

## Flavour Issues in Leptogenesis

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### Abstract

We study the impact of flavour in thermal leptogenesis, including the quantum oscillations of the asymmetries in lepton flavour space. In the Boltzmann equations we find different numerical factors and additional terms which can affect the results significantly. The upper bound on the CP asymmetry in a specific flavour is weaker than the bound on the sum. This suggests that – when flavour dynamics is included – there is no model-independent limit on the light neutrino mass scale, and that the lower bound on the reheat temperature is relaxed by a factor  $\sim (3 - 10)$ .

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**Note added:** The equations (11) and (19) of the published paper, which include the decaying oscillations in flavour space, do not reflect the flavour structure derived in the Appendix, or reproduce the behaviour discussed in the text. In this version, we modify equations (11) and (19), and add a discussion, in Appendix A, on how to obtain these improved equations. We thank Georg Raffelt for drawing our attention to this, (and also to the paper by N.F.Bell *et al*, *Phys. Lett.* **B500** (2001) 16, which indicates that fast gauge interactions should not affect the coherence of flavour oscillations). We also remove the errant factor of  $z$  multiplying the charged lepton Yukawa terms of (11) and (19). Equations (20,21) are solutions to the flavour structure of the Appendix, so should be unaffected by the change of form of equations (11) and (19). However, (20,21) are wrong by the factor  $z$ .

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# 1 Introduction

Leptogenesis [1] is an attractive mechanism to explain the baryon asymmetry of the Universe (BAU) [2]. It consists of the dynamical production of a lepton asymmetry in the context of a given model, which then can be converted to a baryon asymmetry due to  $(B + L)$ -violating sphaleron interactions [3] which exist in the Standard Model (SM).

A simple model in which this mechanism can be implemented is “Seesaw” (type I) [4], consisting of the Standard Model (SM) plus 3 right-handed (RH) Majorana neutrinos. In this simple extension of the SM, the usual scenario that is explored consists of a hierarchical spectrum for the RH neutrinos, such that the lightest of the RH neutrinos is produced by thermal scattering after inflation, and decays out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov’s constraints. In recent years, a lot of work [5, 6], has been devoted to a thorough analysis of this model, the constraints that can be imposed, the relationships that arise with low energy neutrino physics, etc. For instance, a lower bound on the reheating temperature  $T_{\text{RH}}$  and an upper bound on the scale of light neutrino masses can be obtained [7, 8, 9, 10].

However, in order to obtain an accurate value of the produced lepton asymmetry it is necessary to solve (in a first approach at least) the system of coupled Boltzmann equations (BE) which relate the abundance of the lightest RH neutrino with the asymmetry produced in lepton number. The effects of the charged lepton Yukawa coupling matrix  $[h]$  are usually not considered. In this study, we take into account these effects in deriving the evolution equations for the asymmetries in each flavour, and study the implications for leptogenesis. Flavour effects have been also discussed in [11]. More recently, the generation of the family lepton asymmetries has been discussed in [12] and [13], where the case of leptogenesis in the minimal seesaw model with two heavy Majorana neutrinos was considered; the role of flavours in the leptogenesis scenario from  $N_2$  decay is discussed in [14] and for “resonant leptogenesis” (degenerate  $N_J$ ) in [15]. In this work, we find three effects: a qualitative difference in the equations, which allows quantum oscillations of the asymmetry in flavour space. We naively anticipate this could change the final asymmetry by a factor  $\sim 2$  [16]. Secondly, we find different numerical factors compared to the case when no flavour issues are included in the Boltzmann equations, and additional terms when the flavour-dependent decay rates and asymmetries are unequal. Finally, we find that the CP asymmetries in individual flavours can be larger than the sum, which affects the leptogenesis bounds on  $T_{\text{RH}}$  and makes the bound on the light neutrino mass scale disappear.

In section 2 we review our notation and summarise the usual way the calculation is performed. In section 3, we present equations for the lepton asymmetry, represented as a matrix in flavour space: the diagonal elements are the flavour asymmetries, and the off-diagonals encode quantum correlations. These could be relevant, because, as will be explained, the muon Yukawa coupling comes into thermal equilibrium around  $T \sim 10^9$  GeV, allowing the possibility that the asymmetry “oscillates” in flavour space. To motivate the equations used in section 3 we study a quantum mechanical toy model of simple harmonic oscillators. This study is explained in appendix A of this note. We concentrate on the BE for the three flavour asymmetries in section 4, and compare the resulting equations for the lepton asymmetry with the usual approximation. We present results in section 5, which aims to be comparatively self-contained. Appendix B contains a brief discussion on the  $\Delta L = 2$  terms, and Appendix C relates the asymmetries in  $B/3 - L_i$  to those in the lepton doublets.

## 2 Notation, Review and the “Standard” Calculation

We consider the Lagrangian for the lepton sector

$$\mathcal{L} = h^{ik} H \bar{e}_{Ri} \ell_k + \lambda^{Jj} H^c \bar{N}_J \ell_j + \frac{M_K}{2} N_K N_K + \text{h.c.} \quad (1)$$

in the mass eigenstate bases of the singlet (“right-handed”) particles  $N$  and  $e_R$ . The mass eigenstate basis for the doublet leptons will be temperature dependent, so we do not fix it here. We assume from now on a hierarchical spectrum for the RH neutrinos,  $M_1 \ll M_2 < M_3$ .

The light neutrino mass matrix, in the charged lepton mass eigenstate basis, is

$$m_\nu = U^* D_{m_\nu} U^\dagger = \lambda^T M^{-1} \lambda v^2, \quad (2)$$

where  $U$  is the PMNS matrix,  $D_A$  is a diagonal matrix of real eigenvalues of  $A$  and  $v$  the Higgs vacuum expectation value.

The physical process of interest is the production of a lepton asymmetry in the decay of the lightest singlet neutrino  $N_1$ , in the thermalised early Universe. We make the usual assumption, that interactions that are much faster than the expansion rate  $H$  — such as gauge interactions,  $(B+L)$ -violation, and (possibly) the  $\tau$ -Yukawa coupling — are in equilibrium, which imposes certain conditions [17] on the distribution functions of particles. For instance [18, 19], when

$$\Gamma_\tau \sim 5 \times 10^{-3} h_\tau^2 T \gg H \quad (3)$$

the chemical potentials of the singlet and doublet  $\tau$ s satisfy the relation  $\mu_H + \mu_{\tau_L} = \mu_{\tau_R}$ . We neglect interactions whose rates are much smaller than  $H$ , *e.g.* those induced by the electron Yukawa coupling. Interactions that are of order the expansion rate, and those that are lepton number violating, are included in the evolution equations for the number density (eventually, Boltzmann equations). These are the  $N_1$ -interactions, the  $\Delta L = 1$  and  $\Delta L = 2$  scatterings, and possibly the interactions with strength set by the  $\mu$ -Yukawa coupling,  $h_\mu$ .

It is straightforward to write down Boltzmann equations for the conserved asymmetries of the Standard Model (SM). The SM interactions which are in equilibrium define the conserved quantum numbers. For instance, below  $T \sim 10^5$  GeV, when all interactions are in equilibrium, the conserved quantum numbers are the  $\{B/3 - L_i\}$  for  $i = e, \mu, \tau$ . The Boltzmann equations are obtained by identifying processes that change the asymmetries, and summing their rates. The only delicate point arises in the temperature range where the interactions induced by  $h_\mu$  come into equilibrium, because this changes the relation between  $Y_{L_\mu}$  and  $Y_{B/3-L_\mu}$ .

The Boltzmann equations for number densities neglect the possibility of quantum effects, such as oscillations, due to interference among different processes. Such effects could appear in the equations of motion for the (flavour-dependent) number operator

$$\hat{f}_\Delta^{ij}(\vec{p}) = a_+^{i\dagger}(\vec{p}) a_+^j(\vec{p}) - a_-^{j\dagger}(\vec{p}) a_-^i(\vec{p}), \quad (4)$$

which counts the asymmetry in lepton doublets<sup>1</sup>. Notice the inverted flavour order between the particle and antiparticle number operators [20]. The operator is a matrix in flavour space, analogous to the density matrix of quantum mechanics, and is sometimes called a density matrix. When the flavour indices  $ij$  are the charged lepton mass eigenstates, then the diagonal elements are the flavour asymmetries stored in the lepton doublets, and should satisfy Boltzmann(-like) Equations. The trace, which is flavour-basis-independent, is the total lepton asymmetry. The off-diagonals encode (quantum) correlations between the different flavour asymmetries, and should decay away when the charged Yukawa couplings are in equilibrium [11]. Variants on such an operator have been studied in the context of neutrino oscillations in the early Universe [20, 21, 22, 23], and in particular the generation of a lepton asymmetry by active sterile oscillations [24, 25]. This operator makes brief appearances in [11] where the issue of flavour in the context of thermal leptogenesis was first discussed.

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<sup>1</sup>  $\hat{f}$  wears a hat as an operator, to distinguish it from its expectation value which we will usually discuss in the Appendix. The operator  $a_+^{i\dagger}$  ( $a_-^{i\dagger}$ ) creates particles (anti-particles) of flavour  $i$ .

In appendix A, we will motivate equations of motion for these flavour-dependent lepton number operators using quantum mechanical oscillators. For simplicity, we separate the (possibly quantum) flavour effects from the (assumed classical) particle dynamics. That is, we extract the flavour structure from the toy model, and input the particle dynamics and the Universe expansion, by analogy with the Boltzmann equations. This is an elegant way to obtain flavour-dependent Boltzmann equations, because the tensor structure of the operators in flavour space is clear. In a later publication, we will derive equations of motion for  $f_{\Delta}^{ij}$  in field theory. Indeed, its dynamics could be obtained from the transport equations deduced from the finite temperature Schwinger-Dyson equations involving flavoured fermionic propagators. In this set-up [26], one makes use of the closed time-path formalism to describe nonequilibrium phenomena in field theory leading to a complete nonequilibrium quantum kinetic theory. The operators  $f_{\Delta}^{ij}$  are associated to propagators which may mix the flavour and which are “sourced” in the Schwinger-Dyson equations by the flavour-dependent CP asymmetry  $\epsilon_{ij}$  from the decay of the right-handed neutrino.

