Nonlinear Analysis: Modelling and Control, 2018, Vol. 23, No. 6, 904–920 ISSN 1392-5113
https://doi.org/10.15388/NA.2018.6.6

Fixed-time control of delayed neural networks with impulsive perturbations*

Jingting Hu[a,1], **Guixia Sui**[b], **Xiaoxiao Lv**[c,d], **Xiaodi Li**[c]

[a]School of Mathematical Sciences, University of Jinan,
Ji'nan 250022, Shandong, China
ss_hujt@ujn.edu.cn

[b]Primary Education Department, Jinan Preschool Education College,
Ji'nan 250307, Shandong, China
1227796126@qq.com

[c]School of Mathematics and Statistics, Shandong Normal University,
Ji'nan 250014, Shandong, China
lxd@sdnu.edu.cn

[d]School of Mathematics, Southeast University,
Nanjing 210096, Jiangsu, China
lxx_math@163.com

Received: February 16, 2018 / **Revised:** June 29, 2018 / **Published online:** October 31, 2018

Abstract. This paper is concerned with the fixed-time stability of delayed neural networks with impulsive perturbations. By means of inequality analysis technique and Lyapunov function method, some novel fixed-time stability criteria for the addressed neural networks are derived in terms of linear matrix inequalities (LMIs). The settling time can be estimated without depending on any initial conditions but only on the designed controllers. In addition, two different controllers are designed for the impulsive delayed neural networks. Moreover, each controller involves three parts, in which each part has different role in the stabilization of the addressed neural networks. Finally, two numerical examples are provided to illustrate the effectiveness of the theoretical analysis.

Keywords: fixed-time stability, delayed neural networks, impulsive perturbations, settling time, linear matrix inequality.

1 Introduction

In the past few decades, neural networks have received considerable attention due to their various applications in many fields such as associative memories, image processing, speech recognition, automatic control engineering, etc. [2, 10, 39]. In implementations of

*This work was supported by National Natural Science Foundation of China (61673247, 11501333) and the Research Fund for Distinguished Young Scholars and Excellent Young Scholars of Shandong Province (JQ201719, ZR2016JL024). The paper has not been presented at any conference.

[1]Corresponding author.

artificial neural networks, time delays are unavoidable owing to the finite switching speed of amplifiers and the inherent communication time between neurons [16, 32]. Moreover, time delays always influence the dynamic properties of delayed neural networks, which may cause instability, oscillation, chaos, and so on [5, 14, 17, 20]. In recent years, much attention has been given to the analysis of time-delay systems [6, 8, 9, 38]. Impulsive dynamical systems can be viewed as basic models to study the dynamics of processes that are subject to abrupt changes in the system state at certain instants. Impulsive systems have three necessary components, that is, a continuous differential equation that determines the evolution of the system between pulses; an impulse state jumping function that describes how the system state changes at the impulse moments; and a criterion that determines when the states of the system are to be changed. The basic theory for impulsive differential systems has been widely studied in the past several years, see [15, 18, 19, 24, 26, 34, 41, 43] and the references therein. In [18], the authors established general and applicable results for uniform stability, uniform asymptotic stability for nonlinear differential systems with state-dependent delayed impulses. Paper [26] investigated the globally exponential stability for a class of Markovian jumping Cohen–Grossberg BAM neural networks with mixed time delays and impulsive effects. In [15], the authors studied the periodic solutions problem for impulsive differential equations and derived some conditions ensuring the existence and global attractiveness of unique periodic solution.

In recent years, finite-time stability and control of nonlinear dynamical systems have attracted increasing attention [1, 3, 4, 22, 23, 25, 27, 33, 35, 36, 42]. Different from the Lyapunov stability, finite-time stability requires that the state trajectories of the system tend to an equilibrium state in a finite time, and the settling time for finite-time convergence depends on the initial conditions. However, considering that the initial conditions for many real systems may be difficult or impossible to obtain, it is difficult to obtain a good estimate of the settling time. To address this problem, Polyakov in [29] introduced the fixed-time stability, which meaning that the system is globally finite-time stable and the settling time is bounded by some positive constant for any initial values. Although finite-time stability has been investigated extensively, not so much has been developed in the direction of fixed-time stability due to the lacking of the theory of fixed-time stability. Some interesting results have been obtained for fixed-time stability and control problems [5, 7, 11–13, 21, 28, 30, 37, 40, 44]. Paper [30] proposed finite-time and fixed-time observation for linear multiple input and output control systems. In [11], a new theorem of fixed-time stability of coupled discontinuous neural networks was given, and the settling time was accurately estimated. Paper [44] investigated a fixed-time terminal sliding-mode control method for a kind of second-order nonlinear systems with uncertainties and perturbations and obtained a guaranteed closed-loop convergence time by using the Lyapunov functions. In [7], authors designed a novel feedback controller to realize the fixed-time synchronization for complex-valued neural networks with uncertainty and discontinuous activation functions and especially derived criteria of modified controller for fixed-time anti-synchronization. Comparing with the existing literatures on fixed-time stability, there are rare results on the fixed-time stability of neural networks with impulsive perturbations [12, 37]. Even more rare are papers that deal with the fixed-time stability of neural networks with time delays and impulsive perturbations. Since time-delays and impulses can affect the system's dynamic behavior, which will result in oscillations and instabilities, it is necessary to study the effects of impulses and delays on the fixed-time stability of neural networks.

In this paper, motivated by the above discussions, we investigate the fixed-time control of delayed neural networks with impulsive perturbations. By using the Lyapunov stability theory, some sufficient conditions are derived to guarantee the fixed-time stability in terms of linear matrix inequalities, which can be easily verified by the LMI toolbox. The delay-dependent controllers are designed via the established LMIs. Our proposed controllers include three parts: one part is used to stabilize the impulsive delayed system in Lyapunov sense; another one is used to realize the finite-time stability; the other is used to achieve the fixed-time stable. In this paper, two different types of delayed feedback controller are designed for the fixed-time stability of the addressed neural networks. The remainder of the paper is organized as follows. In Section 2, the model of delayed neural networks with impulsive perturbations and some necessary definitions, lemmas, assumptions are given. The fixed-time stability theorems for delayed neural networks with impulsive perturbations are established in Section 3. In Section 4, two numerical simulation examples are provided to show the effectiveness of theoretical analysis.

2 Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of positive numbers, $\mathbb{N}$ the set of positive integer numbers, $\mathbb{R}^n$ the n-dimensional real spaces equipped with the Euclidean norm $|\cdot|$. $A > 0$ or $A < 0$ denotes that the matrix A is a symmetric and positive definite or negative definite matrix. The notations A^{T} and A^{-1} denote the transpose and the inverse of A, respectively. If A, B are symmetric matrices, $A > B$ means that $A - B$ is positive definite matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum eigenvalue and the minimum eigenvalue of matrix A, respectively. I denotes the identity matrix with appropriate dimensions, and $\Omega = \{1, 2, \dots, n\}$. For any interval $D \subseteq \mathbb{R}$, set $H \subseteq \mathbb{R}^l$ $(1 \leqslant l \leqslant n)$, $PC(D, H) = \{\phi : D \to H$ is continuous for all but at most a finite number of points t, at which $\phi(t^+)$, $\phi(t^-)$ exist and $\phi(t^+) = \phi(t)\}$.

