Flight Gate Assignment with a Quantum Annealer

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German Aerospace Center (DLR)
SIAM Annual Meeting, Portland

DLR

Flight Gate Assignment

A day at Frankfurt Airport

- about 1300 aircraft movements (arrival and departure)
- more than 90% are passenger flights
- more than 170000 passengers
- about 60% transfer passengers
- 278 gates

Frankfurt Airport

DLR

Passenger Flows

- F, G sets of flights and gates

gate α

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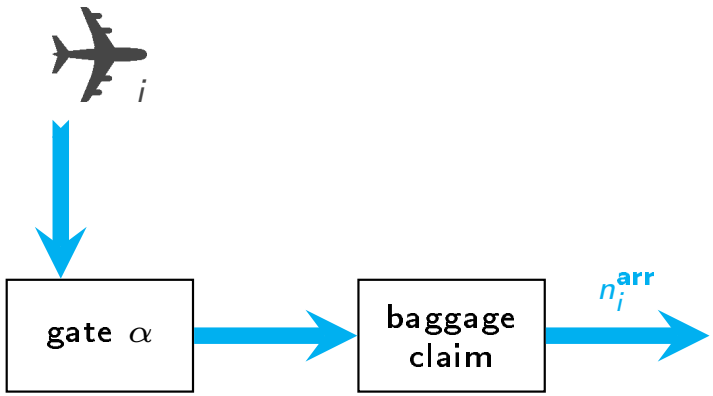

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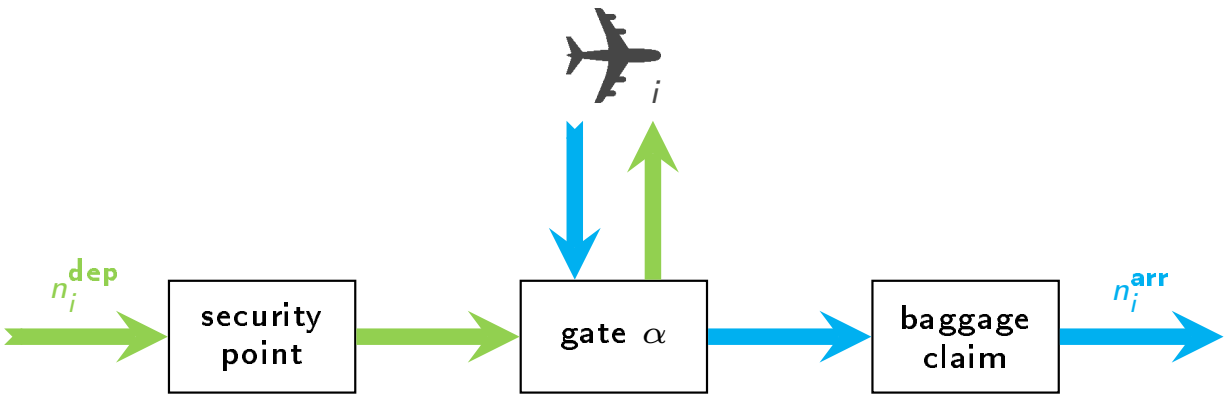

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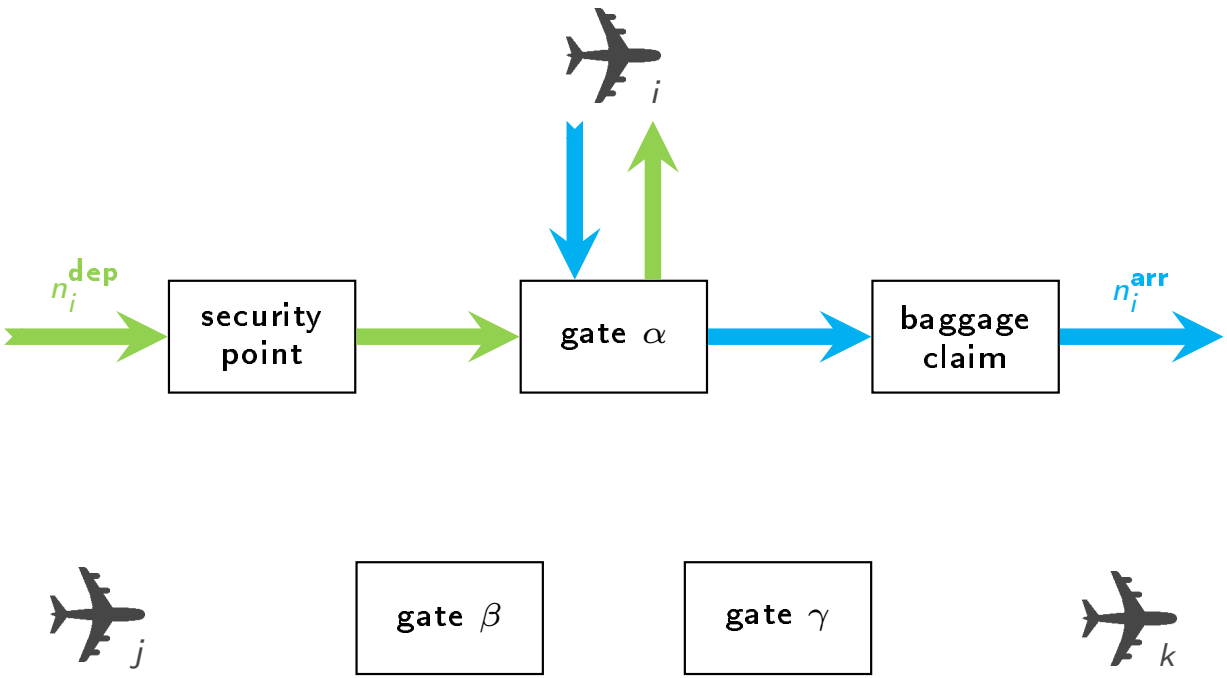

