Flexural Design of Reinforced Concrete Frames Using a Genetic Algorithm

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Abstract: A design procedure implementing a genetic algorithm is developed for discrete optimization of reinforced concrete frames (*RC-GA*). The design procedure conforms to the American Concrete Institute (ACI) Building Code and Commentary. The objective of the *RC-GA* procedure is to minimize the material and construction costs of reinforced concrete structural elements subjected to serviceability and strength requirements described by the ACI Code. Examples are presented demonstrating the efficiency of the *RC-GA* procedure for the flexural design of simply-supported beams, uniaxial columns, and multi-story frames.

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Introduction

Reinforced concrete structures have considerable compressive strength compared to most other materials. In addition to the high compressive strength, reinforced concrete structures are durable, versatile, and have relatively low maintenance cost when compared to steel structures. They also provide good resistance against fire and water damage, and have an excellent potential for a long service life (McCormac 1998).

Material cost is an important issue in designing and constructing reinforced concrete structures. The main factors affecting cost are the amount of concrete and steel reinforcement required. It is, therefore, desirable to make reinforced concrete structures lighter, while still fulfilling serviceability and strength requirements. In addition to material costs, labor and formwork costs are significant. The formwork cost is usually expressed in cost per unit area, but the labor can be difficult to estimate. It is common to combine the labor and the formwork cost to obtain a reasonable estimate of the total construction cost of the structure.

The objective of this research is to design low-cost reinforced concrete frames that satisfy the limitations and specifications of the American Concrete Institute (ACI) Building Code and Commentary using a genetic algorithm (GA). Many researchers have applied traditional optimization techniques to the design of reinforced concrete structures. Krishnamoorthy and Munro (1979) and Krishnamoorthy and Rajeev (1989) used linear programming techniques to optimize reinforced concrete frames. Hoit et al. (1991) designed low-weight two-dimensional frames using the method of augmented Lagrangian multipliers and nonlinear programming techniques. Chung and Sun (1994) used sequential linear programming and the gradient projection method to optimize reinforced concrete beams with a nonlinear material response. Adamu et al. (1994) used a continuum-type optimality criteria method to minimize the cost of reinforced concrete beams. Zielinski et al. (1995) used an internal penalty function algorithm to optimize reinforced concrete short-tied columns. Fadaee and Grierson (1996) optimized the cost of three-dimensional skeletal structures using the optimality criteria. Val et al. (1996) used several iterative methods to evaluate the reliability of reinforced concrete frames. Balling and Yao (1997) optimized three-dimensional frames using a multilevel method that separated the problem into a system optimization problem and a series of individual member optimization problems. More recently, Rajeev and Krishnamoorthy (1998) applied a simple genetic algorithm (SGA) to the cost optimization of two-dimensional frames.

The design of reinforced concrete structures is based upon the restrictions and guidelines found in the ACI Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (ACI 318R-99). The ACI Code provides specifications and information required to design reinforced concrete structures, factors for applied loads, strength reduction factors, and restrictions on the placement of reinforcement.

Goldberg (1986, 1989) is one of the first researchers to use a GA to solve engineering optimization problems. Based on Goldberg's work many other researchers applied GAs to a variety of structural design problems. Jenkins (1991) developed a GA to optimize plane steel frames. Rajeev and Krishnamoorthy (1992) expanded the application of GAs into discrete design variables in obtaining the minimum weight of trusses subjected to stress constraints. Koumousis and Georgiou (1994) used a GA to optimize the mixed layout and sizing problem of a typical steel roof. Adeli and Cheng (1993, 1994) introduced an augmented Lagrangian multiplier into a GA to optimize space trusses. Rajan (1995) developed a GA to optimize truss structures that are subjected to sizing, shape, and topology constraints. Pezeshk and Camp (2001) compiled a comprehensive review of research related to structural optimization using GAs for steel frames.

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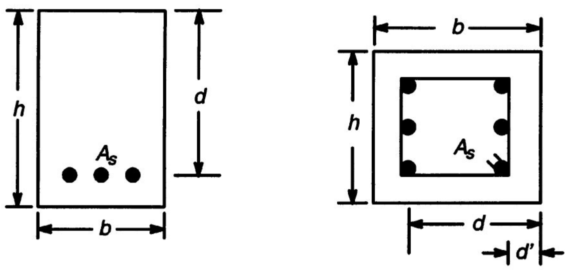

Fig. 1. Typical geometry of reinforced concrete beam and column sections

Structural Optimization

The basic statement in structural optimization problems is the objective function. For reinforced concrete structures, cost is a function of the structural material weight and the formwork. A general statement for the objective function in terms of the properties of both the structure as a whole and the individual structural member is

$$F = f(p_m, p_j, p_s) \tag{1}$$

where F = objective function; p_m = material properties; p_j = connection characteristics; and p_s = structural characteristics.

Objective Function

The objective of this study is to design reinforced concrete frames that minimize the structural cost. Fig. 1 defines the basic geometry of both a reinforced concrete beam and a reinforced concrete column where b is the width of the beam or column section, h is the thickness of the beam or column section, and A_s is the area of the steel reinforcing.

In this study, the design variables are the width of the section, b, the thickness of the section, h, the reinforcing steel bar number, and the number of bars or topology of the reinforcement. An advantage of using the rebar number as a design variable is that both the cross-sectional area and the diameter are intrinsic properties. In this case, values associated with a rebar number variable can be used to compute the total cross-sectional area of the steel reinforcement, A_s, the flexural capacity of a section, and to determine if a reinforcement pattern is consistent with design geometry. The reinforcement topology variable can define both the number and pattern of reinforcement bars within a section.

The mathematical form of the objective function for the design of reinforced concrete frames is

$$\text{Minimize} \quad F = f(p_m, p_j, p_s)$$

$$F = \sum_{\text{elements}} C_c \ell b h + C_s \ell A_s + 2 C_f \ell (b+h) \tag{2}$$

$$\text{Subject to} \quad c_1 \leqslant 0 \quad c_2 \leqslant 0 \quad \ldots \quad c_n \leqslant 0$$

where C_c = cost of the concrete per cubic foot; C_s = cost of steel per cubic foot; C_f = cost of the formwork per square foot (including labor); ℓ = length of the beam or column; and $c_1, c_2, \ldots, c_n$, are constraint functions based on the specifications and limitations of the ACI Code. A simple evaluation of the objective function defined in Eq. (2) reveals that for most cases, the costs of the reinforcing steel and the formwork contribute more to the structural cost estimate than the cost of the concrete. However, the combined costs associated with the geometry of the cross section are typically more significant than the cost of the steel reinforcing.

Penalized Objective Function

In engineering optimization problems, it is vital to satisfy performance constraints. The search strategy implemented in a GA considers the fitness of a solution and is unaffected by any violation of problem constraints. To introduce feasibility into fitness of a solution, penalty functions are used to account for constraints. To form an ‘‘unconstrained problem’’ on which the GA can be applied directly, penalty functions integrate constraints with the objective function.