The usual starting point for leptogenesis is a system of coupled equations. If the asymmetry is due to the decay of the lightest RH neutrino  $N_1$ , and the processes included are decays, inverse decays and lepton number violating scattering ( $\Delta L = 1$  from Higgs exchange, and  $\Delta L = 2$  due to RH neutrinos), then these equations are:

$$\frac{dY_{N_1}}{dz} = -\frac{z}{sH(M_1)} (\gamma_D + \gamma_{\Delta L=1}) \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right), \quad (5)$$

$$\frac{dY_L}{dz} = \frac{z}{sH(M_1)} \left[ \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \epsilon \gamma_D - \frac{Y_L}{Y_L^{eq}} (\gamma_D + \gamma_{\Delta L=1} + \gamma_{\Delta L=2}) \right]. \quad (6)$$

$Y_{N_1}$  is the abundance of the RH neutrino  $N_1$  normalized to the entropy density  $s$ , and  $Y_L$  is the sum over flavour of the difference between normalized abundances of leptons and antileptons. In eqs. (5) and (6),  $Y_i^{eq}$  is the equilibrium number density of a particle  $i$ ,  $z = \frac{M_1}{T}$  and the quantities  $\gamma_D, \gamma_{\Delta L=1}$  and  $\gamma_{\Delta L=2}$  are thermally averaged rates of decays, inverse decays, and scattering processes with  $\Delta L = 1, 2$  and include all contributions summed over flavour ( $s, t$  channel interference etc). The explicit expressions are given in the literature (see for example [5]) and we will discuss them when needed further on. The total CP-asymmetry  $\epsilon$  produced in the decay of the lightest RH neutrino is given by [27]<sup>2</sup>

$$\epsilon = \frac{1}{8\pi[\lambda\lambda^\dagger]_{11}} \sum_{j \neq 1} \Im[\lambda\lambda^\dagger]_{1j}^2 f(M_j^2/M_1^2), \quad (7)$$

where  $f$  is the loop factor [27], given by

$$f(x) = \sqrt{x} \left[ 1 - (1+x) \ln \frac{1+x}{x} + \frac{1}{1-x} \right]. \quad (8)$$

We recall that the rates  $\gamma_D$ <sup>3</sup> and  $\gamma_{\Delta L=1}$  are both proportional to the tree-level decay rate given by

$$\Gamma = \frac{(\lambda\lambda^\dagger)_{11}}{8\pi} M_1 = \frac{\tilde{m}_1 M_1^2}{8\pi v^2}, \quad (9)$$

and the out of equilibrium condition is satisfied when:

$$K \equiv \frac{\Gamma}{H} \Big|_{T=M_1} = \left( \frac{\tilde{m}_1}{\tilde{m}_*} \right) < 1. \quad (10)$$

<sup>2</sup>We exploit this expression in the next section.

<sup>3</sup>For a later use, we give the expression of the reaction density  $\gamma_D = n_{N_1}^{eq} \frac{K_1(z)}{K_2(z)} \Gamma$ , where  $K_n(z)$  are the  $K$  type Bessel functions and  $n_{N_1}^{eq}$  is the equilibrium density.

In other words, comparing  $\tilde{m}_1$  to  $\tilde{m}_* \sim 2.3 \times 10^{-3}$  eV gives a measure of whether the decay is occurring out of equilibrium or not. The rescaled decay rate  $\tilde{m}_1$  is bounded below:  $m_1 \leq \tilde{m}_1$ , and naturally of order  $\sqrt{\Delta m_{atm}^2}$  when the light neutrino spectrum is hierarchical [28].

The term  $\gamma_{\Delta L=2}$ <sup>4</sup> has a more complicated dependence on the masses of the neutrinos, and can be treated in two separate regimes. The contribution in the temperature range from  $z = 1$  to  $z \simeq 10$  is proportional to  $\tilde{m}_1$  while for the range of temperature ( $z \gg 1$ ), the dominant contribution at leading order is proportional to  $\bar{m}^2 \equiv \text{Tr}(m_\nu^\dagger m_\nu) = \sum_i m_i^2$ . Here  $m_i$  denotes the mass eigenvalues of the left-handed neutrinos.

### 3 Flavour Oscillations

In this section we consider thermal leptogenesis for two flavours, which to be explicit, we label to be the  $e$ - and the  $\mu$  leptons. This is interesting in two cases. First, if leptogenesis takes place when  $\Gamma_\mu \ll H$  (see eq. (3)), it will be described by Boltzmann equations for the  $\tau$  and  $\ell'_1$  number densities, where  $\vec{\ell}'_1 = \vec{\ell}_1 - (\vec{\ell}_1 \cdot \hat{\tau})\hat{\tau}$  and  $(\ell_1)_i = \lambda_{1i}$  is the decay direction, in lepton flavour space, of  $N_1$ . So the equations of this section could be applied in this high temperature case, with the relabelling  $\mu \rightarrow \tau$ ,  $e \rightarrow \ell'_1$ . Secondly, if the muon Yukawa coupling comes into equilibrium as the asymmetry is being created, there could be oscillations of the asymmetry, parametrised in off-diagonal elements of the asymmetry density. Focusing on this quantum effect, we neglect the third lepton generation, because we suppose we are in the interesting range of temperatures  $\lesssim 10^{12}$  GeV where the interactions induced by the  $\tau$ -Yukawa coupling are in chemical equilibrium. Therefore no quantum correlations between the lepton asymmetries involving the third family are expected to survive. Adding the third family will be discussed in the next section.

The equations describing the two-flavour system have been obtained in appendix A, to which we refer the reader for more details. To clearly see the effect on flavour oscillations due to the muon Yukawa coupling coming into equilibrium, we map eq. (46) onto two-flavour equations for the asymmetry in the early Universe, neglecting the transformation to  $(B/3 - L_i)$ . As discussed after eq. (46), these equations describe decays and inverse decays of  $N_1$ , and the resonant part of  $\Delta L = 2$  scattering is also included. We work in the charged lepton mass eigenstate basis, and distribute indices by analogy with the model of the appendix ( $f_{\Delta\ell}^{ij} \leftrightarrow Y_L^{ij}$ , and  $\lambda^i \lambda^{*j} \leftrightarrow \gamma^{ij}$ ).

We assume, to obtain these equations, that the flavour-blind gauge interactions can be neglected. However, the oscillation frequency in flavour space depends on the energy of the lepton, and within the oscillation timescale, a lepton will participate in many energy-changing gauge interactions<sup>5</sup>. In this note, we use the thermally averaged energy  $\langle E \rangle$  to estimate the oscillation frequency [24]. That is, we approximate the integral ( $i \int E d\tau$ ) along the path from one lepton-number violating interaction to the next, to be  $i \langle E \rangle \int d\tau$ . We will include the gauge interactions more correctly in a later analysis [16].

The system describing the two flavours is given by

$$\begin{aligned} \frac{d}{dz} \begin{bmatrix} Y_L^{ee} & Y_L^{e\mu} \\ Y_L^{\mu e} & Y_L^{\mu\mu} \end{bmatrix} &= \frac{z}{sH(M_1)} \left( \gamma_D \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \begin{bmatrix} \epsilon^{ee} & \epsilon^{e\mu} \\ \epsilon^{\mu e} & \epsilon^{\mu\mu} \end{bmatrix} - \frac{1}{2Y_L^{eq}} \left\{ \begin{bmatrix} \gamma_D^{ee} & \gamma_D^{e\mu} \\ \gamma_D^{\mu e} & \gamma_D^{\mu\mu} \end{bmatrix}, \begin{bmatrix} Y_L^{ee} & Y_L^{e\mu} \\ Y_L^{\mu e} & Y_L^{\mu\mu} \end{bmatrix} \right\} \right) \\ &\quad - i \left[ [\Lambda_\omega], \begin{bmatrix} Y_L^{ee} & Y_L^{e\mu} \\ Y_L^{\mu e} & Y_L^{\mu\mu} \end{bmatrix} \right] - \frac{1}{2} \left\{ [\Lambda_d], \begin{bmatrix} Y_L^{ee} & Y_L^{e\mu} \\ Y_L^{\mu e} & Y_L^{\mu\mu} \end{bmatrix} \right\} + [\Lambda_d] Y_L^{\mu\mu} \end{aligned} \quad (11)$$

<sup>4</sup>We emphasize that  $-\frac{1}{8}\gamma_{D_j}$  corresponding to RIS (real intermediate states) in the  $\Delta L = 2$  interactions have to be carefully subtracted, to avoid double counting in the Boltzmann equations [5].

<sup>5</sup>We thank A. Strumia for discussions about this point.

where the parenthesis  $[\cdot, \cdot]$  and  $\{\cdot, \cdot\}$  stand for commutators and anti-commutators, respectively. The matrix  $[\gamma_D]$  is defined as

$$\gamma_D^{ij} = \gamma_D \frac{\lambda_{1i} \lambda_{1j}^*}{\sum_k |\lambda_{1k}|^2}, \quad (12)$$

where  $\gamma_D = \sum_i \gamma_D^{ii}$  is the total (thermally averaged) decay rate. For  $i = j$ ,  $\gamma_D^{ii}$  is the decay rate of  $N_1$  to the  $i$ -th flavour. The CP asymmetry in the  $i$ -th flavour is  $\epsilon^{ii}$ ,  $\epsilon = \sum_i \epsilon^{ii}$ , and notice that  $\epsilon^{ij}$  is normalised by the total decay rate (so that  $[\epsilon]$  transforms as a tensor under  $\ell$  basis rotations). The matrix  $[\epsilon]$  is now defined as

$$\begin{aligned} \epsilon_{ij} &= \frac{1}{(16\pi)} \frac{1}{[\lambda\lambda^\dagger]_{11}} \sum_J \Im \left\{ (\lambda_{1i}) [\lambda\lambda^\dagger]_{1J} \lambda_{Jj}^* - (\lambda_{1j}^*) [\lambda^* \lambda^T]_{1J} \lambda_{Ji} \right\} \left[ f \left( \frac{M_J^2}{M_1^2} \right) \right] \\ &\simeq \frac{3}{(16\pi v^2)} \frac{M_1}{[\lambda\lambda^\dagger]_{11}} \text{Im} \{ \lambda_i \lambda_\sigma [m_\nu^*]_{\sigma j} - \lambda_\sigma^* [m_\nu]_{\sigma i} \lambda_j^* \} \quad (\text{for } M_1 \ll M_2, M_3) \end{aligned} \quad (13)$$

where  $f(x)$  is the loop function eq. (8).