Consider the following delayed neural networks with impulsive perturbations:

$$\begin{aligned}
&\dot{x}(t) = -Cx(t) + Af\bigl(x(t)\bigr) + Bg\bigl(x\bigl(t - \tau(t)\bigr)\bigr) + u(t), \quad t \in [t_k, t_{k+1}),\\
&\Delta x(t_k) = -D_k x\bigl(t_k^-\bigr), \quad k \in \mathbb{N},\\
&x(s) = \varphi(s), \quad s \in [-\tau, 0],
\end{aligned} \tag{1}$$

where $\varphi(\cdot) \in PC([-\tau, 0], \mathbb{R}^n)$; $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ stands for the neuron state vector of the neural network. $f(x(t)) = (f_1(x_1(t)), \dots, f_n(x_n(t)))^{\mathrm{T}}$ and $g(x(t - \tau(t))) = (g_1(x_1(t - \tau_1(t))), \dots, g_n(x_n(t - \tau_n(t))))^{\mathrm{T}}$ represent the neuron activation functions. $\tau(t)$ is a time-varying delay, which satisfies $0 \leqslant \tau(t) \leqslant \tau$, and τ is a positive constant. $C = \operatorname{diag}(c_1, \dots, c_n) > 0$ is a positive diagonal matrix. A and B are the connection weight matrix and the delayed connection weight matrix. For all $k \in \mathbb{N}$, $\Delta x(t_k) = x(t_k) - x(t_k^-)$, in which $x(t_k^-) = \lim_{x \to t_k^-} x(t)$ denotes the state jumps at the impulse moments t_k. Without loss of generalization, in this paper, we suppose that $x(t_k^+) = \lim_{x \to t_k^+} x(t) = x(t_k)$, i.e., the solution $x(t)$ is right continuous at impulse point t_k. The impulsive sequence $\{t_k\}$ satisfies $0 \leqslant t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k \to +\infty} t_k = +\infty$. $u(t)$ is the control input, which will be designed in the following form $u(t) = u_1(t) + u_2(t) + u_3(t)$, where the term $u_1(t)$ we will design can stabilize the impulsive neural networks in Lyapunov sense, $u_2(t)$ will realize the system finite-time stability, and $u_3(t)$ will be designed such that the neural networks is fixed-time stable.

Assumption 1. There exist constants $F_i > 0$, $G_i > 0$ such that the functions f_i, g_i ($i \in \Omega$) satisfy the following Lipschitz conditions:

$$\left|f_i(x) - f_i(y)\right| \leqslant F_i|x - y|, \quad \left|g_i(x) - g_i(y)\right| \leqslant G_i|x - y| \quad \forall x, y \in \mathbb{R},$$

and $f_i(0) = g_i(0) = 0$, $F = \operatorname{diag}(F_1, \ldots, F_n)$, $G = \operatorname{diag}(G_1, \ldots, G_n)$.

In the following, we will introduce some definitions and lemmas, which will play important roles in deriving the main results.

Definition 1. (See [29].) The origin of system (1) is said to be globally finite-time stable if it is Lyapunov stable and finite-time convergent. The finite-time convergence means for any initial state $x_0 \in \mathbb{R}^n$, there is a function $T : \mathbb{R}^n \setminus \{0\} \to (0, +\infty)$, called the settling time function, such that $\lim_{t \to T(x_0)} x(t, x_0) = 0$, and $x(t, x_0) = 0$ for all $t \geqslant T(x_0)$.

Definition 2. (See [29].) The origin of system (1) is said to be globally fixed-time stable if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded, i.e., there exists $T_{\max} > 0$ such that $T(x_0) \leqslant T_{\max}$ for all $x_0 \in \mathbb{R}^n$.

Definition 3. For any vector $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$, functions $S(x)$ and $\Gamma(x)$ are defined as

$$S(x) = \left(\operatorname{sign}(x_1), \ldots, \operatorname{sign}(x_n)\right)^{\mathrm{T}} \quad \text{and} \quad \Gamma(x) = \operatorname{diag}\left(|x_1|^{\mu}, \ldots, |x_n|^{\mu}\right).$$

Definition 4. (See [18].) Function $V : [t_0 - \tau, \infty) \times R^n \to \mathbb{R}_+$ is said to belong to class $\mathcal{V}$ if

(i) V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n$ for $x \in \mathbb{R}^n$, $k \in \mathbb{N}$, and $\lim_{(t,u) \to (t_k^-, v)} V(t, u) = V(t_k^-, v)$ exists,

(ii) V is locally Lipschitzian in x and $v(t, 0) \equiv 0$ for all $t \in \mathbb{R}_+$.

Lemma 1. (*See* [12].) *If there exists a positive definite, radially unbounded function* $V(x) : \mathbb{R}^n \to \mathbb{R} \in \mathcal{V}$ *such that any solution* $x(t)$ *of* (1) *satisfies the following inequalities*:

$$\begin{aligned} &\dot{V}\left(x(t)\right) \leqslant -\mu V^{\gamma}\left(x(t)\right) - \lambda V^{\delta}\left(x(t)\right), \quad t \in [t_k, t_{k+1}),\ t \in \mathbb{R}_+, \\ &V\left(x(t_k)\right) \leqslant V\left(x(t_k^-)\right), \quad k \in \mathbb{N}, \end{aligned}$$

where $\lambda > 0$, $\mu > 0$, $0 < \delta < 1$, $\gamma > 1$, *then system* (1) *is globally fixed-time stable, and the settling time* T *is estimated by* $T = 1/(\lambda(1 - \delta)) + 1/(\mu(\gamma - 1))$.

Lemma 2. (*See* [38].) *For any vectors $x, y \in \mathbb{R}^n$, a scalar $\varepsilon > 0$, if the matrix $Q \in \mathbb{R}^n$, $Q > 0$, then the following inequality holds*:

$$x^{\mathrm{T}}y + y^{\mathrm{T}}x \leqslant \varepsilon^{-1}x^{\mathrm{T}}Q^{-1}x + \varepsilon y^{\mathrm{T}}Qy.$$

Lemma 3. (*See* [31].) *For any $a_i \in \mathbb{R}$, $i \in \mathbb{N}$, and any real numbers $0 < p \leqslant 1$, $0 < q < 2$, we have the following inequalities*:

$$|a_1|^q + |a_2|^q + \cdots + |a_n|^q \geqslant \left(|a_1|^2 + |a_2|^2 + \cdots + |a_n|^2\right)^{q/2},$$
$$\left(|a_1| + |a_2| + \cdots + |a_n|\right)^p \leqslant |a_1|^p + |a_2|^p + \cdots + |a_n|^p.$$

3 Main results

In this section, we shall present some sufficient conditions for the fixed-time stability of delayed impulsive neural networks (1) by employing Lyapunov stability theory.

Theorem 1. *Suppose that Assumption* 1 *holds. If there exist $n \times n$ symmetric positive definite matrix $P > 0$, a positive definite diagonal matrix $Q > 0$, an $n \times n$ real matrix L, and some constants $k > 0$, $\delta > 0$, $0 \leqslant \mu < 1$, $\bar{\mu} > 0$, $\gamma > 1$ such that*

$$\begin{pmatrix} -PC - C^{\mathrm{T}}P - L - L^{\mathrm{T}} & PA \\ A^{\mathrm{T}}P & -Q \end{pmatrix} \leqslant 0,$$

$$(I - D_k)^{\mathrm{T}}P(I - D_k) - P \leqslant 0,$$

then system (1) *is fixed-time stable with the controller $u(t)$ given by*

$$\begin{aligned} u_1(t) &= -0.5\lambda_{\max}(Q)P^{-1}F^2x(t) - 0.5P^{-1}\left(L + L^{\mathrm{T}}\right)x(t) \\ &\quad - 0.5k\lambda_{\max}\left(PBB^{\mathrm{T}}P\right)P^{-1}S(x) \\ &\quad - 0.5k^{-1}P^{-1}S(x)x^{\mathrm{T}}\left(t - \tau(t)\right)G^2x\left(t - \tau(t)\right), \\ u_2(t) &= -0.5\delta\lambda_{\max}^{(1+\mu)/2}(P)P^{-1}\Gamma(x)S(x), \\ u_3(t) &= -0.5\bar{\mu}x(t)\left[x^{\mathrm{T}}(t)Px(t)\right]^{\gamma-1}. \end{aligned}$$

