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Fixed point theory for nonself generalized contractions in Kasahara spaces

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Dedicated to the memory of Professor Ştefan Măruşter

Abstract. In this paper we extend some results of V. Berinde, Şt. Măruşter and I.A. Rus (*Saturated contraction principles for nonself operators, generalizations and applications*, Filomat, 31:11(2017), 3391-3406), which were given in metric spaces, to Kasahara spaces. Some research directions are also presented.

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1 Introduction and preliminaries

The aim of this paper is to extend some results given in [4], in a metric space, to a Kasahara space.

1.0 Notations

Throughout this paper we follow the notations given in [18] and [8].

1.1 L-spaces

Definition 1.1 (M. Fréchet [9]). *Let X be a nonempty set. Let*

$$s(X) := \big\{(x_n)_{n\in\mathbb{N}} \mid x_n \in X,\ n \in \mathbb{N}\big\}.$$

Let $c(X) \subset s(X)$ *be a subset of* $s(X)$ *and* $Lim : c(X) \to X$ *be an operator. By definition, the triple* $(X, c(X), Lim)$ *is called an* L*-space if the following conditions are satisfied:*

(*i*) *If* $x_n = x$, *for all* $n \in \mathbb{N}$, *then* $\{x_n\}_{n\in\mathbb{N}} \in c(X)$ *and* $Lim\{x_n\}_{n\in\mathbb{N}} = x$.

(*ii*) *If* $\{x_n\}_{n\in\mathbb{N}} \in c(X)$ *and* $Lim\{x_n\}_{n\in\mathbb{N}} = x$, *then for all subsequences* $\{x_{n_i}\}_{i\in\mathbb{N}}$ *of* $\{x_n\}_{n\in\mathbb{N}}$ *we have that* $\{x_{n_i}\}_{i\in\mathbb{N}} \in c(X)$ *and* $Lim\{x_{n_i}\}_{i\in\mathbb{N}} = x$.

By definition, an element $\{x_n\}_{n\in\mathbb{N}}$ of $c(X)$ is a convergent sequence and $x = Lim\{x_n\}_{n\in\mathbb{N}}$ is the limit of this sequence and we shall write

$$x_n \overset{F}{\to} x \text{ as } n \to \infty.$$

We denote an L-space by $(X, \overset{F}{\to})$.

Example 1.1. In general, an L-space is any set endowed with a structure implying a notion of convergence for sequences. For example, Hausdorff topological spaces, metric spaces, $\mathbb{R}^m_+$-metric spaces, generalized metric spaces in Luxemburg' sense (i.e. $d(x,y) \in \mathbb{R}_+ \cup \{+\infty\}$), K-metric spaces (i.e. $d(x,y) \in K$, where K is a cone in an ordered Banach space), gauge spaces, 2-metric spaces, D-R-spaces, probabilistic metric spaces, syntopogenous spaces, are relevant examples of such L-spaces.

1.2 Kasahara spaces

In this paper, by a Kasahara space we understand a triple $(X, \overset{F}{\to}, \rho)$ where (i.e., a large Kasahara space in the terminology of [18] and [8]):

(1) $(X, \overset{F}{\to})$ is an L-space;

(2) $\rho : X \times X \to \mathbb{R}_+$ is a dislocated metric, i.e.,

   (*i*) $\rho(x,y) = 0 \Rightarrow x = y$;

   (*ii*) $\rho(x,y) = \rho(y,x)$, for all $x, y \in X$;

   (*iii*) $\rho(x,y) \le \rho(x,z) + \rho(z,y)$, for all $x, y, z \in X$;

(3) if $\{y_n\}_{n\in\mathbb{N}} \subset X$ is such that

$$\rho(y_n, y_m) \to 0 \text{ as } n, m \to \infty,$$

then $\{y_n\}_{n\in\mathbb{N}}$ is convergent in $(X, \overset{F}{\to})$.

For examples of such Kasahara spaces see [18] and [8]. For dislocated metric spaces see [15] and the references therein.

A relevant example of Kasahara space is the following one.

Example 1.2. Let X be a nonempty set, $d : X \times X \to \mathbb{R}_+$ be a complete metric on X and $\rho : X \times X \to \mathbb{R}_+$ be a dislocated metric on X. We suppose that there exists $c > 0$ such that

$$d(x,y) \leq c\rho(x,y), \text{ for all } x, y \in X.$$

Then, $(X, \overset{d}{\to}, \rho)$ is a Kasahara space.

