Fixed Gain Amplify-and-Forward Relaying with Co-Channel Interference

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***Abstract*—We investigate the outage probability and the average bit error rate (BER) of a dual-hop fixed gain relaying system in the presence of co-channel interference and thermal noise at the relay and the destination. Our analysis assumes Rayleigh fading for the source-relay and relay-destination channels and Rician fading for the interfering channels. We present new closed-form/series expressions for the outage probability and the average BER, which allow for a rapid performance evaluation of fixed-gain relaying in interference-impaired environments. The achievable diversity order is also studied. It is further shown through numerical results that the Rician K-factor of the interfering channel has a negative effect on the overall outage probability and BER, yet this effect is very small.**

I. Introduction

Amplify-and-forward (AF) relaying has now become a well studied protocol under thermal noise-limited conditions, (see e.g. [1]–[3] and references therein). However, in practice, the interference present in relay networks can cause severe performance degradation [4].

Several recent works have studied the performance of AF and decode-and-forward (DF) relaying for interference-limited (negligible thermal noise) relay(s) or destination(s), see e.g. [4]–[10]. In [4], the impact of interference on the performance of a dual-hop channel state information (CSI)-assisted AF relay network has been investigated. The authors of [5] have analyzed the performance gains of a half-duplex multi-user network where the relay-destination slot is reused, causing interference at the destination nodes. In [6], [7], the performance of different DF relaying systems in the presence of interference has been investigated. Assuming an interference-limited destination, in [8], the outage probability of a fixed gain AF relay system over Rayleigh fading channels has been investigated. In [9], the performance of a dual-hop CSI-assisted AF system with interference-limited relays has been studied. In [10], the outage probability of dual-hop CSI-assisted AF relaying with interference-limited relays and destinations has been derived.

In cellular applications, the case of interference at the relay has practical significance since relays may operate at the cell edge providing extended coverage. Thus, due to the reuse of frequency bands it is likely that relays are affected by line-of-sight (LoS)/non LoS co-channel interference and thermal noise. Nonetheless, the joint effects of interference and thermal-noise on the performance of dual-hop fixed gain relay networks has not been investigated in the existing literature. Motivated by the absence of such an analysis, in this paper, we study the outage probability and the average bit error rate (BER) of dual-hop fixed gain AF relaying with interference and thermal noise at the relay. In addition, the case where there exists interference at the destination is also considered, and analyzed separately. Assuming Rayleigh fading for the source-relay and relay-destination channels and Rician fading for the interfering channel, we derive new outage and BER expressions upon statistically characterizing the signal-to-interference plus noise ratio (SINR). The Rician fading assumption on the interference link is motivated by the fact that it is used to model wireless propagation comprising a LoS component and a scattered component. Therefore, the derived analytical expressions quantify the impact of interference on relay system performance for a large set of environments. The analysis of Rician channels is generally more difficult and as a special case, includes the commonly assumed Rayleigh fading.

II. System Model

A. Mode of Operation

We consider a communication system, where a source, S, communicates with a destination, D, through a relay, R. All nodes are equipped with a single antenna. It is assumed that S does not have a direct link to D. The communication in the system is divided into two orthogonal time intervals. In the first time interval, S sends its symbol s_0 to R. In the second time interval, R communicates with D.

At R, the received signal in the presence of a single interferer, with an average power, P_1, and additive white Gaussian noise (AWGN) can be expressed as

$$y_r = \sqrt{P_s} h_{sr} e^{j\theta_{sr}} s_0 + \sqrt{P_1} h_1 e^{j\theta_1} s_1 + n_r, \qquad (1)$$

where P_s is the transmit power, s_1 is the interference symbol and h_1 is the channel amplitude of an interferer located in the proximity of R; h_{sr} and h_{rd} are the Rayleigh fading amplitudes of the S-R and R-D links, respectively, with average powers $E[|h_{sr}|^2] = \sigma_{sr}^2$ and $E[|h_{rd}|^2] = \sigma_{rd}^2$, respectively, with $E[\cdot]$ denoting expectation. The phase of the useful and the interfering signal at R, are denoted by θ_{sr} and θ_1, respectively. It is further assumed that h_1 is a Rician distributed random variable (RV) with $E[|h_1|^2] = \sigma_1^2$, and is independent of h_{sr} and h_{rd}. Finally, n_r denotes the AWGN at R satisfying $E[|n_r|^2] = N_{01}$.

B. Considered Scenarios

Depending on whether or not there exists interference at D, two cases are considered in this paper which are referred to as

Scenario (a) and *Scenario (b)*, respectively. Specifically:

- ***Scenario (a)***: R experiences interference and AWGN while D is interference-free and subject to AWGN only. Reference [4] first considered *Scenario (a)* in an analysis of CSI-assisted AF relays.
- ***Scenario (b)***: R experiences interference and AWGN while D is interference-limited. An interference-limited D was also considered in [8], however, no interference was assumed at R.