The matrix  $\Lambda$  is given by

$$\Lambda = \begin{bmatrix} \Lambda^{ee} & 0 \\ 0 & \Lambda^{\mu\mu} \end{bmatrix}, \quad (14)$$

where  $\Lambda^{ee}$  and  $\Lambda^{\mu\mu}$  indicate the *complex* thermal masses due to the electron and muon Yukawa couplings:

$$\Lambda^{ii} \equiv \Lambda_\omega^{ii} - i \Lambda_d^{ii} = \frac{\omega_{ii} - i \Gamma_{ii}}{H(M_1)} \Big|_{T=M_1}, \quad i = e, \mu. \quad (15)$$

These will cause the decaying flavour oscillations of the asymmetries. Since the interactions mediated by the electron Yukawa coupling are out-of-equilibrium, we may safely set  $\Lambda^{ee} = 0$ . On the other hand [19]

$$\omega_{\mu\mu} \simeq \frac{h_\mu^2}{16} T, \quad \Gamma_{\mu\mu} \simeq 5 \times 10^{-3} h_\mu^2 T, \quad (16)$$

leading to

$$\Lambda_d^{\mu\mu} \simeq 3 \times 10^{-4} h_\mu^2 \frac{M_P}{M_1}, \quad (17)$$

where  $M_P$  is the Planck mass. Interactions are mediated by the muon Yukawa coupling only for  $z \gtrsim z_d \simeq 1/\Lambda_d^{\mu\mu}$ . Therefore, as thermal leptogenesis takes place at temperatures on the order of  $M_1$ , we conclude that – roughly – interactions involving the muon Yukawa coupling are out-of-equilibrium in the primeval plasma if  $M_1 \gtrsim 10^9$  GeV.

Following Stodolsky [21], we may now write eq. (11) in a more compact form by expanding the various matrices on the basis provided by the  $\sigma$ -matrices,  $\sigma^\mu = (I, \vec{\sigma})$ . A generic  $2 \times 2$  matrix  $A$  can be written as

$$A = A^\mu \sigma_\mu, \quad A^\mu = \frac{1}{2} \text{Tr} \{ A \sigma^\mu \}. \quad (18)$$

In this basis, the Boltzmann equation (11) becomes a system of equations of the form

$$\begin{aligned} \frac{dY_L^0}{dz} &= \frac{z}{sH(M_1)} \left( \gamma_D \left( \frac{Y_{N_1}}{Y^{eq}} - 1 \right) \epsilon^0 - \frac{1}{Y_L^{eq}} \gamma_D^0 Y_L^0 - \frac{1}{Y_L^{eq}} \vec{\gamma}_D \cdot \vec{Y}_L \right), \\ \frac{d\vec{Y}_L}{dz} &= \frac{z}{sH(M_1)} \left( \gamma_D \left( \frac{Y_{N_1}}{Y^{eq}} - 1 \right) \vec{\epsilon} - \vec{\gamma}_D \frac{Y_L^0}{Y_L^{eq}} - \frac{1}{Y_L^{eq}} \gamma_D^0 \vec{Y}_L \right) + \vec{\Lambda}_\omega \times \vec{Y}_L + \hat{\Lambda}_d \times \vec{\Lambda}_d \times \vec{Y}_L. \end{aligned} \quad (19)$$

Notice, in particular, that  $Y_L^0 = (1/2)\text{Tr} Y_L$  and therefore it represents half of the total lepton asymmetry. On the other hand,  $Y_L^z = (1/2)(Y_{ee} - Y_{\mu\mu})$  indicates half of the difference of the asymmetries in the electron and muon flavour densities. The components  $Y_L^x$  and  $Y_L^y$  parametrize the off-diagonal entries of the asymmetries and therefore account for the quantum correlations between the lepton asymmetries. As the matrix  $\Lambda$  in eq. (14) is diagonal, only the  $\Lambda^0$  and  $\Lambda^z$  components are different from zero.

A simple inspection of eqs. (19) tells us that the components  $Y_L^x$  and  $Y_L^y$  precess around the  $z$ -direction with an angular velocity set by the thermal mass  $\omega_{\mu\mu}$ . At the same time, such a precession is damped by the interactions mediated by the muon-Yukawa coupling at a rate  $\sim \Gamma_{\mu\mu}$ . Notice that  $\omega_{\mu\mu} \sim \Gamma_{\mu\mu}$ , so unlike the case of neutrino oscillations in matter (MSW), the decoherence and oscillation timescales are the same. The term  $\vec{\Lambda} \times \vec{Y}_L$  contains all the information about the action of the decoherent plasma onto the coherence of the flavour oscillations: if the damping rate  $\Gamma_{\mu\mu}$  is much larger than the expansion rate of the Universe, the quantum correlations among the flavours asymmetries are quickly damped away. Therefore, we expect that, if leptogenesis takes place at values of  $z \gg z_d$ , quantum correlations play no role in the dynamics of leptogenesis [11].

If thermal leptogenesis occurs well before the muon-Yukawa coupling enters into equilibrium (that is at  $z \ll z_d$ ,  $T \sim M_1 \gg 10^9$  GeV), then  $|\vec{\Lambda}|z \ll 1$  and the last term of the second equation of (19) can be safely dropped. In the absence of the charged lepton Yukawa coupling, we may rotate the system in such a way to put all the asymmetry generated by the decay of the right-handed neutrino into a single flavour, *e.g.*  $Y^{ee}$ . Under these circumstances,  $Y_L^0 = Y_L^z = (Y^{ee}/2)$ ,  $Y_L^x = Y_L^y = 0$  and similarly for the CP-asymmetries,  $\epsilon^0 = \epsilon^z = (\epsilon^{ee}/2)$ ,  $\epsilon^x = \epsilon^y = 0$ . By simply summing the corresponding equations for  $Y_L^0$  and  $Y_L^z$ , one immediately finds the Boltzmann equation for the single flavour case.

To get some insight about the dynamics of the system, let us now discuss the solution of eqs. (19) in the case in which thermal leptogenesis takes place nearly in equilibrium which, given the low-energy neutrino parameters, is likely to be the one realized in Nature. We parametrise the inverse decays with rates  $\gamma_D^{ij} = n_N^{eq}(K_1(z)/K_2(z))\Gamma^{ij}$  through the parameters (defined in eq. (10))  $K^{ij} \equiv (\Gamma^{ij}/H)_{T=M_1}$  and work in the limit in which every  $K^{ij} \gtrsim 1$ . By defining the combinations  $Y_L^\pm = (Y_L^x \pm iY_L^y)/\sqrt{2}$  and by exploiting various saddle-point approximations, we find

$$\begin{aligned}
Y_L^0 &\simeq \frac{\epsilon^0}{g_*} \sqrt{\frac{\pi}{2}} \frac{1}{K^0 z_f} - \frac{1}{K^0} (K^z Y_L^z + K^+ Y_L^+ + K^- Y_L^-) (z_f), \\
Y_L^z &\simeq \frac{\epsilon^z}{g_*} \sqrt{\frac{\pi}{2}} \frac{1}{K^0 z_f} - \frac{K^z}{K^0} Y_L^0 (z_f), \\
Y_L^\pm &\simeq \frac{\epsilon^\pm}{g_*} \sqrt{\frac{\pi}{2}} z_d^{3/2} e^{-z_d} \cos \left[ \frac{\Lambda_\omega^{\mu\mu}}{2} (z_f^2 - z_d^2) \right] e^{-\frac{1}{2}\Lambda_d^{\mu\mu} (z_f^2 - z_d^2)} \\
&\quad - K^\pm Y_L^0 (z_d) z_d^{5/2} e^{-z_d} \cos \left[ \frac{\Lambda_\omega^{\mu\mu}}{2} (z_f^2 - z_d^2) \right] e^{-\frac{1}{2}\Lambda_d^{\mu\mu} (z_f^2 - z_d^2)}, \tag{20}
\end{aligned}$$

where  $z_f \simeq \ln K^0 + (5/2)\ln \ln K^0$ . These solutions have been obtained in the limit in which  $z_d \lesssim z_f$ , that is when the muon-Yukawa coupling induces interactions that are in thermal equilibrium at temperatures larger than the effective temperature at which inverse decays go out-of-equilibrium. In the limit  $z_d \ll z_f$ , eqs. (20) clearly show the decoupling of the quantum correlations from the dynamics in the limit in which muon Yukawa coupling induced interactions have come into equilibrium long before thermal leptogenesis,  $\Lambda^{\mu\mu} z_f \gg 1$ . In this limit, we find

$$\text{Tr} Y_L = 2 Y_L^0 \simeq \sqrt{\frac{\pi}{2}} \frac{1}{g_* z_f} \left( \frac{K^0 \epsilon^0 - K^z \epsilon^z}{(K^0)^2 - (K^z)^2} \right) = \sqrt{\frac{\pi}{2}} \frac{1}{g_* z_f} \left( \frac{\epsilon^{11}}{K^{11}} + \frac{\epsilon^{22}}{K^{22}} \right). \tag{21}$$

As a last comment, let us notice that in the limit  $z_f \simeq z_d$ , the quantum correlations are not efficiently damped out, and appear as oscillations in the washout terms of eq. (19). We expect that the effect on the final total lepton asymmetry,

$$Y_L^0 \propto \frac{K^0 \epsilon^0 - \vec{K} \cdot \vec{\epsilon}}{(K^0)^2 - |\vec{K}|^2}, \quad (22)$$

will be analogous to multiplying the washout rate by a neutrino survival probability, and integrating over time (an  $\mathcal{O}(1)$  effect). This case deserves further investigation and a detailed numerical study [16].

