Flexible and Dynamic Power Allocation in Broadband Multi-Beam Satellites

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***Abstract*—For multi-beam broadband satellites operating at 10 GHz and above frequencies, rain attenuation is the dominant impairment factor. Using a stochastic model for rain attenuation prediction and a greedy approach, dynamic power allocation has been recently shown to increase the number of users served than the static technique. This letter proposes a new dynamic power allocation algorithm the novelty of which lies in treating users with similar power requirement as a group, instead of individuals. Thus, without resorting to exhaustive search we are able to serve more number of users than the existing technique.**

***Index Terms*—Multi-beam antennas, rain attenuation, satellite communications.**

I. Introduction

THE use of high frequency bands is common in broadband satellites. Attenuation due to atmospheric precipitation (rain, gases, clouds, fog etc.) heavily degrade the signals at 10 GHz and above frequencies. Out of these, rain attenuation being highly space and time varying [2], is the dominant fading mechanism. This has given rise to the need for designing algorithms for allocating power amongst the beams of a multi-beam satellite.

In [9], a power allocation algorithm was proposed to maximize the throughput of the system. In [10], the problem of optimal power allocation based on different traffic demands was addressed. Later, in [11], the problem was revisited after taking into consideration the Quality of Service and Service Level Agreement requirements. Subsequently, in [12], the trade off between maximum total capacity and proportional fairness was considered. Recently in [1], the problem of maximizing the number of users served has been addressed. It has been shown that using a stochastic model for rain attenuation prediction and a simple dynamic power allocation algorithm, more number of users can be served than the existing static techniques. In this letter, we propose an improved dynamic power allocation algorithm which can serve even more number of users.

II. System And Channel Model

We will consider the same system and channel model as described in [1]. Consider a communication satellite that employs M beams for providing broadcast services to N geographically separated users, where N $\gg$ M. Every user requires a minimum threshold power for being served and without loss of generality let it be same for all the users and denote it as P_{thr}. The received power of user i, $P_{r,i}$ from beam j is related to the input power of beam j, $P_{s,j}$ as:

$$P_{r,i} = h_{i,j}P_{s,j}, (i = 1, \ldots, N, j = 1, \ldots M) \quad (1)$$

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where $h_{i,j}$ denotes the power gain of the channel from beam j to user i. There can be two scenarios for the channel: a clear sky or a rain attenuated channel. The clear sky gain is given by:

$$h_{i,j,cs} = \frac{G_{s,ij}G_{r,ij}}{(4\pi d_{ij})^2}10^{\frac{-A_{c,ij}}{10}} \quad (2)$$

where d_{ij} is the length of the satellite link while $G_{s,ij}$, $G_{r,ij}$ denote the gains of the satellite transmitter antenna and the user receiver antenna respectively. The attenuations $A_{c,ij}$ due to clouds, fog, gases and melting layer absorption show very little variation with time and are assumed to be constant [1].

For any user i served by beam j, the condition $h_{ij}P_{s,j} \geq P_{thr}$ must hold for reliable operation. Hence the minimum power required by beam j in order to serve user i is $P_{thr}/h_{i,j}$. From now onwards we will use "power required by a user" as the minimum input power required by the beam which serves this user i.e. $P_{thr}/h_{i,j}$ where i is the user number and j is the beam number. For rainy users, the time varying attenuation can be incorporated in the calculation of time dependent channel gains $h_{ij}(t)$ as:

$$h_{ij}(t) = h_{ij,cs}10^{\frac{-A_{r,ij}(t)}{10}} \quad (3)$$

where $A_{r,ij}(t)$ can be found using the following stochastic model for rain attenuation [4]:

$$\frac{dA_{r,ij}(t)}{dt} = A_{r,ij}(t)dA_{ij}[S^2_{\alpha,ij} - (\ln(A_{r,ij}(t)) - \ln(A_{m.ij}))] + S_{\alpha,ij}A_{r,ij}(t)\sqrt{2dA_{i,j}}n(t) \quad (4)$$

where $A_{m,ij}$ and $S_{\alpha,ij}$ are statistical parameters of the longterm lognormal distribution of the rain attenuation, in the decibel scale, dA_{ij} is a statistical parameter describing the dynamic properties of rain attenuation along the propagation path [3] and $n(t)$ is a white noise process. These parameters can be evaluated using the methodology described in [3]. The initial value for $A_{r,ij}(t)$ can be evaluated using the relation [3],[5]:

$$A_{r,ij}(t = 0) = A_o(z, t = 0).L_{eff} \quad (5)$$

where $A_o = aR^b_{ij}$ and

$$L_{eff} = \frac{L_{real}}{a_1 + L_{real}.\frac{(a_2.R_{ij}+a_3.L_{real}+a_4)}{a_5}} \quad (6)$$

where R_{ij} is point rainfall rate in mm/h, L_{eff} is effective path length which accounts for spatial inhomogeneity of rain medium. The parameters a and b depend on frequency, polarization and elevation angle and can be obtained by the method described in [6]. The parameters $a_1 \ldots a_5$ (the constants of Garcia-Lopez Model [8]) and L_{real} - the equivalent path length [8] can be found using the methodology described in [5].