Although there are many penalty function schemes proposed for structural design and optimization (Camp et al. 1998 and Foley and Schinler 2000), in this study, an implementation of linear and quadratic penalty functions is used to account for constraint violations. The general form of the penalty function is

$$\Phi = \prod_{i=1}^{n} (1 + c_i)^{k_i} \tag{3}$$

where Φ = penalty factor; n = total number of constraints; c_i = value that reflects the degree of violation of constraint i ($c_i > 0$); and k_i = exponential factor associated with constraint i. The objective function or structural cost of a particular design is penalized as follows:

$$F' = \Phi F \tag{4}$$

where F' = penalized objective function.

A structural analysis is performed to evaluate the objective function and determine the feasibility of a design. If there are no violations of constraints, the penalty factor is one. When one or more of the constraints are violated, the solution is infeasible to some degree, and the value of the corresponding objective function is meaningless. To retain some form of useful information from infeasible solutions, the value of the penalty factor is used to quantitatively describe the degree of constraint violation and to provide a relative measure of the solution’s performance.

Reinforced Concrete Frames

In general, reinforced concrete structures are designed and engineered to be durable, serviceable, and attractive. Structural elements composing a reinforced concrete system may be broadly classified into floor slabs, beams, columns, walls, and foundations (Nawy 1996). When designing a reinforced concrete frame, consideration of the interaction between beams and columns is vital to developing safe and cost-efficient structures.

Beam

Beams are generally defined as the structural elements that transmit tributory loads from floor slabs and walls to vertical supporting columns (Nawy 1996). Fig. 1 defines the geometry of a general rectangular singly reinforced concrete beam where d is the effective depth to the tensile steel reinforcement. Values of d and A_s are computed from the thickness of the section, the topology and amount reinforcing steel, and the amount of concrete cover.

Beam Columns

Columns are typically vertical elements that support the structural floor systems (Nawy 1996). Columns are to be subjected to both

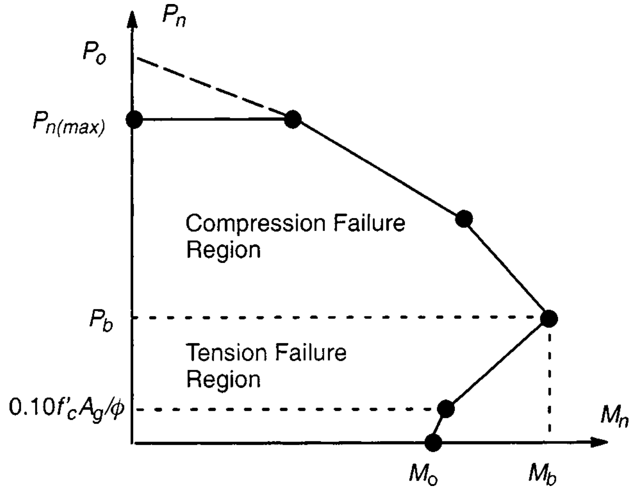

Fig. 2. Column strength interaction diagram

axial compressive loads (column action) and bending moments (beam action). In a multi-story rigid frames, lateral loads cause horizontal compression in each story. Beam-column elements combine beam action, involving bending and lateral torsional buckling, with the column action, which considers compression buckling.

There are many different kinds of columns: circular and square concrete sections with a steel tubing on the outside, circular and square spiral columns with steel reinforcement, and rectangular tied columns with steel reinforcement. Rectangular tied columns are common in construction of reinforced concrete building and are used in all designs presented in this study. Fig. 1 defines the geometry of a typical rectangular column where d' is the distance from the extreme compression fibers in the concrete to the centroid of the compression steel.

Column Strength Interaction Diagram

A column interaction diagram is used to determine the capacity and suitability of a column. To obtain a representation of the interaction diagram, important transitional points on the diagram are computed and connected using linear relationships. The strength capacity of the column is compared to the applied loading and moment. If the applied axial force and bending moment fall inside of the interaction diagram, the capacity of the column is satisfactory. Fig. 2 illustrates the shape and form of the beam-column strength interaction diagram used in this study.

The rectilinear approximation of the strength interaction diagram has three specific regions: the maximum axial compression region permitted by the ACI Code, the compression failure region, and the tension failure region. The maximum axial compression permitted by the ACI Code, $P_{n(\max)}$, is $0.8P_0$ for tied columns where P_0 = nominal axial load strength at zero eccentricity. The compression failure region is delineated from the tension failure region by the balanced strain condition (M_b, P_b). Within the compression failure region an immediate point is computed midway between P_b and $0.8P_0$. The tensile failure region consists of two portions. The upper portion is bounded by the balanced condition and the axial capacity $0.10f'_cA_g/\phi$ with $\phi = 0.7$. In lower portion of the tension failure region, the strength reduction factor varies $0.7 \leqslant \phi \leqslant 0.9$ with a lower bound of zero axial capacity.

Sway and NonSway Unbraced Frames with Beam Columns

Nonsway loads are uniformly distributed loads that cause little, or no, sway in the frame. These loads are typically dead and live loads. Lateral wind loads and earthquake loads, on the other hand, are capable of making the frame sway horizontally.

Frame Stability

The total moment, M, generated by the combination of sway and nonsway applied loads is

$$M = M_{ns} + \delta_s M_s \tag{5}$$

where M_{ns} = moment created by the nonsway loads; M_s = moment generated by the sway loads; and δ_s = moment magnification factor for frames not braced against sidesway.

A story within a structure can be treated as nonsway if

$$Q = \frac{\Sigma P_u \Delta_0}{V_u \ell_c} \leqslant 0.05 \tag{6}$$

where Q = stability index for a story; ΣP_u and V_u = total vertical load and the story shear, respectively, in the story in question; Δ_0 = first-order relative deflection between the top and bottom of that story due to V_u; and ℓ_c = length of the compression member in a frame, measured from center to center of the joints in the frame. If $Q \leqslant 0.05$, the sway moment will not be magnified and δ_s will be equal to one. If $Q > 0.05$, the moment magnification factor is

$$\delta_s = \frac{1}{1-Q} \tag{7}$$

Slenderness

When a column is considered slender, the moment will be magnified. The ACI Code states that for compression members not braced against sidesway, effects of slenderness may be neglected when

$$\frac{k\ell_u}{r} < 22 \tag{8}$$

where k = effective length factor for compression members; ℓ_u = unsupported length of a compression member; and r = radius of gyration. The effective length factor is a function of the stiffness at a joint in a frame. If the slenderness value is greater than 22, the moment has to be magnified. Eq. (3) is then applied to calculate the magnified sway moment.