1) Scenario (a): Let $\mathcal{G}_\mathrm{F}$ denote the fixed gain employed by R. In the absence of interference at D, the received signal can be expressed as

$$y_d = h_{rd}e^{j\theta_{rd}}\mathcal{G}_\mathrm{F}\left(\sqrt{P_s}h_{sr}e^{j\theta_{sr}}s_0 + \sqrt{P_1}h_1e^{j\theta_1}s_1 + n_r\right)+n_d \quad (2)$$

where n_d is the AWGN at D satisfying $E[|n_d|^2] = N_{02}$, and θ_{rd} denotes the phase of the R-D channel. Therefore, we obtain the overall SINR as

$$\gamma_{eq1} = \frac{P_s|h_{sr}|^2|h_{rd}|^2}{|h_{rd}|^2N_{01} + P_1|h_{rd}|^2|h_1|^2 + \frac{N_{02}}{\mathcal{G}_\mathrm{F}^2}}. \quad (3)$$

2) Scenario (b): The received signal at D is given by

$$\begin{aligned} y_d &= h_{rd}e^{j\theta_{rd}}\mathcal{G}_\mathrm{F}\left(\sqrt{P_s}h_{sr}e^{j\theta_{sr}}s_0 + \sqrt{P_1}h_1e^{j\theta_1}s_1\right) \quad (4)\\ &+ h_{rd}e^{j\theta_{rd}}\mathcal{G}_\mathrm{F}n_r + \sqrt{P_2}h_2e^{j\theta_2}s_2, \end{aligned}$$

where h_2 is the Rician faded channel from an interferer located in the proximity of D with average power $E[|h_2|^2] = \sigma_2^2$; s_2 is the signal from the interferer in the proximity of D and θ_2 denotes the phase of the interfering channel at D. The overall SINR can be thus expressed as

$$\gamma_{eq2} = \frac{P_s|h_{sr}|^2|h_{rd}|^2}{|h_{rd}|^2N_{01} + P_1|h_{rd}|^2|h_1|^2 + \frac{P_2|h_2|^2}{\mathcal{G}_\mathrm{F}^2}}. \quad (5)$$

III. Outage Probability Analysis

In this section, the outage probability under *Scenarios (a)* and *(b)* is studied. In the case of outage probability, it is reasonable to assume that s_0, s_1 and s_2 are complex Gaussian distributed RVs with unity average power. Since the relay's output power is constrained, $\mathcal{G}_\mathrm{F}$ takes the form:

$$\mathcal{G}_\mathrm{F} = \sqrt{\frac{P_r}{P_s\sigma_{sr}^2 + P_1\sigma_1^2 + N_{01}}}. \quad (6)$$

1) Scenario (a): Using (6) we can simplify (3) as

$$\gamma_{eq1} = \frac{\gamma_1\gamma_2}{(1+u)\gamma_2 + C}, \quad (7)$$

where $\gamma_1 = \frac{P_s}{N_{01}}|h_{sr}|^2$, $\gamma_2 = \frac{P_r}{N_{02}}|h_{rd}|^2$, and $u = \frac{P_1}{N_{01}}|h_1|^2$. The constant $C = \bar{\gamma}_1 + \eta_1 + 1$, where we have defined $\bar{\gamma}_1 = \frac{P_s\sigma_{sr}^2}{N_{01}}$ and $\eta_1 = \frac{P_1\sigma_1^2}{N_{01}}$.

To derive the outage probability of γ_{eq1}, conditioning on γ_2 and u, we first express the cumulative distribution function (cdf) of γ_{eq1} as

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &= \Pr\left\{\frac{\gamma_1\gamma_2}{(1+u)\gamma_2 + C} < \gamma_T\right\} \\ &= \int_0^\infty\int_0^\infty \Pr\left\{\gamma_1 < (1+x)\gamma_T + \frac{C\gamma_T}{y}\right\} f_u(x)f_{\gamma_2}(y)dxdy \end{aligned} \quad (8)$$

where $\Pr\{\cdot\}$ denotes probability and γ_T represents the outage threshold SNR. The cdf of γ_1, $F_{\gamma_1}(x)$ and the probability density function (pdf) of γ_2, $f_{\gamma_2}(x)$ are given by $F_{\gamma_1}(x) = 1 - e^{-\frac{x}{\bar{\gamma}_1}}$ and $f_{\gamma_2}(x) = \frac{1}{\bar{\gamma}_2}e^{-\frac{x}{\bar{\gamma}_2}}$, respectively, where $\bar{\gamma}_2 = \frac{P_r\sigma_{rd}^2}{N_{02}}$. Since the interferer at R, is subject to Rician fading, the pdf of u is given by

$$f_u(x) = \frac{(1+\omega_1)e^{-\omega_1 - \frac{(1+\omega_1)x}{\eta_1}}}{\eta_1} I_0\left(2\sqrt{\frac{\omega_1(1+\omega_1)x}{\eta_1}}\right) \quad (9)$$

where ω_1 is the Rician K-factor, defined as the ratio of the powers of the LoS component to the scattered components, and $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind defined in [11, Eq. (8.431.1)].

Substituting the pdfs and the cdf into (8) we obtain

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &= 1 - \frac{(1+\omega_1)e^{-\omega_1 - \frac{\gamma_T}{\bar{\gamma}_1}}}{\eta_1\bar{\gamma}_2}\int_0^\infty e^{-\left(\frac{1+\omega_1}{\eta_1} + \frac{\gamma_T}{\bar{\gamma}_1}\right)x} \\ &\times I_0\left(2\sqrt{\frac{\omega_1(1+\omega_1)x}{\eta_1}}\right)dx\int_0^\infty e^{-\frac{C\gamma_T}{\bar{\gamma}_1 y} - \frac{y}{\bar{\gamma}_2}}dy. \end{aligned} \quad (10)$$

