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ABSTRACT

We investigate the welfare properties of fixed and floating exchange rate regimes in a two-country, dynamic, infinite-horizon model with agents optimizing in an environment of uncertainty created by monetary shocks. The optimal exchange rate regime may depend on whether prices are set in the currency of producers or the currency of consumers. When prices are set in consumers' currency, the variance of home consumption is not influenced by foreign monetary variance under floating exchange rates, while there is transmission of foreign disturbances under floating rates if prices are set in producers' currencies, or under fixed exchange rates. An important feature of the model is the exchange rate regime affects not just the variance of consumption and output, but also their average levels. When prices are set in producer's currency, as in the traditional framework, we find that there is a trade-off between floating and fixed exchange rates. Exchange rate adjustment under floating rates allows for a lower variance of consumption, but exchange rate volatility itself leads to a lower average *level* of consumption. When prices are set in consumer's currency, floating exchange rates always dominate fixed exchange rates.

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Introduction

The optimal choice of exchange rate system is a long-standing problem in open-economy macroeconomics. Modern analysis of the issue traces back to Friedman (1953), who argued that flexible exchange rates are preferable to fixed exchange rates on the grounds that they provide greater insulation from foreign shocks. Under fixed exchange rates a country must validate a foreign monetary shock if it is to maintain the peg. But floating rates blunt the impact of the foreign shock. A foreign monetary expansion will not be translated to the home country because the appreciation of the home currency drives up the relative price of home goods, thereby offsetting the potentially expansionary transmission.

Friedman wrote at a time in which there was little capital mobility among even the richest countries. Floating exchange rates maintained a zero current account balance, thus shutting off any channel for transmission of foreign shocks. In a series of papers, Mundell (1960, 1961a, 1961b, 1963) demonstrated that the insulation properties of floating exchange rates are diminished in the presence of capital mobility. The answer to the question of which is better – fixed or floating exchange rates – became more complicated, depending on whether the source of shocks was monetary or real; the degree of capital and other factor mobility; and the relative size of countries.

Both Friedman and Mundell assumed nominal prices are sticky in the short run. Friedman argued that the choice of exchange rate system would be irrelevant if all nominal prices adjusted instantaneously to shocks. Subsequent to Friedman and Mundell, a large number of authors examined the choice of exchange-rate regime under the assumption of some sort of nominal price or wage stickiness[1]. These studies extended Mundell by incorporating expectations of future price and

[1] See for instance, Turnovsky (1976, 1983), Fischer (1977), Hamada and Sakurai (1978), Flood (1979), Flood and Marion (1983), Weber (1981), Kimbrough (1983), Aizenman and Frenkel (1985), and Glick and Wihlborg (1990).

exchange rate changes. In that analysis, the choice of exchange rate regime was based on ad hoc criteria, such as minimization of the variance of output.

Here, we investigate the optimality of exchange rate regimes from a welfare maximization standpoint. We are not the first to study the welfare properties of alternative exchange-rate systems. Prior studies include Lapan and Enders (1980), Helpman (1981), Helpman and Razin (1982), Eaton (1985), Aizenman (1994), Chinn and Miller (1998), and Neumeyer (1998). These papers, however, do not assume any sort of nominal price stickiness, and therefore do not follow directly in the tradition of Friedman and Mundell.

The framework we adopt is a two-country, infinite-horizon model of optimization under uncertainty. Consumers have utility over consumption, leisure and real balances. We assume that there is perfect capital mobility, in the sense that there is a complete market for state-contingent nominal bonds. The source of uncertainty is from random monetary shocks at home and abroad. We assume that imperfectly competitive firms must set nominal prices prior to the realization of monetary shocks. Prices are adjusted fully after one period.

Welfare analysis turns out to be important. We shall see that under some circumstances, floating rates may do a better job of stabilizing output than fixed exchange rates but still may not be optimal. For one thing, output may be stabilized by flexible rates but consumption may be destabilized. But a further in important feature of the model is that the choice of exchange rate system may actually influence the average levels of consumption and output, not just their variances. The intuition for this arises from the fact that profit-maximizing firms will set prices to maximize expected utility of their owners. As a result, the stochastic characteristics of alternative exchange rate regimes will have implications for expected marginal costs and expected marginal revenues. Therefore, average price-cost markups will in general depend on the exchange rate regime, implying that average consumption and output will also depend on the exchange rate regime.

The primary focus of our investigation is how the nature of price setting affects the optimal choice of exchange-rate regime. Friedman and Mundell assumed that producers set prices in their own currency, and that those prices do not adjust when exchange rates change. Indeed, in their models, the law of one price holds for all goods.

Recent empirical research demonstrates that among wealthy countries: 1) real exchange rates (based on consumer price indexes) are very volatile; 2) the law of one price fails broadly; and, 3), almost all of the short-run to medium-run volatility in real exchange rates arises from the failure of the law of one price.[2] Coupled with the observation that nominal and real exchange rates are highly correlated, one can conclude that a better description of price setting is that nominal prices are sticky in the currency of the consumer. We refer to this type of price stickiness as "pricing to market".

The type of price stickiness may be of critical importance in the analysis of fixed versus floating exchange rates. The intuition that underlies analyses reaching back to Friedman is that floating exchange rates play a stabilizing role by allowing the relative price of home goods to foreign goods to adjust, even when nominal prices remain rigid. But, the empirical evidence is that consumers witness relatively little relative price adjustment. The adjustment occurs to a greater extent in the profit margins of exporting firms.

We find that under floating exchange rates and pricing to market, foreign monetary shocks do not affect domestic consumption.[3] In contrast, when prices are set in the producers' currencies, the prices paid by home residents for foreign goods changes as the exchange rate changes. This

[2] See especially Engel (1993, 1998), Rogers and Jenkins (1996), and Engel and Rogers (1996).
[3] However, we shall see that foreign monetary variance may influence the expected level of home consumption.

introduces a channel through which the foreign monetary shocks can affect domestic consumption. The larger the share of foreign goods in consumption, the more vulnerable will consumption be to foreign money shocks.

The optimal exchange rate regime may depend on the currency of price setting. When prices are set in producer's currencies, we find that there is a trade-off between floating and fixed exchange rates. The variance of domestic consumption is lower under floating exchange rates, as suggested by Friedman's argument. But there is a second factor, absent in Friedman's model. The volatility of the exchange rate under floating rates will actually reduce the *average level* of consumption. The reason is that exchange rate volatility raises expected marginal costs facing price-setting firms, leading them to set higher average markups, and producing lower average consumption. Thus, while floating exchange rates allow for greater consumption stabilization, they also impost a welfare cost of exchange rate volatility.

By contrast, we find that floating exchange rates under pricing to market will always be preferable to fixed exchange rates. Under pricing to market, consumption is fully insulated from foreign monetary disturbances. In addition, the variance of the exchange rate does not affect domestic prices or wages, and so does not directly impact on the firm's price setting decision.

The model in this paper of floating exchange rates when prices are set in producers' currency is borrowed directly from Obstfeld and Rogoff (1998). That model, in turn, is heavily influenced by the non-stochastic models of Corsetti and Pesenti (1998) and Obstfeld and Rogoff (1995). These models are examples of recent international models with optimizing agents and prices which are sticky in producers' currencies which include Svensson and van Wijnbergen (1989), Kollman (1996) and Hau (1998).

We develop a new model here – the pricing to market model. It bears some similarities to the investigation by Bacchetta and van Wincoop (1998) of how exchange rate regime affects the volume of trade and capital flows, although their analysis is in a two-period model of pricing to market. Our model of pricing to market is also closely related to the work of Betts and Devereux (1996, 1998a, 1998b). Other recent general equilibrium models in which prices are sticky in consumers' currencies include Chari, Kehoe and McGrattan (1997), Tille (1998a, 1998b) and Engel (1996).

The general models are laid out in section 1. In section 2, we investigate one form of the models in which we can derive closed-form solutions. A more general version of the models, directly comparable to Obstfeld and Rogoff (1998) is examined in section 3. The concluding section points to some potential weaknesses of our analysis and directions for future research.

1. The Model

Here we outline the features of the models we examine. Consumers maximize expected lifetime utility. They take prices and wages as given. Home-country agents own home-country firms (and similarly for foreign country agents), but delegate firm decisions to firm managers. Each consumer receives a share of profits from every firm in his country. Monetary authorities in each country increase the money supply randomly in the floating exchange rate models. Monetary randomness is the only source of uncertainty in the model. In the fixed exchange rate model, the foreign monetary authority changes the foreign money supply randomly, while the domestic central bank alters the domestic money supply in order to keep the exchange rate fixed.

There are sticky prices in all of our models. Producers must set prices prior to the realization of monetary shocks. This is an institutional constraint in our model, though one could justify this constraint with an appropriate menu-cost model. Prices fully adjust to monetary shocks after one period; i.e., there is no persistence to the price-adjustment process. We consider two separate types of price stickiness. In the first type, producers must set prices in terms of their own currency. For example, the home currency price of home goods is set, and unresponsive to monetary shocks. This implies that the price for home goods paid by foreign consumers changes when the exchange rate changes in response to monetary shocks. In the other type of model, producers set prices in consumers' currencies. For example, home firms set one price for home-country consumers in the home currency and another price for foreign-country consumers in the foreign currency. In the fixed exchange rate model, the currency in which prices are set is irrelevant since the exchange rate is permanently fixed. In all of the models, the objective of the firm managers is to maximize the expected utility of the representative owner of the firm.