For a rainy user i to receive adequate power from beam j, it is sufficient to satisfy $P_{s,j} \geq P_{thr}/h_{ij}(t)$. This means that if power is allocated to beam j such that user i^* with channel gain $h_{i^*j}(t)$ is served then every user i covered by this beam with channel gain $h_{ij}(t) > h_{i^*j}(t)$ is automatically served. As $A_{r,ij}(t)$ is a positive quantity, from (3) $h_{ij}(t)$ is always less than $h_{ij,cs}$ and thus serving any rainy user will ensure that all the clear sky users corresponding to that beam are automatically served.

Let m ($m \leq M$) be the total number of beams that serve rainy users. Without loss of generality, we assume them to be $1, 2, \ldots m$ with m being the last rainy beam. Our aim is to serve the maximum possible number of users. Let us define the optimal solution to the power allocation problem as the power allocation $\overline{P} = (P_{s,1}, P_{s,2}, \ldots, P_{s,M})$ to the beams $1, 2, \ldots M$ which serves maximum number of users. The problem can be mathematically stated as: we have to search for the values of $P_{s,j}$ for each $j = 1, 2, \ldots, M$ that maximize the function:

$$J(\overline{P}) \triangleq \sum_{i=1}^{N} I(P_{r,i} \geq P_{thr}) \tag{7}$$

subject to

$$\sum_{j=1}^{M} P_{s,j} \leq P_{avail} \tag{8}$$

where P_{avail} is the total available power and $I(P_{r,i} \geq P_{thr})$ is the indicator function i.e. it is unity if the expression in parenthesis holds and zero otherwise. Equation (7) provides the total number of users that get adequate power and hence can be considered as served. The optimization problem stated above is not convex. This can be shown using a simple counter example. Let us convert the optimization problem into a minimization problem by considering $-J(\overline{P})$. Consider a satellite with $P_{avail} = P$, having only two beams each serving 30 users, with 10 users having power requirement less than or equal to $P/2$ and 20 users between $P/2$ and P. It can be seen that for $\overline{P_1} = (P, 0)$, $\overline{P_2} = (0, P)$ the condition for convexity $J(\,\theta\overline{P_1} + (1-\theta)\,\overline{P_2}) \geq \theta\, J(\overline{P_1}) \,+\, (1-\theta)\, J(\overline{P_2})$ is violated for $\theta = \frac{1}{2}$, as LHS=20 and RHS=30.

III. Exhaustive And Greedy Approaches

In the exhaustive search, all the rainy users that can be served by a beam are sorted in increasing order of their power requirement. Then we select one user from each of the sorted sequences. For each combination we assign powers to the corresponding beams just sufficient to serve the users in the combination. This process is continued till all the possible combinations are exhausted. The combination which satisfies the maximum number of users and also satisfies the power constraint (8) is the optimal solution. Thus exhaustive search can provide a solution to the optimization problem defined in (7) and (8) but its computational complexity will be $\mathcal{O}((\frac{N}{M})^m)$ which is very high as $N \gg M$.

The algorithm in [1] employs a greedy approach. The greedy approach forms a set by sorting all the users in increasing order of their power requirement without any consideration of the beam from which they are being served. Then using an iterative process the users with minimum power requirements are removed from the set one by one and their power requirement is fulfilled by allocating the required power to the corresponding beams. The algorithm terminates at the iteration at which the power constraint (8) gets violated. The major drawbacks of this algorithm can be illustrated with a simple example. Let there be only two beams serving users with power requirements {50,60,. . . ,120} and {65,75,. . . ,135}. Let the total available power be 120 units. The sorted set will be formed as {50,60,65,70,75,. . . ,135}. Now the greedy approach will serve only the first two users in the set both of which correspond to beam 1 and 60 units of power will remain unallocated. Thus it does not consider the fact that serving the user with power requirement 70 of beam 1 after having served the one with 60 from the same beam requires only 10 units of extra power, while serving the user with power requirement 65 from beam 2 requires 65 units of extra power. As a result the greedy approach will not serve users with power requirement 70,80,. . . 120 while they can be actually served since the total available power is 120 units.