Moreover, the settling time is estimated by

$$T = \frac{1}{\delta(1 - \frac{1+\mu}{2})} + \frac{1}{\bar{\mu}(\gamma - 1)}.$$

Proof. Let $x(t) = x(t, 0, \varphi)$ be the solution of system (1) through $(0, \varphi)$. Choose the following Lyapunov function:

$$V(t) = x^{\mathrm{T}}(t)Px(t).$$

Taking the derivative of $V(t)$ along the trajectories of system (1) for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$,

$$\begin{aligned} D^+V(t) &= 2x^{\mathrm{T}}(t)P\dot{x}(t) \\ &= 2x^{\mathrm{T}}(t)P\bigl[-Cx(t) + Af\bigl(x(t)\bigr) + Bg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + u(t)\bigr] \\ &= x^{\mathrm{T}}(t)\bigl(-PC - C^{\mathrm{T}}P\bigr)x(t) + 2x^{\mathrm{T}}(t)PAf\bigl(x(t)\bigr) \\ &\quad + 2x^{\mathrm{T}}(t)PBg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + 2x^{\mathrm{T}}(t)Pu_1(t) \\ &\quad + 2x^{\mathrm{T}}(t)Pu_2(t) + 2x^{\mathrm{T}}(t)Pu_3(t). \end{aligned} \tag{2}$$

It follows from Lemma 2 that

$$\begin{aligned} 2x^{\mathrm{T}}(t)PAf\bigl(x(t)\bigr) &\leqslant x^{\mathrm{T}}(t)PAQ^{-1}A^{\mathrm{T}}Px(t) + f^{\mathrm{T}}\bigl(x(t)\bigr)Qf\bigl(x(t)\bigr) \\ &\leqslant x^{\mathrm{T}}(t)PAQ^{-1}A^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t)FQFx(t) \\ &\leqslant x^{\mathrm{T}}(t)PAQ^{-1}A^{\mathrm{T}}Px(t) + \lambda_{\max}(Q)x^{\mathrm{T}}(t)F^2x(t). \end{aligned} \tag{3}$$

Note that

$$x^{\mathrm{T}}(t)x(t) \leqslant \bigl[x^{\mathrm{T}}(t)S(x)\bigr]^2 \quad \forall x \in \mathbb{R}^n.$$

When $|x(t)| \neq 0$, it then follows from Assumption 1 and Lemma 2 that

$$\begin{aligned} &2x^{\mathrm{T}}(t)PBg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) \\ &\quad\leqslant k\frac{x^{\mathrm{T}}(t)PBB^{\mathrm{T}}Px(t)}{x^{\mathrm{T}}(t)S(x)} + k^{-1}g^{\mathrm{T}}\bigl(x\bigl(t-\tau(t)\bigr)\bigr)g\bigl(x\bigl(t-\tau(t)\bigr)\bigr)x^{\mathrm{T}}(t)S(x) \\ &\quad\leqslant k\lambda_{\max}\bigl(PBB^{\mathrm{T}}P\bigr)\frac{x^{\mathrm{T}}(t)x(t)}{x^{\mathrm{T}}(t)S(x)} + k^{-1}x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)x^{\mathrm{T}}(t)S(x) \\ &\quad\leqslant k\lambda_{\max}\bigl(PBB^{\mathrm{T}}P\bigr)x^{\mathrm{T}}(t)S(x) + k^{-1}x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)x^{\mathrm{T}}(t)S(x). \end{aligned}$$

Moreover, when $|x(t)| = 0$, it is easy to derive that

$$\begin{aligned} 2x^{\mathrm{T}}(t)PBg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) &= k\lambda_{\max}\bigl(PBB^{\mathrm{T}}P\bigr)x^{\mathrm{T}}(t)S(x) \\ &\quad + k^{-1}x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)x^{\mathrm{T}}(t)S(x) = 0. \end{aligned}$$

Hence, the following inequality holds for any $x \in \mathbb{R}^n$:

$$\begin{aligned} 2x^{\mathrm{T}}(t)PBg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) &\leqslant k\lambda_{\max}\bigl(PBB^{\mathrm{T}}P\bigr)x^{\mathrm{T}}(t)S(x) \\ &\quad + k^{-1}x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)x^{\mathrm{T}}(t)S(x). \end{aligned} \tag{4}$$

Substituting (3) and (4) into (2) and using Lemma 3, it then can be derived that

$$\begin{aligned} D^+V(t) &\leqslant x^{\mathrm{T}}(t)\bigl[-PC - C^{\mathrm{T}}P - L - L^{\mathrm{T}} + PAQ^{-1}A^{\mathrm{T}}P \\ &\quad + \lambda_{\max}(Q)F^2\bigr]x(t) + k\lambda_{\max}\bigl(PBB^{\mathrm{T}}P\bigr)x^{\mathrm{T}}(t)S(x) \\ &\quad + k^{-1}x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)x^{\mathrm{T}}(t)S(x) + x^{\mathrm{T}}(t)\bigl(L + L^{\mathrm{T}}\bigr)x(t) \\ &\quad + 2x^{\mathrm{T}}(t)Pu_1(t) + 2x^{\mathrm{T}}(t)Pu_2(t) + 2x^{\mathrm{T}}(t)Pu_3(t) \end{aligned}$$

$$\begin{aligned}
&\leqslant 2x^{\mathrm{T}}(t)Pu_2(t) + 2x^{\mathrm{T}}(t)Pu_3(t)\\
&\leqslant -\delta\lambda_{\max}^{(1+\mu)/2}(P)x^{\mathrm{T}}(t)\Gamma(x)S(x) - \bar{\mu}x^{\mathrm{T}}(t)Px(t)\big[x^{\mathrm{T}}(t)Px(t)\big]^{\gamma-1}\\
&\leqslant -\delta\big[x^{\mathrm{T}}(t)Px(t)\big]^{(1+\mu)/2} - \bar{\mu}\big[x^{\mathrm{T}}(t)Px(t)\big]^{\gamma}\\
&= -\delta V^{(1+\mu)/2}(t) - \bar{\mu}V^{\gamma}(t).
\end{aligned}$$

On the other hand, when $t = t_k$, it can be deduced that

$$\begin{aligned}
V(t_k) = x^{\mathrm{T}}(t_k)Px(t_k) &= \big[(I - D_k)x\big(t_k^-\big)\big]^{\mathrm{T}}P(I - D_k)x\big(t_k^-\big)\\
&= x^{\mathrm{T}}\big(t_k^-\big)(I - D_k)^{\mathrm{T}}P(I - D_k)x\big(t_k^-\big) \leqslant x^{\mathrm{T}}\big(t_k^-\big)Px\big(t_k^-\big) = V\big(t_k^-\big).
\end{aligned}$$

By Lemma 1, we obtain that system (1) is fixed-time stable via the controller u in Theorem 1, and the settling time is estimated by

$$T = \frac{1}{\delta(1 - \frac{1+\mu}{2})} + \frac{1}{\bar{\mu}(\gamma - 1)}.$$

This completes the proof. □

Remark 1. As we all known, the fixed-time stability is a special case of finite-time stability. One may observe from Theorem 1 that the term u_1 in controller u is used to stabilize the impulsive system in Lyapunov sense, u_2 to realize the system finite-time stability, and u_3 to achieve the fixed-time stable. Moreover, the settling time for fixed-time stability can be derived by choosing different parameters δ, μ, $\bar{\mu}$, and γ.