Consumers

The representative consumer in the home country is assumed to maximize

$$U_t = E_t\left(\sum_{s=t}^{\infty} \beta^{s-t} u_s\right), \quad 0 < \beta < 1$$

where

$$u_s = \frac{1}{1-\rho} C_s^{1-\rho} + \frac{\chi}{1-\varepsilon}\left(\frac{M_s}{P_s}\right)^{1-\varepsilon} - \eta V(L_s), \quad \rho > 0,\ \varepsilon > 0,\ V' > 0,\ V'' \geq 0.$$

C is a consumption index that is a geometric average of home and foreign consumption:

$$C = \frac{C_h^n C_f^{1-n}}{n^n (1-n)^{1-n}}.$$

We assume that there are n identical individuals in the home country, $0 < n < 1$. C_h and C_f are indexes over consumption of goods produced at home and in the foreign country, respectively:

$$C_h = \left[n^{-1/\lambda} \int_0^n C_h(i)^{\lambda-1/\lambda} di \right]^{\lambda/\lambda-1}; \qquad C_f = \left[(1-n)^{-1/\lambda} \int_n^1 C_f(i)^{\lambda-1/\lambda} di \right]^{\lambda/\lambda-1}$$

The elasticity of substitution between goods produced within a country is λ, which we assume to be greater than 1. There is a unit elasticity of substitution between the home goods and foreign goods indexes. M/P are domestic real balances, and L is the labor supply of the representative home agent.

We analyze a model with utility of this form so that we can compare welfare results in the pricing-to-market model with welfare results derived in the model with prices sticky in producers' currencies. Obstfeld and Rogoff (1998) derive welfare results in the latter model with this utility function. The separability of the utility function may be problematic for analyzing some aspects of exchange-rate and current-account dynamics, although it is very helpful in making the model more tractable analytically. Bacchetta and van Wincoop (1998) have emphasized (in a two-period framework) the importance of non-separabilities of leisure and consumption for the impact of the choice of exchange-rate regime on the volume of trade.

The price index, P, is defined by

(1.1) $\quad P = P_h^n P_f^{1-n}$,

where

$$P_h = \left[\frac{1}{n} \int_0^n P_h(i)^{1-\lambda} di \right]^{1/1-\lambda}, \qquad P_f = \left[\frac{1}{1-n} \int_n^1 P_f(i)^{1-\lambda} di \right]^{1/1-\lambda}.$$

There are $1-n$ identical individuals in the foreign country. Their preferences are similar to home country residents' preferences. The terms in the utility function involving consumption are identical in the home and foreign countries. The functional form for real balances and labor are the same as for the home country residents, but, for foreign residents, they are functions of foreign real balances and foreign labor supply.

The optimal intratemporal consumption choices take on particularly neat forms:

$$C_h(i) = \frac{1}{n}\left[\frac{P_h(i)}{P_h}\right]^{-\lambda} C_h, \qquad C_f(i) = \frac{1}{1-n}\left[\frac{P_f(i)}{P_f}\right]^{-\lambda} C_f;$$

$$P_h C_h = nPC, \qquad P_f C_f = (1-n)PC;$$

and,

$$\int_0^n P_h(i)C_h(i)di = P_h C_h, \qquad \int_n^1 P_f(i)C_f(i)di = P_f C_f.$$

We assume that there are complete asset markets. Specifically, we assume that residents of each country can purchase state-contingent nominal bonds. As Obstfeld and Rogoff (1998) emphasize, the structure of the utility functions ensures that consumption risk is completely shared when the law of one price holds.[4] But in the pricing-to-market model, the law of one price does not hold. We consider the assumption of complete asset markets to be one of convenience, which approximates the assumption of perfect capital mobility.

Given the intratemporal consumption choices above, we can write the budget constraint of the representative home agent as:

$$P_t C_t + M_t + \sum_{z^{t+1}} q(z^{t+1}, z^t)B(z^{t+1}) = W_t L_t + \pi_t + M_{t-1} + B_t + T_t.$$

[4] See Cole and Obstfeld (1991) for an analysis of why terms of trade changes can serve as a substitute for capital mobility.

Here, $B(z^{t+1})$ are contingent home-currency denominated nominal bonds whose prices at time t are $q(z^{t+1}, z^t)$, where z^t represents the state at time t. π_t is the representative agent's share of profits from home firms. T_t are monetary transfers from the government. W_t is the wage rate.[5]

In addition to the consumption demand equations listed above, we can derive the money demand equation for the representative home-country resident:

$$(1.2) \quad \frac{M_t}{P_t} = \frac{\chi^{1/\varepsilon} C_t^{\rho/\varepsilon}}{(1-d_t)^{1/\varepsilon}},$$

where d_t is the inverse of the gross nominal interest rate, given by

$$d_t = E_t\left(\beta \frac{C_{t+1}^{-\rho} P_t}{C_t^{-\rho} P_{t+1}}\right).$$

Also, the trade-off between consumption and leisure is given by:

$$(1.3) \quad \frac{W_t}{P_t C_t^{\rho}} = \eta V'(L_t).$$

It is easy to see that optimal risk sharing implies

$$(1.4) \quad \frac{S_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*}\right)^{\rho}$$

in equilibrium. Consumption will differ across the two countries only to the extent that there are changes in the real exchange rate. In the PCP model, since purchasing power parity holds, we have, as Obstfeld and Rogoff (1998) derive, $C_t = C_t^*$.[6]

[5] To avoid excess notation, we will not continue to use this state-contingent notation from here on. The important point to remember is that complete asset markets are necessary to sustain the full risk-sharing condition in equation (1.4) below.

Government

Government increases the money supply with direct transfers. The government budget constraint (in per capita terms) is simply

$$M_t = M_{t-1} + T_t .$$

Firms

The firms are monopolistic competitors. The production function for firm i is given by:

$$Y(i) = L(i) .$$

The objective of the domestic firms is to set prices to maximize the expected utility of the owners, who are the domestic residents. Firms must set prices before information about the random domestic and foreign money supplies is known. We will consider three models:

PCP: In this model, there is producer-currency pricing. That is, producers set the price in their own currency. The price that foreigners pay for domestic goods, and the price that home residents pay for foreign goods fluctuates when the exchange rate changes. This is the model examined by Obstfeld and Rogoff (1995, 1998).

PTM: In this model, there is pricing to market. That is, producers set the price in the consumers' currency. Prices consumers face do not respond at all to exchange rate changes.

FER: In this model, exchange rates are fixed with no possibility of changing. Prices are set ahead of time, but the choice of currency is irrelevant.

No state-contingent pricing is allowed in any of the three models.

The optimization problem can be expressed as maximizing the expected present value of profits using the market nominal discount factor for the owners of the firm. Given there is no intertemporal aspect to the firms' optimization problems (see, Obstfeld and Rogoff (1998)), this reduces to maximizing in the PCP case:

$$nE_{t-1}\left[d_{t-1}\pi_t(i)\right] = E_{t-1}\left[d_{t-1}\left((P_{ht}(i) - W_t)(X_{ht}(i) + X^*_{ht}(i))\right)\right],$$

where $X_{ht}(i) = nC_{ht}(i)$ is total sales of firm i to home residents and $X^*_{ht}(i) = (1-n)C^*_{ht}(i)$ is total sales to foreign residents.

The optimal price set by the home firm is:

$$(1.5) \quad P_{ht}(i) = P_{ht} = \frac{\lambda}{\lambda - 1}\frac{E_{t-1}(C_t^{1-\rho}W_t)}{E_{t-1}(C_t^{1-\rho})}.$$

In a world of certainty, the price would simply be a mark-up over unit labor costs. Here, there is an additional mark-up arising from the covariance of nominal wages with $C_t^{1-\rho}$. The term represents a risk premium arising from the covariance of the firm's profits with the marginal utility of consumption. Specifically, the ex post increase in costs from lowering prices by one unit is proportional to $P_{ht}^{-\lambda-1}C_tW_t$, while the increase in revenue is proportional to $P_{ht}^{-\lambda}C_t$. Bearing in mind that P_{ht} is in the time t-1 information set, we can write equation (1.5) as

$$P_{ht} = \frac{\lambda}{\lambda - 1}\frac{E_{t-1}(C_tW_t)E_{t-1}(C_t^{-\rho}) + Cov_{t-1}(C_tW_t, C_t^{-\rho})}{E_{t-1}(C_t)E_{t-1}(C_t^{-\rho}) + Cov_{t-1}(C_t, C_t^{-\rho})}.$$

The covariance terms represent the risk premiums in price setting. If agents were risk-neutral, the covariance terms would be zero, and the price would be a mark-up over the ratio of ex ante marginal cost to marginal revenue.[7]

The law of one price holds for the price charged to foreigners by the home firm:

$$(1.6) \quad P^*_{ht} = P_{ht}/S_t,$$

where S is the home currency price of foreign currency.

Analogous relationships hold for the prices set by the foreign firms:

[7] Bacchetta and van Wincoop (1998) discuss the risk premium in goods prices.

(1.7) $P_{ft}^{*} = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}(C_t^{*1-\rho} W_t^{*})}{E_{t-1}(C_t^{*1-\rho})}$,

(1.8) $P_{ft} = S_t P_{ft}^{*}$.

In the PTM model, the firm chooses two different prices – one to charge residents of its own country, and one to charge residents of the other country. The typical home firm maximizes:

$$E_{t-1}\left[d_{t-1}\left(P_{ht}(i)X_{ht}(i) + S_t P_{ht}^{*}(i)X_{ht}^{*}(i) - W_t(X_{ht}(i) + X_{ht}^{*}(i))\right)\right].$$

The price charged by the home firm to the home residents is given in equation (1.5), but the price charged to foreign residents is given by:

(1.9) $P_{ht}^{*} = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}\left(W_t C_t^{*1-\rho} / S_t\right)}{E_{t-1}(C_t^{*1-\rho})}$.

Likewise, the price charged by foreign firms to its own residents is given by equation (1.7), but the price charged to home-country consumers in the PTM model is:

(1.10) $P_{ft} = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}(S_t C_t^{1-\rho} W_t^{*})}{E_{t-1}(C_t^{1-\rho})}$.

In the FER model, the exchange rate is fixed at all times, so the pricing rule is the same whether the firms state prices in their own currency or the foreign currency.

Given equal consumption at home and abroad, in the PCP model, goods-market equilibrium condition in the home country can be written as:

(1.11) $L_t = \frac{P_t C_t}{P_{ht}}$.

In the PTM model, the law of one price does not hold. The goods-market equilibrium in the home country is written as:

(1.12) $L_t = n\frac{P_t C_t}{P_{ht}} + (1-n)\frac{P_t^{*} C_t^{*}}{P_{ht}^{*}}$.