The effective length factor, k, is dependent on the ratio of the compression members to the flexural members in a plane at the end of a compression member. The ratio, ψ, is

$$\psi = \frac{\Sigma(EI/\ell)_c}{\Sigma(EI/\ell)_g} \tag{9}$$

where I = moment of inertia of the section; E = modulus of elasticity; and c indicates the properties of a column, and g indicates the properties of a girder. Once ψ has been computed for each end of a column, the two values are averaged. The effective length factor can then be calculated based on the following criteria:

$$\psi_m < 2 \quad k = \frac{20-\psi_m}{20}\sqrt{1+\psi_m} \tag{10}$$

$$\psi_m \geqslant 2 \quad k = 0.9\sqrt{1+\psi_m} \tag{11}$$

where ψ_m = average of ψ values from each end of the compression member.

Beam and Column Weight

The total weight of the frame is a function of the length and shape of beams and columns. In the design of reinforced concrete frames the weight of the concrete itself is the most significant factor in determining the overall structural weight. However, in this study, the objective is to minimize the cost of reinforced concrete frames as defined by Eq. (2). In this case, the cost of the concrete material and the cost of the formwork are equally important and outweight the cost of the reinforcing steel.

Frame Analysis

To compute the fitness or the cost of each frame design, the variation of axial and shear force and bending moment in the frame are required. These structural response quantities are computed for each frame design using the finite element method. In general, the properties of the beams and columns for a particular design affect the structural response of an indeterminate frame. The moment of inertia of each element is defined as

$$I_{\text{beam}}=0.35I_g \quad \text{and} \quad I_{\text{column}}=0.7I_g \tag{12}$$

where I_g=moment of inertia of the gross section. The area for a beam or a column is defined as the gross area of the section. The ACI Code states that the modulus of elasticity can be taken as $57{,}000\sqrt{f'_c}$ psi. In addition, the weight of each structural element, which varies depending on the design geometry, is included in the dead load acting on the frame.

The frame analysis includes checks for column slenderness and moment amplification. If a column is found to be slender, the applied moment will be magnified as discussed earlier. The moment caused by the applied loading will also be magnified if the limit for the stability index for a story is violated.

ACI Code Beam Constraints

A reinforced concrete beam must have a structural capacity greater than the factored applied loading and meet specifications defined in the ACI Code. If the shear or moment capacity is below the required strength, the beam is penalized. In addition, the ACI Code has restrictions and limitations on the cross-sectional geometry of a beam and the position and quantity of steel reinforcement. Structural designs that do not satisfy the ACI Code have their fitness values (structural cost) penalized by an amount that quantitatively reflects the degree of constraint violation.

The basic form of the constraints, c_i, are

$$c_i=\begin{cases}0 & \text{if } m_i\leqslant 0\\ m_i & \text{if } m_i>0\end{cases} \tag{13}$$

where m_i=normalized degree of violation of constraint c_i.

Moment Capacity

The penalty for the normalized moment capacity is

$$m_1=\left|\frac{\phi M_n-M_u}{M_u}\right| \tag{14}$$

where M_u=maximum moment in the beam due to the applied loading; and ϕ=strength reduction factor ($\phi=0.9$ for flexure). The moment capacity of a singly reinforced and rectangular beam section, M_n, is defined as

$$M_n=A_s f_y\left(d-\frac{a}{2}\right) \tag{15}$$

where a=depth of the equivalent rectangular stress block

$$a=\frac{A_s f_y}{0.85 f'_c b} \tag{16}$$

and f_y=strength of the steel reinforcement; and f'_c=compressive strength of concrete.

If the cross-sectional geometry and the reinforcing steel of a given design require the depth of the equivalent rectangular stress block, a, to be greater than the effective depth of the beam, d, the moment capacity violates the assumptions inherent in Eq. (15). In such a case, the normalized moment capacity penalty is set to a relatively large value ($m_1=10$).

For simplicity, the computation of the moment capacity of a beam is separated into two stages. First, the maximum positive moment due to the applied loading is compared to the moment capacity of the section considering only the reinforcement at the bottom of the section. The constraints, c_i, are computed to account for any violations. Second, if required, the maximum negative moment is compared to the capacity of the section considering only the reinforcement at the top of the section. The constraints, c_i, are reevaluated to reflect any infeasibility in the design. This approach provides a conservative analysis and is simple to implement, especially for large framed structures.

Shear Capacity

The shear strength of a beam is computed and compared to the maximum shear caused by the applied loading. The penalty for the normalized shear capacity is

$$m_2=\frac{M_{\max}-V_u}{V_u} \tag{17}$$

V_u=shear capacity of the beam section due to the loading and where $V_{\max}$ is defined as

$$V_{\max}=\left(1.9f'_c+2500\varrho\frac{V_u d}{M_u}\right)bd\leqslant 3.5\sqrt{f'_c}bd \quad \text{where } \frac{V_u d}{M_u}\leqslant 1 \tag{18}$$

and ϱ=ratio of nonprestressed reinforcement, given as

$$\varrho=\frac{A_s}{bd} \tag{19}$$

Minimum Reinforcement Ratio

According to the ACI Code, the minimum amount of reinforcement placed in a beam is:

$$\varrho_{\min}=3\frac{\sqrt{f'_c}}{f_y} \quad \text{and} \quad \varrho_{\min}\geqslant\frac{200}{f_y} \tag{20}$$

If ϱ is less than $\varrho_{\min}$, the minimum reinforcement penalty is

$$m_3=\varrho_{\min}-\varrho \tag{21}$$

Table 1. Minimum Thickness of Beams

Support condition	$h_{\min}$
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilevered	$\ell/8$

Note: Span length, ℓ, is in inches.

Table 2. Ordered Steel Areas

No.	Reinforcement bar combinations	A_s (in.^{2})
1	3	0.1105
2	4	0.1963
3	3,3	0.2210
4	5	0.3068
...	...	...
33	9,9,9,9	4.000
34	11,11,11	4.6845
35	10,10,10,10	5.0672
36	11,11,11,11	6.2460

Maximum Reinforcement Ratio

To insure that the beam will fail in tension, an upper limit on the reinforcement ratio is defined as

$$\varrho_b=\frac{0.85\beta_1 f_c'}{f_y}\left(\frac{87{,}000}{87{,}000+f_y}\right) \tag{22}$$

where β_1 is

$$\beta_1=0.85-0.05\left(\frac{f_c'-4{,}000}{1{,}000}\right) \tag{23}$$

If $f_c'<4{,}000$ psi, $\beta_1=0.85$ and if $f_c'>8{,}000$ psi, $\beta_1=0.65$. The amount of steel placed in the beam is not allowed to exceed $0.75\varrho_b$. The maximum reinforcement penalty is

$$m_4=\varrho-0.75\varrho_b \tag{24}$$

Minimum Thickness

Since the deflection of a beam is a function of the loading and the time when the loading is applied, it can be difficult to determine an accurate beam deflection. The ACI Code specifies a minimum thickness for nonprestressed beams for various support conditions. Table 1 lists values of the minimum thickness for the different support conditions.