## 4 Flavour in the Boltzmann Equations

In this section, we consider Boltzmann equations (BE) for the diagonal elements of  $[Y_L]$  with 3 flavours. As discussed in the previous section and in [11], BE are appropriate when the interactions of the charged lepton Yukawa couplings are much faster, or much slower, than the expansion  $H$ —provided one works in the physical basis. To be explicit, we work in the charged lepton mass eigenstate basis, so these equations apply when the  $\tau$  and  $\mu$  Yukawa couplings are in equilibrium (see eq. (3)), and have caused the off-diagonal elements of  $[Y_L]$  to be irrelevant. This corresponds to  $T \lesssim 10^9$  GeV for the SM (and to  $T \lesssim 10^9 \times \tan^2 \beta$ , where  $\tan \beta$  is the ratio of the vacuum expectation values of the two Higgs bosons, in the supersymmetric version of the SM).

Let us start out by considering the Boltzmann equation as a matrix equation in flavour space, with only decays and inverse decays, as we did in the previous section, and neglecting<sup>6</sup> the transformation to  $B/3 - L_i$

$$\frac{sH(M_1)}{z} \frac{d}{dz} \begin{bmatrix} Y_L^{ee} & & \\ & Y_L^{\mu\mu} & \\ & & Y_L^{\tau\tau} \end{bmatrix} = \gamma_D \begin{pmatrix} \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \end{pmatrix} \begin{bmatrix} \epsilon^{ee} & & \\ & \epsilon^{\mu\mu} & \\ & & \epsilon^{\tau\tau} \end{bmatrix} - \frac{1}{2Y_L^{eq}} \left\{ \begin{bmatrix} \gamma_D^{ee} & & \\ & \gamma_D^{\mu\mu} & \\ & & \gamma_D^{\tau\tau} \end{bmatrix}, \begin{bmatrix} Y_L^{ee} & & \\ & Y_L^{\mu\mu} & \\ & & Y_L^{\tau\tau} \end{bmatrix} \right\} \quad (23)$$

where  $\epsilon^{ii}$  and  $\gamma_D^{ii}$  are defined as in eqs. (12, 13). Since the off-diagonal elements of the matrices have been neglected, this equation is not invariant under flavour rotations, and only applies in the physical basis where the off-diagonals are suppressed.

We can illustrate the problem that arises when one sums over flavour by taking the trace of eq. (23). The BE for the sum is

$$\sum_i \frac{dY_L^{ii}}{dz} = \frac{z}{sH} \left( \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \epsilon \gamma_D - \sum_i \gamma_D^{ii} \frac{Y_L^{ii}}{Y_L^{eq}} \right) \quad (24)$$

where  $\sum_i \epsilon^{ii} = \epsilon$  is the total CP asymmetry. Like  $\gamma_D$  it is the trace of a matrix in flavour space, see eq. (12).

In the “single flavour or dominant state approximation” (see eq. (6)), the trace of the product of matrices in the last term on the RHS of eq. (24) is approximated as the product of the traces: this term is replaced by  $\sum_j Y_L^{jj} \sum_i \gamma_D^{ii}$ . This is correct in the absence of charged lepton Yukawa couplings;

<sup>6</sup>Including this transformation would mildly reduce the washout term, because, *e.g.*, an  $\ell_\mu$  produced is a unit of  $B/3 - L_\mu$ , but not all the  $B/3 - L_\mu$  is in  $\ell_\mu$ s. In appendix C we give the explicit transformation from  $L_i$  to  $B/3 - L_i$  for different cases.



if  $\hat{\ell}$  is the flavour combination into which  $N_1$  decays, then there is a basis where  $[\gamma_D] = \text{diag}(\gamma_D, 0, 0)$  and  $[Y_L]_{11} = Y_L$ . The trace is invariant under basis transformations in flavour space, so for any unitary  $V$

$$\frac{1}{2}\text{Tr}\{V[\gamma_D]V^\dagger, V[Y_L]V^\dagger\} = \gamma_D Y_L \quad (\text{no charged lepton Yukawas}). \quad (25)$$

In particular, this is the washout term evaluated in the charged lepton mass eigenstate basis, see eq. (6).

If the charged lepton Yukawa couplings are included, then new terms appear in the equations for the asymmetry [11], as discussed in the previous section. These new terms suppress the off-diagonals of  $[Y_L]$  in the charged lepton mass eigenstate basis, so the Boltzmann equations (without the off-diagonals) are only appropriate in the charged lepton mass eigenstate basis, and in general it is no longer true that  $\text{Tr}([\gamma_D][Y_L]) = \text{Tr}[\gamma_D] \text{Tr}[Y_L]$ .

One way to see what is being neglected, if the single flavour approximation is used when the charged lepton Yukawa couplings are in equilibrium, is to expand the diagonal matrices  $[Y_L]$ ,  $[\epsilon]$  and  $[\gamma_D]$  on the identity matrix  $I$  and the diagonal SU(3) generators  $\Lambda_3 = \text{Diag}\{1, -1, 0\}$  and  $\Lambda_8 = \frac{1}{\sqrt{3}}\text{Diag}\{1, 1, -2\}$ . For instance,

$$[Y_L] = \frac{Y_L}{3}I + Y_3\Lambda_3 + Y_8\Lambda_8 \quad (26)$$

where  $Y_3 = \frac{1}{2}\text{Tr}(Y\Lambda_3)$  is the flavour difference  $(Y^{ee} - Y^{\mu\mu})/2$ . Taking the trace of eq. (23), we obtain the BE for the total asymmetry  $Y_L$ :

$$\frac{dY_L}{dz} = \frac{z}{sH} \left( \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \gamma_D \epsilon - \left( \frac{1}{3} \gamma_D \frac{Y_L}{Y_L^{eq}} + 2\gamma_{D3} \frac{Y_3}{Y_L^{eq}} + 2\gamma_{D8} \frac{Y_8}{Y_L^{eq}} \right) \right). \quad (27)$$

In the absence of  $\Delta L = 2$  interactions, eq. (23) consists of three decoupled equations, so the final asymmetry can be obtained easily by solving for each flavour, then adding the solutions. However, we can also obtain Boltzmann equations for the asymmetries  $Y_3$  and  $Y_8$ , by multiplying eq. (23) by  $\Lambda_3$  (or  $\Lambda_8$ ), and then taking the trace:

$$\frac{dY_3}{dz} = \frac{z}{sH} \left( \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \gamma_D \epsilon_3 - \left( \frac{\gamma_D}{3} \frac{Y_3}{Y_L^{eq}} + \frac{\gamma_{D3}}{3} \frac{Y_L}{Y_L^{eq}} + \frac{\gamma_{D3}}{\sqrt{3}} \frac{Y_8}{Y_L^{eq}} + \frac{\gamma_{D8}}{\sqrt{3}} \frac{Y_3}{Y_L^{eq}} \right) \right), \quad (28)$$

$$\frac{dY_8}{dz} = \frac{z}{sH} \left( \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \gamma_D \epsilon_8 - \left( \frac{\gamma_D}{3} \frac{Y_8}{Y_L^{eq}} + \frac{\gamma_{D8}}{3} \frac{Y_L}{Y_L^{eq}} + \frac{\gamma_{D3}}{\sqrt{3}} \frac{Y_3}{Y_L^{eq}} - \frac{\gamma_{D8}}{\sqrt{3}} \frac{Y_8}{Y_L^{eq}} \right) \right). \quad (29)$$

The factor of 1/3, multiplying  $\gamma_D Y_L$  in eq. (27) is interesting. One can see from [29] that in many models,  $N_1$  decays about equally to all flavours, and produces about equal asymmetry in all flavours. One can therefore neglect the terms  $\gamma_{D3} Y_3 + \gamma_{D8} Y_8$  in eqs. (27) and (29). Thus, leptogenesis is described by the usual equation, with the washout reduced by a factor of 1/3. This should not be surprising: 1/3 of the asymmetry is in each flavour, and for each flavour, the (inverse) decay rate is 1/3 of the total. Summing over flavour,  $3 \times 1/3 \times 1/3 = 1/3$ . We will study leptogenesis in the context of these flavoured Boltzmann equations in a subsequent publication [16].

## 5 Implications of flavours

In this section, we study the modifications due to flavour of the leptogenesis bounds on  $T_{\text{RH}}$  and the light neutrino mass scale  $\bar{m}$ . In the ‘‘single flavour approximation’’, an  $N_1$ -parameter-independent bound on the light neutrino mass scale, of  $\frac{\bar{m}}{\sqrt{3}} \lesssim 0.15$  eV, can be obtained [9, 10]. We will argue below that this bound does not hold when flavour effects are included. The bound  $m_\nu \lesssim 4\sqrt{10^{10}\text{GeV}/T_{B-L}}$

eV [30], from requiring  $\Delta L = 2$  processes to not wash out the asymmetry, would apply, but remains a function of the (unknown) temperature  $T_{B-L}$  at which leptogenesis takes place.

The limit  $m_\nu \lesssim 0.15$  eV, can be understood to arise from the lower bound on the total decay rate  $m_1 \leq \tilde{m}_1$ , and the upper bound on the total CP asymmetry [7, 8]:

$$\epsilon \leq \frac{3M_1 \Delta m_{atm}^2}{8\pi v^2 m_3}, \quad (30)$$

where  $m_3$  is the largest neutrino mass, and  $m_1$  the smallest. We assume the light neutrinos are degenerate, so  $|m_1| \simeq |m_2| \simeq |m_3| \equiv \tilde{m}/\sqrt{3}$  — but the masses can have different Majorana phases. Leptogenesis takes place in the strong washout regime, due to the lower bound on the total decay rate. The final baryon asymmetry can be roughly approximated as  $\eta_B \sim 10^{-3}\epsilon/K \propto \Delta m_{atm}^2/\tilde{m}^2$ . As the light neutrino mass scale is increased,  $M_1$  and the temperature of leptogenesis must increase to compensate the  $\Delta m_{atm}^2/\tilde{m}^2$  suppression. However, this temperature is bounded from above, from the requirement of having the  $\Delta L = 2$  processes out of equilibrium when leptogenesis takes place:

$$\frac{\tilde{m}^2 T^3}{12\pi v^4} \lesssim \frac{10T^2}{M_{\text{P}}}, \quad (31)$$

so  $M_1 \lesssim 10^{10}(\text{eV}/\tilde{m})^2$  GeV. There is therefore an upper bound on the baryon asymmetry which scales as  $1/\tilde{m}^4$ , and with our rough estimates, one finds  $\tilde{m}/\sqrt{3} \lesssim 0.1$  eV.