In the FER model these two relationships are equivalent.

In the PCP model, the 13 variables, C, C^*, P_h, P_h^*, P_f, P_f^*, P, P^*, S, W, W^*, L, and L^*, are determined by the 13 equations (1.1) and its foreign equivalent, (1.2) and its foreign equivalent, (1.3) and its foreign equivalent, (1.11) and its foreign equivalent, (1.4), (1.5), (1.6), (1.7) and (1.8).

In the PTM model these 13 variables are determined by equations (1.1) and its foreign equivalent, (1.2) and its foreign equivalent, (1.3) and its foreign equivalent, (1.12) and its foreign equivalent, (1.4), (1.5), (1.7), (1.9) and (1.10).

In the FER model, the exchange rate is fixed. Then equation (1.4) determines the home money supply as a function of the foreign money supply.

Before proceeding to the solutions to these models, there is one interesting observation to make about the current account. Given that in the PCP model $C_t = C_t^*$, then if consumers in either country have no initial debt, the current account is always zero. The same is not true in the PTM model. Because nominal prices are fixed, current account balance requires that C_t moves proportionately to $S_t C_t^*$. But, equation (1.4) shows that this will not generally occur unless $\rho = 1$.

2. A Special Case with Closed-Form Solutions

In this section, we consider the special case in which money enters logarithmically into the utility function (so $\varepsilon = 1$), and leisure enters linearly. In particular, the felicity function is given by

(2.1) $$u_t = \frac{C_t^{1-\rho}}{1-\rho} + \chi \cdot \ln\left(M_t/P_t\right) - \eta L_t .$$

In addition, we assume the money supply follows a random walk (with drift):

(2.2) $$E_t\left(M_t/M_{t+1}\right) = \mu .$$

These two assumptions permit closed-form solutions for all three models – PCP, PTM and FER. This is useful for developing intuition. Furthermore, as Corsetti and Pesenti (1998) have emphasized, a closed-form solution allows us to interpret the effects of large shocks to exogenous variables, while the approximations derived in the next section (some of which are employed by Obstfeld and Rogoff (1995, 1998)) are only valid for small changes in exogenous variables. However, if prices are sticky because of menu costs, then for large enough shocks firms may decide to adjust prices and the sticky-price solutions no longer are applicable.

Under these assumptions, it is easy to verify from the money demand equation, (1.2), that consumption is a function only of the real money supply:

$$(2.3)\quad C_t^\rho = (\frac{1-\mu\beta}{\chi})\frac{M_t}{P_t}.$$

An analogous equation holds for the foreign country.

This equation implies that the nominal interest rate at home and abroad must be constant. It is worthwhile to develop some intuition for this result. Recall that the gross nominal interest rate is given by the inverse of d_t, which is defined is section 1. The ex post real interest rate is determined by the rate of change of consumption. Higher consumption growth is associated with higher real interest rates.

Consider the effect of a monetary expansion in the home country under flexible exchange rates. In each of our models, as in most Keynesian models, a monetary expansion lowers the real interest rate. (Money is neutral in the long-run (i.e., after one period), but current consumption increases, so the real interest rate must fall.). But a monetary expansion also leads to expected inflation. In each of our models, the future domestic price level increases more than the current price level. In the case in which money enters the utility function logarithmically, the increase in expected inflation exactly offsets the decline in the real interest rate, leaving the nominal interest rate unchanged.

We can now deduce immediately that in both the PCP and PTM models, the exchange rate must follow a random walk. That is, there is no exchange rate overshooting in response to monetary shocks. In our model, uncovered interest parity may not hold exactly, because there may be a foreign exchange risk premium. We will see, however, under assumptions that will be introduced shortly (specifically, that the variance of monetary shocks is constant over time), that the risk premium must be constant. Since domestic and foreign interest rates are also constant, it follows that the expected change in the (log of) the nominal exchange rate must be zero. Since money is neutral in the long run, then it must be the case that the current change in the exchange rate is proportional to changes in domestic money (with a positive sign) and foreign money (with a negative sign.) In fact, it follows directly from equations (1.4) and (2.3) that:

$$(2.4) \quad S_t = \frac{M_t(1-\mu\beta)}{M_t^*(1-\mu^*\beta)}.$$

We can now arrive directly at one of the chief results of this section: in the PTM model, foreign monetary shocks have no effect on domestic consumption, but they do affect domestic consumption in the PCP model. To see this, recall that $P = P_h^n P_f^{1-n}$. In the PTM model, P is predetermined, so it is not influenced by foreign (or domestic) money shocks. It follows from equation (2.3) that domestic consumption is entirely determined in the short-run by the domestic money supply. Changes in the foreign money supply can have no influence on domestic consumption. On the other hand, in the PCP model, P_f increases when the price of the foreign currency increases, since that price is fixed in foreign currency terms. So, a one-percent increase in the foreign money supply leads to a one-percent decrease in S, a one-percent decrease in P_f, and a $1-n$ percent decline in P. Thus, a one-percent increase in the foreign money supply induces a $\frac{1-n}{\rho}$ percent increase in C.

So far, we have been comparing the two floating exchange rate models (PCP and PTM.) How do these models compare to the fixed-exchange rate (FER) model? In order to keep exchange rates fixed, equation (2.4) demonstrates that the domestic money supply must move in proportion to the foreign money supply. Since all prices are fixed, it follows directly from equation (2.3) that a one-percent shock to the foreign money supply leads to a one-percent shock in the domestic money supply, and, therefore, a $\frac{1}{\rho}$ percent change in domestic consumption.

If the only welfare consideration were how the variance of foreign money shocks affected the variance of domestic consumption, it is clear that floating rates dominate fixed exchange rates. Under fixed rates, one percent shocks to foreign money lead to $\frac{1}{\rho}$ percent changes in domestic consumption. By comparison, in the PCP model, the change in domestic consumption is only $\frac{1-n}{\rho}$ percent; and, it is zero percent in the PTM model. Table 1 shows how the variance of (the log of) consumption is related to the variance of (the log of) the foreign money supply in each model.[8]

[8] These results are derived formally in Appendix 1 in equations (A1.10), (A1.18) and (A1.26)

Table 1

Variance of Domestic Consumption
(holding domestic money supply constant)

	σ_c^2
PCP	$\sigma_{m^*}^2 (1-n)^2/\rho^2$
PTM	0
FER	$\sigma_{m^*}^2/\rho^2$

As we shall see, this is not the only welfare consideration. Even if we ignore the effect on welfare coming from money in the utility function, the variance of foreign monetary shocks has further effects on utility. Note that in formulation (2.1), leisure enters utility linearly. So, greater variance of output (and hence leisure) does not directly influence welfare. But, the variance of foreign monetary shocks has other effects on welfare because it influences the means of both consumption and leisure. This channel arises from the effects that monetary variances have on the mark-ups incorporated in firms' prices.

We shall derive our welfare results assuming that shocks to money supplies are log-normally distributed. Using equation (2.2), it follows that

(2.5) $m_{t+1} - m_t = -\ln(\tilde{\mu}) + \upsilon_{t+1}$,

where m_t is the log of M_t (we will follow the convention that lower-case letters are logs of upper-case letters), υ_{t+1} is the white-noise shock to domestic money, and σ_m^2 is the variance of υ_{t+1}. We define:

$$\tilde{\mu} = \mu \cdot \exp\left(-\frac{1}{2}\sigma_m^2\right)$$

An analogous equation holds for the foreign money supply process.

We can write equations (2.3) and (2.4) as

(2.6) $\rho c_t = m_t - p_t + \ln(1-\mu\beta/\chi)$,

(2.7) $s_t = m_t - m_t^* + \ln(1-\mu\beta) - \ln(1-\mu^*\beta)$.

With leisure entering linearly into the utility function, we have from equation (1.3)

(2.8) $C_t = \left(\frac{W_t}{\eta P_t}\right)^{1/\rho}$,

or

(2.9) $w_t = p_t + \rho c_t + \ln(\eta)$,

and analogously for the foreign country.

The solution for goods prices, and ultimately for consumption and leisure, depend on the particular model of price-setting behavior. Appendix 1 derives solutions in terms of the exogenous variables – the domestic and foreign money supplies – for each of our models.

The expected level of consumption can be different depending on the exchange-rate regime. This may seem surprising, since it implies that average long-run consumption depends on a monetary policy choice – the choice of fixed or floating exchange rates. The intuition for this outcome can be seen from equation (2.8). The level of consumption is directly related to the mark-up of prices over wages. When the mark-up is smaller, the level of consumption is higher. The level of prices can be affected by the exchange-rate regime because the risk premium incorporated in prices differs between regimes.

We can compare the effects of the variance of foreign money shocks on the expected level of consumption by setting the variance of domestic money equal to zero in equations (A1.11), (A1.20) and (A1.28). These values are reported in Table 2.

Table 2

Expected Level of Domestic Consumption
(holding domestic money supply constant)

	$E(C)$
PCP	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho}\exp\left[-\left(\frac{(1-n)^2+\rho(1-n)(1-2(1-n))}{2\rho^2}\right)\sigma_{m^*}^2\right]$
PTM	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho}$
FER	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho}\exp\left[-\left(\frac{1-\rho}{2\rho^2}\right)\sigma_{m^*}^2\right]$

We see from Table 2 that the expected level of consumption is higher in the FER model as compared to the PCP model when $\rho > \frac{2-n}{3-2n}$. A sufficient condition is $\rho > 1$. The condition of $\rho > 1$ is necessary and sufficient for the expected level of consumption in the FER model to be higher than in the PTM model. Although there is some dispute empirically about the correct value for ρ, virtually all studies agree that $\rho > 1$. So, while fixed exchange rates are worse than either floating exchange rate model in terms of the variance of consumption, it is better than both when considering the average level of consumption.

This implies a trade-off according to the welfare criterion. Focusing for the moment on the consumption term alone, using the fact that consumption is log-normal, we can write:

$$(2.10)\quad \frac{1}{1-\rho}E(C^{1-\rho}) = \frac{1}{1-\rho}(E(C))^{1-\rho}\cdot\exp\left(\frac{-\rho(1-\rho)}{2}\sigma_c^2\right).$$

So, welfare is positively related to the expected level of consumption, but falls with increases in the variance of consumption.

