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FIXED POINT RESULTS IN LOCALLY CONVEX SPACES WITH τ-KREIN-ŠMULIAN PROPERTY AND APPLICATIONS

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Abstract. In this paper, we present some new fixed point theorems in a locally convex space X with the so called τ-Krein-Šmulian property considering the concept of Φ_Λ^τ-measures of noncompactness, where τ is a weaker Hausdorff locally convex topology of X. Further, we apply our results to discuss the existence of solutions for a nonlinear functional integral equation in the Lebesgue space L^1.
Key Words and Phrases: Φ_Λ^τ-measure of noncompactenss, τ-sequentially continuous, τ-Krein-Šmulian property, angelic space.
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