The individual flavour asymmetries,  $\epsilon_{ii}$  do not satisfy the bound of eq. (30). This can be guessed from the “Jarlskog invariant” for leptogenesis [31]<sup>7</sup>, which is the trace:

$$\begin{aligned} I_1 &= \Im\{\text{Tr}[m_\nu m_\nu^\dagger (\lambda^T \lambda^*)^{-1} m_\nu (\lambda^\dagger \lambda)^{-1} m_\nu^\dagger]\} \\ &= \Im\left\{\sum_i [U^* D_{m_\nu}^2 W^T D_\lambda^{-2} W^* D_{m_\nu} W^\dagger D_\lambda^{-2} W D_{m_\nu} U^T]_{ii}\right\}, \end{aligned} \quad (32)$$

where the unitary matrix  $W$  transforms from the basis where  $m_\nu$  is diagonal to the one where  $\lambda^\dagger \lambda$  is diagonal (*e.g.* in the  $D_\lambda$  basis,  $m_\nu = W^* D_{m_\nu} W^\dagger$ ). The total asymmetry  $\epsilon$  is proportional to this trace, which vanishes when the light neutrinos are exactly degenerate in magnitude. However, if the invariant is “cut open” on lepton flavour indices, one can show that the flavour asymmetries satisfy

$$|\epsilon_{ii}| \leq \frac{\sqrt{3}M_1 \tilde{m}}{8\pi v^2}, \quad (33)$$

and only the sum vanishes. So if we can arrange  $-\epsilon_{\mu\mu} = \epsilon_{\tau\tau} \sim \sqrt{3}M_1 \tilde{m}/(8\pi v^2)$ , and decay rates  $\gamma_D^{\mu\mu} = s^2 \gamma_D$ ,  $\gamma_D^{\tau\tau} = c^2 \gamma_D$ , then in the approximation of strong washout and neglecting  $\Delta L = 2$  processes, eq. (21) gives the total lepton asymmetry to be

$$Y_L \lesssim \frac{1}{g_* z_f} \frac{3M_1 \tilde{m}_* c^4 - s^4}{8\pi v^2 c^2 s^2}. \quad (34)$$

So we obtain that the baryon asymmetry is independent of the light neutrino mass scale, under the assumption that  $\Delta L = 2$  processes are negligible. In passing<sup>8</sup>, one can notice that  $Y_L$  increases, when one of the partial decay widths decreases, provided that it remains in the strong washout regime:  $s^2, c^2 \gg \tilde{m}_*/\tilde{m}$ .

<sup>7</sup>This invariant is only applicable when the right-handed neutrinos  $N_I$  are hierarchical.

<sup>8</sup>We will study the final asymmetry as a function of similar angles in a subsequent publication [16].

We first show that flavour asymmetries are constrained by eq. (33), when light neutrinos are degenerate at mass  $\bar{m}/\sqrt{3}$ , as opposed to the stronger bound on the total asymmetry of eq. (30). For hierarchical  $N_J$ , with exactly degenerate light neutrinos

$$\epsilon_{ii} = \frac{3}{8\pi v^2} \frac{M_1}{|\vec{\lambda}|^2} \text{Im}\{\lambda_i(\vec{\lambda} \cdot m_\nu^*)_i\} = \frac{\sqrt{3}M_1\bar{m}}{8\pi v^2} \frac{\text{Im}\{\lambda_i(\vec{\lambda} \cdot UU^T)_i\}}{|\vec{\lambda}|^2} \leq \frac{\sqrt{3}M_1\bar{m}}{8\pi v^2}, \quad (35)$$

where there is no sum on  $i$ ,  $(\vec{\lambda})_i = \lambda_{1i}$ , and  $m_\nu^* = UD_m U^T = \frac{\bar{m}}{\sqrt{3}}UU^T$ .

We can verify that a large neutrino mass scale is consistent with leptogenesis by explicit construction of a model. It is convenient to use the Casas-Ibarra [32] parametrisation, in terms of  $m_i$ ,  $M_J$ ,  $U$  and the complex orthogonal matrix  $R \equiv vD_M^{-1/2}\lambda UD_m^{-1/2}$ . This ensures we obtain the correct low-energy parameters. We take a two-flavour model, and write  $R$  as a rotation through the complex angle  $\theta = \varphi + i\eta$ . The flavour asymmetries, for maximal atmospheric mixing and no phase in  $U$ , are

$$\begin{aligned} \epsilon_{\mu\mu} &= \frac{\sqrt{3}M_1\bar{m}}{8\pi v^2} \frac{1}{\sum_\alpha |\sum_j R_{1j}U_{\alpha j}^*|^2} \Im\{R_{1k}U_{\mu k}^*R_{1p}U_{\mu p}\} = -\frac{\sqrt{3}M_1\bar{m}}{8\pi v^2} \frac{\sinh\eta \cosh\eta}{\sinh^2\eta + \cosh^2\eta} \cos 2\varphi, \\ \epsilon_{\tau\tau} &= \frac{\sqrt{3}M_1\bar{m}}{8\pi v^2} \frac{1}{\sum_\alpha |\sum_j R_{1j}U_{\alpha j}^*|^2} \Im\{R_{1k}U_{\tau k}^*R_{1p}U_{\tau p}\} = \frac{\sqrt{3}M_1\bar{m}}{8\pi v^2} \frac{\sinh\eta \cosh\eta}{\sinh^2\eta + \cosh^2\eta} \cos 2\varphi. \end{aligned} \quad (36)$$

The flavour dependent decay rates  $\gamma_{\mu\mu}$  and  $\gamma_{\tau\tau}$  are proportional to

$$\begin{aligned} |\lambda_{1\mu}|^2 &= \frac{M_1\bar{m}}{2v^2\sqrt{3}} |R_{11} + R_{12}|^2 = \frac{M_1\bar{m}}{2v^2\sqrt{3}} (\sinh^2\eta + \cosh^2\eta + \sin 2\varphi), \\ |\lambda_{1\tau}|^2 &= \frac{M_1\bar{m}}{2v^2\sqrt{3}} |R_{11} - R_{12}|^2 = \frac{M_1\bar{m}}{2v^2\sqrt{3}} (\sinh^2\eta + \cosh^2\eta - \sin 2\varphi). \end{aligned} \quad (37)$$

We see that we can choose  $\eta$  and  $\varphi$  as required to obtain eq. (34) with  $c^4 - s^4 \sim s^2c^2$ . We conclude that, if  $M_1 \lesssim 10^{12}$  GeV and flavour effects are relevant, the absolute bound on the light neutrino masses is the one inferred in ref. [30],  $m_\nu \lesssim 4\sqrt{10^{10}\text{GeV}/T_{B-L}}$  eV. Of course, if  $M_1 \gtrsim 10^{12}$  GeV, flavour effects are irrelevant and we are back to the single flavour case where the bound  $\frac{\bar{m}}{\sqrt{3}} \lesssim 0.15$  eV applies<sup>9</sup>.

The minimum required reheat temperature of the Universe,  $T_{\text{RH}}$ , will depend on the flavour structure of the lepton asymmetry. For thermal leptogenesis to take place, an adequate number density of  $N_1$  must be produced by thermal scattering in the plasma, suggesting  $T_{\text{RH}} \gtrsim M_1/10$  [8, 5]. There is a lower bound on  $M_1$ , from requiring a large enough CP asymmetry; the corresponding bound on  $T_{\text{RH}}$  depends on the fine details of reheating, whose dynamics have been addressed in ref. [5]. The lower bound of [5],  $T_{\text{RH}} \gtrsim (2-3) \times 10^9$  GeV, was obtained with hierarchical light neutrinos and strong washout. We include flavour effects by rescaling this bound.

In many popular seesaw models, the light neutrinos are hierarchical, leptogenesis takes place in the strong washout regime  $K > 1$ , and the asymmetries and decay rates are similar for all flavours:  $\gamma_D^{ii} \sim \gamma_D^{jj}$  and  $\epsilon^{ii} \sim \epsilon^{jj}$ . In this case,  $\epsilon$  is bounded by (30), but the washout is reduced by the factor of 1/3 from equation (27), so we estimate that

$$T_{\text{RH}} \gtrsim 6 \times 10^8 \text{GeV} . \quad (38)$$

Thermal leptogenesis could work with a lower reheat temperature if the washout rate in one flavour is suppressed. If  $K > 1$ , a thermal distribution of  $N_1$ s will be created, but the asymmetry in flavour  $j$  is only washed out by  $\gamma_D^{jj}$ , so  $^{10} Y_L^{jj} \sim 10^{-3} \epsilon^{jj}/K^{jj}$ . For instance, in our ‘‘degenerate’’ two

<sup>9</sup>We thank E. Nardi, Y. Nir and E. Roulet for discussions about this point.

<sup>10</sup>This formula is valid in the strong washout regime, so requires  $K^{jj} = \gamma^{jj}K/\gamma_D > 1$ .

generation model, we can tune the couplings<sup>11</sup> so that  $\lambda_{1\mu}/\lambda_{1\tau} \simeq s$  is of order  $\sqrt{\bar{m}_*/\bar{m}}$  where  $\bar{m} \sim \text{eV}$ , in a generous interpretation of the cosmological bound [33]. This suppresses  $\epsilon^{\mu\mu}$  with respect to eq. (34) by  $s$ , so the final asymmetry  $Y_L \sim Y_L^{\mu\mu}$  scales as  $1/s$  and we obtain

$$T_{\text{RH}} \gtrsim 2s \times 10^9 \text{ GeV} \simeq 10^8 \text{ GeV} \quad \text{for tuned, degenerate light neutrinos.} \quad (39)$$

This appears theoretically unmotivated, compared to other leptogenesis scenarios at low  $T_{\text{RH}}$  [15, 34].

Our results can therefore relax the tension between thermal leptogenesis and the gravitino bound [35] in supersymmetric extensions of the SM which impose an upper bound on  $T_{\text{RH}}$  of about  $10^9 \text{ GeV}$ .

## 6 Summary

Including flavour in the analysis of leptogenesis modifies the equations of motion (Boltzmann Equations) for the asymmetry, and changes some of the relations between parameters of the calculation.