Using [11, Eq. (3.471.9)] and [12, Eq. (9)]

$$\int_0^\infty xe^{-\frac{p^2x^2}{2}}I_0(ax)dx = \frac{1}{p^2}e^{\frac{a^2}{2p^2}} \quad (11)$$

in (10) yields

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &= 1 - 2(1+\omega_1)\frac{e^{-\omega_1 - \frac{\gamma_T}{\bar{\gamma}_1} + \frac{\omega_1}{1 + \frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}}}{1 + \omega_1 + \frac{\eta_1\gamma_T}{\bar{\gamma}_1}} \\ &\times \sqrt{\frac{C\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}}K_1\left(2\sqrt{\frac{C\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \end{aligned} \quad (12)$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind defined in [11, Eq. (8.432.6)]. As expected, we notice in (12) that the existence of interference at the relay increases the overall outage probability. Furthermore, taking the first order derivative of (12) with respect to ω_1 we obtain

$$\begin{aligned} \frac{\partial F_{\gamma_{eq1}}(\gamma_T)}{\partial\omega_1} &= 2(\eta_1\gamma_T)^2\sqrt{\frac{C\bar{\gamma}_1\gamma_T}{\bar{\gamma}_2}}K_1\left(2\sqrt{\frac{C\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \\ &\times \frac{\omega_1}{\left((1+\omega_1)\bar{\gamma}_1 + \eta_1\gamma_T\right)^3}e^{-\omega_1 - \frac{\gamma_T}{\bar{\gamma}_1} + \frac{\omega_1}{1 + \frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}}. \end{aligned} \quad (13)$$

Since $\frac{\partial F_{\gamma_{eq1}}(\gamma_T)}{\partial\omega_1} > 0$ holds, we infer that the outage probability is an increasing function of ω_1. As a result, an interferer with a high Rician K-factor increases the outage probability, as compared to an interferer with a low Rician K-factor.

2) Scenario (b): In this scenario, we can express the overall SINR as

$$\gamma_{eq2} = \frac{\gamma_1\gamma_2}{(1+u)\gamma_2 + Cv}, \quad (14)$$

where $v = \frac{P_2}{N_{02}}|h_2|^2$.

The outage probability can be expressed as

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= \Pr\left(\frac{\gamma_1\gamma_2}{(1+u)\gamma_2 + Cv} < \gamma_T\right) \\ &= \int_0^\infty\int_0^\infty\int_0^\infty \Pr\left(\gamma_1 < (1+x)\gamma_T + \frac{C\gamma_T z}{y}\right) \\ &\times f_u(x) f_{\gamma_2}(y) f_v(z) dx dy dz. \end{aligned} \tag{15}$$

Since the interferer at D is subject to Rician fading, the pdf of v can be expressed as

$$f_v(x) = \frac{(1+\omega_2)e^{-\omega_2 - \frac{(1+\omega_2)x}{\eta_2}}}{\eta_2} I_0\left(2\sqrt{\frac{\omega_2(1+\omega_2)x}{\eta_2}}\right), \tag{16}$$

where $\eta_2 = \frac{P_2\sigma_2^2}{N_{02}}$ and ω_2 is the Rician K-factor.

Substituting the cdf of γ_1 and the pdfs of γ_2, u, and v we obtain

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= 1 - \frac{(1+\omega_1)(1+\omega_2)e^{-\omega_1-\omega_2-\frac{\gamma_T}{\bar{\gamma}_1}}}{\eta_1\eta_2\bar{\gamma}_2} \\ &\times \int_0^\infty e^{-\left(\frac{1+\omega_1}{\eta_1}+\frac{\gamma_T}{\bar{\gamma}_1}\right)x} I_0\left(2\sqrt{\frac{\omega_1(1+\omega_1)x}{\eta_1}}\right) dx \\ &\times \int_0^\infty e^{-\frac{(1+\omega_2)z}{\eta_2}} I_0\left(2\sqrt{\frac{\omega_2(1+\omega_2)z}{\eta_2}}\right) \\ &\times \int_0^\infty e^{-\frac{C\gamma_T z}{\bar{\gamma}_1 y} - \frac{y}{\bar{\gamma}_2}} dy dz. \end{aligned} \tag{17}$$

With the help of (11) and [11, Eq. (3.471.9)] we solve the integrals with respect to x and y as follows

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= 1 - 2\sqrt{\frac{C\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}}\frac{(1+\omega_1)(1+\omega_2)}{\left(1+\omega_1+\frac{\eta_1\gamma_T}{\bar{\gamma}_1}\right)\eta_2} \\ &\times e^{-\omega_1-\omega_2-\frac{\gamma_T}{\bar{\gamma}_1}+\frac{\omega_1}{1+\frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}} \int_0^\infty \sqrt{z} e^{-\frac{(1+\omega_2)z}{\eta_2}} \\ &\times I_0\left(2\sqrt{\frac{\omega_2(1+\omega_2)z}{\eta_2}}\right) K_1\left(2\sqrt{\frac{C\gamma_T z}{\bar{\gamma}_1\bar{\gamma}_2}}\right) dz. \end{aligned} \tag{18}$$

Unfortunately, a closed-form expression for the outage probability of fixed gain relaying under the assumption of *Scenario (b)* is very difficult, if not impossible, to be obtained. Nonetheless, the integral in (18) can be evaluated in a series form by making the change of variables, $t^2 = z$, and using [11, Eq. (8.447.1)], [11, Eq. (6.631.3)], [11, Eq. (9.220.4)], [11, Eq. (9.220.2)], and [11, Eq. (9.210.2)], yielding for the case of $\omega_2 \neq 0$