Remark 2. A important problem for fixed-time stability of delayed neural networks is to deal with the time delay. In this paper, we introduce the function $x^{\mathrm{T}}(t)S(x)$ to ensure that the inequality (4) holds, which provides an effective way to balance the relationship between the delay term and non-delay term. Moreover, the fixed-time stability conditions are derived in terms of linear matrix inequalities, which can make the controller easier to implement in practices.

In particular, if we only consider the finite-time stability of system (1), then the following corollary can be derived based on Theorem 1.

Corollary 1. (*See* [35].) *Suppose that Assumption* 1 *holds. If there exist* $n \times n$ *matrix* $P > 0$*, a positive definite diagonal matrix* Q*, an* $n \times n$ *real matrix* L*, and some constants* $k > 0$, $\delta > 0$, $0 \leqslant \mu < 1$ *such that*

$$\begin{pmatrix} -PC - C^{\mathrm{T}}P - L - L^{\mathrm{T}} & PA\\ A^{\mathrm{T}}P & -Q \end{pmatrix} \leqslant 0,$$

$$(I - D_k)^{\mathrm{T}}P(I - D_k) - P \leqslant 0.$$

Then system (1) *is finite-time stable with the controller* $u(t) = u_1(t) + u_2(t)$*, where* $u_1(t)$ *and* $u_2(t)$ *are the same as Theorem* 1*, and the settling time is estimated by*

$$T = t_0 + \frac{V^{1-(1+\mu)/2}(t_0)}{\delta(1 - \frac{1+\mu}{2})}.$$

On the other hand, the fixed-time stability of system (1) is investigated by designing another controller, then we obtain the following theorem.

Theorem 2. *Assume that there exist $n \times n$ diagonal matrix $P > 0$, an $n \times n$ diagonal matrix L, and some constants $\delta > 0$, $0 \leqslant \mu < 1$, $\bar{\mu} > 0$, $\gamma > 1$, k_1, $k_2 > 0$ such that*

$$\sum_{j=1}^{n} p_j |d_{ij}| \leqslant p_i, \qquad 2p_i c_i + l_i \geqslant 0,$$

where $(I - D_k)^{\mathrm{T}} = (d_{ij})_{n\times n}$, $L = \operatorname{diag}(l_1, l_2, \dots, l_n)$, $P = \operatorname{diag}(p_1, p_2, \dots, p_n)$, $p_i > 0$, $i \in \Omega$. Then system (1) *is fixed-time stable with the controller $u(t)$ given by*

$$\begin{aligned} u_1(t) &= -0.5\bigl[P^{-1}Lx(t) + AA^{\mathrm{T}}PS(x)k_1S^{\mathrm{T}}(x)Fx(t) + P^{-1}Fk_1^{-1}x(t) \\ &\quad + k_2BB^{\mathrm{T}}PS(x) + k_2^{-1}P^{-1}S(x)x^{\mathrm{T}}\bigl(t-\tau(t)\bigr)G^2x\bigl(t-\tau(t)\bigr)\bigr], \\ u_2(t) &= -0.5\delta P^{-1}S(x)\bigl[2x^{\mathrm{T}}(t)PS(x)\bigr]^{(1+\mu)/2}, \\ u_3(t) &= -\bar{\mu}x(t)\bigl[2x^{\mathrm{T}}(t)PS(x)\bigr]^{\gamma-1}. \end{aligned}$$

Moreover, the settling time is estimated by

$$T = \frac{1}{\delta(1-\frac{1+\mu}{2})} + \frac{1}{\bar{\mu}(\gamma-1)}. \tag{5}$$

Proof. Let $x(t) = x(t, 0, \varphi)$ be the solution of system (1) through $(0, \varphi)$. Consider the Lyapunov function as follows:

$$V(t) = 2x^{\mathrm{T}}(t)PS(x).$$

When $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, taking the derivative of $V(t)$ along the solution of system (1), we have

$$\begin{aligned} D^+V(t) &= 2x^{\mathrm{T}}(t)P\dot{S}(x) + 2\dot{x}^{\mathrm{T}}(t)PS(x) = 2S^{\mathrm{T}}(x)P\dot{x}(t) \\ &= 2S^{\mathrm{T}}(x)P\bigl[-Cx(t) + Af\bigl(x(t)\bigr) + Bg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + u(t)\bigr] \\ &= S^{\mathrm{T}}(x)\bigl(-PC - C^{\mathrm{T}}P\bigr)x(t) + 2S^{\mathrm{T}}(x)PAf\bigl(x(t)\bigr) \\ &\quad + 2S^{\mathrm{T}}(x)PBg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + 2S^{\mathrm{T}}(x)Pu_1(t) \\ &\quad + 2S^{\mathrm{T}}(x)Pu_2(t) + 2S^{\mathrm{T}}(x)Pu_3(t). \end{aligned} \tag{6}$$

When $|x(t)| \neq 0$, based on Assumption 1 and Lemma 2, one can obtain the following inequality:

$$\begin{aligned} &2S^{\mathrm{T}}(x)PAf\bigl(x(t)\bigr) \\ &\quad\leqslant S^{\mathrm{T}}(x)PAA^{\mathrm{T}}PS(x)k_1x^{\mathrm{T}}(t)FS(x) + f^{\mathrm{T}}\bigl(x(t)\bigr)f\bigl(x(t)\bigr)k_1^{-1}\frac{1}{x^{\mathrm{T}}(t)FS(x)} \\ &\quad\leqslant S^{\mathrm{T}}(x)PAA^{\mathrm{T}}PS(x)k_1x^{\mathrm{T}}(t)FS(x) + x^{\mathrm{T}}(t)F^2x(t)k_1^{-1}\frac{1}{x^{\mathrm{T}}(t)FS(x)} \\ &\quad\leqslant S^{\mathrm{T}}(x)PAA^{\mathrm{T}}PS(x)k_1x^{\mathrm{T}}(t)FS(x) + x^{\mathrm{T}}(t)FS(x)k_1^{-1}. \end{aligned}$$

While $|x(t)| = 0$, it is easy to derive that

$$2S^{\mathrm{T}}(x)PAf\big(x(t)\big) = S^{\mathrm{T}}(x)PAA^{\mathrm{T}}PS(x)k_1x^{\mathrm{T}}(t)FS(x) + x^{\mathrm{T}}(t)FS(x)k_1^{-1} = 0.$$

Hence, the following inequality holds for any $x \in \mathbb{R}^n$:

$$2S^{\mathrm{T}}(x)PAf\big(x(t)\big) \leqslant S^{\mathrm{T}}(x)PAA^{\mathrm{T}}PS(x)k_1x^{\mathrm{T}}(t)FS(x) + x^{\mathrm{T}}(t)FS(x)k_1^{-1}. \tag{7}$$

Note that

$$S^{\mathrm{T}}(x)S(x) = \begin{cases} \Theta \in [1, n], & |x| \neq 0, \\ 0, & |x| = 0. \end{cases} \tag{8}$$

It then follows from Assumption 1, Lemma 2, and (8) that

$$\begin{aligned}
&2S^{\mathrm{T}}(x)PBg\big(x\big(t-\tau(t)\big)\big) \\
&\quad \leqslant k_2S^{\mathrm{T}}(x)PBB^{\mathrm{T}}PS(x) + k_2^{-1}g^{\mathrm{T}}\big(x\big(t-\tau(t)\big)\big)g\big(x\big(t-\tau(t)\big)\big) \\
&\quad \leqslant k_2S^{\mathrm{T}}(x)PBB^{\mathrm{T}}PS(x) + k_2^{-1}x^{\mathrm{T}}\big(t-\tau(t)\big)G^2x\big(t-\tau(t)\big)S^{\mathrm{T}}(x)S(x).
\end{aligned} \tag{9}$$

