Hrycyna and M. Szydlowski, *From model dynamics to oscillating dark energy parametrisation*, [arXiv:0805.4005 [astro-ph]].

[23] S. Dutta and R.J. Scherrer, *Evolution of Oscillating Scalar Fields as Dark Energy*, *Phys. Rev.* **D78** (2008) 083512 [arXiv:0805.0763 [astro-ph]].

[24] C. Clarkson, B.A. Bassett and T.H.-C. Lu, *A general test of the Copernican Principle*, *Phys. Rev. Lett.* **101** (2008) 011301 [arXiv:0712.3457 [astro-ph]].

[25] S. Räsänen, *Light propagation in statistically homogeneous and isotropic dust universes*, JCAP02(2009)011 [arXiv:0812.2872 [astro-ph]].

[26] R.H. Cyburt, B.D. Fields, K.A. Olive and E. Skillman, *New BBN limits on physics beyond the Standard Model from He-4*, *Astropart. Phys.* **23** (2005) 313 [arXiv:astro-ph/0408033].

[27] C. Amsler *et al.*, *Review of Particle Physics*, *Phys. Lett.* **B667** (2008) 1.

[28] V. Barger, J.P. Kneller, P. Langacker, D. Marfatia and G. Steigman, *Hiding relativistic degrees of freedom in the early universe*, *Phys. Lett.* **B569** (2003) 123 [arXiv:hep-ph/0306061].

[29] S. Watson and R.H. Brandenberger, *Linear perturbations in brane gas cosmology*, JHEP03(2004)045 [hep-th/0312097].

[30] S. Watson, *UV perturbations in brane gas cosmology*, *Phys. Rev.* **D70** (2004) 023516 [hep-th/0402015].

[31] T.J. Battefeld, S.P. Patil and R.H. Brandenberger, *On the transfer of metric fluctuations when extra dimensions bounce or stabilize*, Phys. Rev. **D73** (2006) 086002 [hep-th/0509043].

[32] R. Jimenez and A. Loeb, *Constraining Cosmological Parameters Based on Relative Galaxy Ages*, Astrophys. J. **573** (2002) 37 [arXiv:astro-ph/0106145];

J. Simon, L. Verde and R. Jimenez, *Constraints on the redshift dependence of the dark energy potential*, Phys. Rev. **D71** (2005) 123001 [arXiv:astro-ph/0412269].

[33] T.M. Davis *et al.*, *Scrutinizing exotic cosmological models using ESSENCE supernova data combined with other cosmological probes*, Astrophys. J. **666** (2007) 716 [arXiv:astro-ph/0701510].

[34] M. Kowalski *et al.* (Supernova Cosmology Project), *Improved Cosmological Constraints from New, Old and Combined Supernova Datasets*, [arXiv:0804.4142 [astro-ph]].

[35] D.J. Eisenstein *et al.* [SDSS Collaboration], *Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies*, Astrophys. J. **633** (2005) 560 [arXiv:astro-ph/0501171].

[36] G. Efstathiou and J.R. Bond, *Cosmic Confusion: Degeneracies among Cosmological Parameters Derived from Measurements of Microwave Background Anisotropies*, Mon. Not. Roy. Astron. Soc. **304** (1999) 75 [arXiv:astro-ph/9807103].

[37] Ø. Elgarøy and T. Multamäki, *On using the CMB shift parameter in tests of models of dark energy*, Astron. & Astrophys. **471** (2007) 65 [arXiv:astro-ph/0702343];

Y. Wang and P. Mukherjee, *Observational Constraints on Dark Energy and Cosmic Curvature*, Phys. Rev. **D76** (2007) 103533 [arXiv:astro-ph/0703780].

[38] E. Komatsu *et al.* [WMAP Collaboration], *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation* [arXiv:0803.0547 [astro-ph]].

[39] P. Crotty, J. Lesgourgues and S. Pastor, *Measuring the cosmological background of relativistic particles with WMAP*, Phys. Rev. **D67** (2003) 123005 [arXiv:astro-ph/0302337];

E. Pierpaoli, *Constraints on the cosmic neutrino background*, Mon. Not. Roy. Astron. Soc. **342** (2003) L63 [arXiv:astro-ph/0302465];

S. Hannestad, *Neutrino masses and the number of neutrino species from WMAP and 2dFGRS*, JCAP05 (2003) 004 [arXiv:astro-ph/0303076];

V. Barger, J.P. Kneller, H.-S. Lee, D. Marfatia and G. Steigman, *Effective number of neutrinos and baryon asymmetry from BBN and WMAP*, Phys. Lett. **B566** (2003) 8 [arXiv:hep-ph/0305075];

K.-i. Umezu, K. Ichiki and M. Yahiro, *Cosmological constraints on Newton's constant*, Phys. Rev. **D72** (2005) 044010 [arXiv:astro-ph/0503578].

[40] L.M. Krauss and B. Chaboyer, *Age Estimates of Globular Clusters in the Milky Way: Constraints on Cosmology*, *Science* **299** (2003) 65.

[41] Jackson N, *The Hubble Constant*, *Living Reviews in Relativity* **10** (2007) 4 [arXiv:0709.3924 [astro-ph]].

[42] B.A. Bassett, M. Kunz, J. Silk and C. Ungarelli, *A late-time transition in the cosmic dark energy?*, *Mon. Not. Roy. Astron. Soc.* **336** (2002) 1217 [arXiv:astro-ph/0203383];

U. Alam, V. Sahni and A.A. Starobinsky, *Is there Supernova Evidence for Dark Energy Metamorphosis?*, *Mon. Not. Roy. Astron. Soc.* **354** (2004) 275 [arXiv:astro-ph/0311364];

U. Alam, V. Sahni and A.A. Starobinsky, *The case for dynamical dark energy revisited* JCAP06(2004)008 [arXiv:astro-ph/0403687];

J. Jönsson, A. Goobar, R. Amanullah and L. Bergström, *No evidence for Dark Energy Metamorphosis?* JCAP09(2004)007 [arXiv:astro-ph/0404468];

P.S. Corasaniti, M. Kunz, D. Parkinson, E.J. Copeland and B.A. Bassett, *The foundations of observing dark energy dynamics with the Wilkinson Microwave Anisotropy Probe*, *Phys. Rev.* **D70** (2004) 083006 [arXiv:astro-ph/0406608];

U. Alam, V. Sahni and A.A. Starobinsky, *Rejoinder to "No Evidence of Dark Energy Metamorphosis"*, [arXiv:astro-ph/0406672];

S. Hannestad and E. Mörtsell, *Cosmological constraints on the dark energy equation of state and its evolution* JCAP09(2004)001 [arXiv:astro-ph/0407259];

B.A. Bassett, P.S. Corasaniti and M. Kunz, *The essence of quintessence and the cost of compression*, *Astrophys. J.* **617** (2004) L1 [arXiv:astro-ph/0407364];

C. Stephan-Otto, *Optimized supernova constraints on dark energy evolution*, *Phys. Rev.* **D74** (2006) 023507 [arXiv:astro-ph/0605403].