The interactions related to the charged Yukawa couplings may induce quantum oscillations amongst the lepton asymmetries. This interesting result, in combination with the temperature dependent relation between chemical potentials, makes  $\mathcal{O}(1)$  changes to the final value of the baryon asymmetry. At temperatures such that the quantum oscillations can be neglected, we find that the usual washout terms in the Boltzmann Equations are modified, and that there are additional terms proportional to flavour asymmetries.

The upper bound on the CP asymmetry produced for each flavour is weaker than the bound applicable to the total CP asymmetry. In particular, it increases with the neutrino mass scale. By judicious choice of flavour-dependent washout rates, one can obtain a final baryon asymmetry which is independent of the light neutrino mass scale. Finally, given the experimental value of the Baryon Asymmetry of the Universe, we find a reduced lower bound on the lightest right-handed neutrino mass and on the reheating temperature.

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## A From a toy model to equations of motion for $f_{\Delta}^{ij}$

The aim of this appendix is to obtain equations of motion for a system of quantum harmonic oscillators, which carry two “flavours”. The oscillators are coupled to match the interactions of the Lagrangian in eq. (1). By taking expectation values of the number operator in a “thermal” state, this model can be reduced to a two-state quantum system. We then extrapolate the flavour structure of these equations of motion, to obtain “flavoured” equations of motion for the lepton asymmetry number operator in the early Universe. We will discuss the field theory derivation of such equations in a later publication [16].

Consider a flavour-dependent number operator, which we call  $\hat{f}$ , for a system consisting of two simple harmonic oscillators labelled  $e$  and  $\mu$ :

$$\hat{f} = \begin{bmatrix} a_e^\dagger a_e & a_e^\dagger a_\mu \\ a_\mu^\dagger a_e & a_\mu^\dagger a_\mu \end{bmatrix}, \quad (40)$$

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<sup>11</sup>For complex  $U_{\mu 3} = U_{\tau 3} = e^{i\delta}/\sqrt{2}$ , with  $\phi$  very close to  $-\pi/4$  and  $\sinh \eta = -\cosh \eta \sin \delta / (1 + \cos \delta)$  we obtain small  $\lambda_{1\mu}$ .

where the commutation<sup>12</sup> relations are  $[a_e, a_e^\dagger] = 1$ ,  $[a_\mu, a_\mu^\dagger] = 1$ , and the hamiltonian is  $H_0 = (\omega_e a_e^\dagger a_e + \omega_\mu a_\mu^\dagger a_\mu)I$ . The number operator evolves according to the Hamiltonian equations of motion:  $\frac{df}{dt} = +i[H, \hat{f}]$ . Since our Hamiltonian conserves particle number, we can take expectation values—for instance, in a thermal bath—and reduce our toy model to a two state system, described by  $f \equiv \langle \hat{f} \rangle$ , satisfying

$$\frac{df}{dt} = -i \begin{bmatrix} 0 & (\omega_e - \omega_\mu) f^{e\mu} \\ (\omega_\mu - \omega_e) f^{\mu e} & 0 \end{bmatrix}. \quad (41)$$

Notice that quantum mechanically,  $f$  is the density matrix of the two-state system. If at  $t = 0$  the system is created in some state, then the probability to be found at a time  $t$  later in this state is  $\text{Tr}\{f(0)f(t)\}$ , which can oscillate.

The excitations of the oscillators can be imagined to be the lepton *asymmetry*, carried by particles of energies  $\omega_\mu \sim \omega_e \sim E$ . Then  $\omega_\mu - \omega_e \simeq m_\mu^2(T)/E$ , where  $m_\mu^2(T) = h_\mu^2 T^2/16$  is the contribution of  $h_\mu$  to the “thermal mass” of  $\ell_\mu$  in the early Universe. Suppose the production and washout of the lepton asymmetry are treated as initial condition and subsequent measurement on the system. That is, we imagine to produce the asymmetry in some linear combination of  $e$  and  $\mu$ , allow it to evolve, and then at a later time turn on the inverse decays. Then the inverse decay rate could be reduced (or increased) because the asymmetry changed flavour during time evolution (analogous to a survival probability in neutrino oscillations).

We have neglected the thermal masses  $\propto \lambda_i \lambda_j^*$  because we anticipate that its effects are negligible at temperatures  $\sim 10^9$  GeV: if  $|h_\mu|^2 \gg |\vec{\lambda}|^2$ , the  $\lambda$ -contribution to  $H_0$  is small and can be ignored. If  $|h_\mu|^2 \ll |\vec{\lambda}|^2$ , then the asymmetry is produced in the flavour direction  $\hat{\lambda}$ , so a thermal mass for this direction will not cause oscillations (asymmetry in a mass eigenstate). For a detailed discussion of “neutrino flavour effects” (due to  $\lambda$ ) in resonant leptogenesis, see [15].

The decays and inverse decays due to the Yukawa interactions can be included by perturbing in an interaction Hamiltonian,  $H_I$ . We first consider the production and washout of leptons, due to  $\lambda$ , in a model of four harmonic oscillators:  $N, H$ , and two leptons flavours  $\{\ell_i\}$ . Later we will treat the  $N$  and  $H$  number densities as background fields, and take the difference with respect to the equation for anti-lepton flavours. We are looking for behaviours analogous to the lepton number production and washout in the decay/inverse decay  $N \rightarrow H\ell$ .

We suppose the  $H$  and  $\ell$  are massless and  $N$  is massive, and ignore the free Hamiltonian (considered previously) which does not change the particle numbers. The interaction Hamiltonian is

$$H_I = \lambda_k a_N^\dagger a_H a_\ell^k + \lambda_k^* a_N a_H^\dagger a_\ell^{k\dagger}. \quad (42)$$

We perturbatively expand the Heiseberg equations of motion to get that

$$\begin{aligned} \frac{\partial}{\partial t} f_\ell^{ij} = -[H_I, [H_I, f_\ell^{ij}]] &= -\lambda_k^* \lambda_i [a_N a_H^\dagger a_\ell^{k\dagger}, a_N^\dagger a_H a_\ell^j] + \lambda_k \lambda_j^* [a_N^\dagger a_H a_\ell^k, a_N a_H^\dagger a_\ell^{j\dagger}] \\ &= -\lambda_i \lambda_k^* (f_H f_\ell^{kj} - f_N f_H \delta^{kj} - f_N f_\ell^{jk} - f_N \delta^{jk}) \\ &\quad + (f_N \delta^{ik} + f_N f_H \delta^{ik} + f_N f_\ell^{ik} - f_H f_\ell^{ik}) \lambda_k \lambda_j^*. \end{aligned} \quad (43)$$

This looks like inverse decays and decays of  $N$ , and the middle two terms of each parentheses we drop because they would not be allowed kinematically for  $N$  decay in the early Universe. A similar calculation<sup>13</sup> can be performed for the muon Yukawa coupling  $h$ , with

$$H_{I,h} = h_k a_H^\dagger a_{\mu^c} a_\ell^k + h_k^* a_H a_{\mu^c}^\dagger a_\ell^{k\dagger}, \quad (44)$$

<sup>12</sup>In studying flavour, we consider “bosonic” leptons. The generalization to the realistic case of fermionic leptons is straightforward.

<sup>13</sup>We neglect possible interference terms between  $H_I$  and  $H_{I,h}$ , because the expectation value of terms such as  $a_{\mu^c}^\dagger a_N$  will vanish.

and using the kinematic constraint that the processes  $H \leftrightarrow \ell_\mu \mu^c$  are allowed, because the thermal mass of Higgs has contributions from the top. Taking expectation values, this gives in the charged lepton mass eigenstate basis:

$$\begin{aligned} \frac{\partial}{\partial t} \begin{bmatrix} f_\ell^{ee} & f_\ell^{e\mu} \\ f_\ell^{\mu e} & f_\ell^{\mu\mu} \end{bmatrix} &= -f_{\mu^c} \begin{bmatrix} 0 & 0 \\ 0 & |h_\mu|^2 \end{bmatrix} \begin{bmatrix} f_\ell^{ee} & f_\ell^{e\mu} \\ f_\ell^{\mu e} & f_\ell^{\mu\mu} \end{bmatrix} - f_{\mu^c} \begin{bmatrix} f_\ell^{ee} & f_\ell^{e\mu} \\ f_\ell^{\mu e} & f_\ell^{\mu\mu} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & |h_\mu|^2 \end{bmatrix} \\ &+ 2f_H \begin{bmatrix} 0 & 0 \\ 0 & |h_\mu|^2 \end{bmatrix}. \end{aligned} \quad (45)$$

It is reassuring that the  $t$ -independent equilibrium solution of this equation imposes  $f_{\mu^c} f_\ell^{\mu\mu} - f_H = 0$ . Combined with eq. (41), this gives damped oscillations for the off-diagonals:

$$\partial f_\ell^{e\mu} / \partial t = -i(\omega_e - \omega_\mu - i|h_\mu|^2 f_{\mu^c}) f_\ell^{e\mu},$$

so we can extrapolate that exchanging doublet with singlet leptons destroys the coherence between different doublet flavours.

Equations for the anti-lepton density can similarly be derived. For simplicity, we take  $H$  and  $N$  to be their own anti-particles, so there are now eight coupled oscillators:  $N, H, \ell^i, \bar{\ell}^i \mu^c$  and  $\bar{\mu}^c$ . The expectation value of the difference between the two equations, again in the charged lepton mass eigenstate basis, is

$$\begin{aligned} \frac{\partial}{\partial t} f_\Delta^{ij} &= -\lambda_i \lambda_k^* f_H f_\Delta^{kj} - f_H f_\Delta^{ik} \lambda_k \lambda_j^* + 2(f_N - f_N^{eq}) \epsilon^{ij} \\ &- i(\omega_i - \omega_j) f_\Delta^{ij} - \frac{|h_\mu|^2}{2} (f_{\mu^c} + f_{\bar{\mu}^c}) (f_\Delta^{i\mu} \delta_{\mu j} + \delta_{\mu i} f_\Delta^{\mu j}) \\ &- \frac{|h_\mu|^2}{2} (f_{\mu^c} - f_{\bar{\mu}^c}) ((f_\ell + f_\ell^T)^{i\mu} \delta_{\mu j} + \delta_{\mu i} (f_\ell + f_\ell^T)^{\mu j}), \end{aligned} \quad (46)$$

where  $k, i, j$  can be  $e$  or  $\mu$ , and  $i, j$  are not summed. The purpose of this equation is to motivate the flavour index structure we use in the text. We now briefly discuss the various terms.