Substituting (7), (9) into (6), it can be deduced that

$$\begin{aligned}
D^+V(t) &\leqslant S^{\mathrm{T}}(x)\big[-PC - C^{\mathrm{T}}P - L + PAA^{\mathrm{T}}PS(x)k_1S^{\mathrm{T}}(x)F + Fk_1^{-1}\big]x(t) \\
&\quad + k_2S^{\mathrm{T}}(x)PBB^{\mathrm{T}}PS(x) + k_2^{-1}S^{\mathrm{T}}(x)S(x)x^{\mathrm{T}}\big(t-\tau(t)\big)G^2x\big(t-\tau(t)\big) \\
&\quad + S^{\mathrm{T}}(x)Lx(t) + 2S^{\mathrm{T}}(x)Pu_1(t) + 2S^{\mathrm{T}}(x)Pu_2(t) + 2S^{\mathrm{T}}(x)Pu_3(t) \\
&\leqslant 2S^{\mathrm{T}}(x)Pu_2(t) + 2S^{\mathrm{T}}(x)Pu_3(t) \\
&= -\delta S^{\mathrm{T}}(x)S(x)\big[2x^{\mathrm{T}}(t)PS(x)\big]^{(1+\mu)/2} \\
&\quad + 2S^{\mathrm{T}}(x)P(-\bar{\mu})x(t)\big[2x^{\mathrm{T}}(t)PS(x)\big]^{\gamma-1} \\
&\leqslant -\delta V^{(1+\mu)/2}(t) - \bar{\mu}V^{\gamma}(t).
\end{aligned}$$

On the other hand, when $t = t_k$, it can be deduced that

$$\begin{aligned}
V(t_k) &= 2x^{\mathrm{T}}(t_k)PS\big(x(t_k)\big) = 2\big[(I-D_k)x\big(t_k^-\big)\big]^{\mathrm{T}}PS(x(t_k)) \\
&= 2x^{\mathrm{T}}(t_k^-)(I-D_k)^{\mathrm{T}}PS\big(x(t_k)\big) \\
&= 2\big(x_1\big(t_k^-\big), \dots, x_n\big(t_k^-\big)\big)(I-D_k)^{\mathrm{T}}P\big[\operatorname{sign}\big(x_1(t_k)\big), \dots, \operatorname{sign}\big(x_n(t_k)\big)\big]^{\mathrm{T}} \\
&= 2\big[p_1d_{11}x_1\big(t_k^-\big)\operatorname{sign}\big(x_1(t_k)\big) + p_2d_{12}x_1\big(t_k^-\big)\operatorname{sign}\big(x_2(t_k)\big) + \cdots \\
&\quad + p_nd_{1n}x_1\big(t_k^-\big)\operatorname{sign}\big(x_n(t_k)\big)\big] + \cdots + 2\big[p_1d_{n1}x_n\big(t_k^-\big)\operatorname{sign}\big(x_1(t_k)\big) \\
&\quad + p_2d_{n2}x_n\big(t_k^-\big)\operatorname{sign}\big(x_2(t_k)\big) + \cdots + p_nd_{nn}x_n\big(t_k^-\big)\operatorname{sign}\big(x_n(t_k)\big)\big] \\
&\leqslant 2\big[p_1|d_{11}|\big|x_1\big(t_k^-\big)\big| + p_2|d_{12}|\big|x_1\big(t_k^-\big)\big| + \ldots + p_n|d_{1n}|\big|x_1\big(t_k^-\big)\big|\big] + \cdots \\
&\quad + 2\big[p_1|d_{n1}|\big|x_n\big(t_k^-\big)\big| + p_2|d_{n2}|\big|x_n\big(t_k^-\big)\big| + \cdots + p_n|d_{nn}|\big|x_n\big(t_k^-\big)\big|\big]
\end{aligned}$$

$$\begin{aligned}
&= 2\big(p_1|d_{11}| + p_2|d_{12}| + \cdots + p_n|d_{1n}|\big)\big|x_1\big(t_k^-\big)\big| + \cdots \\
&\quad + 2\big(p_1|d_{n1}| + p_2|d_{n2}| + \cdots + p_n|d_{nn}|\big)\big|x_n\big(t_k^-\big)\big| \\
&\leqslant 2p_1\big|x_1\big(t_k^-\big)\big| + \cdots + 2p_n\big|x_n\big(t_k^-\big)\big| = 2\big(p_1\big|x_1\big(t_k^-\big)\big| + \cdots + p_n\big|x_n\big(t_k^-\big)\big|\big) \\
&= 2x^{\mathrm{T}}\big(t_k^-\big)PS\big(t_k^-\big) = V\big(t_k^-\big).
\end{aligned}$$

By Lemma 1, we obtain that system (1) is fixed-time stable by the controller u in Theorem 2, and the settling time is estimated in (5). This completes the proof. □

Remark 3. Dealing with impulsive effects and time delays is a challenging problem for fixed-time stability. In this paper, the addressed neural network involves the impulsive effects and time delays. In recent years, some interesting results about fixed-time stability have been reported, such as those in [7, 11–13, 21, 28, 30, 37, 40, 44]. Among them, the impulse and time delay are excluded in [7, 11]. Paper [37] considered fixed-time synchronization of complex networks with impulsive effects by designing non-chattering controller, and paper [12] studied the fixed-time stabilization for impulsive Cohen–Grossberg BAM neural networks by designing two different controllers. However, the addressed systems in [12,37] do not consider the effect of time delays. In this sense, our development results improve and extend the existing results in [7, 11–13, 21, 28, 30, 37, 40, 44].

As a special case, if there is no impulsive perturbation, then the following corollary for fixed-time stability can be derived.

Corollary 2. *Assume that there exist $n \times n$ diagonal matrix $P > 0$ and $n \times n$ diagonal matrix L, some constants $\delta > 0$, $0 \leqslant \mu < 1$, $\bar{\mu} > 0$, $\gamma > 1$, k_1, $k_2 > 0$ such that $2p_ic_i + l_i \geqslant 0$, where $L = \operatorname{diag}(l_1, l_2, \ldots, l_n)$, $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$, $p_i > 0$, $i \in \Omega$. Then ystem* (1) *without impulsive perturbations is fixed-time stable. Moreover, the estimated settling time is the same as Theorem* 2.

4 Examples

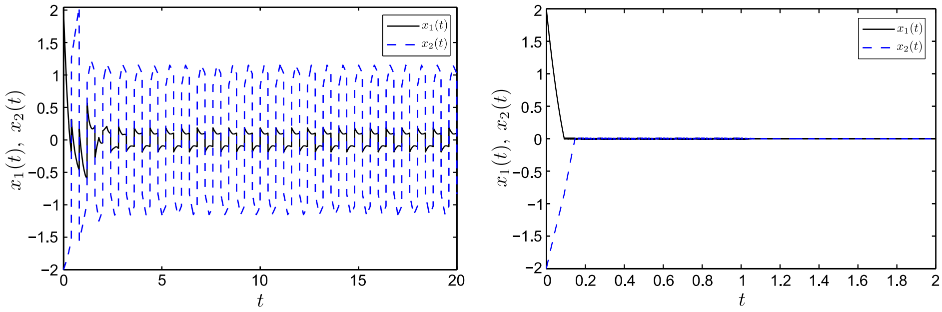
In this section, two examples are presented to demonstrate the effectiveness and applicability of the proposed design schemes.