In comparing welfare under fixed and floating exchange rates, we will focus on the effects of the variance of foreign money shocks on domestic welfare. Clearly, a fixed exchange rate system eliminates the possibility of domestic money shocks, but even under floating rates central banks can choose a zero variance. So, we will set $\sigma_m^2 = 0$ in our welfare analysis.

Real money balances enter the utility function. It may not be wise to evaluate exchange rate systems in terms of how they contribute to the real balance part of the utility function. Money in the utility function is a convenient way to generate demand for an asset, money, that would otherwise be dominated by other assets. But there are other ways to model demand for money that may be more realistic and more complicated that do not involve welfare being directly influenced by holdings of real balances. In making welfare comparisons, we will look at the case in which real balances are not important in welfare ($\chi \to 0$).

We have already discussed how foreign money shocks affect the mean and variance of consumption. In model 1, disutility from labor supply enters utility linearly. So, to complete the welfare analysis, we also need to consider the expected level of labor supply in the three models.

PCP vs. FER models

In the PCP and FER models we can use equations (1.11) and (A1.1) to derive the relationship:

$$(2.11) \qquad E(L) = \frac{\lambda - 1}{\eta\lambda} E(C^{1-\rho}).$$

This allows us to write the welfare expression simply as a function of the mean and variance of consumption:

$$(2.12) \qquad E(u) = \frac{1+\rho(\lambda-1)}{\lambda(1-\rho)} E(C^{1-\rho}) = \frac{1+\rho(\lambda-1)}{\lambda(1-\rho)} (E(C))^{1-\rho} \cdot \exp\left(\frac{-\rho(1-\rho)}{2}\sigma_c^2\right).$$

There is a trade-off in choosing between fixed and floating exchange rates. Fixed exchange rates have a higher expected level of consumption (when $\rho > \frac{2-n}{3-2n}$), but under floating rates the variance of consumption is lower.

We find that welfare is higher under floating rates when $\rho \leq \frac{2-n}{1-n}$. When $\rho \leq 2$, floating rates are always better. As n approaches 1, so that the home country is getting very large relative to the small country, then it is almost surely the case that floating rates dominate. This accords with the intuition that smaller countries may find it desirable to fix their exchange rate to the currency of a larger country, but large countries are probably better off with their own independent currency.

As ρ increases, the fixed exchange rate system's advantage in terms of expected consumption increases. Its disadvantage in terms of variance also falls as the square of ρ increases, although that is offset by the fact that the importance of the variance in welfare rises with the square of ρ (see equation (2.12).) On net, for large values of ρ, fixed exchange rates become more desirable.

PTM vs. FER Models

We have seen from Table 1 that foreign monetary variance does not affect either the mean or variance of consumption in the PTM model. Compared to the FER model, the variance of consumption is lower, but the mean of consumption is also lower when $\rho > 1$.

In addition, expected output, and hence welfare through the expected leisure term, is influenced by the variance of foreign monetary shocks. From equation (A1.22), setting σ_m^2 to zero, in the PTM model:

$$(2.13) \quad E(L) = \left(\frac{\lambda-1}{\eta\lambda}\right)^{1/\rho} \left(n + (1-n)\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\right).$$

If $\rho > 1$, then greater foreign monetary variance may raise expected output and lower welfare.

By comparison, using the fact that $E(L) = \frac{\lambda - 1}{\eta\lambda} E(C^{1-\rho})$, we can derive from equation (A1.28) in the FER model:

$$(2.14) \quad E(L) = \left(\frac{\lambda - 1}{\eta\lambda}\right)^{1/\rho} \exp\left(\frac{\rho - 1}{2\rho^2}\sigma_{m^*}^2\right).$$

Comparing expressions (2.13) and (2.14), we see that when $\rho > 1$, expected labor supply is higher under the FER model, and welfare correspondingly lower.

So, the welfare comparison seems complicated here. Floating rates are better on the grounds that the variance of consumption is lower and expected leisure is greater (when $\rho > 1$), but expected consumption is higher under fixed exchange rates (again, when $\rho > 1$.)

Adding equations (A1.21) and (A1.22), and setting σ_c^2 to zero, we have welfare in the PTM model given by:

$$(2.15) \quad E(u_{PTM}) = \left(\frac{\lambda - 1}{\eta\lambda}\right)^{1-\rho/\rho} \left[\frac{1}{1-\rho} - \frac{\lambda - 1}{\lambda}\left(n + (1-n)\exp(\frac{\rho - 1}{2\rho^2}\sigma_{m^*}^2)\right)\right].$$

This can be compared directly to welfare in the FER model:

$$(2.16) \quad E(u_{FER}) = \left(\frac{\lambda - 1}{\eta\lambda}\right)^{1-\rho/\rho} \left(\frac{1}{1-\rho} - \frac{\lambda - 1}{\lambda}\right)\exp(\frac{\rho - 1}{2\rho^2}\sigma_{m^*}^2).$$

A small amount of algebra using equations (2.15) and (2.16) shows that

$$E(u_{PTM}) > E(u_{FER})$$

for all admissible parameter values. Even though, when $\rho > 1$, there may be less expected consumption under floating exchange rates and PTM, the fact that home consumption is completely insulated from foreign money shocks is enough to insure that floating rates always dominate fixed exchange rates in this model.

3. The General Preferences Case

While the model of section 2 offers an exact solution, it is a special case in that preferences are linear in labor supply and the utility of real balances is of the log form. Here we extend the model to the case of more general preferences. Let the felicity function now be given by

$$(3.1) \quad u_s = \frac{1}{1-\rho} C_s^{1-\rho} + \frac{\chi}{1-\varepsilon}\left(\frac{M_s}{P_s}\right)^{1-\varepsilon} - \frac{\eta}{2} L_s^2$$

This is the function used in Obstfeld and Rogoff (1995, 1998). The definition of the consumption composite and price indices is just as before.

This more general case complicates the solution of the model. Because $\varepsilon \neq 1$, it is no longer true that nominal interest rates are constant. This means that the money market equilibrium condition cannot be solved exactly in logarithmic form. Also, the disutility of labor is now increasing in labor, so it is necessary to solve explicitly the labor market clearing conditions in each country.

As before, we proceed by describing the properties of the three cases of PCP, PTM, and FER. Under all three models however, money market equilibrium condition (1.2) must hold (with a similar condition for the foreign country). Condition (1.3) now becomes

$$(3.2) \quad \frac{W_t}{P_t C_t^{\rho}} = \eta L_t$$

With complete risk-sharing, condition (1.4) must also hold as before. Again, in both the PCP and the FER model, purchasing power parity will hold, so that $C_t = C_t^*$.

The full set of equilibrium conditions for each case is described at the end of section 1. The PCP model in this form is essentially equivalent to that of Obstfeld and Rogoff (1998).

We will continue to maintain the assumptions regarding money supply shocks described in equation (2.5). Thus, in logs, money supply in each country is a random walk[9].

The solution of the money market equilibrium condition requires an approximation. Following the method described by Obstfeld and Rogoff (1998), we approximate around a non-stochastic steady state where the trend growth rates of money supply are the same in each country.

$$(3.3)\ m_t - p_t = \frac{\psi}{i\varepsilon} + \frac{\rho}{\varepsilon} c_t - \frac{\rho}{i\varepsilon}(E_t c_{t+1} - c_t) - \frac{1}{i\varepsilon}(E_t p_{t+1} - p_t) + \frac{\rho^2}{2i\varepsilon}\sigma_c^2 + \frac{1}{2i\varepsilon}\sigma_p^2 + \frac{\rho}{i\varepsilon}\sigma_{cp}$$

where $\psi = \ln\beta + \ln(1+i) - i\ln(i/(1+i)(1/\chi))$, and i represents the non-stochastic steady state equilibrium nominal interest rate in the model. Assume that this is the same in both countries. If the trend growth rate of the money supply is μ, then the steady state nominal interest rate is

$$i = \frac{(1-\beta\mu)}{\beta\mu}.$$ [10]

3.1 The PCP model

As in the previous section, all variances and covariances are constant. Take the difference between the home country money market clearing condition (3.3) and its counterpart for the foreign economy, use PPP, and the assumption that the log money supply is a random walk. Then we may derive the solution for the exchange rate as

$$(3.4)\qquad s_t = m_t - m_t^* - \frac{(1-2n)}{2i\varepsilon}\sigma_s^2 - \frac{\rho}{i\varepsilon}\sigma_{sc}$$

[9] Since the results of this section are derived by an logarithmic approximation around a non-stochastic steady state (as in Obstfeld and Rogoff 1998), the stochastic properties of the *levels* of the money supply do not need to be specified.
[10] In the deterministic steady state, $\mu = \tilde{\mu}$.

Now take the weighted (by n) sum of (3.3) and its foreign counterpart, and subtract the unconditional expectation, we find

$$(3.5) \qquad c_t = \phi(n\upsilon_t + (1-n)\upsilon_t^*) + Ec$$

where $\phi = \dfrac{(1+i\varepsilon)}{\rho(1+i)}$.

With preset prices, output is determined by equation (1.11). Expressing this in logs, and using the previous solutions for c_t and s_t, we have:

$$(3.6) \qquad \ell_t = E\ell + (1-n)(\upsilon_t - \upsilon_t^*) + \phi(n\upsilon_t + (1-n)\upsilon_t^*)$$

As in the previous example, in the PCP model consumption depends positively upon both home and foreign monetary shocks. While a foreign monetary expansion will improve the domestic terms of trade, reducing demand for domestic goods, the magnitude of the direct effect on consumption may be either greater ($\phi > 1$) or less than ($\phi < 1$) the expenditure switching effect. Thus, a foreign monetary expansion may lead domestic output to rise or fall.