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= 1 - \frac{(1+\omega_1)e^{-\omega_1-\omega_2-\frac{\gamma_T}{\bar{\gamma}_1}+\frac{\omega_1}{1+\frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}}}{\left(1+\omega_1+\frac{\eta_1\gamma_T}{\bar{\gamma}_1}\right)} \\ &\times \sum_{m=0}^{M} (m+1)\,\omega_2^m \Psi\left(m+1, 0, \frac{C\gamma_T}{\left(\frac{1+\omega_2}{\eta_2}\right)\bar{\gamma}_1\bar{\gamma}_2}\right) \end{aligned} \tag{19}$$

where $\Psi(\cdot,\cdot,\cdot)$ denotes the Tricomi confluent hypergeometric function defined in [11, Eq. (9.210.2)]; M represents the sufficiently large number of terms required for the series in (19) to converge, which in practical cases takes values between one and 20. We also note that for the special case when h_2 experiences Rayleigh fading ($\omega_2 = 0$), using [13, Eq. (2.16.8.5)] in (18) yields

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= 1 - \frac{(1+\omega_1)C\eta_2\gamma_T}{\left(1+\omega_1+\frac{\eta_1\gamma_T}{\bar{\gamma}_1}\right)\bar{\gamma}_1\bar{\gamma}_2} \\ &\times e^{-\omega_1-\left(1-\frac{C\eta_2}{\bar{\gamma}_2}\right)\frac{\gamma_T}{\bar{\gamma}_1}+\frac{\omega_1}{1+\frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}}\,\Gamma\left(-1, \frac{C\eta_2\gamma_T}{\bar{\gamma}_1\bar{\gamma}_2}\right) \end{aligned} \tag{20}$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete Gamma function defined in [11, Eq. (8.350.2)].

A. Diversity Order

*1) **Scenario (a)**:* Using approximation [14, Eq. (9.6.9)] followed by the McLaurin series representation of the exponential function in (12) and taking only the first order terms, (12) yields for high signal-to-noise ratio (SNR)

$$F_{\gamma_{eq1}}(\gamma_T) \approx \frac{\gamma_T}{\bar{\gamma}_1} + \eta_1\frac{\gamma_T}{\bar{\gamma}_1}. \tag{21}$$

Then, expressing η_1 as $\eta_1 = \mu\bar{\gamma}_1$, where μ is a finite non-zero constant, and using the definition of the diversity order ($d = -\lim_{\bar{\gamma}_1\to\infty}\log\left(F_{\gamma_{eq1}}(\gamma_T)\right)/\log(\bar{\gamma}_1)$) it follows that the diversity order of the interference-limited fixed gain relaying scheme with interference-free reception at D, is zero.

*2) **Scenario (b)**:* For asymptotically high $\bar{\gamma}_1/\gamma_T$ and for $\omega_2 \neq 0$, using the approximation $\Psi\left(m+1, 0, C\eta_2\gamma_T/\left[(1+\omega_2)\bar{\gamma}_1\bar{\gamma}_2\right]\right) \approx 1/(m+1)$ [14, Eq. (13.5.11)] and the infinite series expansion of the exponential function, (19) yields

$$F_{\gamma_{eq2}}(\gamma_T) \approx 1 - \frac{(1+\omega_1)e^{-\omega_1-\frac{\gamma_T}{\bar{\gamma}_1}+\frac{\omega_1}{1+\frac{\eta_1\gamma_T}{(1+\omega_1)\bar{\gamma}_1}}}}{\left(1+\omega_1+\frac{\eta_1\gamma_T}{\bar{\gamma}_1}\right)}. \tag{22}$$

Taking the first order terms of the Taylor series expansion of (22), we arrive at (21). Therefore, similarly to Section III-A1, it follows that the diversity order of the interference-limited fixed gain relaying scheme with interference-limited reception at D, is zero. For the special case of $\omega_2 = 0$, we may infer zero diversity order from (20) and [11, Eq. (8.351.4)].

IV. Average BER Analysis

In this section we derive the system's exact error performance using a characteristic function (CF) method. Moreover, we assume that both the desired signal, s_0 and the interference signals at R and D, s_1 and s_2, are binary phase-shift keying (BPSK) modulated [4]. In the special case of Rayleigh faded interferer/s, we present new exact closed-form BER expressions.

A. *Scenario (a)*

After demodulation, matched filtering and sampling, the decision statistic for the data symbol can be expressed as

$$\mathcal{D}_0 = h_{rd}\mathcal{G}_{\rm F}\left(\sqrt{P_s}h_{sr}s_0 + \sqrt{P_1}h_1\cos(\theta_1-\theta_{sr})s_1 + \tilde{n}_r\right) + \tilde{n}_d \tag{23}$$

where $\tilde{n}_r$ and $\tilde{n}_d$ are zero mean Gaussian RVs with variances $\frac{N_{01}}{2}$ and $\frac{N_{02}}{2}$ respectively. The average BER conditioned on h_{sr} can be written as

$$\widetilde{P}_b = \Pr\Bigg[h_{rd}\mathcal{G}_{\rm F}\sqrt{P_s}h_{sr}s_0 + h_{rd}\mathcal{G}_{\rm F}\left(I+\tilde{n}_r\right)+\tilde{n}_d<0\Bigg|s_0=+1,h_{sr}\Bigg] \quad (24)$$

where $I = \sqrt{P_1}h_1\cos\theta s_1$ and $\theta = (\theta_1 - \theta_{sr})$ modulo 2π. Following [15], [16] we assume that θ is uniformly distributed over $(0, 2\pi]$. Simplifying further yields

$$\widetilde{P}_b = \Pr\Bigg[\sqrt{P_s}h_{sr}s_0 + I + \tilde{n}_r + \frac{\tilde{n}_d}{\mathcal{G}_{\rm F}h_{rd}}<0\Bigg|s_0=+1,h_{sr}\Bigg]. \quad (25)$$