Example 1. Consider the 2D system (1) with the following parameters:

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} -3 & 0.4 \\ -0.8 & 2 \end{pmatrix},$$

$$B = \begin{pmatrix} -2.5 & 0.6 \\ 0.2 & -1.5 \end{pmatrix}, \qquad D_k = \begin{pmatrix} 1.8 & 0.1 \\ 0.2 & 1.8 \end{pmatrix},$$

$\tau = 1$, $f_i(x) = g_i(x) = \tanh(x)$ $(i = 1, 2)$, $\varphi(s) = (2, -2)^{\mathrm{T}}$, $t_k = 0.4k$, $k \in \mathbb{N}$. We then consider the fixed-time control of system (1) with the above parameters. By simple calculation, one can get $F = G = I$. Choose $\mu = 0.5$, $\bar{\mu} = 0.5$, $\gamma = 2$, and $k = \delta = 1$.



(a) (b)

Figure 1. State trajectories of system (1) with the initial condition $\varphi(s) = [2, -2]$: (a) without control; (b) under the controller $u(t)$ in (10).

It then follows that there exist feasible solutions for LMIs in Theorem 1. The controller $u(t)$ is designed by

$$\begin{aligned}
u_{11}(t) &= -4.3x_1(t) - 1.4x_2(t) - 2.3\,\mathrm{sign}\big(x_1(t)\big) - 0.7\,\mathrm{sign}\big(x_2(t)\big)\\
&\quad - \big[2\,\mathrm{sign}\big(x_1(t)\big) + 0.6\,\mathrm{sign}\big(x_2(t)\big)\big]x^{\mathrm{T}}(t-1)x(t-1),\\
u_{12}(t) &= -1.5x_1(t) - 2.8x_2(t) - 0.7\,\mathrm{sign}\big(x_1(t)\big) - 1.7\,\mathrm{sign}\big(x_2(t)\big)\\
&\quad - \big[0.6\,\mathrm{sign}\big(x_1(t)\big) + 1.5\,\mathrm{sign}\big(x_2(t)\big)\big]x^{\mathrm{T}}(t-1)x(t-1),\\
u_{21}(t) &= -1.1\big|x_1(t)\big|^{0.5}\,\mathrm{sign}\big(x_1(t)\big) - 0.4\big|x_2(t)\big|^{0.5}\,\mathrm{sign}\big(x_2(t)\big),\\
u_{22}(t) &= -0.4\big|x_1(t)\big|^{0.5}\,\mathrm{sign}\big(x_1(t)\big) - 0.8\big|x_2(t)\big|^{0.5}\,\mathrm{sign}\big(x_2(t)\big),\\
u_{31}(t) &= \big[-0.1x_1^2(t) + 0.1x_1(t)x_2(t) - 0.1x_2^2(t)\big]x_1(t),\\
u_{32}(t) &= \big[-0.1x_1^2(t) + 0.1x_1(t)x_2(t) - 0.1x_2^2(t)\big]x_2(t).
\end{aligned} \tag{10}$$

It then follows from Theorem 1 that system (1) is globally fixed-time stable by controller (10). The settling time is estimated by $T \approx 6$. From Fig. 1(a) one can see that the impulsive system (1) without control protocol is unstable. Then based on the designed controller (10), it becomes globally fixed-time stable, which is shown in Fig. 1(b).

Example 2. Consider the 3D system (1) with the following parameters:

$$C = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 0.3 & 2 & 1.5\\ 1 & 0.5 & 1\\ 2.5 & 3 & 0.2 \end{pmatrix},$$

$$B = \begin{pmatrix} -3 & 0.3 & 1\\ 0.5 & 0.8 & 0.5\\ -3 & 1.5 & -2 \end{pmatrix}, \qquad D_k = \begin{pmatrix} 1.5 & 0.2 & 0.1\\ 0.2 & 1.6 & 0.1\\ 0.1 & 0.1 & 1.4 \end{pmatrix},$$

$\tau = 0.5$, $f_i(x) = \tanh(x)$, $g_i(x) = 0.5[|x+1| - |x-1|]$ $(i = 1, 2, 3)$, $\varphi(s) = (2, 1, -2)^{\mathrm{T}}$, $t_k = 0.3k$, $k \in \mathbb{N}$. Next, we investigate the fixed-time control of system (1). Note that

$F = G = I$. Choose $\mu = 0.5$, $\bar{\mu} = 0.5$, $\gamma = 2$, and $k = \delta = 1$. It then follows from Theorem 1 that a feasible solution to LMIs can be obtained via the Matlab LMI toolbox. Then the controller $u(t)$ is designed by

$$\begin{aligned}
u_{11}(t) &= -3.1x_1(t) - 1.1x_2(t) - 2.5x_3(t) - 3.5\,\mathrm{sign}\big(x_1(t)\big) - \mathrm{sign}\big(x_2(t)\big) \\
&\quad - 2.3\,\mathrm{sign}\big(x_3(t)\big) - \big[1.6\,\mathrm{sign}\big(x_1(t)\big) + 0.5\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 1.1\,\mathrm{sign}\big(x_3(t)\big)\big]x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{12}(t) &= -1.2x_1(t) - 1.4x_2(t) - 1.5x_3(t) - \mathrm{sign}\big(x_1(t)\big) - 2.1\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 1.4\,\mathrm{sign}\big(x_3(t)\big) - \big[0.5\,\mathrm{sign}\big(x_1(t)\big) + \mathrm{sign}\big(x_2(t)\big) \\
&\quad + 0.7\,\mathrm{sign}\big(x_3(t)\big)\big]x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{13}(t) &= -2.4x_1(t) - 1.5x_2(t) - 6x_3(t) - 2.3\,\mathrm{sign}\big(x_1(t)\big) - 1.4\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 6.1\,\mathrm{sign}\big(x_3(t)\big) - \big[1.1\,\mathrm{sign}\big(x_1(t)\big) \\
&\quad + 0.7\,\mathrm{sign}\big(x_2(t)\big) + 2.8\,\mathrm{sign}\big(x_3(t)\big)\big]x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{21}(t) &= -1.2\big|x_1(t)\big|^{0.5}\mathrm{sign}\big(x_1(t)\big) - 0.4\big|x_2(t)\big|^{0.5}\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 0.8\big|x_3(t)\big|^{0.5}\mathrm{sign}\big(x_3(t)\big), \\
u_{22}(t) &= -0.4\big|x_1(t)\big|^{0.5}\mathrm{sign}\big(x_1(t)\big) - 0.7\big|x_2(t)\big|^{0.5}\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 0.5\big|x_3(t)\big|^{0.5}\mathrm{sign}\big(x_3(t)\big), \\
u_{23}(t) &= -0.8\big|x_1(t)\big|^{0.5}\mathrm{sign}\big(x_1(t)\big) - 0.5\big|x_2(t)\big|^{0.5}\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 2.1\big|x_3(t)\big|^{0.5}\mathrm{sign}\big(x_3(t)\big), \\
u_{31}(t) &= \big[-0.1x_1^2(t) - 0.2x_2^2(t) - 0.1x_3^2(t) + 0.1x_1(t)x_2(t) + 0.1x_1(t)x_3(t) \\
&\quad + 0.1x_2(t)x_3(t)\big]x_1(t), \\
u_{32}(t) &= \big[-0.1x_1^2(t) - 0.2x_2^2(t) - 0.1x_3^2(t) + 0.1x_1(t)x_2(t) + 0.1x_1(t)x_3(t) \\
&\quad + 0.1x_2(t)x_3(t)\big]x_2(t), \\
u_{33}(t) &= \big[-0.1x_1^2(t) - 0.2x_2^2(t) - 0.1x_3^2(t) + 0.1x_1(t)x_2(t) + 0.1x_1(t)x_3(t) \\
&\quad + 0.1x_2(t)x_3(t)\big]x_3(t).
\end{aligned} \tag{11}$$

It then follows from Theorem 1 that system (1) is globally fixed-time stable by controller (11). The settling time is estimated by $T \approx 6$. The state trajectories of the impulsive system (1) with and without controller u are shown in Figs. 2(a) and 2(b), where Fig. 2(a) shows the state trajectories without control, and Fig. 2(b) shows state trajectories with the controller (11).