- The first two terms describe washout by inverse decays. By construction, there is no washout by  $\Delta L = 2$  scattering (we will include this by hand).
- The third term, which is  $\propto \epsilon$ , has been obtained artificially. It is well-known in field theory that no asymmetry is created at second order in  $\lambda$ ; the difference  $\Gamma(N \rightarrow H\ell) - \Gamma(N \rightarrow \bar{H}\bar{\ell})$  is proportional to the imaginary parts of the loop amplitude times the loop coupling constants. We write the latter as  $(\delta\lambda)_i$ , replace the ‘‘tree-level’’  $\lambda_i$  by a ‘‘tree + loop’’ vertex  $\lambda'_i = \lambda_i + (\delta\lambda)_i$ , so  $\Gamma - \bar{\Gamma}$  becomes the matrix in flavour space  $[(\delta\lambda)_i \lambda_j^* - \lambda_i (\delta\lambda)_j^*]$ . This gives

$$\epsilon_{ij} = \Im\{\lambda_i (\delta\lambda)_j^* - (\delta\lambda)_i \lambda_j^*\} = \Im\{\lambda_i \kappa_{jk}^* \lambda_k - \kappa_{ik} \lambda^{*k} \lambda_j^*\} \quad (47)$$

where  $[\kappa]_{ij} = [m_\nu]_{ij}/v^2$ . Using the equilibrium condition

$$2f_N^{eq}[\epsilon] = \frac{f_H}{2} ([\epsilon][f_\ell + f_\ell^T] + [f_\ell + f_\ell^T][\epsilon]),$$

(where square brackets are matrices in flavour space), we obtain a production term  $2(f_N + f_N^{eq})\epsilon^{ij}$ . This is incomplete, because it would produce an asymmetry in thermal equilibrium. The resonant part of  $\Delta L = 2$  scattering should be subtracted [36], which ensures that  $\text{Tr } f_\Delta$  vanishes in thermal equilibrium. We work in the charged lepton mass eigenstate basis, and follow [36] to obtain eq. (46).

- The remaining terms of eq. (46) describe the effects of the muon Yukawa coupling. The asymmetry oscillates in flavour space, as described at the beginning of the section, because the off-diagonal flavour matrix elements have time-dependent phases.
- The muon Yukawa coupling also causes the off-diagonal elements to decay away, via the second to last term. To get the correct relative normalisation between oscillation and decay in the early Universe, we will extract the rates from the real and imaginary parts of the thermal propagator.
- The last term has minimal effect on the off-diagonal terms, as one can see by solving eq. (45). However, combined with the second to last term, it drives the diagonal asymmetry  $Y_{\Delta\ell_\mu}$  to the correct relation with  $Y_{B/3-L_\mu}$ , when the muon Yukawa coupling is in equilibrium<sup>14</sup>. For simplicity, we drop this term, assuming its effects could be approximated as time-dependent components in the  $A$ -matrix [11] relating  $Y_{\Delta\ell_j} = A^{jj}Y_{B/3-L_i}$  :

$$A^{ij}(t) = A_{T>}^{ij} e^{-\Gamma_\mu t} + A_{T<}^{ij} (1 - e^{-\Gamma_\mu t}), \quad (48)$$

where  $A_{T>}^{ij}$  are the matrix elements before  $h_\mu$  comes into equilibrium,  $A_{T<}^{ij}$  are the matrix elements afterwards, and  $\Gamma_\mu$  is the interaction rate associated to the muon Yukawa coupling, see also appendix C.

## Added Discussion

The effect of the charged lepton Yukawa in equation (46) can be clarified by studying the evolution of  $F^{ij} = f_\Delta^{ij} - \frac{h_i h_j^*}{|h|^2} (f_{\mu^c} - f_{\bar{\mu}^c})$ , where  $h_i$  is the muon Yukawa coupling in some arbitrary basis for lepton doublet space. The trace of  $F$  is the total lepton number stored in  $e_L$ ,  $\mu_L$  and  $\mu_R$ , so studying  $F$  in the toy model is analogous to replacing the asymmetry in lepton doublets with the asymmetry  $F$  in  $B/3 - L_\alpha$  [11] in usual leptogenesis calculations. Neglecting the first four terms of eqn (46), which describe the interactions of  $N$  and the oscillations due to the muon Yukawa, the last two terms of eqn (46) can be written

$$\frac{\partial}{\partial t} [f_\Delta] = - \left\{ [\vec{h}\vec{h}^\dagger], [f^\Delta] \right\} \frac{(f_{\mu^c} + f_{\bar{\mu}^c})}{2} - \left\{ [\vec{h}\vec{h}^\dagger], [f_\ell + f_\ell^T] \right\} \frac{(f_{\mu^c} - f_{\bar{\mu}^c})}{2}. \quad (49)$$

where  $\vec{h}$  is a column vector, and objects in square brackets are matrices in doublet space. The rate of change of the  $\mu^c$  asymmetry  $f_{\Delta\mu^c} = f_{\mu^c} - f_{\bar{\mu}^c}$  is

$$\frac{\partial}{\partial t} f_{\Delta\mu^c} = -|h|^2 (f_{\mu^c} + f_{\bar{\mu}^c}) f_\Delta^{\mu\mu} - |h|^2 f_{\Delta\mu^c} (f_\ell + f_\ell^T)^{\mu\mu}$$

So the equation for the lepton asymmetry  $F^{ij}$ , in the flavour basis, is

$$\begin{aligned} \frac{\partial}{\partial t} [F] = & - \left( \left\{ [\vec{h}\vec{h}^\dagger], [f^\Delta] \right\} - 2[\vec{h}\vec{h}^\dagger] \hat{h}^\dagger \cdot [f^\Delta] \cdot \hat{h} \right) \frac{(f_{\mu^c} + f_{\bar{\mu}^c})}{2} \\ & - \left( \left\{ [\vec{h}\vec{h}^\dagger], [f_\ell + f_\ell^T] \right\} - 2[\vec{h}\vec{h}^\dagger] \hat{h}^\dagger \cdot [(f_\ell + f_\ell^T)] \cdot \hat{h} \right) \frac{(f_{\mu^c} - f_{\bar{\mu}^c})}{2}. \end{aligned} \quad (50)$$

where the second line is of the same order as the first (see eqn (46)). As discussed in the last point after eqn (46), an approximate equation for  $F^{jj}$  can be obtained by expressing  $f_{\Delta\mu^c}$  and  $f_\Delta^{jj}$  in terms

<sup>14</sup>In our toy model, where  $H$  is a real scalar, this means  $(f_{\mu^c} - f_{\bar{\mu}^c}) = f_\Delta^{\mu\mu}$ . In the SM, there is Higgs asymmetry [17, 11].

of  $F^{jj}$  using a time-dependent A-matrix. To obtain equation (11), we simply drop the second line. In the flavour basis, eqn (50) can be written as

$$\frac{\partial}{\partial t} \begin{bmatrix} f_{\Delta}^{ee} & f_{\Delta}^{e\mu} \\ f_{\Delta}^{\mu e} & f_{\Delta}^{\mu\mu} - f_{\Delta\mu^c} \end{bmatrix} = -\frac{|h_{\mu}|^2}{2} \begin{bmatrix} 0 & f_{\Delta}^{e\mu} \\ f_{\Delta}^{\mu e} & 0 \end{bmatrix} (f_{\mu^c} + f_{\bar{\mu}^c}) - \frac{|h_{\mu}|^2}{2} \begin{bmatrix} 0 & f_{\ell}^{e\mu} + f_{\bar{\ell}}^{\mu e} \\ f_{\ell}^{\mu e} + f_{\bar{\ell}}^{e\mu} & 0 \end{bmatrix} f_{\Delta\mu^c}$$

As expected, the Yukawa coupling  $h$  causes the flavour off-diagonal asymmetries to decay away, as it did for the off-diagonal number densities in eqn (45). It has no effect on the asymmetry in either flavour. If the matrices  $F$  and  $[\vec{h}\vec{h}^{\dagger}] = H_a\sigma^a$  are expanded on Pauli matrices, as in equation (18), this behaviour can be modelled as a triple cross product:  $\hat{H} \times \vec{H} \times \vec{F}$ . A single cross product, as in the original version of the paper, obviously does not cause the off-diagonals to be damped as  $\partial F^{\perp}/\partial t \propto |h|^2 F^{\perp}$ .

## B $\Delta L = 2$ terms

Following the same procedure of section 4 we can include the contributions arising from the  $\Delta L = 2$  exchange mediated by RH neutrinos. In matrix form, the additional terms on the RHS of eq. (23) arising from the  $s$ -channel contribution would be

$$\begin{aligned} \text{RHS of eq. (23)} &= \frac{1}{2Y_L^{eq}} \left\{ \begin{bmatrix} \gamma_{\Delta L=2}^{eeee} & & \\ & \gamma_{\Delta L=2}^{\mu\mu ee} & \\ & & \gamma_{\Delta L=2}^{\tau\tau ee} \end{bmatrix}, \begin{bmatrix} Y_L^{ee} & & \\ & Y_L^{\mu\mu} & \\ & & Y_L^{\tau\tau} \end{bmatrix} \right\} \\ &+ \frac{1}{2Y_L^{eq}} \left\{ \begin{bmatrix} \gamma_{\Delta L=2}^{ee\mu\mu} & & \\ & \gamma_{\Delta L=2}^{\mu\mu\mu\mu} & \\ & & \gamma_{\Delta L=2}^{\tau\tau\mu\mu} \end{bmatrix}, \begin{bmatrix} Y_L^{ee} & & \\ & Y_L^{\mu\mu} & \\ & & Y_L^{\tau\tau} \end{bmatrix} \right\} \\ &+ \frac{1}{2Y_L^{eq}} \left\{ \begin{bmatrix} \gamma_{\Delta L=2}^{ee\tau\tau} & & \\ & \gamma_{\Delta L=2}^{\mu\mu\tau\tau} & \\ & & \gamma_{\Delta L=2}^{\tau\tau\tau\tau} \end{bmatrix}, \begin{bmatrix} Y_L^{ee} & & \\ & Y_L^{\mu\mu} & \\ & & Y_L^{\tau\tau} \end{bmatrix} \right\} \\ &+ \frac{1}{Y_L^{eq}} \begin{bmatrix} \text{Tr}[\gamma_{\Delta L=2}^{eejj} Y_L^{jj}] & & \\ & \text{Tr}[\gamma_{\Delta L=2}^{\mu\mu jj} Y_L^{jj}] & \\ & & \text{Tr}[\gamma_{\Delta L=2}^{\tau\tau jj} Y_L^{jj}] \end{bmatrix}, \end{aligned} \quad (51)$$

where

$$\gamma_{\Delta L=2}^{ijjj} = \sum_J \gamma_{\Delta L=2}^{ijjJ} = \sum_J |\lambda_{Ji}|^2 |\lambda_{Jj}|^2 F_{JJ}. \quad (52)$$

