# Flexible 3-D shape measurement using projector defocusing

**Shuangyan Lei and Song Zhang***

*Department of Mechanical Engineering, Iowa State University, Ames, Iowa 50011, USA*
**Corresponding author: song@iastate.edu*

Received July 28, 2009; revised September 4, 2009; accepted September 12, 2009;
posted September 21, 2009 (Doc. ID 114888); published October 6, 2009

We present a 3-D shape-measurement technique using a defocused projector. The ideal sinusoidal fringe patterns are generated by defocusing binary structured patterns, and the phase shift is realized by shifting the binary patterns spatially. Because this technique does not require calibration of the gamma of the projector, it is easy to implement and thus is promising for developing flexible 3-D shape measurement systems using digital video projectors. © 2009 Optical Society of America

*OCIS codes:* 120.0120, 120.2650, 100.5070.

3-D shape measurement is very important to numerous disciplines; over the years, a number of techniques have been developed, including some with real-time capability [1–5]. With recent advancement in digital display technology, 3-D shape measurement based on digital sinusoidal fringe projection techniques is rapidly expanding. However, developing a system with an off-the-shelf projector for high-quality 3-D shape measurement remains challenging. One of the major issues is the nonlinear gamma effect of the projector.

To perform high-quality 3-D shape measurement using a digital fringe projection and phase-shifting method, projector gamma calibration is usually mandatory. This is because the commercial video projector is usually a nonlinear device that is purposely designed to compensate for human vision. A variety of techniques have been studied, including the methods to actively change the fringe to be projected [6,7] and those to passively compensate for the phase errors [8–11]. Moreover, because the output light intensity does not change much when the input intensity is close to 0 and/or 255 [6], it is impossible to generate fringe images with a full intensity range (0–255). In addition, our experiments found that the projection nonlinear gamma actually changes over time and thus needs to be recalibrated frequently. All these problems hinder its applications, especially for precision measurement. On the contrary, if a technique can generate ideal sinusoidal fringe images without worrying about nonlinear gamma, it would significantly simplify the 3-D shape measurement system development.

This Letter will present a flexible 3-D shape measurement technique without requiring gamma calibration. The idea came from our two observations: (1) seemingly sinusoidal fringe patterns often appear on the ground when the light shines through open window blinds and (2) the sharp features of an object are blended together in a blurring image that is captured by an out-of-focus camera. The former gives the insight that an ideal sinusoidal fringe image could be produced from a binary structured pattern, and the latter provides the hint that if the projector is defocused, the binary structured pattern might become ideal sinusoidal. Because only binary patterns are needed, the nonlinear response of the projector would not be a problem, because only 0 and 255 intensity values are used. Moreover, phase shifting can be introduced by spatially moving the binary structured patterns. Therefore, if this hypothesis is true, a flexible 3-D shape measurement system based on a digital fringe-projection technique can be developed without nonlinear gamma calibration. Experiments will be presented to verify the performance of the proposed technique.

Sinusoidal phase-shifting methods are widely used in optical metrology because of their measurement accuracy [12]. In this research, we use a three-step phase-shifting algorithm with a phase shift of $2\pi/3$; the intensities of these three fringe images are

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos(\phi - 2\pi/3), \tag{1}$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos(\phi), \tag{2}$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos(\phi + 2\pi/3). \tag{3}$$

Solving these three equations, this phase can be obtained:

$$\phi(x,y) = \tan^{-1}\left[\frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3}\right]. \tag{4}$$

This equation provides the wrapped phase with $2\pi$ discontinuities. A spatial phase-unwrapping algorithm can be applied to obtain continuous phase [13], which can be used to retrieve 3-D coordinates [14].

For a 3-D shape measurement system using a sinusoidal phase-shifting algorithm, ideal sinusoidal fringe images are required. To generate ideal sinusoidal fringe images with a projector, one approach is to directly send the computer-generated sinusoidal patterns to an in-focused projector; the other approach, as proposed in this Letter, is to send the binary patterns to a defocused projector. The former has proven successful with nonlinear gamma corrections. The latter does not have the problems related to nonlinear gamma but is not trouble free. This is because, intuitively, if the degree of defocusing is too small, the fringe stripes are not sinusoidal, while there are no high-contrast fringes if the projector is defocused too much.

Mathematically, a binary pattern generated by a computer can be regarded as a square wave horizontally, $s(x)$, and the imaging system can be regarded as a point-spread function (PSF), $p(x)$ [15]. The defocusing of the projector will generate blurred images. The degree of blur can be modeled as a different breadth of PSF. The PSF can be approximated as a Gaussian smoothing filter [16]. If a filter is applied so that only the first harmonics is kept, an ideal sinusoidal waveform will be produced. In the Fourier domain, because the square wave has only odd harmonics without even ones, it is easier to design a filter to suppress the higher-frequency components. Our simulation shows that by applying the Gaussian filter to a square wave, ideal sinusoidal waveform can indeed be generated, and the phase error will be less than rms 0.0003 rad if a three-step phase-shifting algorithm is applied. This phase error is negligible in comparison with random noise.