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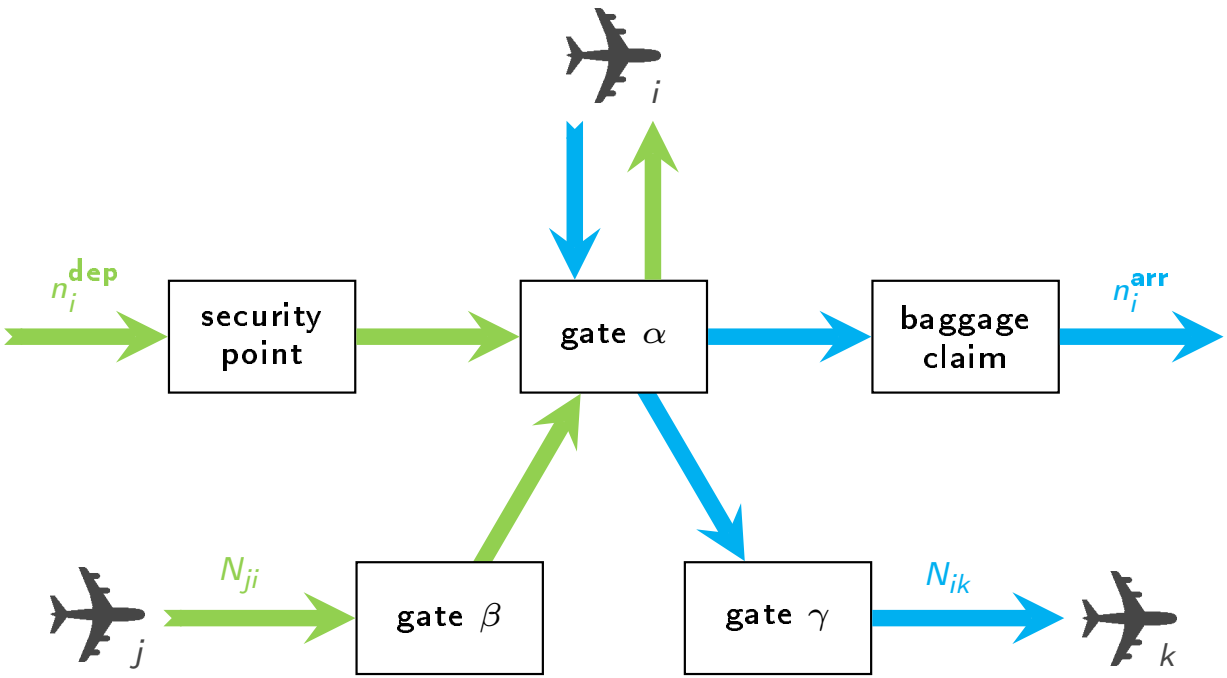

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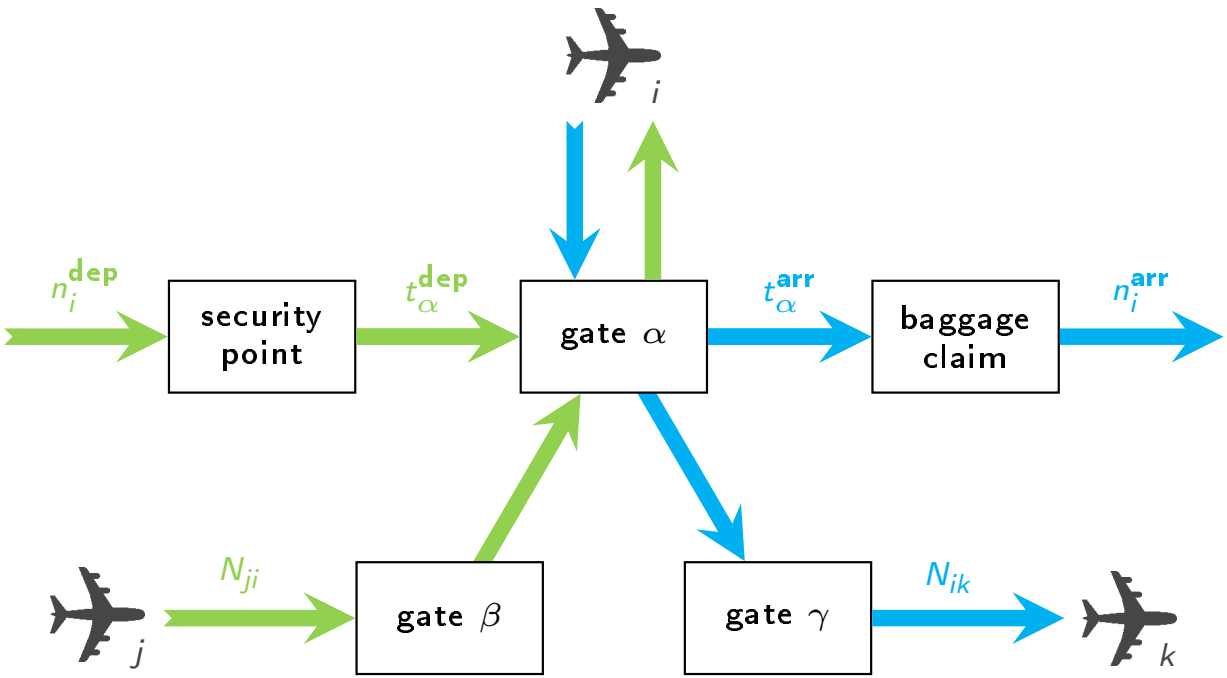

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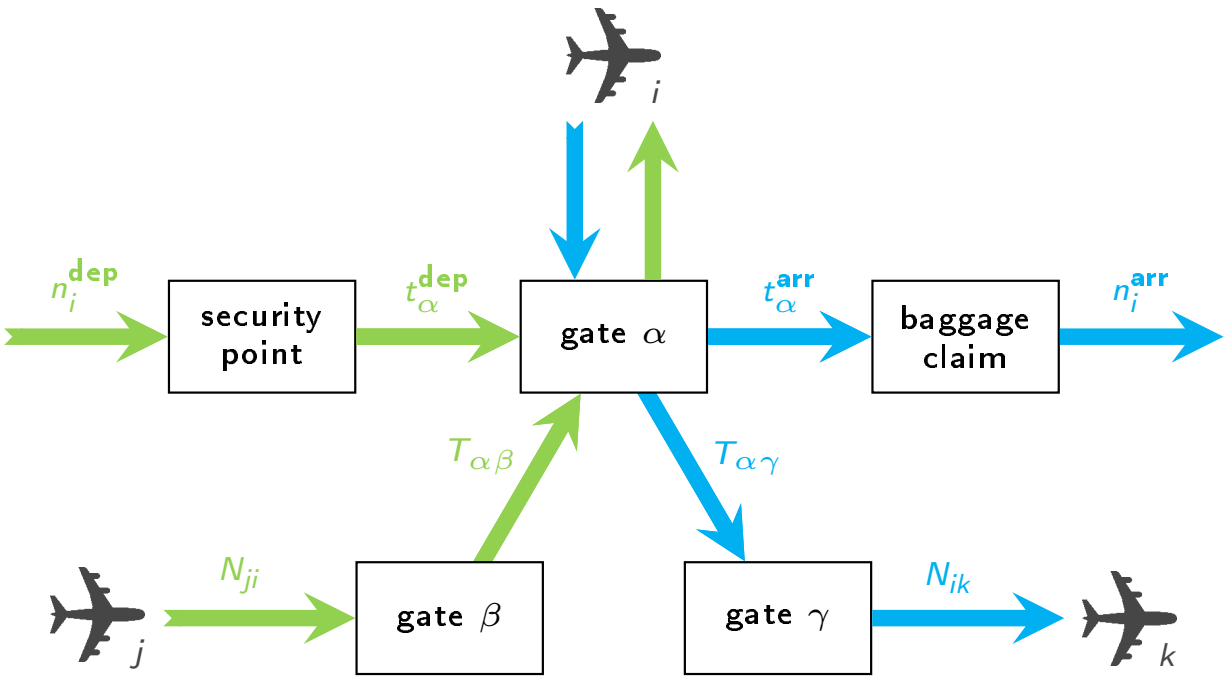