To conduct welfare analysis we need to derive the expected consumption level. Take the home country pricing equation (1.5), the labor supply equation (3.2), and the labor market clearing equation (1.11). This gives

$$(3.7) \quad P_h^2 = \frac{\lambda\eta}{\lambda-1} P_h^{2n} P_f^{*2(1-n)} \frac{E(C^2 S^{2(1-n)})}{E(C^{1-\rho})}.$$

A similar equation applies for the foreign country price P_f^*. Then using both expressions, weighting them by n and $1-n$ respectively, and using the properties of the lognormal distribution, we may establish that

$$(3.8) \quad EC = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-1/1+\rho} \exp\left(-\frac{(2-\rho)}{2}\sigma_c^2 - \frac{2n(1-n)}{1+\rho}\sigma_s^2\right)$$

where, from above

$$\sigma_c^2 = \phi^2 (n^2 \sigma_m^2 + (1-n)^2 \sigma_{m*}^2)$$

and

$$\sigma_s^2 = \sigma_m^2 + \sigma_{m*}^2 .$$

Expected consumption depends negatively (positively) on consumption variance when $\rho < 2$ ($\rho > 2$). Intuitively, from the pricing equation (1.5), for a given variance of the exchange rate, consumption variance increases the firms expected marginal cost and expected marginal revenue. If $\rho < 2$ the expected marginal cost dominates, which raises the firms price cost markup, reducing equilibrium expected employment and consumption.

But expected consumption also depends independently on exchange rate variance. A higher variance of the exchange rate will raise expected marginal costs facing either country's firm, (since exchange rates directly affect aggregates CPI's), therefore raise prices and reduce expected consumption.

Of course, both consumption and exchange rate variance depend ultimately on domestic and foreign monetary variability.

3.2 The PTM Model

Since price levels are predetermined in the PTM model, we may solve the money market clearing condition (3.3) directly for consumption. This gives

(3.9) $c_t - Ec = \phi \upsilon_t$

Domestic consumption is independent of foreign money supply shocks with pricing-to-market. Using (1.4), it then follows that

(3.10) $s_t - Es_t = \rho\phi(\upsilon_t - \upsilon_t^*)$

In contrast to (3.4), the exchange rate does not necessarily respond in one for one proportion to money supply shocks. Given the definition of ϕ, it is easy to see that the response of the exchange rate to a home country monetary expansion will be more than proportionate if (and only if) $\varepsilon > 1$. This establishes also that the exchange rate may either `overshoot' or `undershoot' in response to money supply shocks. It is therefore clear that nominal interest rates must be affected by monetary shocks.

To determine expected consumption, again we need to employ the pricing equations (1.5), (1.7), (1.9) and (1.10), labor supply (3.2), and labor market clearing (1.12), with their counterparts for the foreign economy. In Appendix 2, it is shown that within a country, home and foreign firms will set identical prices. Thus $P_{ht} = P_{ft}$ and $P^*_{ht} = P^*_{ft}$, and we obtain the following two conditions

$$(3.11) \qquad EC^{1-\rho} = \frac{\lambda\eta}{\lambda-1}E(CL)$$

$$(3.12) \qquad EC^{*(1-\rho)} = \frac{\lambda\eta}{\lambda-1}E(C^*L)$$

A notable fact about (3.11) and (3.12) is that they do not depend directly on the exchange rate, unlike the analogous expression (3.7) for the PCP case. Exchange rate volatility has no direct effect on expected marginal costs in the PTM economy, and therefore will not directly influence expected consumption levels.

Using the demand condition $L = nC + (1-n)C^*$, we may solve these equations for EC and EC^*, given σ_c^2 and $\sigma_{c^*}^2$ from (3.8) and its foreign counterpart. Appendix 2 shows that the solutions are

$$(3.13)\quad EC = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-1/_{1+\rho}} \times \exp\left(-[\frac{(3-\rho)n}{2} + \frac{n(1-n)}{\rho(1+\rho)} + (1-n)(1-\frac{\rho}{2}) - \frac{1}{2}]\phi^2\sigma_m^2 + \frac{(1-n)}{\rho(1+\rho)}(1-n+\frac{\rho}{2}-\frac{\rho^2}{2})\phi^2\sigma_{m*}^2 \right)$$

$$(3.14)\quad EC^* = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-1/_{1+\rho}} \times \exp\left(-[\frac{(3-\rho)(1-n)}{2} + \frac{n(1-n)}{\rho(1+\rho)} + n(1-\frac{\rho}{2}) - \frac{1}{2}]\phi^2\sigma_{m*}^2 + \frac{n}{\rho(1+\rho)}(n+\frac{\rho}{2}-\frac{\rho^2}{2})\phi^2\sigma_m^2 \right)$$

Although the variance of consumption is unaffected by foreign monetary variability under PTM, expected consumption will depend on the variability of foreign money. Intuitively this is because, from equation (3.11) expected consumption will depend on the properties of output, which does depend on foreign monetary shocks. The influence of monetary variability on expected consumption again depends upon the size of ρ. When ρ is small, home monetary variability reduces expected home consumption, but foreign monetary variability *raises* home consumption[11].

3.3 The FER model

As before, the fixed exchange rate regime is maintained by the home country monetary authorities, which will adjust the domestic money supply so as to respond one-for-one to changes in the foreign money supply. It is then immaterial whether firms use PTM pricing or PCP pricing. Again, let the exchange rate be fixed at $S = 1$.

[11] Loosely speaking, when ρ is small, foreign monetary variability tends to reduce expected foreign consumption, so reducing expected output. From (3.11), this has the effect of raising home country expected consumption.

In the absence of exchange rate uncertainty there are no ex-ante or ex-post deviations from the law of one price, and PPP will hold by default. Thus, from (1.4), $C_t = C_t^*$. Then from the money market clearing condition, we may derive

(3.15) $$c_t - Ec = \phi \upsilon_t^*$$

With no exchange rate movements, prices of domestic and foreign goods will be equal, and the terms of trade will be constant at 1. It then follows that $L_t = C_t$. Using this in conjunction with (1.5) we derive the equilibrium expected consumption level under fixed exchange rates as

(3.16) $$EC = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-1/1+\rho} \exp(-\frac{(2-\rho)}{2}\phi^2\sigma_{m*}^2)$$

Tables 3 and 4 summarize the implications for consumption variance and expected consumption for each of the three models.

Table 3

Variance of Domestic Consumption (holding domestic money supply constant)

	σ_c^2
PCP	$\phi^2(1-n)^2\sigma_{m*}^2$
PTM	0
FER	$\phi^2\sigma_{m*}^2$

Table 4

Expected Level of Domestic Consumption (holding domestic money supply constant)

	$E(C)$
PCP	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/1+\rho}\exp\left[-\left(\frac{(2-\rho)(1-n)^2\phi^2}{2}+\frac{2n(1-n)}{1+\rho}\right)\sigma_{m^*}^2\right]$
PTM	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/1+\rho}\exp(\frac{(1-n)}{\rho(1+\rho)}(1-n+\frac{\rho}{2}-\frac{\rho^2}{2})\phi^2\sigma_{m^*}^2)$
FER	$\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/1+\rho}\exp\left[-\left(\frac{2-\rho}{2}\right)\phi^2\sigma_{m^*}^2\right]$

3.4 Welfare Comparisons

As before, the normative approach is to investigate what is the maximum possible welfare that the monetary authority of a country could achieve, within each regime. This leads us to focus on the case where, under a floating exchange rate, domestic monetary disturbances are eliminated.

PCP vs. FER

First, we compare the properties of PCP and fixed exchange rates. From a comparison of (3.5) and (3.16), consumption variance must be lower under the PCP regime (when $\upsilon_t = 0$). But expected consumption may be lower under PCP, due to the effect of exchange rate variability.

Welfare however depends upon both consumption and employment (again we omit the utility of real money balances in our welfare calculations). But we may use the fact that from the optimal pricing decision

$$EC^{(1-\rho)} = (\lambda/(\lambda-1))\eta EL^2$$

Thus expected utility exclusive of the utility of real money balances, becomes

$$E(u)=\frac{(1+\lambda+\rho(\lambda-1))}{2\lambda}\frac{EC^{(1-\rho)}}{1-\rho}$$

With PCP, this becomes

$$E(u_{PCP})=\frac{\varsigma}{1-\rho}\exp(-(1-\rho)(\phi^2(1-n)^2+\frac{2(1-n)n}{(1+\rho)})\sigma_{m*}^2) \tag{3.18}$$

where

$$\varsigma=\frac{1+\lambda+\rho(\lambda-1)}{2\lambda}\left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{1+\rho}}.$$

Under fixed exchange rates, expected utility is

$$E(u_{FER})=\frac{\varsigma}{1-\rho}\exp(-(1-\rho)\phi^2\sigma_{m*}^2)$$

Therefore, with PCP, floating exchange rates are more desirable when

$$\phi^2(1-n)^2+\frac{2(1-n)n}{(1+\rho)}<\phi^2$$

Using the definition of $\phi=\frac{(1+i\varepsilon)}{\rho(1+i)}$, we may express this as a condition on the value of ρ. Let

$$\bar{\rho}=\frac{1}{2}(\frac{(1+i\varepsilon)^2}{(1+i)^2}\frac{2-n}{2(1-n)}+\left[\frac{(1+i\varepsilon)^4}{(1+i)^4}\frac{(2-n)^2}{4(1-n)^2}+4\frac{(1+i\varepsilon)^2}{(1+i)^2}\frac{2-n}{2(1-n)}\right]^{\frac{1}{2}})$$

Then floating exchange rates under PCP dominate fixed exchange rates if $\rho\leq\bar{\rho}$. Note that in the case $\varepsilon\geq 1$, the critical value of ρ will exceed 1. Thus, the higher is ε, the more likely it is that this condition is satisfied, and floating exchange rates dominate fixed exchange rates.

Foreign money variability directly increases the variance of domestic consumption, and through that channel reduces expected consumption under both regimes. With a fixed exchange rate there cannot be any offsetting exchange rate movements in face of a foreign money shock, so the variance of (log) consumption is higher than under floating rates (i.e. $\phi^2\sigma_{m^*}^2$ as opposed to $(1-n)^2\phi^2\sigma_{m^*}^2$). On this count alone, floating exchange rates would always dominate. But exchange rate variability has an additional cost, independent of the impact of money shocks on consumption. This is represented by the term $\frac{2(1-n)n}{(1+\rho)}$ in the above condition. This captures the fact that higher exchange rate variance will raise producer prices and depress expected consumption.