Figure 1 shows examples of the fringe images captured when the projector is defocused at different levels. The degree of defocusing is controlled by manually adjusting the focal length of the projector. The image in Fig. 1(c) shows sinusoidal fringe stripes, thus it seems to be feasible to generate ideal sinusoidal patterns by properly defocusing binary patterns. However, if the projector is defocused too much, the contrast of the fringe images is low, as shown in Fig. 1(d).

The performance of the proposed approach was verified with a structured light system comprised of a Dell LED projector (M109S), and The Imaging Source digital universal serial bus CCD camera (DMK 21BU04) with a Computar M3514-MP lens $F/1.4$ with $f=35$ mm. The camera resolution is $640\times480$, with a maximum frame rate of 60 frames/s. The projector has a resolution of $858\times600$, and the projection lens with $F/2.0$ and $f=16.67$ mm.

A three-step phase-shifting algorithm is used for this experiment. By shifting the binary patterns by one third of the period, the phase-shifted fringe images with a phase shift of $2\pi/3$ can be generated. Three spatially shifted fringe images under the defocusing level 3 [shown in Fig. 1(c)] are projected onto an uniform white flat board and are captured by the camera. Figure 2(a) shows the 200th cross sections of these fringe images. It shows the desired phase-shifted fringe images can be generated by shifting binary patterns spatially.

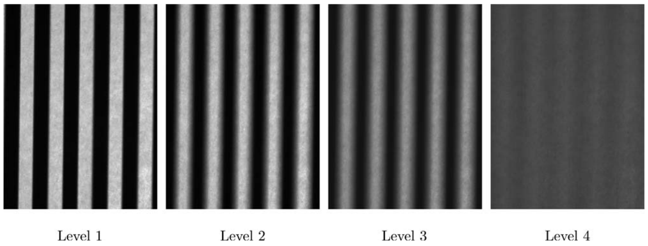
Level 1 Level 2 Level 3 Level 4

Fig. 1. Binary structured patterns projected with a projector at different defocusing levels, where level 1 is in focus and level 4 is severely defocused.

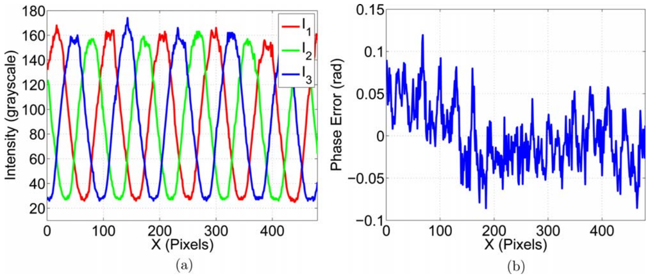

Fig. 2. (Color online) (a) 200th row of the fringe images; (b) 200th row of the phase error.

Applying Eq. (4) to these fringe images, the wrapped phase map can be obtained. The phase is then unwrapped by applying a spatial phase-unwrapping algorithm [13]. Figure 2(b) shows the 200th row cross section of the phase after removing the unwrapped phase slope. Because the nonsinusoidal waveforms usually result in periodical phase errors while no obvious periodical patterns appear in this phase map, ideal sinusoidal fringe images are actually generated. It should be noted that the camera is always in focus to capture surface details.

To compare the performance of the proposed approach against the traditional method, 13 levels of defocusing are tested. The projector starts in focus and then increases its degree of defocusing. For the traditional method, the gamma of the projector is calibrated and the associated phase error is compensated. Figure 3(a) shows the phase error. This experiment indicates that when the projector is in focus, the traditional method works better. When the projector is defocused to a degree, the proposed method starts outperforming the traditional one. It is interesting to know that both methods produce similar phase error under their own best conditions. Moreover, another experiment is also performed without nonlinear gamma correction; the phase map is shown in Fig. 3(b). This figure clearly shows that the traditional method is much worse than the proposed one without gamma correction.

A complex object is then measured with this proposed approach. Figure 4 shows the measurement result. To measure a larger range, a Computar

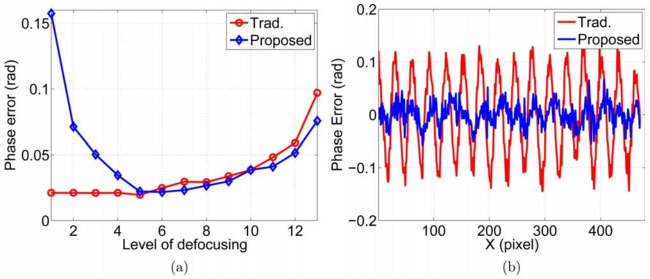

Fig. 3. (Color online) Comparison between the traditional and proposed methods: (a) phase error (rms) under different level of defocusing; (b) phase error without projector's gamma calibration.