1.3 Partial metric spaces as Kasahara spaces

Let (X,p) be a partial metric space (see [1], [7]-[17], [22], ...; for an heuristic introduction to the partial metric spaces, see [5]). Let us consider the following functionals induced by a partial metric on X:

$$d_p^s : X \times X \to \mathbb{R}_+,\ d_p^s(x,y) := 2p(x,y) - p(x,x) - p(y,y)$$

and

$$\tilde{d}_p : X \times X \to \mathbb{R}_+,\ \tilde{d}_p(x,y) := \begin{cases} p(x,y), \text{ if } x \neq y, \\ 0, \text{ if } x = y. \end{cases}$$

These two functionals are metrics on X.

Moreover we have:

- $d_p^s(x,y) \leq 2p(x,y)$, for all $x, y \in X$;
- $\tilde{d}_p(x,y) \leq p(x,y)$, for all $x, y \in X$.

It is clear that if d_p^s is complete then $(X, \overset{d_p^s}{\to}, p)$ is a Kasahara space.

It is also clear that if $\tilde{d}_p$ is complete then $(X, \overset{\tilde{d}_p}{\to}, p)$ is a Kasahara space.

2 Theorems of equivalent statements

The basic problem for a nonself operator f is to give conditions which imply that $F_f \neq \emptyset$. For a better understanding of this problem, in what follows we shall present some of such conditions.

Theorem 2.1 (Theorem of equivalent statements)**.** *Let $(X, \xrightarrow{F}, \rho)$ be a Kasahara space, $Y \in P_{cl}(X, \xrightarrow{F})$ and $f : Y \to X$ be an operator. We suppose that:*

(i) if $x_n \xrightarrow{F} x^*$, $y_n \xrightarrow{F} y^*$ and $\rho(x_n, y_n) \to 0$ as $n \to \infty$, then $x^* = y^*$;

(ii) $f : (Y, \xrightarrow{F}) \to (X, \xrightarrow{F})$ is continuous;

(iii) $f : (Y, \rho) \to (X, \rho)$ is an l-contraction.

Then the following statements are equivalent:

(a) $F_f = \{x^*\}$;

(b) There exists a sequence $\{y_n\}_{n\in\mathbb{N}}$ in Y such that $\rho(y_n, f(y_n)) \to 0$ as $n \to \infty$;

(c) There exists a sequence $\{y_n\}_{n\in\mathbb{N}} \subset P_b(Y, \rho)$ such that $\rho(y_{n+1}, f(y_n)) \to 0$ as $n \to \infty$;

(d) There exists $U \in P_{cl}(Y, \xrightarrow{F})$ such that $f(U) \subset U$;

(e) There exists $U \in P_{cl}(Y, \xrightarrow{F})$ and a nonexpansive retraction $r : (X, \rho) \to (U, \rho)$ such that $f : U \to X$ is retractible with respect to r.

Proof. First, we remark that (a) implies all of the statements (b)-(e). Indeed, by choosing the sequence $\{y_n\}_{n\in\mathbb{N}} \subset Y$, $y_n := x^*$, for all $n \in \mathbb{N}$, and the set $U = \{x^*\} \subset Y$, the conclusions follow.

$(b) \Rightarrow (a)$. Let $\{y_n\}_{n\in\mathbb{N}}$ in Y such that $\rho(y_n, f(y_n)) \to 0$ as $n \to \infty$. Let $m \in \mathbb{N}$, $m > n$. We have

$$\begin{aligned}\rho(y_n, y_m) &\le \rho(y_n, f(y_n)) + \rho(f(y_n), f(y_m)) + \rho(f(y_m), y_m) \le \\ &\le \rho(y_n, f(y_n)) + l\rho(y_n, y_m) + \rho(f(y_m), y_m),\end{aligned}$$

which implies further that

$$\rho(y_n, y_m) \le \frac{1}{1-l}[\rho(y_n, f(y_n)) + \rho(f(y_m), y_m)] \to 0 \text{ as } n, m \to \infty.$$

Hence, $\{y_n\}_{n\in\mathbb{N}^*}$ is convergent in $(X, \xrightarrow{F})$. So, there exists $y^* \in X$ such that $y_n \xrightarrow{F} y^*$ as $n \to \infty$. By (ii), we have that $f(y_n) \xrightarrow{F} f(y^*)$ as $n \to \infty$. Since $\rho(y_n, f(y_n)) \to 0$ as $n \to \infty$, we have:

$$\lim_{n\to\infty} \rho(y_n, f(y_n)) = \rho(y^*, f(y^*)) = 0.$$

So $y^* \in F_f$. The uniqueness of the fixed point y^* is assured by (iii). Indeed, if $x^* \in Y$ is another fixed point for f, then $\rho(x^*, y^*) = \rho(f(x^*), f(y^*)) \leq l\rho(x^*, y^*)$, i.e., $(1 - l)\rho(x^*, y^*) \leq 0$, so $\rho(x^*, y^*) = 0$ which implies that $x^* = y^*$.