If the thickness of the beam is less than the allowable thickness, $h_{\min}$ the beam thickness penalty is

$$m_5=\frac{h_{\min}-h}{h_{\min}} \tag{25}$$

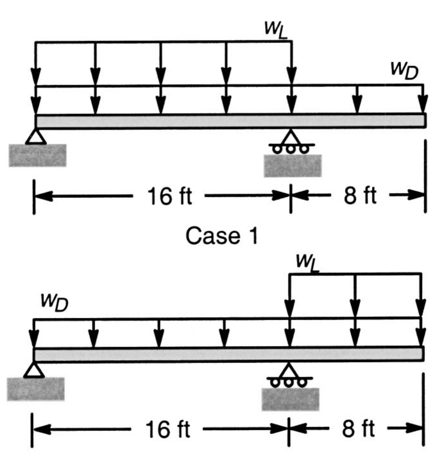

Fig. 3. Reinforced concrete beam loading cases

Table 3. Feasible Beam Cross Sections

Index No.	b (in.)	h (in.)	Bar combination (Table 2)	Moment (k in.)
1	8	18	6	393.9
2	9	18	6	395.5
3	8	19	6	417.9
...	...	...	...	...
3,376	19	33	36	10,363.9
3,377	20	33	36	10,436.2
3,378	21	33	36	10,501.6

Minimum/Maximum Width

The ACI Code specifies the allowable spacing in a beam. The minimum clear spacing between parallel bars in a layer should be the diameter of the reinforcing bar, but not less than 1 in. Beam sections may contain reinforcement with different bar sizes; the biggest bar size in a row is used in computing the minimum clear space. The minimum width of the beam $b_{\min}$ is

$$b_{\min}=2x_c+2x_t+d_c(n_b-1)+t_d \tag{26}$$

where x_c=specified clear cover for reinforcement, x_t=diameter of the tie reinforcement, d_c=diameter of the largest bar (not less than 1 in.), n_b=total number of bars in the row; and t_d= total width (the sum of all the bar diameters in the row). The minimum beam width penalty is

$$m_6=\frac{b_{\min}-b}{b_{\min}} \tag{27}$$

The maximum allowable width of the beam is limited to the thickness of the beam section $b_{\max}=h$. The maximum width penalty is

$$m_7=\frac{b-h}{h} \tag{28}$$

Maximum Thickness

It is also common practice in design of reinforced concrete beams to fix the maximum ratio of the thickness to the width of the beam. Typically, $h_{\max}/b$ varies from 2 to 3. In this study, an average value of $h_{\max}/b=2.5$ is used. The maximum thickness penalty is

$$m_8=\frac{h-h_{\max}}{h_{\max}} \tag{29}$$

where $h_{\max}$=maximum allowable thickness ($h_{\max}=2.5b$).

Spacing Limits on Reinforcement

The ACI Code has a series of spacing limits for the placement of steel reinforcing. The difference in bar sizes within a single row of reinforcement should not exceed two bar sizes. The horizontal bar difference penalty is

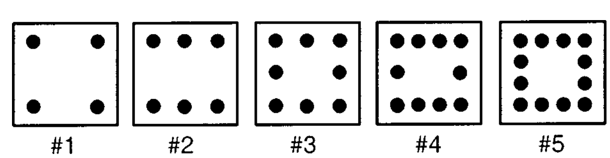

Fig. 4. Possible topologies for column design

Table 4. Design Properties for Short-Tied Column

Design example	d' (in.)	f_y (psi)	f'_c (psi)	P_f (lb)	M_f (ft-lb)
1	2.56	58,015	3,626	553,030	326,740
2	2.76	58,015	4,351	400,160	266,997
3	2.95	58,015	4,351	449,618	414,510

$$m_9 = \text{bar size}_{\max} - \text{bar size}_{\min} \tag{30}$$

Vertical spacing limitations must be considered for beams with more than two rows of tensile or compressive reinforcement. If the difference between the largest bar in the outer row and the smallest bar in the inner row exceeds two bar sizes, the vertical bar difference penalty is $m_{10}=1$.

In addition, if the difference between the largest bar in the inner row and the smallest bar in the outer row exceeds two bar sizes, the vertical bar sizing penalty is $m_{10}=2$.

If two rows of reinforcing are used, it is common practice not to let the total area of the inner row be greater than the outer row. The double row spacing penalty is

$$m_{11} = A_{s(\text{inner row})} - A_{s(\text{outer row})} \tag{31}$$

As an additional check, if the difference between the largest bar in the inner row and the largest bar in the outer row is greater than zero the bar size penalty is $m_{11}=1$.

ACI Code Column Constraints

A reinforced concrete column must have sufficient structural capacity to withstand the combined effect of axial force and bending moment while meeting specifications defined in the ACI Code for the position and quantity of steel reinforcement. Beam-column designs that do not satisfy the ACI Code have their fitness values (structural cost) penalized by an amount that quantitatively reflects the degree of constraint violation.

Table 5. Design Results for Short-Tied Column Examples

Design example	Design variable	Zielinski et al. (1995)	*RC-GA* designs
	b (in.)	15.58	8.5
	h (in.)	26.91	29.5
1	A_s (in.^{2})	4.26	4.00
	ϱ	1.02	1.60
	Cost ($/ft)	40.64	34.31
	b (in.)	12.58	12
	h (in.)	23.37	25
2	A_s (in.^{2})	4.00	3.14
	ϱ	1.36	1.05
	Cost ($/ft)	37.14	32.18
	b (in.)	18.66	12
	h (in.)	19.65	29.5
3	A_s (in.^{2})	11.35	4.00
	ϱ	3.09	1.13
	Cost ($/in.)	60.78	38.01

Load-Moment Interaction Diagram

A load-moment interaction diagram, as shown in Fig. 2, is constructed for each column in a frame. If the axial force and the bending moment for a column fall inside the load-moment interaction diagram, the designed beam column is feasible. If not, the beam column is not adequate and is penalized. The load-moment interaction diagram penalty is

$$c_{12} = \begin{cases} 0 & \text{Load-Moment Interaction Satisfied} \\ m_{12} & \text{Load-Moment Interaction Not Satisfied} \end{cases} \tag{32}$$

where m_{12}=measure of the degree of violation from the load-moment interaction diagram. The capacity of a column is checked to determine if it lies within the load-moment diagram using the residue theorem technique (Gipson 1986; Camp and Gipson 1990). The load-moment interaction penalty is

$$m_{12} = \frac{r_1}{r_0} - 1 \tag{33}$$

where r_1=radial distance from the load-moment combination to the origin of the interaction diagram; and r_0=radial distance from the intersection of the r_1 vector and the load-moment curve to the origin.

Reinforcement Ratio

The ACI Code has limits for reinforcement of compression members, and states that the area of longitudinal reinforcement shall not be less than 1%, nor more than 8% of the gross or total area of the section. The reinforcement ration penalty is

$$c_{13} = \begin{cases} 0.01 - \varrho_g & \text{if } \varrho_g \leqslant 0.01 \\ \varrho_g - 0.08 & \text{if } \varrho_g > 0.08 \end{cases} \quad \varrho_g = \frac{A_s}{A_g} \tag{34}$$

Spacing Limits on Reinforcement

The ACI Code states that in tied reinforced compression members, the clear distance between longitudinal bars shall not be less than 1.5 times bar diameter or 1.5 in. The longitudinal bar spacing penalty is

$$m_{14} = \frac{d_{\min} - d_c}{d_{\min}} \tag{35}$$

where $d_{\min}$=allowable clear spacing between the bars according to the ACI Code; and d_c=spacing between the longitudinal reinforcement in a particular column design.