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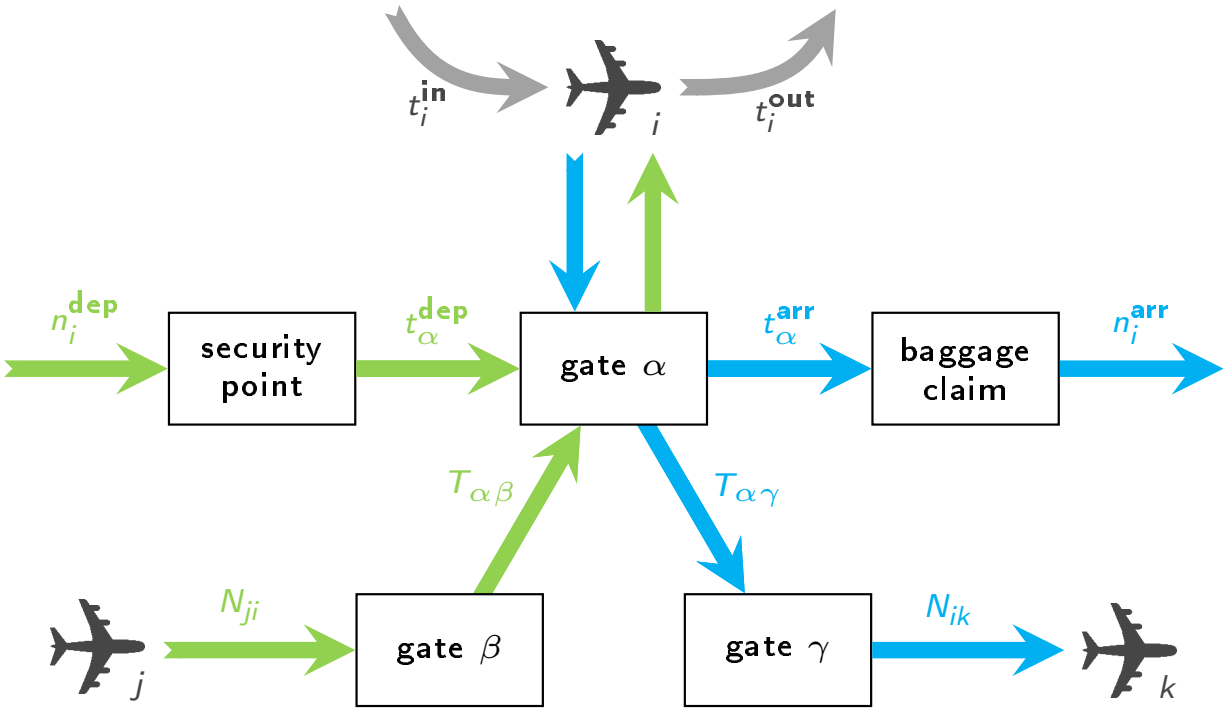

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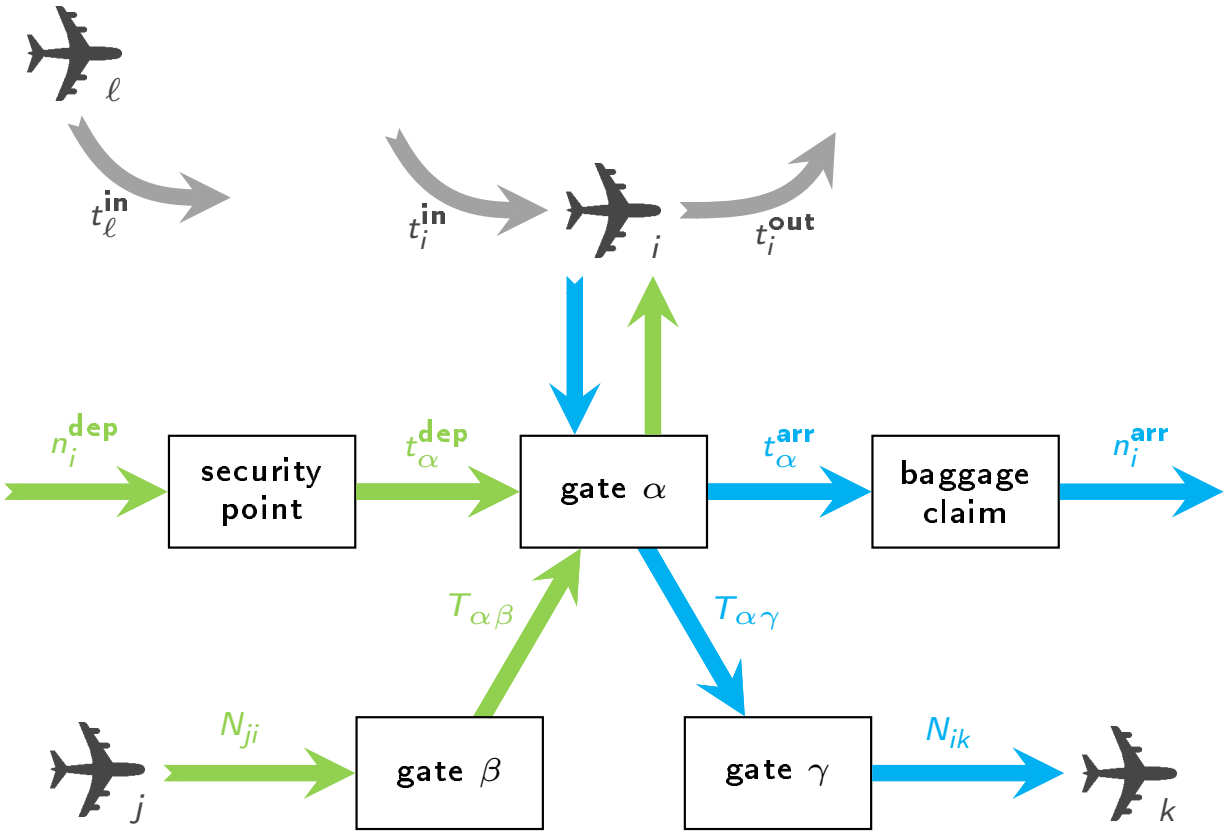