Let $X_1 = \tilde{n}_r + \frac{\tilde{n}_d}{\mathcal{G}_{\rm F}h_{rd}}$ and $\Lambda = I + X_1$. Therefore, (25) can be written as

$$\widetilde{P}_b = \Pr\left(I+W<-\sqrt{P_s}h_{sr}\right) = 1 - F_\Lambda\left(\sqrt{P_s}h_{sr}\right). \quad (26)$$

Since the cdf, $F_\Lambda(x)$ can be written as [17, Eq. (3-23)]

$$F_\Lambda(x) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\sin(xz)}{\omega}\Phi_\Lambda(z)dz \quad (27)$$

where $\Phi_\Lambda(z)$ is the CF of the RV, Λ, given by $\Phi_\Lambda(z) = \Phi_I(z)\Phi_{X_1}(z)$. Hence (26) can be re-expressed as

$$= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin(\sqrt{P_s}rz)}{z}\Phi_\Lambda(z)dz. \quad (28)$$

Using the pdf of the Rayleigh distributed RV, h_{sr}

$$f_{h_{sr}}(x) = \frac{2x}{\sigma_{sr}^2}e^{-\frac{x^2}{\sigma_{sr}^2}}, \quad (29)$$

the unconditional average BER, P_b can be written as

$$P_b = \frac{1}{2} - \frac{2}{\pi\sigma_{sr}^2}\int_0^\infty\int_0^\infty r\sin(\sqrt{P_s}rz)e^{-\frac{r^2}{\sigma_{sr}^2}}\frac{\Phi_\Lambda(z)}{z}drdz. \quad (30)$$

Using [11, Eq. (3.952.1)] we can simplify (30) as

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{P_s\sigma_{sr}^2}{\pi}}\int_0^\infty \Phi_\Lambda(z)e^{-\frac{P_s\sigma_{sr}^2z^2}{4}}dz\right). \quad (31)$$

Now consider $\Phi_I(z)$. Using [15, Appendix A] we can write

$$\Phi_I(z) = E\left[e^{jzI}\right] = \int_0^\infty J_0\left(\sqrt{P_1}rz\right)f_{h_1}(r)dr \quad (32)$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. Substituting $f_{h_1}(r)$ into (32) yields

$$\begin{aligned}\Phi_I(z) &= \frac{2(1+\omega_1)e^{-\omega_1}}{\sigma_1^2}\int_0^\infty re^{-\frac{(1+\omega_1)r^2}{\sigma_1^2}} \\ &\quad\times I_0\left(2r\sqrt{\frac{\omega_1(1+\omega_1)}{\sigma_1^2}}\right)J_0\left(\sqrt{P_1}rz\right)dr \\ &= e^{-\frac{P_1\sigma_1^2z^2}{4(1+\omega_1)}}J_0\left(\sqrt{\frac{\omega_1P_1\sigma_1^2}{1+\omega_1}}z\right),\end{aligned} \quad (33)$$

where the last line follows with the help of [11, Eq. (6.633.4)].

Using [11, Eq. (8.432.6)], $\Phi_{X_1}(z)$ is expressed as

$$\begin{aligned}\Phi_{X_1}(z) &= e^{-\frac{N_{01}z^2}{4}}\left(\frac{1}{\sigma_{rd}^2}\int_0^\infty e^{-\frac{N_{02}z^2}{4\mathcal{G}_{\rm F}^2h}}e^{-\frac{h}{\sigma_{rd}^2}}dh\right) \\ &= \frac{\sqrt{N_{02}}}{\mathcal{G}_{\rm F}\sigma_{rd}}ze^{-\frac{N_{01}z^2}{4}}K_1\left(\frac{\sqrt{N_{02}}}{\mathcal{G}_{\rm F}\sigma_{rd}}z\right).\end{aligned} \quad (34)$$

Hence, substituting (34) and (33) into (31), we obtain

$$\begin{aligned}P_b &= \frac{1}{2}\Bigg(1-\sqrt{\frac{P_sN_{02}\sigma_{sr}^2}{\pi\mathcal{G}_{\rm F}^2\sigma_{rd}^2}}\int_0^\infty ze^{-\left(P_s\sigma_{sr}^2+\frac{P_1\sigma_1^2}{1+\omega_1}+N_{01}\right)\frac{z^2}{4}} \\ &\quad\times J_0\left(\sqrt{\frac{\omega_1P_1\sigma_1^2}{1+\omega_1}}z\right)K_1\left(\frac{\sqrt{N_{02}}}{\mathcal{G}_{\rm F}\sigma_{rd}}z\right)dz\Bigg).\end{aligned} \quad (35)$$