On the other hand, based on Theorem 2, one may design another controller $u(t)$. In fact, choose $\delta = 1$, $\gamma = 2$, and $\mu = \bar{\mu} = k_1 = k_2 = 0.5$. Based on Theorem 2, one can obtain a feasible solution to LMIs via the Matlab LMI toolbox. Then the controller $u(t)$

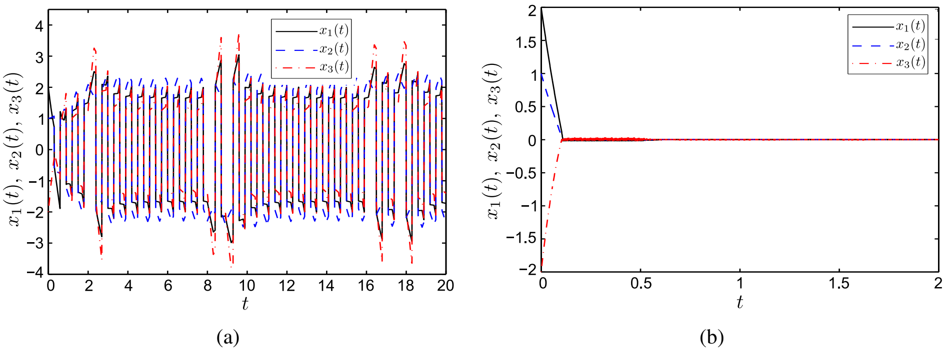


Figure 2. State trajectories of system (1) with the initial condition $\varphi(s) = [2, 1, -2]$: (a) without the control; (b) under the controller $u(t)$ in (11).

is designed by

$$
\begin{aligned}
u_{11}(t) &= -0.1x_1(t) - \big[9.3\,\mathrm{sign}\big(x_1(t)\big) + 4.8\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 7.3\,\mathrm{sign}\big(x_3(t)\big)\big] S^{\mathrm{T}}(x)x(t) - 14.7\,\mathrm{sign}\big(x_1(t)\big) + 1.3\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 7.7\,\mathrm{sign}\big(x_3(t)\big) - 0.2\,\mathrm{sign}\big(x_1(t)\big)x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{12}(t) &= -0.1x_2(t) - \big[4.1\,\mathrm{sign}\big(x_1(t)\big) + 3.8\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 4.4\,\mathrm{sign}\big(x_3(t)\big)\big] S^{\mathrm{T}}(x)x(t) + 1.1\,\mathrm{sign}\big(x_1(t)\big) - 1.9\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 1.3\,\mathrm{sign}\big(x_3(t)\big) - 0.2\,\mathrm{sign}\big(x_2(t)\big)x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{13}(t) &= -0.2x_3(t) - \big[10.3\,\mathrm{sign}\big(x_1(t)\big) + 7.1\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 15.8\,\mathrm{sign}\big(x_3(t)\big)\big] S^{\mathrm{T}}(x)x(t) - 10.9\,\mathrm{sign}\big(x_1(t)\big) + 2.2\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 15.8\,\mathrm{sign}\big(x_3(t)\big) - 0.2\,\mathrm{sign}\big(x_3(t)\big)x^{\mathrm{T}}(t-0.5)x(t-0.5), \\
u_{21}(t) &= -0.1\,\mathrm{sign}\big(x_1(t)\big)\big[11.7x_1(t)\,\mathrm{sign}\big(x_1(t)\big) + 13.6x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 8.3x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big]^{0.75}, \\
u_{22}(t) &= -0.1\,\mathrm{sign}\big(x_2(t)\big)\big[11.7x_1(t)\,\mathrm{sign}\big(x_1(t)\big) + 13.6x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 8.3x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big]^{0.75}, \\
u_{23}(t) &= -0.1\,\mathrm{sign}\big(x_3(t)\big)\big[11.7x_1(t)\,\mathrm{sign}\big(x_1(t)\big) + 13.6x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad + 8.3x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big]^{0.75}, \\
u_{31}(t) &= x_1(t)\big[-5.8x_1(t)\,\mathrm{sign}\big(x_1(t)\big) - 6.8x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 4.1x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big], \\
u_{32}(t) &= x_2(t)\big[-5.8x_1(t)\,\mathrm{sign}\big(x_1(t)\big) - 6.8x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 4.1x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big], \\
u_{33}(t) &= x_3(t)\big[-5.8x_1(t)\,\mathrm{sign}\big(x_1(t)\big) - 6.8x_2(t)\,\mathrm{sign}\big(x_2(t)\big) \\
&\quad - 4.1x_3(t)\,\mathrm{sign}\big(x_3(t)\big)\big].
\end{aligned} \tag{12}
$$

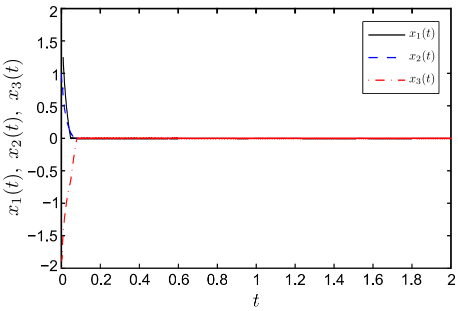


Figure 3. State trajectories of system (1) under the controller $u(t)$ in (12) for Example 2 with the initial condition $\varphi(s) = [2, 1, -2]$.

It then follows from Theorem 2 that system (1) is fixed-time stable under controller (12), and the settling time is estimated by $T \approx 6$, which is shown in Fig. 3.

5 Conclusion

This paper has addressed the fixed-time stability of delayed neural networks with impulsive perturbations. The criteria for fixed-time stability of impulsive neural networks with delay have been obtained. The settling time can be estimated without depending on any initial conditions and only on the designed controller. Two controllers have been designed to guarantee the stabilization of delayed impulsive system, where the related control parameters can be obtained by solving LMIs. Finally, the effectiveness of the derived stability criteria has been tested by several numerical examples. A future work will focus on the fixed-time stability for delayed neural networks with impulsive perturbations and parameter uncertainties.

Acknowledgment. The authors would like to thank the referee for his/her very important comments that improved the results and the quality of the paper.

References

1. M.S. Ali, J. Yogambigai, J. Cao, Synchronization of master-slave Markovian switching complex dynamical networks with time-varying delays in nonlinear function via sliding mode control, *Acta Math. Sci., Ser. B, Engl. Ed.*, **37**(2):368–384, 2017.
2. A. Arbi, J. Cao, A. Alsaedi, Improved synchronization analysis of competitive neural networks with time-varying delays, *Nonlinear Anal. Model. Control*, **23**(1):82–102, 2018.
3. H. Bao, J. Cao, Finite-time generalized synchronization of nonidentical delayed chaotic systems, *Nonlinear Anal. Model. Control*, **21**(3):306–324, 2016.
4. S. Bhat, D. Bernstein, Global finite-time stability of continuous autonomous systems, *SIAM J. Control Optim.*, **38**(3):751–766, 2000.