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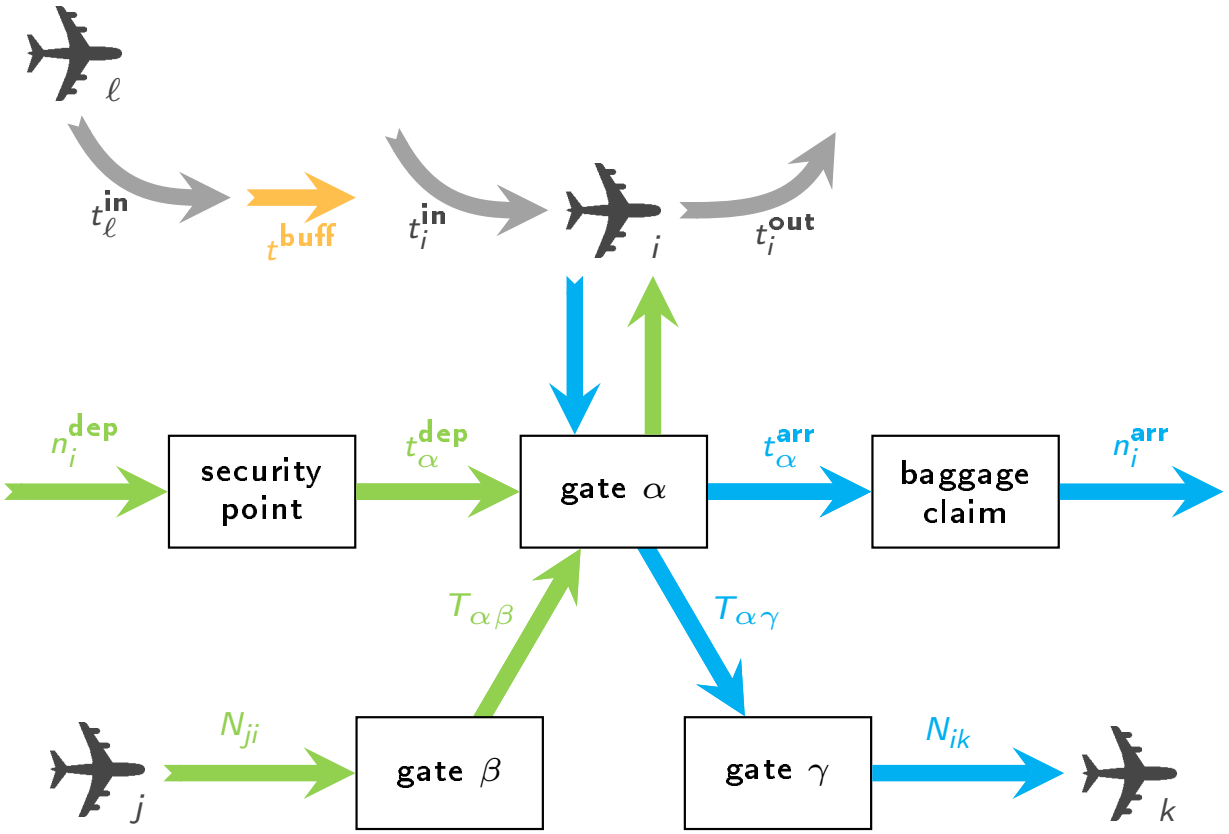

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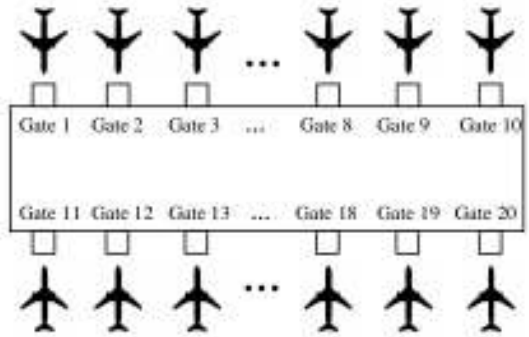
- t^{buff} buffer time between two flights at the same gate

Question

Which flight should be assigned to which gate, such that the total transit time of the passengers in minimal?



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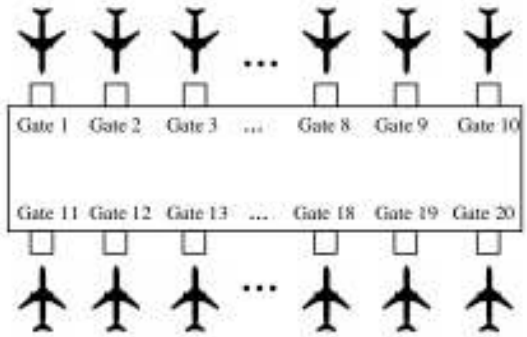

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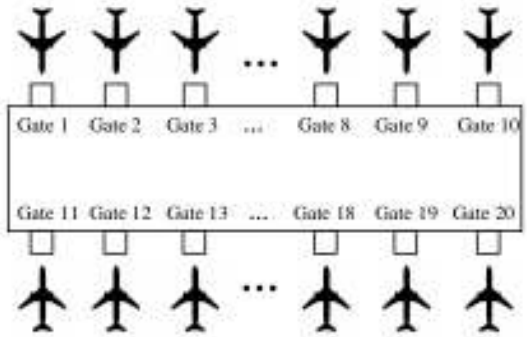

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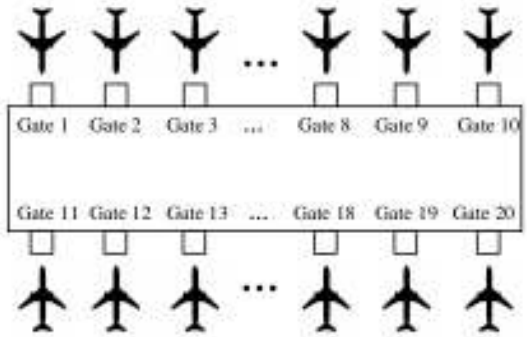

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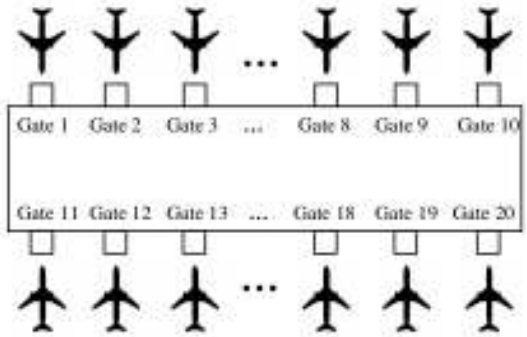

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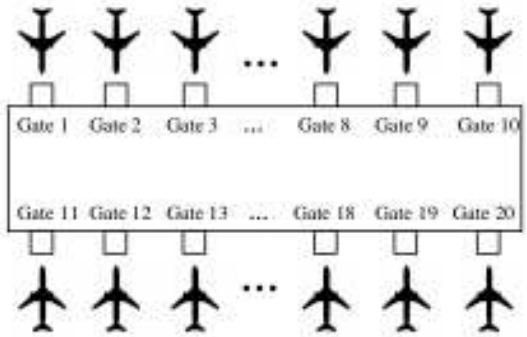
- seems to exploit possible advantages of the D-Wave machine