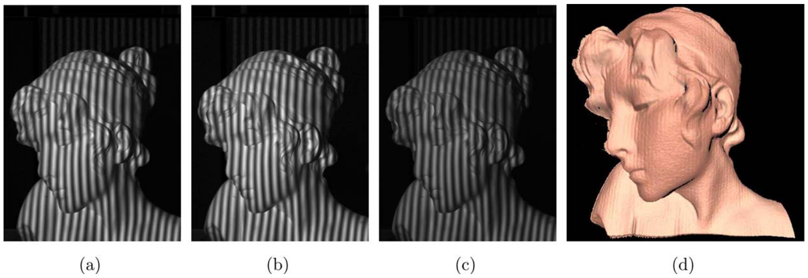
(a) (b) (c) (d)

Fig. 4. (Color online) Measurement result of a complex sculpture with the proposed approach: (a) $I_1$, (b) $I_2$, (c) $I_3$, (d) 3-D shape rendered in shaded mode.

M1208-MP lens ($F/1.4$, $f=8$ mm) was used for the camera. In this research, the phase is converted to coordinates by applying a phase-to-height conversion algorithm introduced in [5], and the 3-D geometry is smoothed by a $5\times 5$ Gaussian filter to reduce the most significant random noises. This experiment shows that the proposed approach can be used for measuring 3-D objects with complicated features.

Compared with a binary structured-light-based 3-D shape measurement method, a phase-shifting based one has the advantage of spatial measurement resolution, because it can reach pixel level with the minimum number of three fringe images. However, one drawback of the phase-shifting based system lies in the complexity of generating ideal sinusoidal fringe patterns. Errors resulting from nonsinusoidal waveforms are significant if there is no gamma correction. On the contrary, the proposed method does not have this problem, because only two intensity levels are used.

The advantage of the proposed approach is to avoid the error caused by nonlinear gamma of a digital video projector while still maintaining the advantage of a phase-shifting-based approach. However, because almost all existing structured-light-system calibration methods require the projector to be in focus, none of them can be adopted to calibrate the proposed system, since the projector is defocused. This research used a standard phase-to-height conversion algorithm using a reference plane, as explained in [5], albeit it is not accurate for large-depth-range measurement. Another possible shortcoming of this approach is that the degree of defocusing must be controlled to a certain range in order to produce high-contrast fringe images. Even with these drawbacks, this technique is still very useful, because it significantly simplifies the problem relating to the ideal fringe generations with a digital video projector.

The major contributions of this paper are that it (1) provides another angle to perform 3-D shape measurement using a digital video projector and (2) offers a flexible way to develop a 3-D shape measurement system using digital fringe projection and phase-shifting methods.

This Letter has presented a flexible 3-D shape measurement technique based on the projector defocusing effect. Experiments have verified the feasibility of this new method. Because only two levels (0s and 255s) are used for sinusoidal fringe generation, there is no need to calibrate the projector's nonlinear response. Therefore it simplified the development of a 3-D shape measurement system using a digital projector. Moreover, because generating binary fringe images is much easier than generating sinusoidal ones, this technique could potentially provide new views for 3-D optical metrology.

We would like to express our sincere gratitude to the reviewers who provided us deep insights on understanding the defocusing effect mathematically through Fourier transform and point spread function.

## References

1. Y. Huang, C. Quan, C. J. Tay, and L. J. Chen, Opt. Eng. **44**, 1552 (2005).
2. M. E. Pawlowski, M. Kujawinska, and M. G. Wgiel, Opt. Eng. **41**, 450 (2002).
3. C. Quan, C. J. Tay, H. M. Shang, and P. J. Bryanston-Cross, Opt. Commun. **119**, 479 (1995).
4. M. Takeda and K. Mutoh, Appl. Opt. **22**, 3977 (1983).
5. S. Zhang and P. S. Huang, Opt. Eng. **45**, 123601 (2006).
6. P. S. Huang, C. Zhang, and F.-P. Chiang, Opt. Eng. **42**, 163 (2002).
7. S. Kakunai, T. Sakamoto, and K. Iwata, Appl. Opt. **38**, 2824 (1999).
8. S. Zhang and P. S. Huang, Opt. Eng. **46**, 063601 (2007).
9. S. Zhang and S.-T. Yau, Appl. Opt. **46**, 36 (2007).
10. H. Guo, H. He, and M. Chen, Appl. Opt. **43**, 2906 (2004).
11. B. Pan, Q. Kemao, L. Huang, and A. Asundi, Opt. Lett. **34**, 2906 (2009).
12. D. Malacara, ed., *Optical Shop Testing* 3rd ed. (John Wiley and Sons, 2007).
13. S. Zhang, X. Li, and S.-T. Yau, Appl. Opt. **46**, 50 (2007).
14. S. Zhang and P. S. Huang, Opt. Eng. **45**, 083601 (2006).
15. Wikipedia, http://en.wikipedia.org/wiki/Point_spread_function (retrieved September 22, 2009).
16. Wikipedia, http://en.wikipedia.org/wiki/Airy_disc (retrieved September 22, 2009).