With a high value of ρ (or a low value of ε), the direct impact of money shocks on the variance of consumption is quite small, and so the cost of exchange rate volatility will dominate. But with lower values of ρ (or a high value of ε), the direct effect of monetary shocks through consumption variability tend to dominate, and floating exchange rates are more desirable to cushion this. As in the special case model of section 2, it is also clear that a large country (high value of n) will find floating exchange rates under PCP more desirable.

PTM vs. FER

Now we compare welfare effects of fixed and floating exchange rates under PTM. From the analysis above we know that with floating exchange rates and PTM, the variance of home consumption is zero. But from (3.13) and (3.16), expected consumption under PTM may be either higher or lower than that under FER. In addition, while consumption variance is eliminated with PTM, employment variance is not, and average employment will also be affected by foreign monetary fluctuations. Thus, the welfare comparison between the two models is not immediately obvious. Nevertheless, we now show that floating exchange rates with PTM will always dominate fixed exchange rates. We show this in two parts; first for the case $\rho < 1$, and then for the case $\rho > 1$.

Under the PTM regime, utility does not collapse in a simple manner to a function of the mean and variance of consumption. Appendix 2 shows that expected utility under PTM may be written approximately as

(3.19)

$$E(u_{PTM}) = \frac{1}{1-\rho} EC^{(1-\rho)} \left[\frac{2\lambda - (\lambda - 1)(1-\rho)\exp\left(-(1-\rho)\theta\phi^2\sigma_{m*}^2\right)}{2\lambda} \right]$$

where

$$EC^{(1-\rho)} = \vartheta \exp\left((1-\rho)\left(\frac{(1-n)}{\rho(1+\rho)}(1-n+\frac{\rho}{2}-\frac{\rho^2}{2})\phi^2\sigma_{m*}^2 \right)\right)$$

$$\vartheta = \left(\frac{\lambda\eta}{\lambda - 1} \right)^{\left(\frac{\rho-1}{1+\rho}\right)}$$

and

$$\theta = \left(\frac{(1-n)}{\rho}(1-n+\frac{\rho}{2}) \right) > 0$$

Under fixed exchange rates we rewrite the analogous welfare expression from the previous subsection as

(3.20) $$E(u_{FER}) = \frac{1}{1-\rho} EC^{(1-\rho)} \left[\frac{\lambda + 1 + \rho(\lambda - 1)}{2\lambda} \right]$$

where

$$EC^{(1-\rho)} = \vartheta \exp(-(1-\rho)\phi^2\sigma_{m*}^2)$$

In comparing (3.19) and (3.20), we see that the term $\frac{1}{1-\rho} EC^{(1-\rho)}$ must always be higher under PTM. Expected utility of consumption is declining in σ_{m*}^2 under fixed exchange rates. When

$\rho < \frac{1}{2} + \frac{1}{2}\sqrt{1+8(1-n)}$ expected utility of consumption is increasing in $\sigma_{m^*}^2$ under PTM. Although when $\rho > \frac{1}{2} + \frac{1}{2}\sqrt{1+8(1-n)}$ the expected utility of consumption is declining in $\sigma_{m^*}^2$ under PTM, it is easy to show that it is always higher than the corresponding expression under fixed exchange rates.

A further comparison of (3.19) and (3.20) however reveals that the term inside the square brackets must be higher for (3.19) than (3.20). To see this, note that this term is increasing in θ for (3.19), and, for $\theta = 0$, the term is identical (and positive) for both equations. Since $\theta > 0$ must always hold, the statement follows. Now assume that $\rho < 1$. It then follows from the two previous comparisons that expected utility is always higher under floating exchange rates with PTM than under fixed exchange rates.

When $\rho > 1$, the same argument does not apply. In this case however, we may write out expected utility in the PTM model and the FER model in terms of their separate consumption and employment components in the following way.

(3.21)

$$E(u_{PTM}) = \frac{\vartheta}{1-\rho}\exp\left((1-\rho)\left(\frac{(1-n)}{\rho(1+\rho)}(1-n+\frac{\rho}{2}-\frac{\rho^2}{2})\phi^2\sigma_{m^*}^2\right)\right)$$

$$-\xi\frac{\eta}{2}\exp\left(-(1-n)(1-\rho)(1-\frac{n}{1+\rho})\phi^2\sigma_{m^*}^2\right)$$

where $\xi = \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{2}{1+\rho}}$

The first line of the above equation represents expected utility of consumption. The second line represents the expected disutility of employment. Foreign monetary variability must raise the variance of employment, which reduces expected utility. But, for low values of ρ, it also *reduces* expected employment, so raising expected utility. For $\rho < 1$ the net effect on expected utility is positive, while for $\rho > 1$ it will be negative.

Under fixed exchange rates we may write the analogous welfare expression as

$$(3.22) \qquad E(u_{FER}) = \frac{\upsilon}{1-\rho}\exp\left((1-\rho)\left[-\phi^2\sigma^2_{m^*}\right]\right)$$

$$-\frac{\eta\xi}{2}\exp\left(2\left[-\frac{(1-\rho)}{2}\phi^2\sigma^2_{m^*}\right]\right)$$

The first line in equation (3.21) will always exceed the first line in equation (3.22). Thus, a *sufficient* condition for floating exchange rates under PTM to dominate fixed exchange rates is that the expected disutility of employment term is lower under the former. This amounts to the condition

$$[(1-n)(1-\rho)(1-\frac{n}{1+\rho})] > \frac{(1-\rho)}{2}$$

which is always true for $\rho > 1$.

We have thus established that expected utility under PTM is always higher than that under FER. There is no trade-off between the two regimes under pricing to market. By choosing floating exchange rates, the policy maker can completely insulate consumption from foreign monetary disturbances, and can achieve a lower variance of leisure. It is true that unlike the special case model of section 2, expected consumption is affected by foreign monetary variability under PTM, due to the impact of foreign money shocks on employment. When ρ is sufficiently high, foreign monetary variability does reduce home country expected consumption under PTM. But this negative welfare effect through average consumption is always outweighed by the positive effect of a lower variance of consumption.

Discussion

Our welfare comparisons indicate that the choice of exchange rate regime will depend critically on the type of commodity pricing arrangement. If prices are set in producer's currency, then floating exchange rates may or may not dominate fixed exchange rates. This price setting arrangement is the one implicit in the original Friedman and Mundell models. Like these papers, we find that floating exchange rates help to stabilize consumption from foreign monetary disturbances. But variance is not the only welfare indicator here. Floating exchange rates also reduce the average level of consumption, by pushing up price-cost markups. In this sense, our model identifies a pure welfare cost of exchange rate volatility. The choice between fixed and floating exchange rates is a trade-off between the higher variance of consumption under fixed exchange rates relative to the possibly lower mean consumption under floating exchange rates.

In the PTM economy, exchange rate volatility does not have an independent negative effect on expected consumption, since the exchange rate does not influence optimal pricing policies, compounding the monopolistic competitive distortion. It is still true that average consumption is influenced by foreign monetary variability, since foreign money shocks influence output, and output fluctuations affect expected marginal costs facing price-setting firms. But the benefits of consumption stabilization under floating exchange rates will always dominate any adverse movements in average consumption or leisure due to foreign monetary variability.

Rather than the extremes of free floating with fixed exchange rates, we could have used an exchange market intervention rule that has floating and fixed exchange rates as special cases. One can then show that for the case of PCP the optimal foreign exchange market intervention rule depends upon the parameters of the model in an intuitive way. The higher is ρ and the lower is ε, the greater the degree of intervention, and the closer we will be to fully fixed exchange rates.

Home Monetary Volatility under Floating Rates

We have been assuming that under floating exchange rates, the monetary authorities have the ability to set home money supply variance equal to zero. Perhaps, however, under floating exchange rates the home country policy makers cannot control the home money supply. Under fixed exchange rates, the policy makers have enough control that they can alter the home money supply to keep the exchange rate fixed. As a final robustness check, we consider the possibility that in the absence of a rule such as fixing the exchange rate, the monetary authority lacks the discipline to control domestic money.

Consider the fixed vs. floating tradeoff in the symmetric case in which the home money supply variance is equal to the foreign money supply variance in the PCP and PTM models: $\sigma^2 \equiv \sigma_m^2 = \sigma_{m^*}^2$, but the home and money shocks are independent. In addition, focus on equally sized countries, so that n=0.5. Table 5 reports the expressions for expected utility in the three models with symmetric monetary disturbances (the derivations are contained in Appendix 2).

Table 5

Expected Utility with Home Country Monetary Shocks

	$E(u)$
PCP	$\frac{1}{1-\rho}\varsigma \exp\left((1-\rho)(-\frac{1}{2}\phi^2\sigma^2 - \frac{\sigma^2}{(1+\rho)})\right)$
PTM	$\frac{1}{1-\rho}\varsigma \exp(-(1-\rho)\frac{1}{2}(1+\frac{\rho}{(1+\rho)})\phi^2\sigma^2)$
FER	$\frac{1}{1-\rho}\varsigma \exp\left((1-\rho)(-\phi^2\sigma^2)\right)$

In comparing PCP and FER in this case, we have the same trade-off as before. Exchange rate variability is still desirable in order to cushion the impact of home or foreign money shocks, since the variance of consumption is lower under floating exchange rates. But exchange rate variability is costly in the same way as before. Again, with a low value of ρ, floating rates are more desirable, while fixed rates are better when ρ is high. Thus, the qualitative nature of the trade off of fixed and floating exchange rates remains unchanged (though it is more likely that fixed exchange rates are preferred when we introduce home monetary variability.)

In the PTM case, the welfare results are also unchanged when home country monetary variability is allowed. Table 5 reveals that expected utility under PTM is always higher than that under FER.