IV. Proposed Algorithm

Instead of doing an exhaustive search we propose an intelligent search by forming groups of users (within a rainy beam) with similar power requirement and then, unlike Section III, sorting the groups (instead of users) in increasing order of their power requirement. This has to be done for each rainy beam. The power requirement of a group is defined as the minimum power required to serve the entire group. Now, just like we had users in exhaustive search, here we have groups. As the number of groups are less than the number of users, the number of combinations of groups are also less than number of combinations of users. In this framework, we have to divide the total power into M parts (need not be equal) such that each part is just sufficient to fulfill the requirement of an integral number of groups for each beam. This can be done in many ways but the number of combinations are significantly reduced. We check only these combinations to find out the one which serves the maximum number of users. Now we will define some variables to enable a mathematical presentation of the proposed algorithm.

Let $P_{j,min}$ and $P_{j,max}$ denote the minimum and the maximum power required by any rainy user from beam j. Thus, $P_{j,min}$ and $P_{j,max}$ are given by $min(P_{thr}/h_{i,j})$ and $max(P_{thr}/h_{i,j})$ respectively, $\forall i$ served by beam j. Let K be the number of groups for each beam and define d_j as:

$$d_j = (P_{j,max} - P_{j,min})/K \tag{9}$$

A particular group of rainy users $g_{q_j,j}$ for beam j is a group of rainy users having power requirement in the range $P_{j,min} + (q_j - 1)d_j$ to $P_{j,min} + q_j d_j$, $q_j \in \{1, 2, \ldots K\}$. Thus, the power allocation to the beam can take only discrete values of the form $P_{j,min} + q_j d_j$, $q_j \in \{1, 2, \ldots K\}$. This will ensure that all the users in the groups $g_{1,j}, g_{2,j}, \ldots g_{q_j,j}$ are served. Let $\Omega_j = \{P_{j,min} + d_j, P_{j,min} + 2d_j, \ldots P_{j,max}\}$.

As we said earlier, the total available power is to be divided into M parts. This power division should be such that every rainy beam is able to get at least clear sky power as we are assuming that whatever be the condition all the clear sky users of all the beams should be served. This entire process of power division is to be done iteratively to consider all the possible combinations of q_j's and choose the one which serves maximum number of users.

Algorithm

Step 1: Determine $h_{min,j,cs}$ where $h_{min,j,cs}$ is the minimum over all the channel gains of clear sky users served by beam j.

Step 2: Evaluate $P_{s,j,cs} = P_{thr}/h_{min,j,cs}\ \forall j$

Step 3: Compute all the channel gains h_{ij} using (2) and (3).

Step 4: Evaluate $P_{j,min}$ and $P_{j,max}$ $\forall j$ serving rainy users.

Step 5: Choose a value of K.

Step 6: Evaluate d_j $\forall j$ serving rainy users.

Step 7: Evaluate Ω_j $\forall j$ serving rainy users..

Step 8: Find W, where W is a set of vectors such that each vector represents a possible power allocation to the M beams. The k^{th} vector is denoted as $\overline{w_k} = (w_{k1}, w_{k2}, w_{k3}, \dots w_{kM})$ where

1. $w_{kj} \in \Omega_j \cup \{P_{s,j,cs}\} \forall j$ serving rainy users and $j \neq m$.
2. $w_{kj} = P_{s,j,cs}$ $\forall j$ not serving any rainy user, within the constraint $\sum_{j=1,j\neq m}^{M} w_{kj} \leq P_{avail} - P_{m,cls}$.
3. $w_{k,m} = P_{avail} - \sum_{j=1,j\neq m}^{M} w_{kj}$.

Step 9: Find $J(\overline{w_k})$ $\forall$ $\overline{w_k} \in W$.

Step 10: Find $\overline{P} = (P_{s1}, P_{s2}, P_{s3}, \dots, P_{sM}) \in W$ such that $J(\overline{P})$ is maximum over all $J(\overline{w_k})$ $\forall$ $\overline{w_k} \in W$.

Now let us discuss how the points 1 - 3 listed in Step 8 above help us in reducing the cardinality of set W, thereby reducing the search space and hence the computational complexity. The points 1 and 2 reduce the cardinality of set W by restricting to only those power allocations which do not violate the power constraint (8). The point 3 allocates the remaining power, if any, to the last rainy beam so that only those power allocations are included in the set W in which the total available power gets exhausted among the beams. This can be explained as follows. Suppose there exists a power allocation which serves the maximum possible users but does not exhaust the total available power, then the remaining power can be allocated to any of the beams without decreasing the number of served users. So, the proposed algorithm restricts its search to only those allocations in which the total available power gets exhausted among the beams and thus tries to reduce the computational complexity as compared to exhaustive search. The computational complexity is further reduced with the help of group formation among the beams.