$(c) \Rightarrow (a)$. Let $\{y_n\}_{n\in\mathbb{N}} \subset P_b(Y, \rho)$ such that $\rho(y_{n+1}, f(y_n)) \to 0$ as $n \to \infty$. Let $p \in \mathbb{N}$. We have

$$\begin{aligned}\rho(y_{n+p+1}, y_{n+1}) &\leq \rho(y_{n+p+1}, f(y_{n+p})) + \rho(f(y_{n+p}), f(y_n)) + \rho(y_{n+1}, f(y_n))\\ &\leq \rho(y_{n+p+1}, f(y_{n+p})) + l\rho(y_{n+p}, y_n) + \rho(y_{n+1}, f(y_n))\\ &\leq \rho(y_{n+p+1}, f(y_{n+p})) + l\rho(y_{n+p}, f(y_{n+p-1})) + \ldots +\\ &\quad + l^{n+1}\rho(y_p, f(y_{p-1})) + l^{n+1}\rho(y_p, y_0) +\\ &\quad + l^n\rho(y_1, f(y_0)) + \ldots + l^1\rho(y_n, f(y_{n-1})) + \rho(y_{n+1}, f(y_n)).\end{aligned}$$

From a Cauchy lemma, we get that

$$\rho(y_{n+p+1}, y_{n+1}) \to 0 \text{ as } n \to \infty, \text{ for all } p \in \mathbb{N} \text{ or } p \to \infty.$$

It follows that $\{y_n\}_{n\in\mathbb{N}^*}$ is convergent in $(X, \xrightarrow{F})$. So, there exists $y^* \in X$ such that $y_n \xrightarrow{F} y^*$ as $n \to \infty$. By (ii) we have that $f(y_n) \xrightarrow{F} f(y^*)$ as $n \to \infty$. Since $\rho(y_{n+1}, f(y_n)) \to 0$ as $n \to \infty$, by (i), we get that $y^* = f(y^*)$. The uniqueness of y^* follows from (iii).

$(d) \Rightarrow (a)$. It follows from the contraction condition (iii).

$(e) \Rightarrow (a)$. Since $f\big|_U$ is retractible with respect to r, it follows that $F_{f|_U} = F_{r\circ f|_U}$. But $r \circ f|_U : U \to U$ is a contraction. □

3 Saturated principle of fixed points

In a Kasahara space we have the following saturated principle of contraction (see [19], [18], [8], [7]).

Theorem 3.1. *Let* $(X, \xrightarrow{F}, \rho)$ *be a Kasahara space and* $f : X \to X$ *be an operator. We suppose that:*

(i) $f : (X, \xrightarrow{F}) \to (X, \xrightarrow{F})$ *is orbitally continuous;*

(ii) $f : (X, \rho) \to (X, \rho)$ *is an* l*-contraction.*

Then we have that:

(a) $F_f = F_{f^n} = \{x^*\}$, *for all* $n \in \mathbb{N}^*$;

(b) $f^n(x) \xrightarrow{F} x^*$ *as* $n \to \infty$, *for all* $x \in X$;

(c) $\rho(f^n(x), x^*) \to 0$ *as* $n \to \infty$, *for all* $x \in X$;

(d) $\rho(x, x^*) \leq \frac{1}{1-l}\rho(x, f(x))$, *for all* $x \in X$;

(e) *if* $\{y_n\}_{n\in\mathbb{N}} \subset X$ *is such that* $\rho(y_n, f(y_n)) \to 0$ *as* $n \to \infty$, *then* $\rho(y_n, x^*) \to 0$ *as* $n \to \infty$, *i.e., the fixed point problem for* f *is well posed in* (X, ρ);

(f) *if* $\{y_n\}_{n\in\mathbb{N}} \subset X$ *is such that* $\rho(y_{n+1}, f(y_n)) \to 0$ *as* $n \to \infty$, *then* $\rho(y_n, x^*) \to 0$ *as* $n \to \infty$, *i.e., the operator* f *has the Ostrowski's property.*

Proof. The proof is similar with the proof of Theorem 2.1.2 in [8]. □

So, our problem is to extend Theorem 3.1 for the case of nonself operators.