To the best of the authors' knowledge, Eq. (35) cannot be simplified to a closed-form expression. However, using the following series expansion of $J_0(x)$ [11, Eq. (8.402)]

$$J_0(x) = \sum_{m=0}^\infty \frac{(-1)^m}{(m!)^2}\left(\frac{x}{2}\right)^{2m}, \quad (36)$$

we can write (35) as

$$\begin{aligned}P_b &= \frac{1}{2}\Bigg(1-\sqrt{\frac{P_sN_{02}\sigma_{sr}^2}{\pi\mathcal{G}_{\rm F}^2\sigma_{rd}^2}}\sum_{m=0}^\infty\frac{(-1)^m}{(m!)^2}\left(\frac{\omega_1\eta_1N_{01}}{4(1+\omega_1)}\right)^m \\ &\quad\times\int_0^\infty z^{2m+1}e^{-\left(P_s\sigma_{sr}^2+\frac{P_1\sigma_1^2}{1+\omega_1}+N_{01}\right)\frac{z^2}{4}}K_1\left(\frac{\sqrt{N_{02}}}{\mathcal{G}_{\rm F}\sigma_{rd}}z\right)dz\Bigg).\end{aligned} \quad (37)$$

Further, using [13, Eq. (2.16.8.4)] and after some algebraic manipulations, we can express the average BER as

$$\begin{aligned}P_b &= \frac{1}{2}\Bigg(1-\left(\frac{(\bar{\gamma}_1+\eta_1+1)\sqrt{\bar{\gamma}_1}}{\sqrt{\pi}\left(\bar{\gamma}_1+\frac{\eta_1}{1+\omega_1}+1\right)^{\frac{3}{2}}\bar{\gamma}_2}\right) \\ &\quad\times\sum_{m=0}^\infty\frac{\left(-\frac{\omega_1\eta_1}{1+\omega_1}\right)^m}{(m!)^2}\frac{\Gamma\left(m+\frac{3}{2}\right)\Gamma\left(m+\frac{1}{2}\right)}{\left(\bar{\gamma}_1+\frac{\eta_1}{1+\omega_1}+1\right)^m} \\ &\quad\times\Psi\left(m+\frac{3}{2},2,\frac{\bar{\gamma}_1+\eta_1+1}{\left(\bar{\gamma}_1+\frac{\eta_1}{1+\omega_1}+1\right)\bar{\gamma}_2}\right)\Bigg).\end{aligned} \quad (38)$$

Note that in the special case of a Rayleigh faded interferer, substituting $\omega_1 = 0$ and $m = 0$ into (38) and using

$$\Psi(a,2a-1,x) = \frac{x^{\frac{3}{2}-a}e^{\frac{x}{2}}}{2(a-1)\sqrt{\pi}}\left(K_{a-\frac{1}{2}}\left(\frac{x}{2}\right)-K_{a-\frac{3}{2}}\left(\frac{x}{2}\right)\right) \quad (39)$$

it can be shown that P_b admits the following closed-form solution

$$P_b = \frac{1}{2}\left(1-\frac{1}{2\bar{\gamma}_2}\sqrt{\frac{\bar{\gamma}_1}{\bar{\gamma}_1+\eta_1+1}}e^{\frac{1}{2\bar{\gamma}_2}}\Theta\left(\frac{1}{2\bar{\gamma}_2}\right)\right), \quad (40)$$

where $\Theta(x) = K_1(x) - K_0(x)$.

B. Scenario (b)

Let us now assume the case of *Scenario (b)*. After demodulation, the decision statistic for s_0 can be expressed as

$$\mathcal{D}_0 = h_{rd}\mathcal{G}_{\mathrm{F}}\left(\sqrt{P_s}h_{sr}s_0 + \sqrt{P_1}h_1\cos\theta s_1 + \tilde{n}_r\right) + \sqrt{P_2}h_2\cos\left(\theta_2 - \theta_{rd} - \theta_{sr}\right)s_2 \quad (41)$$

and the average BER conditioned on h_{sr} can be written as

$$\widetilde{P}_b = \Pr\left(\sqrt{P_s}h_{sr}s_0 + \sqrt{P_1}h_1\cos(\theta)s_1 + \tilde{n}_r + \frac{\sqrt{P_2}h_2\cos\left(\varphi\right)s_2}{h_{rd}\mathcal{G}_{\mathrm{F}}} < 0 \middle| s_0 = +1, h_{sr}\right) \quad (42)$$

where $\varphi = (\theta_2 - \theta_{rd} - \theta_{sr})$. Let $\Upsilon = \sqrt{P_1}h_1\cos(\theta)s_1 + \tilde{n}_r + \frac{\sqrt{P_2}h_2\cos(\varphi)s_2}{h_{rd}\mathcal{G}_{\mathrm{F}}} = I + X_2$. By using a similar approach as in Section IV-A we can re-express (42) as

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{P_s\sigma_{sr}^2}{\pi}}\int_0^{\infty}\Phi_{\Upsilon}(z)e^{-\frac{P_s\sigma_{sr}^2z^2}{4}}dz\right) \quad (43)$$

where the CF of Υ is expressed as

$$\Phi_{\Upsilon}(z) = \Phi_I(z)\Phi_{X_2}(z) = e^{-\frac{P_1\sigma_1^2z^2}{4(1+\omega_1)}}J_0\left(\sqrt{\frac{\omega_1P_1\sigma_1^2}{1+\omega_1}}z\right)\Phi_{X_2}(z). \quad (44)$$

