Fixed Point Theory, 22(2021), No. 2, 625-644 DOI: 10.24193/fpt-ro.2021.2.41 http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

FIXED POINT THEOREMS IN *r*-NORMED AND LOCALLY *r*-CONVEX SPACES AND APPLICATIONS

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Abstract. In this paper we prove some new fixed point theorems in *r*-normed and locally *r*-convex spaces. Our conclusions generalize many well-known results and provide a partial affirmative answer to Schauder's conjecture. Based on the obtained results, we prove the analogue of a Von Neumann's theorem in locally *r*-convex spaces. In addition, an application to game theory is presented. **Key Words and Phrases**: *r*-normed space, locally *r*-convex space, *s*-convex set, fixed point theorems, topological vector space, Schauder's conjecture.

2020 Mathematics Subject Classification: 47H10, 46A16, 46A50.

Acknowledgement. The authors are grateful to the editor and the anonymous referees for their useful comments and suggestions that contributed to the improvement of the original version of this work.

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Received: April 25, 2019; Accepted: October 31, 2019.