Theorem 3.2 (Saturated principle of nonself contractions)**.** *Let* $(X, \xrightarrow{F}, \rho)$ *be a Kasahara space,* $Y \subset P_{cl}(X, \xrightarrow{F})$. *Let* $f : Y \to X$ *be an operator. We suppose that:*

(i) f *is an* l*-contraction;*

(ii) $F_f \neq \emptyset$.

Then:

(a) $F_f = \{x^*\}$. *Moreover, if for some* $y \in Y$ *and* $n \in \mathbb{N}^*$, $f^n(y)$ *is defined and* $f^n(y) = y$ *then* $y = x^*$;

(b) $\rho(x, x^*) \leq \psi(\rho(x, f(x)))$, *for all* $x \in Y$ *where* $\psi(t) = \frac{t}{1-l}$, $t \geq 0$;

(c) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $\rho(y_n, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \xrightarrow{F} x^*$ *as* $n \to \infty$, *i.e., the fixed point problem for* f *is well posed in* (X, ρ);

(d) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $\rho(y_{n+1}, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \xrightarrow{F} x^*$ *as* $n \to \infty$, *i.e., the operator* f *has the Ostrowski's property.*

Proof. (a) The uniqueness of the fixed point follows by the contraction condition. Let $y \in Y$ and $n \in \mathbb{N}^*$ be such that $f^n(y)$ is defined. Since $f^n(y) = y$, we have $f^{n+1}(y) = f(f^n(y)) = f(y)$ and $f^{n+1}(y) = f^n(f(y))$ which imply $y = f(y)$, i.e., $y \in F_f = \{x^*\}$.

(b) It follows by the fact that

$$\rho(x, x^*) \leq \rho(x, f(x)) + \rho(f(x), x^*) \leq \rho(x, f(x)) + l\rho(x, x^*)$$

which yields the retraction-displacement condition, i.e,

$$\rho(x, x^*) \leq \frac{1}{1-l}\rho(x, f(x)), \text{ for all } x \in Y.$$

(c) It follows by (b), by setting $x := y_n$ in the retraction-displacement condition.

(d) By (i), we obtain, in particular, that f is an l-quasicontraction, i.e., $\rho(f(x), f(x^*)) \leq l\rho(x, x^*)$, for all $x \in Y$. We have

$$\begin{aligned} \rho(y_{n+1}, x^*) &\leq \rho(y_{n+1}, f(y_n)) + \rho(f(y_n), x^*) \leq l\rho(y_n, x^*) + \rho(y_{n+1}, f(y_n)) \\ &\leq l[\rho(y_n, f(y_{n-1})) + \rho(f(y_{n-1}), x^*)] + \rho(y_{n+1}, f(y_n)) \\ &\leq l\rho(y_n, f(y_{n-1})) + l^2\rho(y_{n-1}, x^*) + \rho(y_{n+1}, f(y_n)) \\ &\leq \ldots \leq l\rho(y_n, f(y_{n-1})) + l^2\rho(y_{n-1}, f(y_{n-2})) + \ldots \\ &\qquad \ldots + l^{n+1}\rho(y_0, x^*) + \rho(y_{n+1}, f(y_n)) \end{aligned}$$

and by a Cauchy lemma, the conclusion follows. □

Theorem 3.3. *Let X be a nonempty set, $d : X \times X \to \mathbb{R}_+$ be a complete metric on X, $\rho : X \times X \to \mathbb{R}_+$ be a dislocated metric on X, $Y \in P_{cl}(X, d)$ and $f : (Y, \rho) \to (X, \rho)$ be an l-contraction. In addition, we suppose that:*

- (i) *there exists $c > 0$ such that $d(x, y) \leq c\rho(x, y)$, for all $x, y \in X$;*
- (ii) *$f : (Y, d) \to (X, d)$ is continuous;*
- (iii) *$F_f \neq \emptyset$.*

Then we have:

- (a) *$F_f = \{x^*\}$;*
- (b) *$\rho(x, x^*) \leq \frac{1}{1-l}\rho(x, f(x))$, for all $x \in X$;*
- (c) *for each sequence $\{y_n\}_{n\in\mathbb{N}}$ in Y with $\rho(y_n, f(y_n)) \to 0$ as $n \to \infty$, we have that $y_n \xrightarrow{d} x^*$ as $n \to \infty$, i.e., the fixed point problem for f is well posed in (X, ρ);*

(*d*) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $\rho(y_{n+1}, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \overset{d}{\to} x^*$ *as* $n \to \infty$, *i.e., the operator* f *has the Ostrowski's property.*

Proof. We remark that $(X, \overset{d}{\to}, \rho)$ is a Kasahara space and we are in the conditions of Theorem 3.2, with $\overset{F}{\to} = \overset{d}{\to}$. The proof follows from Theorem 3.2. □

4 Partial metric spaces

The notion of partial metric was introduced by S.G. Matthews in [13] as follows:

Definition 4.1. *Let* X *be a nonempty set. A functional* $p : X \times X \to \mathbb{R}_+$ *is a partial metric on* X *if* p *satisfies the following conditions:*

(*i*) $p(x, x) = p(y, y) = p(x, y)$ *if and only if* $x = y$;

(*ii*) $p(x, x) \leq p(x, y)$, *for all* $x, y \in X$;

(*iii*) $p(x, y) = p(y, x)$, *for all* $x, y \in X$;

(*iv*) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$, *for all* $x, y, z \in X$.

The couple (X, p), where X is a nonempty set and p is a partial metric on X, is called a partial metric space.

For examples of partial metric spaces see [13], [17].

Let us consider now the Kasahara spaces $(X, \overset{d_p^s}{\to}, p)$ and $(X, \overset{\tilde{d}_p}{\to}, p)$. We have:

Theorem 4.1. *Let* (X, p) *be a partial metric space,* $Y \in P_{cl}(X, d_p^s)$ *and* $f : (X, p) \to (X, p)$ *be an* l*-contraction. We suppose that:*

(*i*) $f : (Y, d_p^s) \to (Y, d_p^s)$ *is continuous;*

(*ii*) $F_f \neq \emptyset$.

Then we have:

(*a*) $F_f = \{x^*\}$;

(*b*) $p(x, x^*) \leq \frac{1}{1-l} p(x, f(x))$, *for all* $x \in X$;

(c) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $p(y_n, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \overset{d_p^s}{\to} x^*$ *as* $n \to \infty$, *i.e., the fixed point problem for* f *is well posed in* (X, p);

(d) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $p(y_{n+1}, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \overset{d_p^s}{\to} x^*$ *as* $n \to \infty$, *i.e., the operator* f *has the Ostrowski's property.*

Proof. The proof follows from Theorem 3.3. □

Theorem 4.2. *Let* (X, p) *be a partial metric space,* $Y \in P_{cl}(X, d_p^s)$ *and* $f : (X, p) \to (X, p)$ *be an* l*-contraction. We suppose that:*

(i) $f : (Y, \tilde{d}_p) \to (Y, \tilde{d}_p)$ *is continuous;*

(ii) $F_f \neq \emptyset$.

Then we have:

(a) $F_f = \{x^*\}$;

(b) $p(x, x^*) \leq \frac{1}{1-l} p(x, f(x))$, *for all* $x \in X$;

(c) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $p(y_n, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \overset{\tilde{d}_p}{\to} x^*$ *as* $n \to \infty$, *i.e., the fixed point problem for* f *is well posed in* (X, p);

(d) *for each sequence* $\{y_n\}_{n\in\mathbb{N}}$ *in* Y *with* $p(y_{n+1}, f(y_n)) \to 0$ *as* $n \to \infty$, *we have that* $y_n \overset{\tilde{d}_p}{\to} x^*$ *as* $n \to \infty$, *i.e., the operator* f *has the Ostrowski's property.*

Proof. The proof follows from Theorem 3.3. □

5 Research directions

5.1. To extend the results of this paper in the case of some generalized contractions (φ-contractions, Kannan type contractions, strongly demicontractive operators, ...).

References: [12], [21], [1], [8], [14], [16], [17], [20], [22], [3], [21], [6], ...

5.2. To extend the results of this paper to the case of multivalued operators.

References: [12], [21], [8], ...

5.3. To apply these type of results in logic semantic programing and more general, to computer science.

References: [5], [1], [11], [2] ...

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