Genetic Algorithm

The GA used in this study is a modified version of a program originally developed by David Carroll at the University of Illinois. The source code for the GA driver is free for public use and is available over the Internet (Carroll, 1997). Carroll's program is a *FORTRAN* version of a GA driver and can be used for a variety of different problems by simply designing an encoding scheme and supplying routines for estimating the fitness of a individual solution. The main advantage of using the GA drive system is modularity and code reuse. New options can be added to the GA portion of the program with little or no modifications to the fitness evaluation routines.

The GA driver initializes a random sample of individual solutions upon initiation of the algorithm. The GA driver uses binary coding for individual solutions in the population. A modified version of the GA driver has two strategies for choosing solution pairs for mating: tournament selection with a shuffling technique and a partitioning scheme (Camp et al. 1998). There are several crossover techniques in the modified version of the GA driver: single, double, triple point or uniform. In addition, there is an option to randomly vary the crossover method at each application. Reproduction allows for the generation of either a single child or two children from each set of parent solutions. In addition, there is a concurrent option for an elitist operation that guarantees the survival of the best solution into the next generation. Mutation may be applied to either the genotype (jump mutation) or the phenotype (creep mutation). Additional features include: a niching (sharing) operator and an option for the number of children generated per pair of parents. Each operator is designed to either enhance the convergence properties or to slow the process to ensure adequate exploration of the design search space.

The reinforced concrete design computer program (*RC-GA*) integrates a finite element analysis of reinforced concrete structures, accounting for ACI Code specifications and limitations, with Carroll's GA driver. The design variables in *RC-GA* are:

1. Cross-sectional dimensions (h and b);
2. Reinforcement bar number;
3. The number of reinforcing bars per row; and
4. The number of rows of reinforcing bars.

Carroll's GA driver requires a value for the fitness of each solution in the search population. To evaluate the fitness, the structural capacity (shear and moment) of each design is determined using a finite element analysis. The response of the structure is based on section properties, reinforcement configuration, and loading cases. The value of the objective function, defined in Eq. (2), approximates the cost of the structure. If the design is infeasible, as prescribed by the ACI Code, the degree to which constraints are violated is estimated using Eq. (13). The ACI specifications and limitations are checked and the product of any constraint violations, given in Eq. (3), is used to penalize structural cost, as defined in Eq. (4). The resulting penalized cost is the fitness of the design.

Beam Design Examples

Two examples are presented to demonstrate the effectiveness and efficiency of designing reinforced concrete structures using a GA. The first example considers a simply supported beam with one row of tensile reinforcement. The second example is a simply supported beam with a cantilevered section, inducing negative moments in addition to positive moments.

In these examples, the maximum number of bars in a row is limited to four. In addition, bars in a row will be the same size. Based on these values, the smallest area of steel is one ♯3 bar and the maximum area is four ♯11 bars. Table 2 lists values of the steel reinforcing variables for all 36 possible bar combinations for a row (based on increasing steel area).

The cost of concrete, steel, and formwork is estimated as $3.11/ft^3, $468/ft^3, and $2.50/ft^2, respectively (Zielinski et al. 1995). The unit weight of concrete and steel is approximately 145 lbs/ft^3 and 490 lbs/ft^3, respectively. All computations were performed on a Dell XPS 450 MHz computer running Microsoft FORTRAN Powerstation 4.0.

Unless otherwise indicated, the GA parameters for all the design examples are:

1. Population size 100;
2. Partitioning selection scheme with a partition point of 0.5 and a probability of 0.7;
3. Uniform crossover with a probability of 0.5 to 0.7;
4. Jump mutation with a probability of 0.01; and
5. Two children solutions from each parent set.

The exponential values k_i, defined in Eq. (3), for all the examples are

$$k_{1-8}=2 \quad k_{9-14}=1 \tag{36}$$

Simply Supported Beam with One Row of Reinforcing Steel

Consider a simply supported beam loaded by a factored distributed vertical load of 9.6 k/ft which includes an estimate of the beam weight (McCormac 1998). The strength of concrete, $f_c' = 4{,}000$ psi, and the yield strength of steel, $f_y = 60{,}000$ psi. This design utilizes a single row of steel reinforcement. The ranges of the beam dimensions are: $10 \leqslant b \leqslant 25$ inches and $20 \leqslant h \leqslant 35$ inches. The search increment for both the beam width and thickness is 1 inch.

The design presented by McCormac (1998), is a beam section with $b=18$ in., $h=29$ in., four ♯11 bars, and an estimated cost of $1,125.44. An exhaustive search of all 9,216 possible solutions requires about 25 s of computing time and results in a beam section with $b=13$ in., $h=32$ in., four ♯10 bars, and a structural cost of $972.46. In designs developed by the *RC-GA* program, over 100 generations require about the same amount of computing time as the exhaustive search. However, on average, the minimum cost design of $972.46 is found after 13 generations. Over a number of different runs, the *RC-GA* program found the minimum cost design in as few as five generations or as many as 37 generations.

Simply Supported Beam with Compression and Tension Steel

Fig. 3 shows a beam where tensile and compressive reinforcing steel is required (McCormac 1998). A factored uniformly distributed dead load, including an estimate of the beam weight, of 2.8 k/ft is applied to the entire length of the beam. The live load is 5.1 k/ft and there are two cases of live load placement. The maximum negative and positive moments generated by the factored loading cases are used to design the beam. The maximum positive and negative moments are produced in the beam by loading cases 1 and 2, respectively. The beam design is limited to one row of reinforcing steel in tension and one row in compression. The ranges of values for the beam dimensions (in) are: $6 \leqslant b \leqslant 21$ and $18 \leqslant h \leqslant 33$. The search increment for both the beam width and thickness is 1 in. The strength of concrete, $f_c' = 3{,}000$ psi, and the yield strength of steel, $f_y = 60{,}000$ psi.