4. Conclusion

This paper has addressed an old theme in open economy macroeconomics; the optimal choice of exchange rate regime. The novel aspect of the paper is that the comparison of regimes is done within an expected utility maximizing framework in the presence of nominal price rigidity. The exchange rate regime `matters' in this framework for the same reasons that it did in the models of Friedman and Mundell. But we find that the normative analysis of exchange rate regimes is quite different in our framework. Different exchange rate regimes have implications not just for the variability of consumption and output, but also (because of risk-premia in firm's pricing.) for the average value of these variables.

Our main findings indicate that the choice of exchange rate regime depends in important ways on the currencies in which prices are set. Under pricing to market, floating exchange rates will always dominate fixed exchange rates. Under producer currency pricing, floating exchange rates may or may not dominate. When risk aversion is very high, exchange rate variability is so costly in welfare terms that fixed exchange rates tend to dominate. The results indicate generally the sensitivity of normative conclusions about exchange rate regimes to the structural features of pricing arrangements.

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Appendix 1

Recall that the pricing formulas for P_{ht} and P^*_{ft} are the same in all three models. From equations (1.5) and (2.8), we have

(A1.1) $P_{ht} = (\frac{\lambda\eta}{\lambda - 1})\frac{E_{t-1}(P_t C_t)}{E_{t-1}(C_t^{1-\rho})}$.

Using the fact that prices and consumption will be log-normally distributed in equilibrium, we can derive

(A1.2) $p_{ht} = m_{t-1} - \ln(\mu) + \sigma_m^2 + (1-\rho)\sigma_{mc} + \ln(\lambda\eta(1-\mu\beta)/\chi(\lambda - 1))$,

where σ_{mc} is the covariance of m_t and c_t conditional on $t-1$ information. (In general, we will use the notation σ_{xz} to denote $Cov_{t-1}(x_t, z_t)$, and σ_z^2 to denote $Var_{t-1}(z_t)$.) Similar derivations yield:

(A1.3) $p^*_{ft} = m^*_{t-1} - \ln(\mu^*) + \sigma^2_{m^*} + (1-\rho)\sigma_{m^*c^*} + \ln(\lambda\eta(1-\mu^*\beta)/\chi(\lambda - 1))$.

Equations (2.5), (2.6), (2.7), (A1.2) and (A1.3) hold in all three models. We will also use the relationship

(A1.4) $p_t = np_{ht} + (1-n)p_{ft}$,

and the analogous expression for the foreign price level. These equations must be augmented by equations for p_{ft} and p^*_{ht}, as well as equations for output, in order to complete derivations in each model.

A.1 The PCP Model

The law of one price holds in this model, so

(A1.5) $p_{ft} = s_t + p^*_{ft}$,

(A1.6) $p_{ht}^{*} = p_{ht} - s_t$.

Recall, also, the perfect risk-sharing property of this model, so that

$$c_t = c_t^{*}.$$

Then, using (2.7), (A1.2), (A1.4), and (A1.5), we can derive for the domestic price level

(A1.7) $p_t = m_t - n(\upsilon_t - \frac{1}{2}\sigma_m^2 - (1-\rho)\sigma_{mc}) - (1-n)(\upsilon_t^{*} - \frac{1}{2}\sigma_{m^*}^2 - (1-\rho)\sigma_{m^*c}) + \ln(\frac{\lambda\eta(1-\mu\beta)}{\chi(\lambda-1)})$.

From equations (2.6) and (A1.7) we get

(A1.8) $\rho c_t = n(\upsilon_t - \frac{1}{2}\sigma_m^2 - (1-\rho)\sigma_{mc}) + (1-n)(\upsilon_t^{*} - \frac{1}{2}\sigma_{m^*}^2 - (1-\rho)\sigma_{m^*c}) - \ln(\frac{\lambda\eta}{\lambda-1})$.

We can derive expressions for σ_{mc} and σ_{m^*c} directly from equation (A1.8):

$$\sigma_{mc} = \frac{n}{\rho}\sigma_m^2,$$

$$\sigma_{m^*c} = \frac{1-n}{\rho}\sigma_{m^*}^2.$$

Plugging these into (A1.8), we arrive at our expression for domestic consumption:

(A1.9) $c_t = \frac{n}{\rho}\upsilon_t + \frac{1-n}{\rho}\upsilon_t^{*} - (\frac{n\rho + 2n^2(1-\rho)}{2\rho^2})\sigma_m^2 - (\frac{(1-n)\rho + 2(1-n)^2(1-\rho)}{2\rho^2})\sigma_{m^*}^2 - \frac{1}{\rho}\ln(\frac{\lambda\eta}{\lambda-1})$.

The variance of domestic and foreign money shocks affects utility also because they affect the variance of consumption. From equation (A1.8),

(A1.10) $\sigma_c^2 = \frac{n^2}{\rho^2}\sigma_m^2 + \frac{(1-n)^2}{\rho^2}\sigma_{m^*}^2$.

The expected level of consumption is given by:

(A1.11)

$$E(C)=\exp(Ec+\sigma_c^2/2)=\left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho}\exp\left(-\left(\frac{n^2+\rho n(1-2n)}{2\rho^2}\right)\sigma_m^2-\left(\frac{(1-n)^2+\rho(1-n)(1-2(1-n))}{2\rho^2}\right)\sigma_{m^*}^2\right)$$

Expected utility depends on both the expected level and the variance of consumption:

(A1.12)
$$\frac{1}{1-\rho}E(C^{1-\rho})=\frac{1}{1-\rho}E(\exp((1-\rho)c))=\frac{1}{1-\rho}(\exp((1-\rho)Ec+\frac{(1-\rho)^2}{2}\sigma_c^2))=$$
$$\left(\frac{1}{1-\rho}\right)\left(\frac{\lambda-1}{\eta\lambda}\right)^{1-\rho/\rho}\left(\exp\left(-\frac{n(1-\rho)(\rho+n(1-\rho))}{2\rho^2}\sigma_m^2-\frac{(1-n)(1-\rho)(1-n(1-\rho))}{2\rho^2}\sigma_{m^*}^2\right)\right)$$

From equations (1.11) and (A1.1), we can derive the expected level of output easily as:

$$E(L)=\frac{\lambda-1}{\eta\lambda}E(C^{1-\rho}).$$

A.2 The PTM Model

In this model, prices are set in the consumers' currencies. So, P_{ft} and P_{ht}^* are predetermined variables at time t. Using equations (1.9) and (2.8), we have

(A1.13)
$$P_{ht}^*=\frac{\eta\lambda}{\lambda-1}\frac{E_{t-1}(\frac{P_tC_t^\rho C_t^{*1-\rho}}{S_t})}{E_{t-1}(C_t^{*1-\rho})}.$$

Then, using the consumption risk-sharing equation, (1.4), we can rewrite equation (A1.13) as

$$P_{ht}^*=\frac{\eta\lambda}{\lambda-1}\frac{E_{t-1}(P_t^*C_t^*)}{E_{t-1}(C_t^{*1-\rho})}.$$

Symmetrically,

(A1.14)
$$P_{ft}=\frac{\eta\lambda}{\lambda-1}\frac{E_{t-1}(P_tC_t)}{E_{t-1}(C_t^{1-\rho})}.$$

Comparing equations (A1.1) and (A1.14), we see that P_{ft} and P_{ht} are equal. Using this fact, and the definition of the price index in equation (1.1), we have:

$$(\text{A1.15}) \qquad P_{ft} = P_{ht} = P_t = \frac{\eta\lambda}{\lambda-1}\frac{E_{t-1}(P_t C_t)}{E_{t-1}(C_t^{1-\rho})}.$$

But, since P_t is in the time t-1 information set, equation (A1.15) gives us that

$$(\text{A1.16}) \qquad E_{t-1}(C_t^{1-\rho}) = \frac{\eta\lambda}{\lambda-1}E_{t-1}(C_t).$$

Writing this equation in logs:

$$(1-\rho)E_{t-1}(c_t) + \frac{(1-\rho)^2}{2}\sigma_c^2 = \ln\left(\frac{\lambda\eta}{\lambda-1}\right) + E_{t-1}(c_t) + \frac{1}{2}\sigma_c^2.$$

Solving, we find

$$(\text{A1.17}) \qquad E_{t-1}(c_t) = \frac{-1}{\rho}\ln\left(\frac{\lambda\eta}{\lambda-1}\right) + \frac{\rho-2}{2}\sigma_c^2.$$

Since P_t is in the time t-1 information set, we have from equation (2.6)

$$(\text{A1.18}) \qquad \sigma_c^2 = \frac{1}{\rho^2}\sigma_m^2.$$

As compared to the PCP model (equation (A1.10)), domestic monetary variance has a greater effect on the variance of domestic consumption. Substituting into equation (A1.17), we get

$$(\text{A1.19}) \qquad E_{t-1}(c_t) = \frac{-1}{\rho}\ln\left(\frac{\lambda\eta}{\lambda-1}\right) + \frac{\rho-2}{2\rho^2}\sigma_m^2.$$

The expected level of consumption is given by

$$(\text{A1.20}) \qquad E(C_t) = \exp(Ec + \sigma_c^2/2) = \left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho}\exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right).$$

The expected utility term involving consumption is derived directly from equation (A1.16) using equation (A1.20):

(A1.21) $$\frac{1}{1-\rho}E(C^{1-\rho}) = \left(\frac{1}{1-\rho}\right)\left(\frac{\lambda-1}{\eta\lambda}\right)^{1-\rho/\rho}\left(\exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right)\right).$$

From equation (1.12), using the fact that $P_{ht} = P_t$ and $P_{ht}^* = P_t^*$, we have

$$L_t = n\exp(c_t) + (1-n)\exp(c_t^*).$$

This yields the term in the utility function:

(A1.22) $$-\eta E(L) = -\frac{\lambda-1}{\lambda}\left(\frac{\lambda-1}{\eta\lambda}\right)^{1-\rho/\rho}\left[n\exp(\frac{\rho-1}{2\rho^2}\sigma_m^2) + (1-n)\exp(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2)\right].$$

A.3 FER Model

We will assume the exchange rate is fixed at 1, so $s_t = 0$ for all t. From equation (2.7) it follows that

(A1.23) $$m_t + \ln(1-\mu\beta) = m_t^* + \ln(1-\mu^*\beta).$$

The domestic money supply moves in reaction to foreign money supply shocks in order to keep the exchange rate fixed.