FGA Binary Program

α

Variables $x \in \{0,1\}^{F \times G}$ with

β

γ

$$x_{i\alpha} = \begin{cases} 1, & \text{if flight } i \text{ takes gate } \alpha, \\ 0, & \text{otherwise} \end{cases}$$

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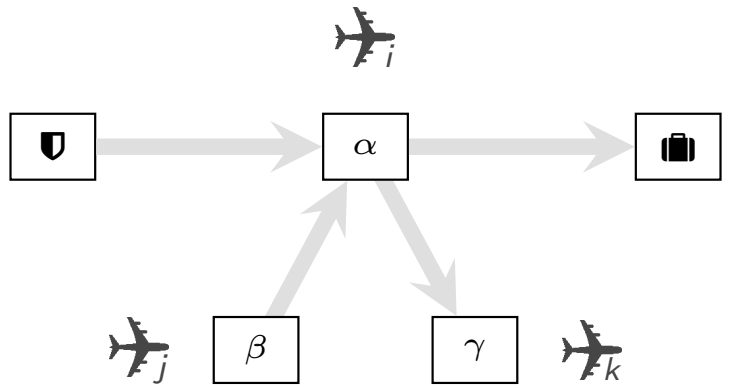

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Minimizing the total transfer time with objective function

$$O(x)$$

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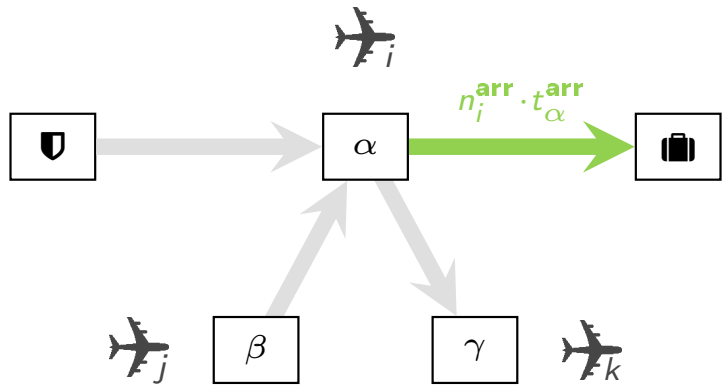

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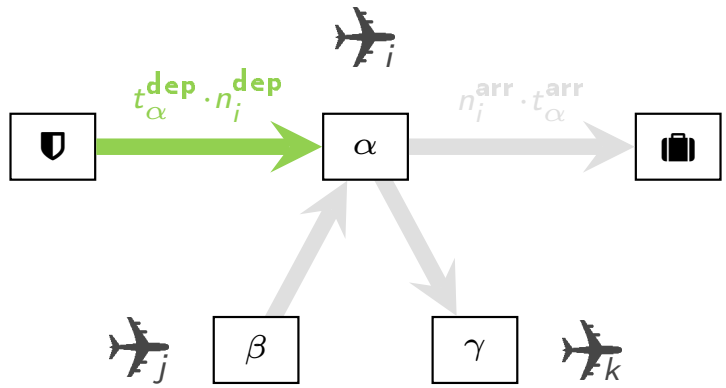

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$$\begin{aligned} O(x) &= O_{\text{arr}}(x) + O_{\text{dep}}(x) \\ &= \sum_{i\alpha} n_i^{\text{arr}} t_\alpha^{\text{arr}} x_{i\alpha} + \sum_{i\alpha} n_i^{\text{dep}} t_\alpha^{\text{dep}} x_{i\alpha} \end{aligned}$$

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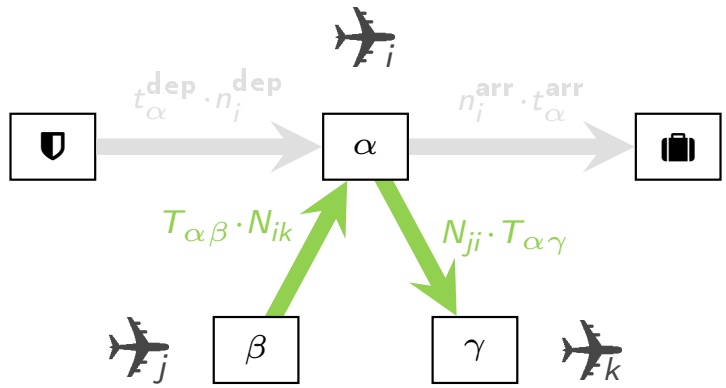

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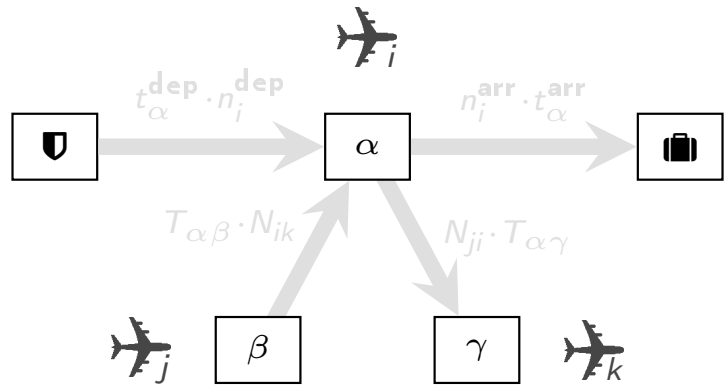

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$$\Updownarrow$$

$$x_{i\alpha} \cdot x_{j\alpha} = 0$$

QUBO Formulation

$$Q(x) = O(x) + \lambda_{\text{one}} C_{\text{one}}(x) + \lambda_{\text{not}} C_{\text{not}}(x)$$

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Contribution of objective function should not exceed penalty!