The *RC-GA* design procedure checks to determine if a design is feasible using a three-phase approach. First, a table is generated of feasible reinforced concrete beam and column sections that satisfy the restrictions and specifications of the ACI Code for reinforcement ratio, cross-sectional geometry, and reinforcement spacing within the prescribed variable limits. The remaining geometrically feasible beam sections are sorted based on the moment capacity of the section. Table 3 lists the ordered beam sections, as well as the corresponding steel pattern number as defined in Table 2. In the second phase, the absolute maximum moment in each beam section is determined and *RC-GA* uses selection, reproduction, and mutation operations to search the geometrically feasible

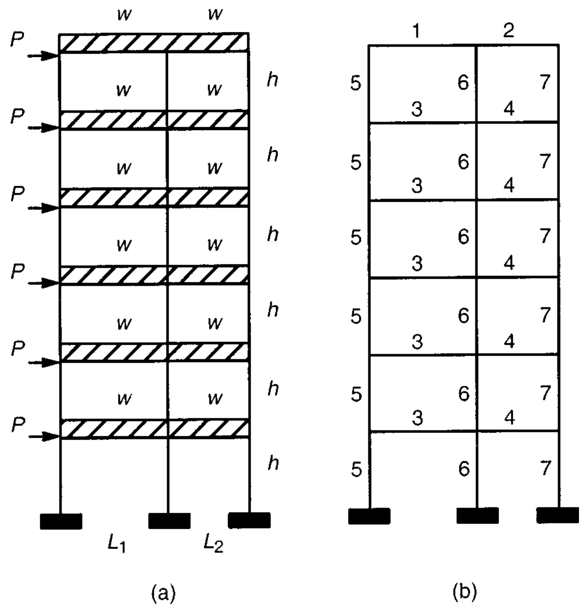

Fig. 5. Two-bay six-story frame: (a) geometry and loading; and (b) beam and column groups

beam space for a singly reinforced design that satisfies the required maximum moment capacity and minimizes the cost. Depending on the direction of the moment, the primary reinforcement is placed in the tension zone, either the bottom or top of the beam. In the final phase, the *RC-GA* procedure analyzes the best design and computes the required secondary reinforcement for each beam.

While generating a list of geometrically feasible beam elements can initially be a time-consuming task, the resulting element feasibility tables can be stored and reused to design other structures. Accumulation of heuristic design information on the feasibility of structural elements can improve the efficiency of the *RC-GA* procedure by focusing the search on the feasible design space. Feasible element tables can also provide a simple mapping scheme where a single value can encode information about many design variables. In addition, the element tables can be sorted to associate the table index number with information not readily available from the values of the design variables. For example, feasible beam elements may be sorted based on the moment capacity of the section, described by the encoded design variable, such that as the index number increases, the moment capacity of the section increases.

The design presented by McCormac (1998), is a beam section with $b=14$ in., $h=26$ in., three ♯8 bars in the bottom, four ♯8 bars in the top, and an estimated cost of $1,017.50. An exhaustive search of all 331,776 possible solutions requires about 12 min of computing time and results in beam section with $b=10$ in., $h=25$ in., three ♯8 bars in the bottom, three ♯9 bars in the top, and a minimum structural cost of $897.36.

The majority of the designs in the specified search space are infeasible. In many of those cases, the reinforcing steel selected from Table 2 does not meet ACI Code spacing limitations and can be eliminated from consideration very quickly. In this example, the number of feasible beam sections that satisfy spacing limitations is 3,378 (listed in Table 3).

In designs developed by the *RC-GA* program, over 100 generations require about the same amount of computing time as the exhaustive search. However, on average, the minimum cost design of $897.36 is found after ten generations.

Uniaxial Short-Tied Column Design Examples

Zielinski et al. (1995) designed reinforced short-tied concrete columns that conformed to the Canadian Standard CSA CAN3-A23.3-M84 using a two-part penalty function optimization technique. The objective of the design procedure is to minimize the cost per unit length of column. In the first part of the design procedure, an initial design is assumed and its structural capacity is calculated and compared to the applied loading. During the second part of the procedure, the cross-sectional dimensions and the amount and position of the reinforcing bars are adjusted until a minimum cost is found.

Each uniaxial column is loaded with a factored axial force, P_f, and a factored bending moment, M_f. In the column design, the depth, b, was not allowed to be greater than the width, h.

Four variables define the design of each short-tied column: the depth and thickness of the column, the topology of the steel reinforcement, and the bar size. In this study, the reinforcement topologies are limited to even numbered bar patterns to avoid possibility of biaxial eccentricity in the column. The column designs consider five different reinforcement patterns consisting of 4, 6, 8, 10, or 12 equal-sized bars. Fig. 4 shows the five reinforcement topologies. Since the designs presented by Zielinski et al. (1995) are for short columns, slenderness is not considered. Values for the cost of concrete, steel, and formwork are the same as the costs assumed in the beam design examples.

The design examples have different loadings and material properties, in addition to the different clear cover specifications. Table 4 lists the geometric, material, and loading conditions of

Table 6. Search Space Parameters for Two-Bay Six-Story Frame

			Reinforcement	
	b (in.)	h (in.)	Number of bars	Bar size
Column				
Min	6	7	4	3
Max	22	22	12	11
Increment	1	1	2	1
Beam				
Min	8	12	1	3
Max	18	33	4	11
Increment	1	1	1	1

Table 7. Ordered Feasible Column Cross Sections

Index No.	b (in.)	h (in.)	Steel Bar Combination: Number of bars	Steel Bar Combination: Bar size
1	7	7	6	3
2	7	7	8	3
3	7	7	4	4
...	...	...	...	...
3,109	22	22	10	11
3,110	21	22	12	11
3,111	22	22	12	11

each example. In each design example, the distance from the outer fiber of the concrete to the centroid of the first reinforcing bar, d', is predetermined (see Fig. 1). The clear cover for each column design will change depending on the size of the longitudinal reinforcing bars. Values for the column dimensions in inches range from $7 \leqslant b \leqslant 22$ and $7 \leqslant h \leqslant 22$. The discrete search increment for both the column width and thickness is 0.5 in.

The design of short-tied columns is dependent on the stresses in the reinforcing steel. These stresses are used to generate the strength load-moment interaction diagram for the column (see Fig. 2). Feasible solutions are generated based on the restrictions and specifications outlined in the ACI Code.

Table 5 lists a comparison between the column designs presented by Zielinski et al. (1995) and those developed by the *RC-GA* procedure. There is no restriction on the column width-to-thickness ratio as long as the column dimensions are within the prescribed range of values for the width and the thickness. Without any restriction on column width-to-depth ratio, the *RC-GA* program develops column sections that are wide and narrow. This characteristic of the design is reasonable considering the uniaxial nature of the analysis. In addition, the *RC-GA* program tends to minimize the amount of reinforcing steel in columns since the unit cost of steel significantly affects the overall structural cost as defined in Eq. (2). The ACI Code requires a minimum reinforcement ratio of 1%. The reinforcement ratios for the *RC-GA* designs range from 1.05 to 1.60%, while the ratios for the designs presented by Zielinski et al. (1995) range from 1.02 to 3.09%.

In all three examples, the *RC-GA* column designs are more cost efficient than the designs presented by Zielinski et al. (1995). *RC-GA* designs reduce the cost per foot for examples 1, 2, and 3 by 15.6, 13.4, and 37.5%, respectively.