The sum is over the RH exchanged neutrino eigenstates and the function  $F_{JJ}$  is the kinematical factor corrected according to [5]. We define the matrices

$$[\gamma_{\Delta L=2}^i] = \begin{bmatrix} \gamma_{\Delta L=2}^{iee} & & \\ & \gamma_{\Delta L=2}^{i\mu\mu} & \\ & & \gamma_{\Delta L=2}^{i\tau\tau} \end{bmatrix}, \quad (53)$$

and

$$[\gamma_{\Delta L=2}^j] = \begin{bmatrix} \gamma_{\Delta L=2}^{eej} & & \\ & \gamma_{\Delta L=2}^{\mu\mu j} & \\ & & \gamma_{\Delta L=2}^{\tau\tau j} \end{bmatrix}. \quad (54)$$



Each of which can be decomposed in terms of the matrices  $I, \Lambda_3, \Lambda_8$ :

$$[\gamma_{\Delta L=2}^i] = \frac{1}{3}\gamma_{\Delta L=2}^i I + \gamma_{\Delta L=2,3}^i \Lambda_3 + \gamma_{\Delta L=2,8}^i \Lambda_8 \quad (55)$$

and similarly for  $\gamma_{\Delta L=2}^j$ . The interference terms can also be dealt with in terms of the quantities

$$\gamma_{\Delta L=2}^{ijj(IJ)} = \sum_{I<J} [\lambda_{Ii}\lambda_{Ij}^*][\lambda_{Ji}^*\lambda_{Jj}]G_{IJ}, \quad (56)$$

where  $G_{IJ}$  is a kinematical factor of the same kind as  $F_{IJ}$  [5]. Thus taking the trace the contribution to eq. (27)

$$\begin{aligned} \text{RHS} &= \frac{1}{3} [\gamma_{\Delta L=2}^e + \gamma_{\Delta L=2}^\mu + \gamma_{\Delta L=2}^\tau + \gamma_{\Delta L=2}^e + \gamma_{\Delta L=2}^\mu + \gamma_{\Delta L=2}^\tau] Y_L \\ &+ \frac{1}{3} [\gamma_{\Delta L=2}^{e(IJ)} + \gamma_{\Delta L=2}^{\mu(IJ)} + \gamma_{\Delta L=2}^{\tau(IJ)} + \gamma_{\Delta L=2}^{e(IJ)} + \gamma_{\Delta L=2}^{\mu(IJ)} + \gamma_{\Delta L=2}^{\tau(IJ)}] Y_L \\ &+ 2 [\gamma_{\Delta L=2,3}^e + \gamma_{\Delta L=2,3}^\mu + \gamma_{\Delta L=2,3}^\tau + \gamma_{\Delta L=2,3}^e + \gamma_{\Delta L=2,3}^\mu + \gamma_{\Delta L=2,3}^\tau] Y_3 \\ &+ 2 [\gamma_{\Delta L=2,3}^{e(IJ)} + \gamma_{\Delta L=2,3}^{\mu(IJ)} + \gamma_{\Delta L=2,3}^{\tau(IJ)} + \gamma_{\Delta L=2,3}^{e(IJ)} + \gamma_{\Delta L=2,3}^{\mu(IJ)} + \gamma_{\Delta L=2,3}^{\tau(IJ)}] Y_3 \\ &+ 2 [\gamma_{\Delta L=2,8}^e + \gamma_{\Delta L=2,8}^\mu + \gamma_{\Delta L=2,8}^\tau + \gamma_{\Delta L=2,8}^e + \gamma_{\Delta L=2,8}^\mu + \gamma_{\Delta L=2,8}^\tau] Y_8 \\ &+ 2 [\gamma_{\Delta L=2,8}^{e(IJ)} + \gamma_{\Delta L=2,8}^{\mu(IJ)} + \gamma_{\Delta L=2,8}^{\tau(IJ)} + \gamma_{\Delta L=2,8}^{e(IJ)} + \gamma_{\Delta L=2,8}^{\mu(IJ)} + \gamma_{\Delta L=2,8}^{\tau(IJ)}] Y_8 . \quad (57) \end{aligned}$$

We next show that for low temperatures ( $z \gg 1$ ), the  $\Delta L = 2$  terms can be neglected with respect to the inverse decays. To simplify the discussion we focus on the ID and  $\Delta L = 2$  contributions to a single flavour, say elements 11 of the correspondings matrices that enter the BE. For  $z \gg 1$

$$\gamma_D^{ee} Y_L^{ee} \sim \frac{e^{-z}}{\pi^{5/2} z^{3/2}} M_1^4 \lambda_{1e} \lambda_{1e}^* Y_L^{ee}, \quad (58)$$

and the  $\Delta L = 2$  terms give,

$$\text{RHS of Eq.(51)} \propto \frac{1}{\pi^5 z^6} M_1^4 \left( 2|\lambda_{1e}|^2 \left( \sum_k |\lambda_{1k}|^2 \right) Y_L^{ee} + |\lambda_{1e}|^2 |\lambda_{1\mu}|^2 |Y_L^{\mu\mu}| + |\lambda_{1e}|^2 |\lambda_{1\tau}|^2 |Y_L^{\tau\tau}| \right). \quad (59)$$

This means that for temperatures  $z \lesssim z_c$ , we can neglect the contribution to the BE from the lepton number violating exchange of the RH neutrinos as the comparative strength of the inverse decays suppression is  $\sim e^{-z}$  while the suppression from the  $\Delta L = 2$  is

$$\sim \frac{1}{\pi^{5/2} z^{9/2}} \left( 2 \sum_k |\lambda_{1k}|^2 + |\lambda_{1\mu}|^2 \frac{Y_L^{\mu\mu}}{Y_L^{ee}} + |\lambda_{1\tau}|^2 \frac{Y_L^{\tau\tau}}{Y_L^{ee}} \right). \quad (60)$$

This implies that for  $\sum_k |\lambda_{1k}|^2 \sim 1$  only for  $z \gtrsim 15$  will the  $\Delta L = 2$  terms dominate over  $\gamma_D$ . For smaller values of  $\lambda_{1k}$ , as constrained by the departure from thermal equilibrium and neutrino masses, the  $\Delta L = 2$  terms are always negligible for the more interesting values of  $z$ . That is, if the freeze out temperature is denoted by  $z_f$ , then for the regions in parameter space for which  $z_f \lesssim z_c$  the produced asymmetry is not affected by the  $\Delta L = 2$  washout terms and thus can be neglected.

## C Chemical Equilibrium

An important point which was pointed out in ref. [11] is the relationship between the lepton asymmetry for each flavour  $L_i$  and the  $B/3 - L_i \equiv \Delta_i$  asymmetry, which is what is effectively conserved by the sphaleron interactions, from chemical equilibrium equations. More recently ref. [37] has also discussed issues related to this. For completeness we include the corresponding matrices which relate  $\Delta_i$  to the chemical potential of the left-handed leptons,  $\mu_{\nu_i}$ , depending on which of the charged Yukawa couplings are in equilibrium as discussed in the main text of the paper. Note that the relationship between  $\mu_{\nu_i}$  and  $L_i$  also varies depending on the corresponding charged Yukawa couplings that are in equilibrium.

When  $\Gamma_\tau \gg H$ , and sphaleron interactions are also in equilibrium, then

$$\begin{pmatrix} \Delta_e \\ \Delta_\mu \\ \Delta_\tau \end{pmatrix} = \begin{pmatrix} -\frac{22}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} & -\frac{22}{9} & -\frac{4}{9} \\ -\frac{4}{9} + \frac{5}{3} \frac{1}{3N+3} & -\frac{4}{9} + \frac{5}{3} \frac{1}{3N+3} & -\frac{4}{9} + \frac{8}{3} \frac{1}{3N+2} - 3 \end{pmatrix} \begin{pmatrix} \mu_{\nu_e} \\ \mu_{\nu_\mu} \\ \mu_{\nu_\tau} \end{pmatrix}, \quad (61)$$

where  $N$  is the number of generations. For the range of temperatures we have focused on in the main part the relation between these quantities is modified to:

$$\begin{pmatrix} \Delta_e \\ \Delta_\mu \\ \Delta_\tau \end{pmatrix} = \begin{pmatrix} -\frac{22}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} + \frac{5}{3} \frac{1}{3N+4} & -\frac{4}{9} - 3 + \frac{8}{3} \frac{1}{3N+4} & -\frac{4}{9} + \frac{8}{3} \frac{1}{3N+4} \\ -\frac{4}{9} + \frac{5}{3} \frac{1}{3N+4} & -\frac{4}{9} + \frac{8}{3} \frac{1}{3N+4} & -\frac{4}{9} - 3 + \frac{8}{3} \frac{1}{3N+4} \end{pmatrix} \begin{pmatrix} \mu_{\nu_e} \\ \mu_{\nu_\mu} \\ \mu_{\nu_\tau} \end{pmatrix}. \quad (62)$$

Finally, when all charged Yukawa couplings are in equilibrium, the relation is simply

$$\Delta_i = \left( -\frac{4}{9} + \frac{8}{3} \frac{1}{4N+2} \right) \sum_j \mu_{\nu_j} - 3\mu_{\nu_i}. \quad (63)$$

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