Moreover, $\Phi_{X_2}(z)$ is expressed as

$$\Phi_{X_2}(z) = e^{-\frac{N_{01}z^2}{4}}\int_0^{\infty}J_0\left(\frac{\sqrt{P_2}yz}{\mathcal{G}_{\mathrm{F}}}\right)f_Y(y)dy \quad (45)$$

where $Y = \frac{h_2}{h_{rd}}$ and $f_Y(y)$ is the pdf of the ratio of a Rician/Rayleigh RV. Using the cdf of h_2 and the pdf of h_{rd} we can express the cdf of Y as

$$F_Y(y) = \int_0^{\infty}F_{h_2}(xy)f_{h_{rd}}(x)dx = 1 - \frac{2}{\sigma_{rd}^2}\int_0^{\infty}x\,Q\left(\sqrt{2\omega_2}, \sqrt{\frac{2(1+\omega_2)}{\sigma_2^2}}xy\right)e^{-\frac{x^2}{\sigma_{rd}^2}}dx \quad (46)$$

where $Q(a,b)$ is the Marcum Q-function [12, Eq. (1)]. Simplifying (46) using [12, Eq. (40)] and [12, Eq. (41)] yields

$$F_Y(y) = \frac{(1+\omega_2)\sigma_{rd}^2y^2}{(1+\omega_2)\sigma_{rd}^2y^2 + \sigma_2^2}e^{-\frac{\omega_2\sigma_2^2}{(1+\omega_2)\sigma_{rd}^2y^2+\sigma_2^2}}. \quad (47)$$

Differentiating (47) with respect to y, we obtain the pdf as

$$f_Y(y) = \frac{2\omega_2(1+\omega_2)^2\sigma_{rd}^4\sigma_2^2y^3}{((1+\omega_2)\sigma_{rd}^2y^2 + \sigma_2^2)^3}e^{-\frac{\omega_2\sigma_2^2}{(1+\omega_2)\sigma_{rd}^2y^2+\sigma_2^2}} + \frac{2(1+\omega_2)\sigma_{rd}^2\sigma_2^2y}{((1+\omega_2)\sigma_{rd}^2y^2 + \sigma_2^2)^2}e^{-\frac{\omega_2\sigma_2^2}{(1+\omega_2)\sigma_{rd}^2y^2+\sigma_2^2}}. \quad (48)$$

Hence, by substituting (48) into (45), $\Phi_{X_2}(z)$ can be evaluated. Unfortunately, however, with $\omega_2 \neq 0$, $\Phi_{X_2}(z)$ can not be evaluated in closed-form. Nevertheless, using (44) in (43), the average BER can be expressed as

$$P_b = \tfrac{1}{2}\left(1 - \sqrt{\tfrac{P_s\sigma_{sr}^2}{\pi}}\int_0^{\infty}\left(\int_0^{\infty}J_0\left(\tfrac{\sqrt{P_2}yz}{\mathcal{G}_{\mathrm{F}}}\right)f_Y(y)dy\right) \times J_0\left(\sqrt{\tfrac{\omega_1P_1\sigma_1^2}{1+\omega_1}}z\right)e^{-\left(P_s\sigma_{sr}^2 + \frac{P_1\sigma_1^2}{1+\omega_1} + N_{01}\right)\frac{z^2}{4}}dz\right). \quad (49)$$

In the special case where h_2 is Rayleigh distributed, we can substitute $\omega_2 = 0$ into (47) and next using (45), $\Phi_{X_2}(z)$ can be written as

$$\Phi_{X_2}(z) = \frac{2\sigma_2^2e^{-\frac{N_{01}z^2}{4}}}{\sigma_{rd}^2}\int_0^{\infty}\frac{y\,J_0\left(\frac{\sqrt{P_2}yz}{\mathcal{G}_{\mathrm{F}}}\right)}{\left(y^2 + \frac{\sigma_2^2}{\sigma_{rd}^2}\right)^2}dy. \quad (50)$$

Using [13, Eq. (2.12.2.28)] and noting that $K_{-1}(x) = K_1(x)$, (50) can be evaluated as

$$\Phi_{X_2}(z) = \frac{\sqrt{P_2}\sigma_2}{\mathcal{G}_{\mathrm{F}}\sigma_{rd}}\,z\,e^{-\frac{N_{01}z^2}{4}}K_1\left(\frac{\sqrt{P_2\sigma_2^2}}{\mathcal{G}_{\mathrm{F}}\sigma_{rd}}z\right). \quad (51)$$

Finally, if h_1 is also Rayleigh distributed, it can be shown that the overall BER takes the form

$$P_b = \frac{1}{2}\left(1 - \frac{\eta_2}{2\bar{\gamma}_2}\sqrt{\frac{\bar{\gamma}_1}{\bar{\gamma}_1 + \eta_1 + 1}}e^{\frac{\eta_2}{2\bar{\gamma}_2}}\Theta\left(\frac{\eta_2}{2\bar{\gamma}_2}\right)\right). \quad (52)$$

V. Numerical Results

In this section, we illustrate the expressions derived in Sections III and IV using numerical examples and examine the effect of interference on the system's performance. It is noted that all results presented here were also verified by simulations.

Fig. 1 depicts the outage probability of dual-hop AF fixed gain relaying for different values of the Rician K-factor, ω_1, and assuming *Scenario (a)*. The power of the interfering signal at R is assumed to be 20 dB and 40 dB lower than the received signal power, while the average SNRs at the S-R and R-D links are assumed equal to each other, so that $\eta_1 = \bar{\gamma}_1 - 20$ dB $= \bar{\gamma}_2 - 20$ dB and $\eta_1 = \bar{\gamma}_1 - 40$ dB $= \bar{\gamma}_2 - 40$ dB. As can be seen, the outage probability is hardly affected by the Rician K-factor of the interferer channel at R, yet any increase in ω_1 results in a small outage probability increase.