5. J. Cao, R. Li, Fixed-time synchronization of delayed memristor-based recurrent neural networks, *Sci. China, Inf. Sci.*, **60**:032201, 2017.
6. J. Cao, K. Yuan, H. Li, Global asymptotic stability of recurrent neural networks with multiple discrete delays and distributed delays, *IEEE Trans. Neural Networks*, **17**(6):1646–1651, 2006.
7. X. Ding, J. Cao, A. Alsaedi, F. Alsaadi, T. Hayat, Robust fixed-time synchronization for uncertain complex-valued neural networks with discontinuous activation functions, *Neural Netw.*, **90**:42–55, 2017.
8. K. Gu, V.L. Kharitonov, J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, MA, 2003.
9. J. Hale, S. Verduyn Lunel, *Introduction to Functional Differential Equations*, Springer, New York, 1993.
10. S. Haykin, *Neural Networks: A Comprehensive Foundations*, Macmillan, Englewood Cliffs, NJ, 1994.
11. C. Hu, J. Yu, Z. Chen, H. Jiang, T. Huang, Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks, *Neural Netw.*, **89**:74–83, 2017.
12. H. Li, C. Li, T. Huang, W. Zhang, Fixed-time stabilization of impulsive Cohen–Grossberg BAM neural networks, *Neural Netw.*, **98**:203–211, 2018.
13. R. Li, J. Cao, A. Alsaedi, F. Alsaadi, Exponential and fixed-time synchronization of Cohen–Grossberg neural networks with time-varying delays, *Applied Math. Comput.*, **313**:37–51, 2017.
14. R. Li, J. Cao, A. Alsaedi, F. Alsaadi, Stability analysis of fractional-order delayed neural networks, *Nonlinear Anal. Model. Control*, **22**(3):505–520, 2017.
15. X. Li, M. Bohner, C. Wang, Impulsive differential equations: Periodic solutions and applications, *Automatica*, **52**:173–178, 2015.
16. X. Li, S. Song, Impulsive control for existence, uniqueness and global stability of periodic solutions of recurrent neural networks with discrete and continuously distributed delays, *IEEE Trans. Neural Networks Learn. Syst.*, **24**(6):868–877, 2013.
17. X. Li, S. Song, Stabilization of delay systems: Delay-dependent impulsive control, *IEEE Trans. Autom. Control*, **62**(1):406–411, 2017.
18. X. Li, J. Wu, Stability of nonlinear differential systems with state-dependent delayed impulses, *Automatica*, **64**:63–69, 2016.
19. X. Li, X. Zhang, S. Song, Effect of delayed impulses on input-to-state stability of nonlinear systems, *Automatica*, **76**:378–382, 2017.
20. J. Liang, Z. Wang, X. Liu, State estimation for coupled uncertain stochastic networks with missing measurements and time-varying delays: The discrete-time case, *IEEE Trans. Neural Networks*, **20**:781–793, 2009.
21. J. Liu, Y. Yu, Q. Wang, C. Sun, Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties, *Neurocomputing*, **260**:497–504, 2017.
22. X. Liu, N. Jiang, J. Cao, S. Wang, Z. Wang, Finite time stochastic stabilization for BAM neural networks with uncertainties, *J. Franklin Inst.*, **350**(8):2109–2123, 2013.

23. X. Liu, J. Park, N. Jiang, J. Cao, Nonsmooth finite-time stabilization of neural networks with discontinuous activations, *Neural Netw.*, **52**:25–32, 2014.
24. J. Lu, C. Ding, J. Lou, J. Cao, Outer synchronization of partially coupled dynamical networks via pinning impulsive controllers, *J. Franklin Inst.*, **352**(11):5024–5041, 2015.
25. X. Lv, X. Li, Finite time stability and controller design for nonlinear impulsive sampled-data systems with applications, *ISA Trans.*, **70**:30–36, 2017.
26. C. Maharajan, R. Raja, J. Cao, G. Rajchakit, A. Alsaedi, Impulsive Cohen–Grossberg BAM neural networks with mixed time-delays: An exponential stability analysis issue, *Neurocomputing*, **275**:2588–2602, 2018.
27. E. Moulay, W. Perruquetti, Finite time stability and stabilization of a class of continuous systems, *J. Math. Anal. Appl.*, **323**(2):1430–1443, 2006.
28. J. Ni, L. Liu, C. Liu, X. Hu, S. Li, Fast fixed-time nonsingular terminal sliding mode control and its application to chaos suppression in power system, *IEEE Trans. Circuits Syst., II, Express Briefs*, **64**:151–155, 2017.
29. A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Trans. Autom. Control*, **57**:2106–2110, 2012.
30. A. Polyakov, D. Efimov, W. Perruquetti, Finite-time and fixed-time stabilization: Implicit lyapunov function approach, *Automatica*, **51**:332–340, 2015.
31. C. Qian, J. Li, Robust exponential stability of uncertain impulsive delays differential systems, *IEEE Trans. Autom. Control*, **50**:885–890, 2005.
32. R. Schalkoff, *Artificial Neural Networks*, McGraw-Hill, New York, 1997.
33. L. Shi, X. Yang, Y. Li, Z. Feng, Finite-time synchronization of nonidentical chaotic systems with multiple time-varying delays and bounded perturbations, *Nonlinear Dyn.*, **83**(1–2):75–87, 2016.
34. I. Stamova, T. Stamov, X. Li, Global exponential stability of a class of impulsive cellular neural networks with supremums, *Int. J. Adapt. Control Signal Process.*, **28**(11):1227–1239, 2014.
35. X. Yang, J. Cao, Finite-time stochastic synchronization of complex networks, *Appl. Math. Modelling*, **34**(11):3631–3641, 2010.
36. X. Yang, J. Cao, C. Xu, J. Feng, Finite-time stabilization of switched dynamical networks with quantized couplings via quantized controller, *Sci. China, Technol. Sci.*, **61**(2):299–308, 2018.
37. X. Yang, J. Lam, D. Ho, Z. Feng, Fixed-time synchronization of complex networks with impulsive effects via non-chattering control, *IEEE Trans. Autom. Control*, **62**:5511–5521, 2017.
38. X. Yang, X. Li, J. Cao, Robust finite-time stability of singular nonlinear systems with interval time-varying delay, *J. Franklin Inst.*, **355**(3):1241–1258, 2018.
39. X. Yang, Q. Song, J. Liang, B. He, Finite-time synchronization of coupled discontinuous neural networks with mixed delays and nonidentical perturbations, *J. Franklin Inst.*, **352**(10):4382–4406, 2015.
40. W. Zhang, C. Li, T. Huang, J. Huang, Fixed-time synchronization of complex networks with nonidentical nodes and stochastic noise perturbations, *Physica A*, **492**:1531–1542, 2018.
41. X. Zhang, X. Lv, X. Li, Sampled-data based lag synchronization of chaotic delayed neural networks with impulsive control, *Nonlinear Dyn.*, **90**(3):2199–2207, 2017.

42. C. Zhou, W. Zhang, X. Yang, C. Xu, J. Feng, Finite-time synchronization of complex-valued neural networks with mixed delays and uncertain perturbations, *Neural Process. Lett.*, **46**(1): 271–291, 2017.
43. Y. Zhou, C. Li, T. Huang, X. Wang, Impulsive stabilization and synchronization of Hopfield-type neural networks with impulse time window, *Neural Comput. Appl.*, **28**(4):775–782, 2017.
44. Z. Zuo, Non-singular fixed-time terminal sliding mode control of non-linear systems, *IET Control Theory Appl.*, **9**:545–552, 2014.