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Comparing coefficients in worst cases for

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$$\lambda_{\mathbf{one}} > \max_{i,\alpha} \left(n_i^{\mathbf{dep}} t_\alpha^{\mathbf{dep}} + n_i^{\mathbf{arr}} t_\alpha^{\mathbf{arr}} + \max_\beta T_{\alpha\beta} \sum_j N_{ij} \right)$$

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- assigning a pair of forbidden flights to the same gate

$$\lambda_{\mathbf{not}} > \max_{i,\alpha,\gamma} \Bigg(\left(n_i^{\mathbf{dep}} t_\alpha^{\mathbf{dep}} - n_i^{\mathbf{dep}} t_\gamma^{\mathbf{dep}} \right) + \left(n_i^{\mathbf{arr}} t_\alpha^{\mathbf{arr}} - n_i^{\mathbf{arr}} t_\gamma^{\mathbf{arr}} \right) + \max_\beta \left(T_{\alpha\beta} - T_{\gamma\beta} \right) \sum_j N_{ij} \Bigg)$$

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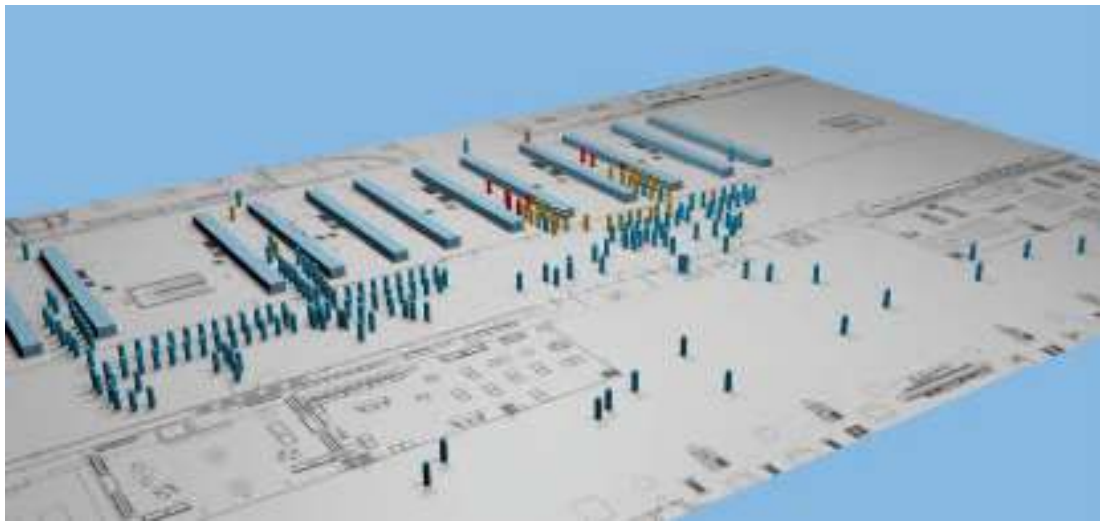

Simulating a multi-airport region to foster individual door-to-door travel, **M. Jung et al.**

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- Flight schedule for one day from a mid-sized European airport
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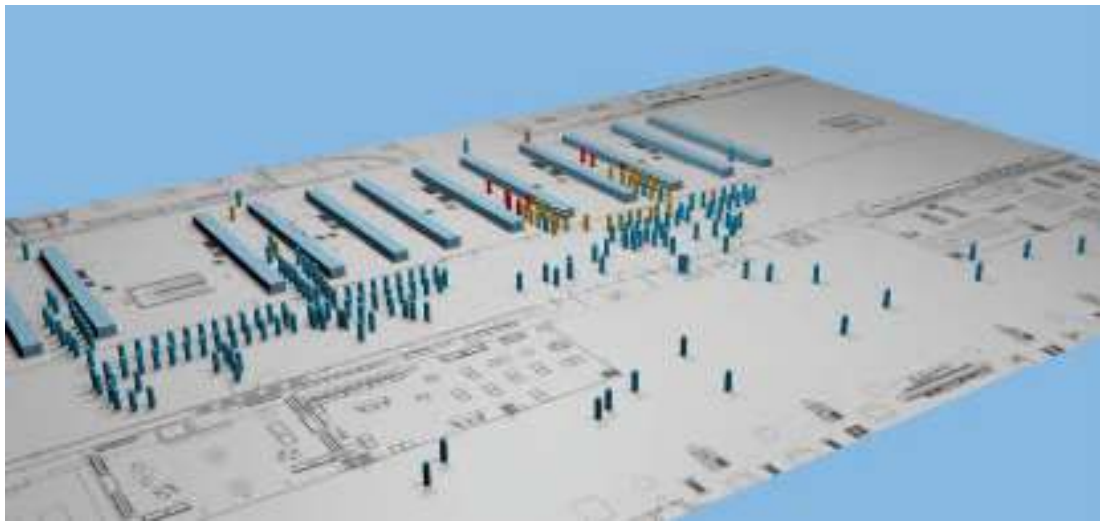
- Extracted total instance: 293 flights and 97 gates



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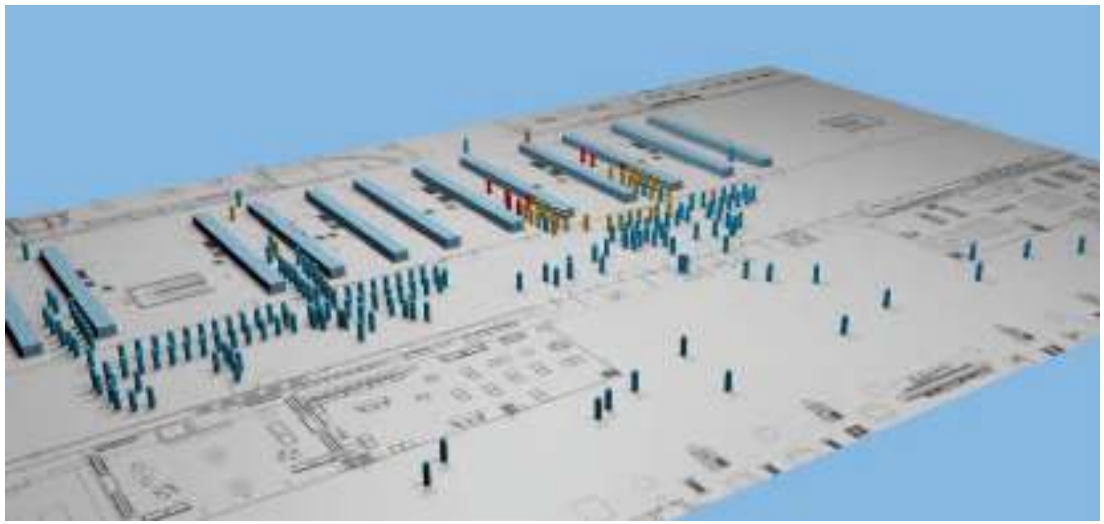
$\Rightarrow$ Over 28000 binary variables with about 400 Mio. couplings



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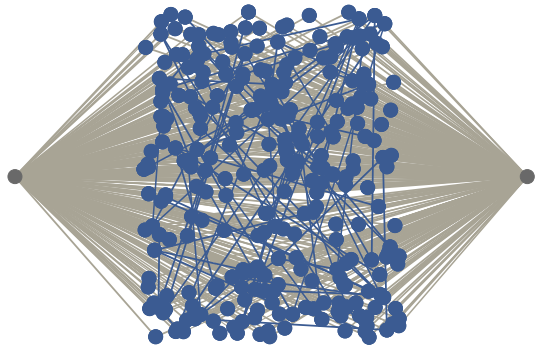
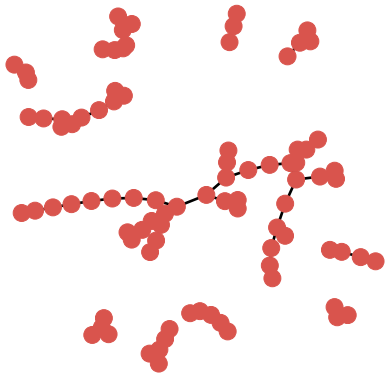
Instance Preprocessing