Two-Bay Six-Story Frame Design Example

Fig. 5 shows a two-bay six-story reinforced concrete frame designed by Rajeev and Krishnamoorthy (1998). The design conforms to the Indian Standard Code of Practice for Reinforced Concrete (IS 1978) design code and does not consider the shear capacity of the beam sections. The dimensions of the frame are: $h = 4$ m (13.12 ft), $L_1 = 6$ m (19.69 ft), and $L_2 = 4$ m (13.12 ft). A factored uniformly distributed vertical load of $w = 30$ kN/m (2,056 lb/ft) is applied to every beam in the frame. In addition, a lateral load of $P = 10$ kN (2,248 lb) is applied to each story. The estimated cost of the frame is

$$F = \sum_{i=1}^{n_b+n_c} \{C_c b_i h_i + C_s A_{s_i} + 2C_f(b_i + h_i)\} \ell_i \qquad (37)$$

where n_b = number of beams in the frame; and n_c = number of columns in the frame.

The cost of concrete, steel, and formwork is estimated as \$735/m^3 (\$20.81/ft^3), \$7.1/kg (\$1,578/ft^3), and \$54/m^2 (\$5.02/ft^2), respectively (Rajeev and Krishnamoorthy 1998). The unit weight of concrete and steel is approximately 145 lbs/ft^3 and 490 lbs/ft^3, respectively. The strength of concrete, $f'_c = 3{,}000$ psi, and the yield strength of steel, $f_y = 60{,}000$ psi.

The structural elements of the frame are divided into three column groups and four beam groups as shown in Fig. 5 (Rajeev and Krishnamoorthy, 1998). All the beams spanning the same frame bay are in element groups 3 or 4, except for the roof beam, which are in element groups 1 or 2. Each column group consists of all colinear columns extending from the ground level to the roof level, element groups 5–7. Table 6 lists the boundaries of the search space for the frame design.

The cost function used by Rajeev and Krishnamoorthy (1998) is slightly different than the cost function used in *RC-GA*-as defined in Eq. (38). The cost function used by Rajeev and Krishnamoorthy accounts for their requirement that there must be two rows of steel in both the compression and the tension portions of the beam. Typically, the outer row of the reinforcement is continuous and has the same number of equal-sized bars throughout colinear spans in the frame. In addition, the cost function incorporates prescribed bar cut-off lengths for the inner row of the reinforcement in both the compression and tension bar groups. The *RC-GA* design uses one row of continuous reinforcement in both the positive and negative moment zones and does not include bar cutoff.

Table 8. Design Results Without Shear Considerations for Two-Bay Six-Story Frame

	Beam Group Number				Column Group Number		
Rajeev and Krishnamoorthy (1998)	1	2	3	4	5	6	7
b (in.)	7.9	7.9	7.9	7.9	9.9	9.9	9.9
h (in.)	13.8	9.9	13.8	11.8	9.9	9.9	11.8
$A_{s\ \mathrm{bottom}}$ (in.^{2})	2 #4	2 #4	2 #5	1 #11	6 #7	6 #7	6 #7
$A_{s\ \mathrm{top}}$ (in.^{2})	2 #5	2 #5	1 #9	1 #9			
Cost	\$26,052						
RC-GA							
b (in.)	11	13	9	8	7	7	7
h (in.)	22	19	22	19	8	18	11
$A_{s\ \mathrm{bottom}}$ (in.^{2})	2 #6	1 #5	4 #4	1 #6	4 #5	4 #7	4 #4
$A_{s\ \mathrm{top}}$ (in.^{2})	2 #8	2 #7	1 #11	2 #5			
Cost	\$24,959						

Bold–Indicates infeasible design with respect to the ACI Code (not including shear).

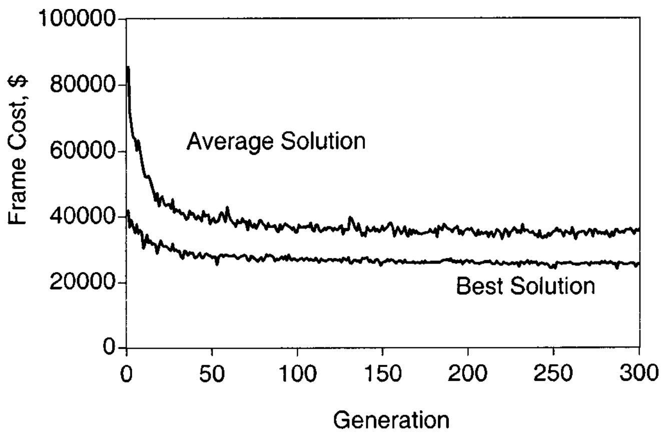

Fig. 6. Typical GA convergence history for two-bay six-story frame

As previously described in the simply supported beam examples, the *RC-GA* design procedure uses the ACI Code limits and restrictions on the reinforcement ratio, cross-sectional geometry, and reinforcement spacing to check for feasible beam elements within the prescribed limits of the design variables. The feasible beams listed in Table 3, generated for the simply supported beam examples, are reused for this frame design. Table 7 lists geometrically feasible column elements, sorted by increasing moment capacity, used in this design.

Table 8 compares the *RC-GA* design, without shear considerations, with the design presented by Rajeev and Krishnamoorthy (1998). The best design developed by *RC-GA* has a cost of $24,959, whereas the Rajeev and Krishnamoorthy design has a cost of $26,052, a cost reduction of 4.2%. While it appears that the cost reduction is small, the *RC-GA* design conforms to the specifications of the ACI Code, whereas the beams in element groups 1, 2 and 3, and the columns in element groups 6 and 7 of the Rajeev and Krishnamoorthy design do not conform to the ACI Code. The factored nominal moment capacities of these sections are between 3.4 and 125% less than the required moment. In order for the Rajeev and Krishnamoorthy design to be acceptable under the ACI Code, 19 beam and column elements must be redesigned to increase their moment capacities. The associated cost of this redesign should significantly increase the cost of the structure.

The cost of beams designed by *RC-GA* is larger than the cost of the beams developed by Rajeev and Krishnamoorthy; however, all the *RC-GA* beams have the moment capacity required by ACI. In contrast, the cost of the columns design by *RC-GA* is smaller than the equivalent columns developed by Rajeev and Krishnamoorthy. Two of the three column groups developed by Rajeev and Krishnamoorthy have square cross-sectional geometries. All column groups in the *RC-GA* design have rectangular cross-sectional properties and develop the required load-moment capacity by employing a more efficient structural section.

Fig. 6 shows a typical convergence history of the *RC-GA* procedure for the example frame design. The computational time for 300 generations with a population size of 300 is about 13 h. A large population was required due to the size of the geometrically feasible design space, in this case, four beam groups with 3,378 beams types and three column groups with 3,111 column types (approximately 1.26×10^{21} possible solutions).

The two-bay six-story frame is re-designed considering the moment and shear capacities of the beams and the load-moment interaction in the columns with moment magnification due to frame stability and column slenderness. The finite element analysis used to evaluate the fitness of *RC-GA* designs has an option for checking the slenderness of columns using Eqs. (5)–(11). The best design developed by *RC-GA*, with column slenderness considered, cost $25,471, an increase of 2% over the no-slenderness design. Table 9 lists the *RC-GA* design details for the beam and column elements.