V. Simulation Results

In this section, we present simulation results for the geostationary satellite Hellas Sat-2 (39^oE) operating at 12 GHz, having 16 beams. The coverage area of the satellite was taken to be from 34^oN to 59.5^oN in latitude and from 14^oE to 39.5^oE in longitude. Each of the 16 beams was supposed to serve a span of 6.375^oN in latitude and 6.375^oE in longitude. The users were assumed to be uniformly distributed with separation of 0.5^o both in latitude and longitude resulting in a total of 2704 users. Six of the sixteen beams were chosen to be rainy and the remaining ten were assumed to serve clear sky users. The number of users per beam turned out to be 169. Thus, there were a total of 1014 rainy users and 1690 clear sky users. For the simulation of the stochastic model described in (4) the parameter values were taken to be same as [4]: $A_m = 0.3524$, $S_\alpha = 1.1401$, $dA = 3.53 \times 10^{-4} sec^{-1}$. The white noise stochastic process $n(t)$ in (4) is taken to be Gaussian with zero mean and unity variance. The boundary condition $A_{r,ij}(t = 0)$ for the stochastic model (4) was evaluated using equations (5)-(6) and the corresponding values of point rainfall rate R_{ij} were obtained using ITU-R rainmaps [7]. The parameter a was obtained by the method described in [6] and came out to be $a = 0.0243$ while the value of the parameter b was taken to be $b = 1$ as suggested by [3]. The parameters $a_1 \dots a_5$ were obtained using [5] and came out to be $a_1 = 0.53, a_2 = 23.76, a_3 = -35.76, a_4 = 307, a_5 = 8000$ while L_{real} was latitude dependent and was found using the methodology described in [5]. The power threshold P_{thr} for every receiver was assumed to be -110 dBm and the clear sky attenuation $A_{c,ij}$ (2) was assumed to be same for all the users. Power allocation is done in such a way that all the clear sky users are served in each case. Hence we will compare only the number of rainy users served.

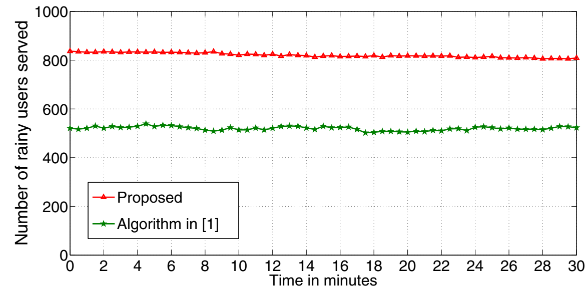

Fig. 1. Number of rainy users served over time for both the approaches.

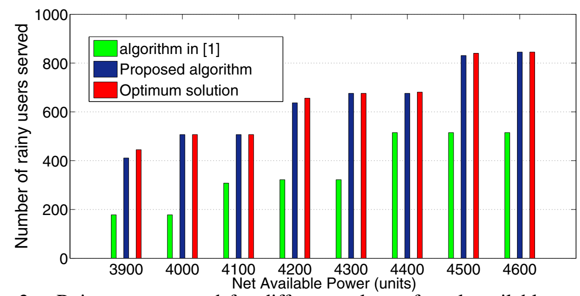

Fig. 2. Rainy users served for different values of total available power.

Since the rain attenuation is time varying, the power requirement of the users are continuously changing with time. Fig. 1 shows the number of rainy users served over a period of 30 minutes with the beam powers configured at the starting of the period i.e. t=0 and then kept constant for the entire 30 minute interval. The rain attenuation is simulated using the stochastic model (4)-(6). The total available power is assumed to be 4600 units and the value of K is chosen to be 12. It can be observed that for both the algorithms there is very little change in the number of rainy users served over this entire period.

Fig. 2 shows the number of rainy users served for the algorithm in [1], proposed algorithm and the maximum number of users (optimum solution) that can be served, for several different values of net available power. The number of groups i.e. K has been taken to be 12. We observe that in all the cases the number of rainy users served by the proposed algorithm are significantly more than the algorithm in [1].