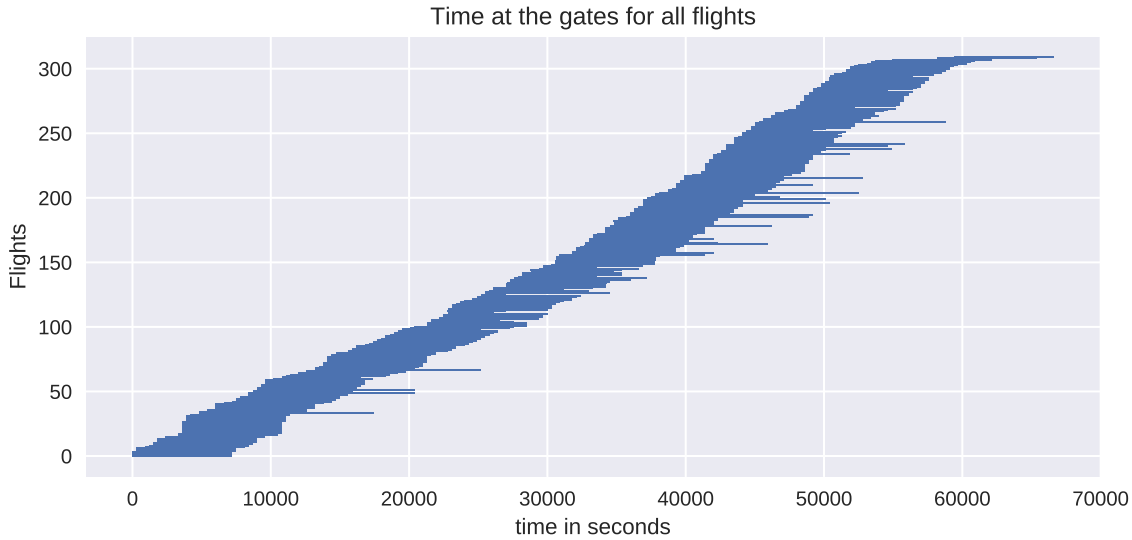
- Removing corrupted data

Total problem



Transfer Passengers

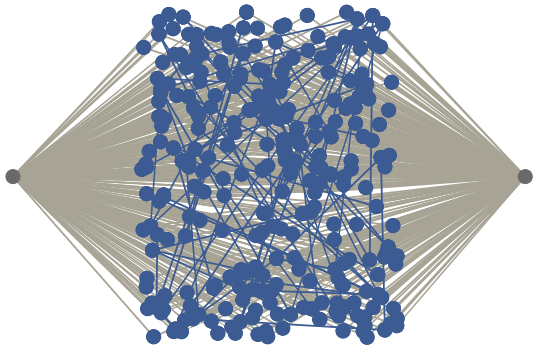




Instance Preprocessing

- Removing corrupted data
- Splitting too long on-block times

Total problem



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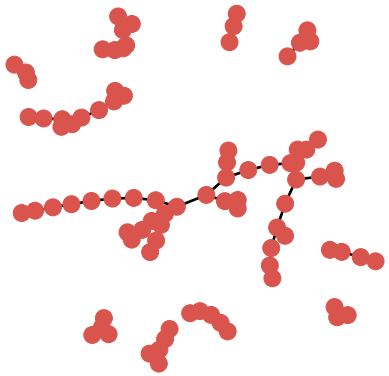
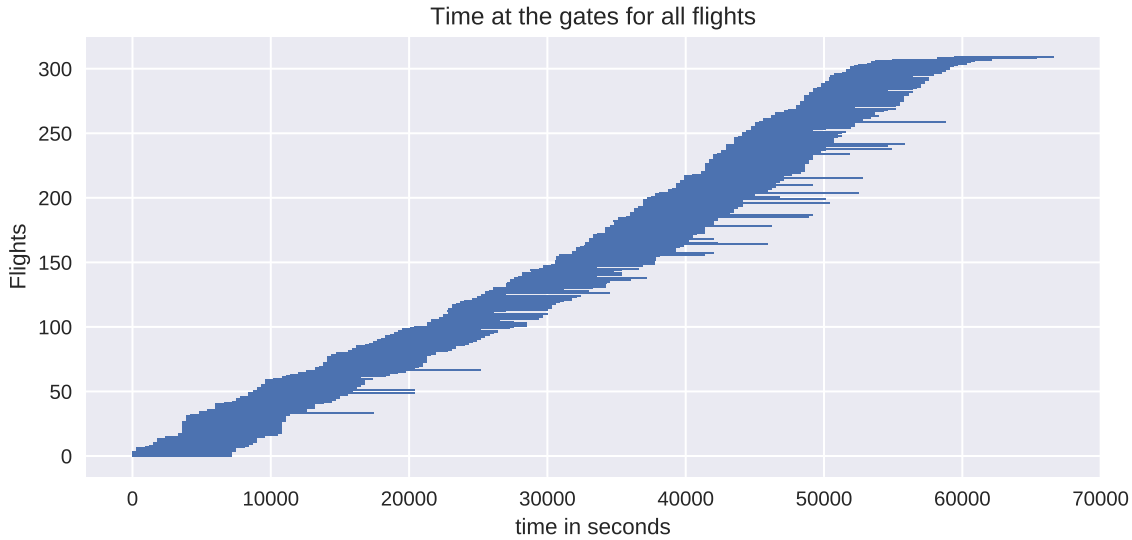
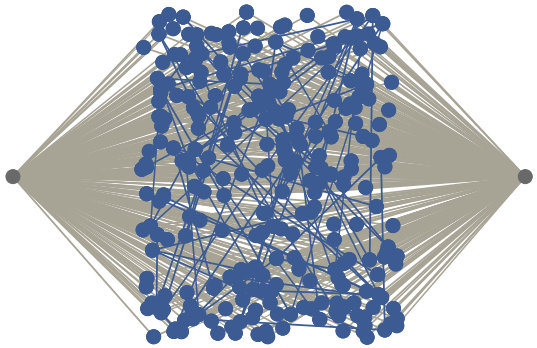