Summary

A procedure for designing low-cost reinforced concrete frames using a genetic algorithm is presented. The *RC-GA* design procedure minimizes the material and construction cost of reinforced concrete while satisfying the limitations and specifications of the ACI Code. Beam elements are evaluated based on their flexural response considering moment magnification factors due to frame stability. A rectilinear column strength interaction diagram is used to evaluate the feasibility of columns with moment magnification due to slenderness effects. The limitations and specifications of the ACI Code are formulated as a series of constraints to the discrete cost optimization problem and applied as penalties on the fitness function of the genetic algorithm. Several design examples are presented to demonstrate the effectiveness and efficiency of the *RC-GA* procedure. While the reduction in structural costs associated with a *RC-GA* design might be viewed as insignificant in the total cost of the structure, the systemic and automatic verification of the ACI Code limitations and restrictions can provide an increased level of confidence in the integrity of the design.

Unit Conversions

$$1 \text{ in.} = 25.4 \text{ mm}$$

$$1 \text{ kip} = 4{,}450 \text{ N}$$

$$1 \text{ k in.} = 113 \text{ N mm}$$

$$1 \text{ ksi} = 6.9 \text{ MPa}$$

Table 9. Design Results Considering Column Slenderness for Two-Bay Six-Story Frame

	Beam Group Number				Column Group Number		
RC-GA	1	2	3	4	1	2	3
b (in.)	10	8	9	9	7	8	7
h (in.)	22	19	18	19	8	22	12
$A_{s\ \mathrm{bottom}}$ (in.^{2})	4 #4	1 #5	4 #4	1 #6	4 #6	4 #6	6 #4
$A_{s\ \mathrm{top}}$ (in.^{2})	1 #9	3 #6	3 #7	2 #5			
Cost	$25,471						

References

Adamu, A., Karihaloo, B. L., and Rozvany, G. I. N. (1994). ''Minimum cost design of reinforced concrete beams using continuum-type optimality criteria.'' *Struct. Optim.*, (7), 91–102.

Adeli, H., and Cheng, N. T. (1993). ''Integrated genetic algorithms for optimization of space structures.'' *ASCE, J. Aerosp. Eng.*, 6(4), 315–328.

Adeli, H., and Cheng, N. T. (1994). ''Augmented Lagrangian genetic algorithm for structural optimization.'' *ASCE, J. Aerosp. Eng.*, 7(1), 104–118.

American Concrete Institute (1999). ''Building code requirements for structural concrete and commentary.'' *ACI 318-99*.

Balling, R. J., and Yao, X. (1997). ''Optimization of reinforced concrete frames.'' *J. Struct. Eng.*, 123(2), 193–202.

Camp, C. V., and Gipson, G. S. (1990). ''A boundary oriented domain fanning quadrature technique for arbitrary multiply-connected regions.'' *Adv. Eng. Software*, 12(1), 27–38.

Camp, C. V., Pezeshk, S., and Cao, G. (1998). ''Optimized design of two-dimensional structures using a genetic algorithm.'' *J. Struct. Eng.*, 124(5), 551–559.

Carroll, D. (1997). ''D. L. Carroll's *FORTRAN* genetic algorithm driver.'' ⟨http://www.uiuc.edu/~carroll/ga.html⟩ (Jan. 6, 1997).

Chung, T. T., and Sun, T. C. (1994). ''Weight optimization for flexural reinforced concrete beams with static nonlinear response.'' *Struct. Optim.*, (8), 174–180.

Fadaee, M. J., and Grierson, D. E. (1996). ''Design optimization of 3D reinforced concrete structures.'' *Struct. Optim.*, (12), 127–134.

Foley, C. M., and Schinler, D. (2000). ''Optimized design of steel frames using distributed plasticity.'' *Proc., 2001 Structures Congress*, Chang, P., ed., ASCE, (CD-ROM).

Gipson, G. S. (1986). ''Use of residue theorem in locating points within an arbitrary multiply-connected region.'' *Adv. Eng. Software*, 8(2), 73–80.

Goldberg, D. E. (1989). *Genetic algorithms in search, optimization and machine learning*, Addison–Wesley, New York.

Goldberg, D. E., and Samtani, M. P. (1986). ''Engineering optimization via genetic algorithm.'' *Proc., 9th Conf. Electronic Computation*, ASCE, New York, 471–482.

Hoit, M., Soeiro, A., and Fagunda, F. (1991). ''Probabilistic design and optimization of reinforced concrete frames.'' *Eng. Optimiz.*, 17, 229–235.

IS:456. (1978). *Code of practice for plain and reinforced concrete*, Bureau of Indian Standards, New Delhi, India.

Jenkins, W. M. (1991). ''Plane frame optimum design environment based on genetic algorithm.'' *J. Struct. Eng.*, 118(11), 3103–3112.

Koumousis, V. K., and Georgiou, P. G. (1994). ''Genetic algorithms in discrete optimization of steel truss roofs.'' *J. Comput. Civ. Eng.*, 8(3), 309–325.

Krishnamoorthy, C. S., and Munro, J. (1979). ''Linear program for optimal design of reinforced concrete frames.'' *Proc. IABSE*, 3(1), 119–141.

Krishnamoorthy, C. S., and Rajeev, S. (1989). ''Computer-aided optimal design of reinforced concrete frames.'' *Proc., Int. Conf. on Engineering Software*, C. V. Ramakrisnan and A, Varadarajan, eds., Narosa, New Delhi, India.

McCormac, J. C. (1998). *Design of reinforced concrete*, 4th Ed., Addison–Wesley Longman, Menlo Park, Calif.

Nawy, E. G. (1996). *Reinforced concrete-A fundamental approach*, 3rd Ed., Prentice–Hall, Upper Saddle River, N. J.

Pezeshk, S. and Camp, C. V. (2001). ''State of the art on the use of genetic algorithms in design of steel structures.'' *Advances in structural optimization*, Scott Burns, ed.

Rajan, S. D. (1995). ''Sizing, shape, and topology design optimization of trusses using genetic algorithm.'' *J. Struct. Eng.*, 121(10), 1480–1487.

Rajeev, S., and Krishnamoorthy, C. S. (1992). ''Discrete optimization of structures using genetic algorithms.'' *J. Struct. Eng.*, 118(5), 1233–1250.

Rajeev, S., and Krishnamoorthy, C. S. (1998). ''Genetic algorithms-based methodology for design optimization of reinforced concrete frames.'' *Computer-Aided Civil Infrastructure Eng.*, 13, 63–74.

Val, D., Bljuger, F., and Yankelevsky, D. (1996). ''Optimization problem solution in reliability analysis of reinforced concrete structures.'' *Comput. Struct.*, 60(3), 351–335.

Zielinski, Z. A., Long, W., and Troitsky, M. S. (1995). ''Designing reinforced concrete short-tied columns using the optimization technique.'' *ACI Struct. J.*, 92(5), 619–626.