Fig. 3 shows the number of rainy users served for different values of K ranging from 1 to 32. In each case the number of rainy users served by the proposed algorithm is greater than the number of rainy users served by the algorithm in [1]. As the value of K increases the number of rainy users

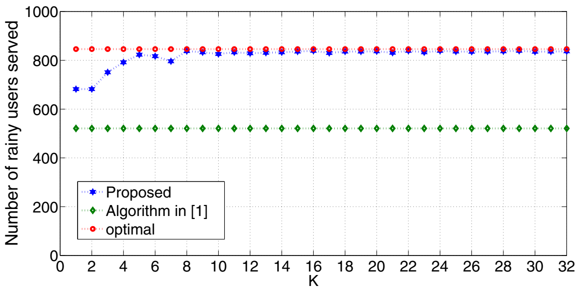

Fig. 3. Rainy users served out of 1014 for different values of K.

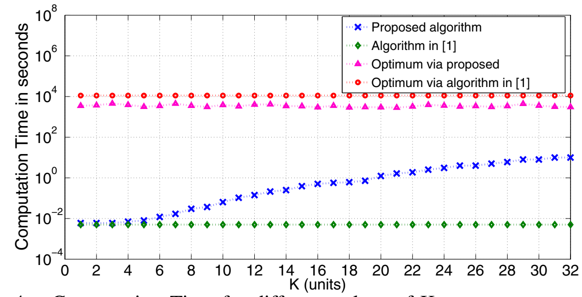

Fig. 4. Computation Time for different values of K.

served approaches the maximum number of users that can be served. However, it may be noted that the increase is not necessarily monotonic except for the following situation. Let us compare two cases - when the number of groups are K and nK i.e. integral multiple of K. The latter can be interpreted as subdividing groups and hence include all the possible power allocations as in the former. Thus, the number of users served in the latter case will be at least the number of users served in the former or even higher. This can also be seen to hold in Fig. 3, by say, comparing the case $K = 3$ with any integral multiple of 3 i.e. $K = 6$, $K = 9$, $K = 12$ etc for which the number of users served is higher than for $K = 3$.

Let us construct an illustrative, even if unrealistic, scenario to understand why the number of users served by the proposed algorithm is more. Thus consider a hypothetical case of only two beams with 100 rainy users and zero clear sky users to be served by each beam. Let the power requirements of the users from the first beam (i.e. $P_{thr}/h_{i,1}$) be 200, 202, 204,...398 while the power requirements of the users from the second beam (i.e. $P_{thr}/h_{i,2}$) be 207, 209, 211,...405 and let the total available power be 400. Then according to the algorithm in [1] the channel gains $h_{i,j}$ will be sorted in decreasing order. As P_{thr} is the same for all, this is equivalent to sorting $P_{thr}/h_{i,j}$ in increasing order. For this example, power requirements in increasing order will be $\{200, 202, 204, 206, 207, 208, 209 \ldots 398, 399, 401, 403, 405\}$. The algorithm in [1] will be able to serve only the first four users because the fifth user with 207 power requirement is from the second beam, and hence if it has to be served, 206+207 i.e. 413 units of total power will be required while only 400 units is available. Now, for the same case let us examine the proposed algorithm with $K = 2$. The groups formed within the first beam will be $\{200, 202, ...298\}$ and $\{300, 302, ...398\}$, each having 50 users. The groups formed in the second beam will be $\{207, 209.....305\}$ and $\{307, 309, ...405\}$, having 50 users each again. The proposed algorithm will serve both the groups of the first beam and thus will end up serving the maximum number of users possible i.e. 100 users which is also the optimum solution.

Fig. 4 shows the computation time of the algorithm in [1] and the proposed algorithm for different values of K. Since K is defined only for the proposed algorithm, the computation time of the algorithm in [1] is independent of K and hence is plotted as a horizontal line in Fig. 4. As the value of K increases, the computation time of the proposed algorithm increases. We would also like to compare this time with the time required for the optimal solution. The optimal solution can be found by taking the solution of the proposed algorithm or the algorithm in [1] as the starting point and searching for more users that can be served within the same power constraint. These curves have been plotted as "optimum via proposed" and "optimum via algorithm in [1]" in Fig. 4. It can be seen from Fig. 4 that the time required for the optimal solution (via proposed or [1]) is very large compared to that of the proposed algorithm. It can be clearly observed from Fig. 2 and 3 that the number of users served is very close to the optimal solution even for small values of K when the time complexity is not so high, say K = 8, 10, 12 etc.

VI. Conclusion

In this letter we proposed a beam power allocation algorithm by forming groups of users with similar power requirement. It is capable of serving more number of users compared to a recently proposed algorithm. The algorithm is flexible in the sense that its runtime can be varied over a large range to make the best use of the available computational resources.

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