Similar results regarding the outage probability of the fixed gain system under consideration of the *Scenario (b)* are shown in Fig. 2, where the relative received power of the interferers at both R and D are assumed 20 dB and 40 dB lower than the received signal power at R and D, respectively. In particular, it is noted that there is hardly any dependence of the outage probability on the Rician K-factor of the interferer on the relay, as well as that the achievable diversity order is zero, as was analytically shown in Section III-A2.

Fig. 3 illustrates the BPSK-modulated BER results for both *Scenarios (a)* and *(b)*. In addition to the aforementioned observations, one may note that in the case of $\eta_2 = \bar{\gamma}_1 - 20$ dB and for low-to-medium SNRs, there is a cross-over point in the average BER curves of *Scenarios (a)* and *(b)*. This interesting observation can be explained from the fact that, in the low SNR region, the BER is dominated by the AWGN power. Therefore,

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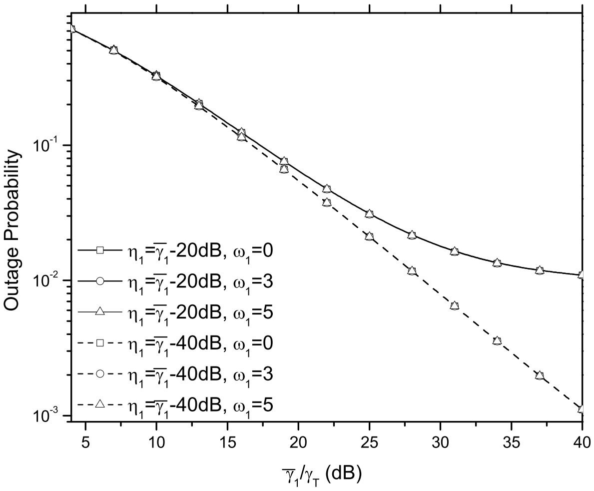

Fig. 1. Outage probability of *Scenario (a)* for $\eta_1 = \bar{\gamma}_1 - 20$ dB $= \bar{\gamma}_2 - 20$ dB and $\eta_1 = \bar{\gamma}_1 - 40$ dB $= \bar{\gamma}_2 - 40$ dB.

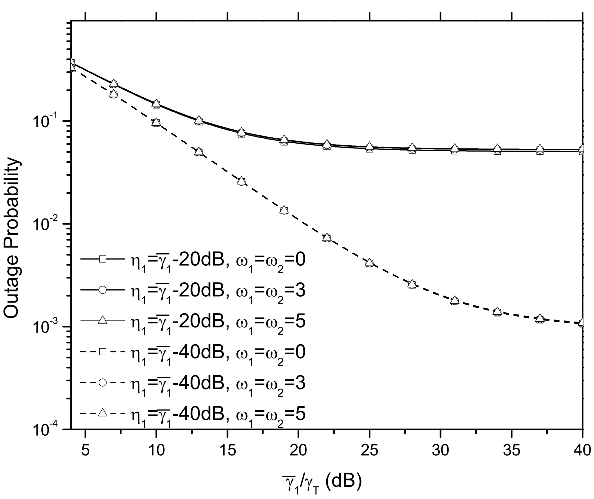

Fig. 2. Outage probability of *Scenario (b)* for $\bar{\gamma}_1 = \bar{\gamma}_2$ and $\eta_1 = \eta_2 = \bar{\gamma}_1 - 20$ dB and $\eta_1 = \eta_2 = \bar{\gamma}_1 - 40$ dB.

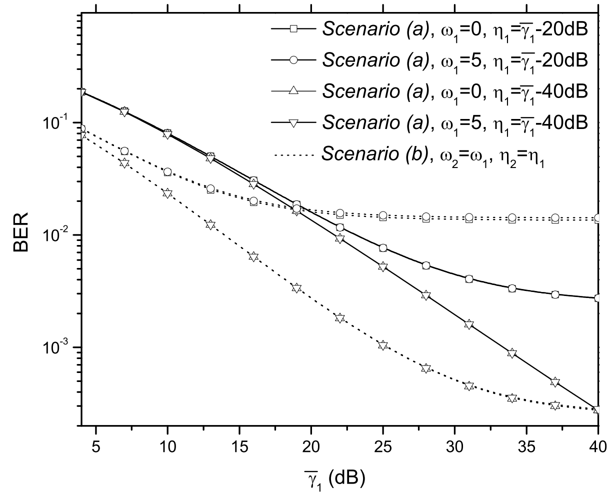

Fig. 3. BER of *Scenarios (a)* and *(b)*, for $\bar{\gamma}_1 = \bar{\gamma}_2$ and $\eta_1 = \eta_2 = \bar{\gamma}_1 - 20$ dB and $\eta_1 = \eta_2 = \bar{\gamma}_1 - 40$ dB.

a low interference power, ($\eta_2 = \bar{\gamma}_1 - 20$ dB) at D in the case of *Scenario (b)* can lead to a lower BER, as compared to high AWGN power at D in case of *Scenario (a)*. We also note that the BER cross-over point in the case of $\eta_2 = \bar{\gamma}_1 - 40$ dB is at $\bar{\gamma}_1 = 40$ dB.

VI. Conclusions

In this paper, we investigated the performance of a dual-hop fixed gain AF relay system with interference. Both interference-free and interference-limited reception at the destination were studied by deriving new expressions for the outage probability and, as well as precise average BER expressions. It was shown that, as expected the presence of interference at the relay and/or the destination significantly degrades the performance. Numerical results, which were verified by simulations, revealed that the overall performance is hardly affected by the Rician-K factor of the interfering channel.

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