Because exchange rates are fixed, $P_{ft} = P_{ft}^*$ and $P_{ht} = P_{ht}^*$. We can use equations (A1.2) and (A1.3) to derive expressions for prices. Noting that all prices are preset, the money demand equation (2.6) (combined with relation (A1.23)) tells us

$$\sigma_{mc} = \sigma_{m^*c^*} = \frac{1}{\rho}\sigma_{m^*}^2.$$

We can then derive:

(A1.24) $$p_{ht} = p_{ft} = p_t = m_{t-1}^* + \frac{1}{\rho}\sigma_{m^*}^2 + \ln(\frac{\lambda\eta(1-\mu^*\beta)}{\mu^*\chi(\lambda-1)}).$$

Then, using equation (2.6) we get the expression for consumption under fixed exchange rates:

(A1.25) $$c_t = \frac{1}{\rho}\upsilon_t^* + \frac{\rho-2}{2\rho^2}\sigma_{m^*}^2 - \frac{1}{\rho}\ln(\frac{\lambda\eta}{\lambda-1}).$$

The variance of consumption in the FER model is given by

(A1.26) $$\sigma_c^2 = \frac{1}{\rho^2}\sigma_{m^*}^2.$$

The expected level of consumption is given by:

(A1.27) $$E(C) = \exp(Ec + \sigma_c^2/2) = \left(\frac{\lambda-1}{\lambda\eta}\right)^{1/\rho} \exp\left(-\left(\frac{1-\rho}{2\rho^2}\right)\sigma_{m^*}^2\right).$$

The expected utility term involving consumption is

(A1.28) $$\frac{1}{1-\rho}E(C^{1-\rho}) = \frac{1}{1-\rho}E(\exp((1-\rho)c)) = \frac{1}{1-\rho}(\exp((1-\rho)Ec + \frac{(1-\rho)^2}{2}\sigma_c^2)) =$$
$$\left(\frac{1}{1-\rho}\right)\left(\frac{\lambda-1}{\eta\lambda}\right)^{1-\rho/\rho}\left(\exp\left(\frac{-(1-\rho)}{2\rho^2}\sigma_{m^*}^2\right)\right)$$

Given the structure of the model, the fixed exchange rate model simply reduces to the PCP model with $n = 0$.

Appendix 2

In log terms, the pricing equations under PTM, given by (1.5), (1.7), (1.9), and (1.10), combined with the labor supply condition (3.2) and the risk-sharing condition (1.4) may be written as

(A2.1) $$P_{ht} = \frac{\lambda\eta}{\lambda-1}\frac{E(CP(\frac{P}{P_h}nC + \frac{P^*}{P_h^*}(1-n)C^*))}{EC^{1-\rho}}.$$

(A2.2) $$P_{ht}^* = \frac{\lambda\eta}{\lambda-1}\frac{E(C^*P^*(\frac{P}{P_h}nC + \frac{P^*}{P_h^*}(1-n)C^*))}{EC^{*1-\rho}}.$$

(A2.3) $$P_{ft} = \frac{\lambda\eta}{\lambda-1}\frac{E(CP(\frac{P}{P_f}nC + \frac{P^*}{P_f^*}(1-n)C^*))}{EC^{1-\rho}}.$$

(A2.4) $$P_{ft}^* = \frac{\lambda\eta}{\lambda-1}\frac{E(C^*P^*(\frac{P}{P_f}nC + \frac{P^*}{P_f^*}(1-n)C^*))}{EC^{*1-\rho}}.$$

Conjecture that a solution to these equations satisfies $P_{ht} = P_{ft}$ and $P_{ht}^* = P_{ft}^*$. Imposing this, we see that the right hand side of (A2.1) and (A2.3) is identical, so the condition $P_{ht} = P_{ft}$ is a solution. Similarly we see that it is easy to see that $P_{ht}^* = P_{ft}^*$ represents a solution for (A2.2) and (A2.4). The home country CPI is just $P = P_h^n P_f^{(1-n)}$, and similarly for the foreign CPI.

To derive the expressions (3.14) and (3.15) of the text, rewrite (3.12) and (3.13) as

(A2.6)

$$\exp((1-\rho)(Ec + \frac{1-\rho}{2}\sigma_c^2)) = \frac{\lambda\eta}{\lambda-1}[n\exp(2(Ec+\sigma_c^2) + (1-n)\exp(Ec + Ec^* + \frac{1}{2}\sigma_c^2 + \frac{1}{2}\sigma_{c^*}^2)]$$

(A2.7)

$$\exp((1-\rho)(Ec^* + \frac{1-\rho}{2}\sigma_{c^*}^2)) = \frac{\lambda\eta}{\lambda-1}[(1-n)\exp(2(Ec^* + \sigma_{c^*}^2) + n\exp(Ec + Ec^* + \frac{1}{2}\sigma_c^2 + \frac{1}{2}\sigma_{c^*}^2)]$$

The right hand side of (A2.6) and (A2.7) is not log linear. But we may take a first-order log approximation around the initial steady state where $\sigma_c^2 = \sigma_{c*}^2 = 0$, as we did for equation (3.3). Then solving the resulting two simultaneous equations for Ec and Ec^*, and transforming back to levels gives us (3.14) and (3.15).

To derive equation (3.19), we may write out expected utility under PTM as

(A2.8)

$$
\begin{aligned}
E(u_{PTM}) &= \frac{1}{1-\rho}\exp((1-\rho)(Ec + \frac{1-\rho}{2}\sigma_c^2)) - \frac{\eta}{2}E(nC + (1-n)C^*)^2 \\
&= \frac{1}{1-\rho}\exp((1-\rho)(Ec + \frac{(1-\rho)}{2}\sigma_c^2)) - \\
&\frac{\eta}{2}[n^2\exp(2(Ec+\sigma_c^2) + 2n(1-n)\exp(Ec + Ec^* + \frac{1}{2}\sigma_c^2 + \frac{1}{2}\sigma_{c*}^2) + (1-n)^2\exp(2(Ec^* + \sigma_{c*}^2))]
\end{aligned}
$$

Now using the fact that $\sigma_c^2 = 0$, for small σ_{c*}^2 the latter expression may be approximated as

$$E(u_{PTM}) = \frac{1}{1-\rho}\exp((1-\rho)Ec) - \frac{\eta}{2}[\exp(2(nEc + (1-n))Ec^* + (1-n)(2-n)\sigma_{c*}^2)]$$

Substituting for Ec, Ec^*, and σ_{c*}^2 and re-arranging gives equation (3.19).

Home Monetary Variability

Let home and foreign money shocks be independent and have identical variance. Also, let $n = \frac{1}{2}$. We have $\sigma_c^2 = \phi^2\frac{1}{4}(2\sigma^2)$ and $\sigma_s^2 = 2\sigma^2$.

From equation (3.8)

(A2.9) $$Ec = \frac{1}{1+\rho}\log\frac{\lambda\eta}{\lambda-1} - \frac{(3-\rho)}{2}\frac{1}{4}\phi^2(2\sigma^2) - \frac{1}{2}\frac{1}{(1+\rho)}(2\sigma^2)$$

Then, noting that $\sigma_c^2 = \phi^2\frac{\sigma^2}{2}$ we have that

$$\frac{1}{1-\rho}EC^{1-\rho} = \frac{1}{1-\rho}\vartheta\exp\left((1-\rho)(-\frac{(3-\rho)}{2}\frac{\phi^2\sigma^2}{2} - \frac{1}{2}\frac{2\sigma^2}{(1+\rho)} + \frac{(1-\rho)}{2}\frac{\phi^2\sigma^2}{2})\right)$$

Then evaluating expected utility in the same way as before gives the expression in Table 5. Using a similar approach, we calculate $E(u_{FER})$.

For the case of PTM, note that the pricing equation for the symmetric case gives

$$EC^{1-\rho} = \frac{\lambda\eta}{\lambda-1}E(CL)$$

$$L = \frac{1}{2}(C+C^*)$$

We know that $Ec = Ec^*$ and $\sigma_c^2 = \sigma_{c^*}^2$. Then take expectations to get

$$\text{(A2.10)} \quad \exp((1-\rho)(Ec + \frac{(1-\rho)}{2}\sigma_c^2)) = \frac{\lambda\eta}{\lambda-1}\frac{1}{2}\exp(2Ec+\sigma_c^2)(1+\exp(\sigma_c^2))$$

Approximate the very last expression

$$\log(1+\exp(\sigma_c^2)) \approx \log 2 + \frac{1}{2}\sigma_c^2$$

Take logs of both sides of (A2.10) using this approximation to get

$$Ec = -\frac{1}{1+\rho}\log\frac{\lambda\eta}{\lambda-1} - \left[1-\frac{\rho^2}{2(1+\rho)}\right]\sigma_c^2$$

Expected utility under PTM is

$$E(u_{ptm}) = \frac{1}{1-\rho}\exp((1-\rho)(Ec+\frac{1-\rho}{2}\sigma_c^2)) - \frac{\eta}{2}EL^2$$

The first expression is

$$\frac{1}{1-\rho}\vartheta\exp(-(1-\rho)\frac{1}{2}(1+\frac{\rho}{1+\rho})\sigma_c^2)$$

The second expression needs to be approximated

$$EL^2 = \frac{1}{2}\exp(2Ec + \sigma_c^2)(1 + \exp(\sigma_c^2))$$

Again, we have

$$\log EL^2 \approx 2Ec + \frac{3}{2}\sigma_c^2$$

So $EL^2 \approx \exp(2Ec + \frac{3}{2}\sigma_c^2) = \exp(-2(1 - \frac{\rho^2}{2(1+\rho)})\sigma_c^2 + \frac{3}{2}\sigma_c^2) = \exp(-(\frac{1}{2} - \frac{\rho^2}{(1+\rho)})\sigma_c^2)$

Putting both parts together gives the expression in Table 5.