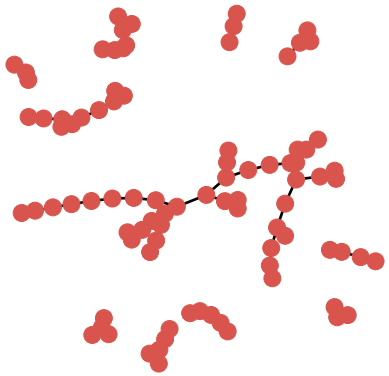
Instance Preprocessing

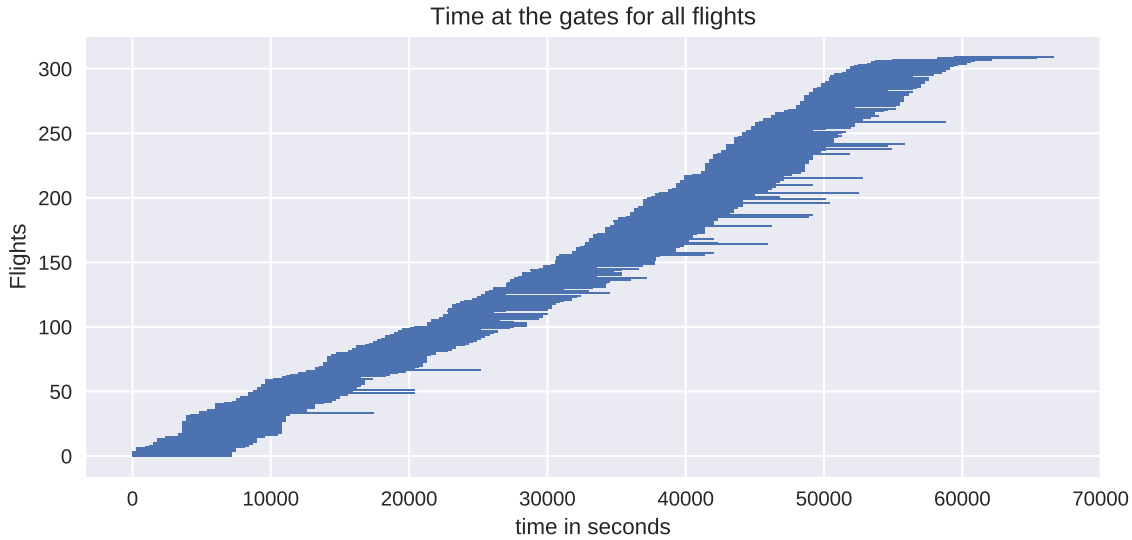
- Removing corrupted data
- Splitting too long on-block times
- Reducing to flights with transfers only

Total problem



Transfer Passengers



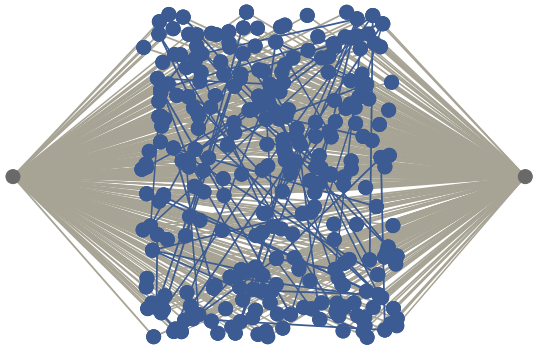


Instance Preprocessing

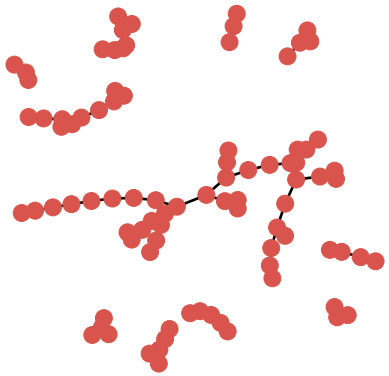
- Removing corrupted data
- Splitting too long on-block times
- Reducing to flights with transfers only

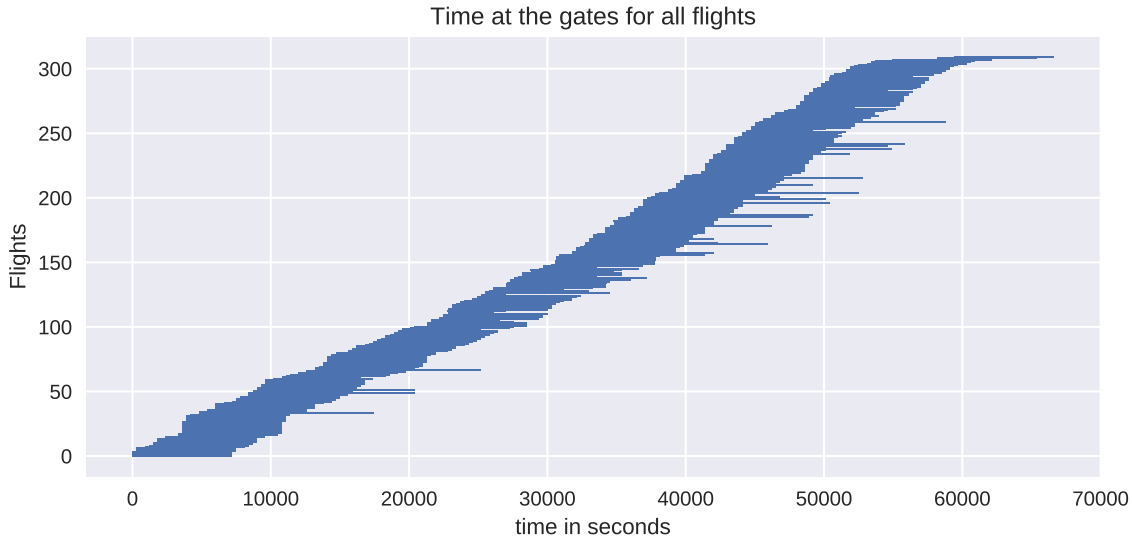
$\Rightarrow$ 89 remaining flights with 80 transfers

Total problem



Transfer Passengers





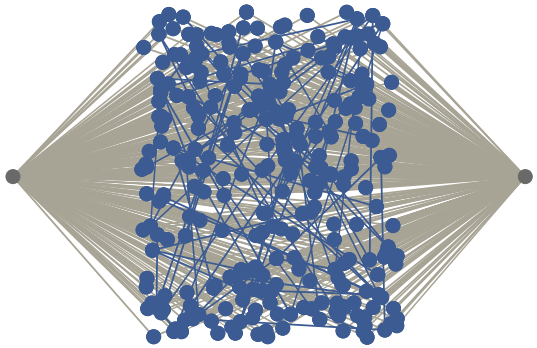
Instance Preprocessing

- Removing corrupted data
- Splitting too long on-block times
- Reducing to flights with transfers only

$\Rightarrow$ 89 remaining flights with 80 transfers

$\Rightarrow$ more than 35 gates

Total problem



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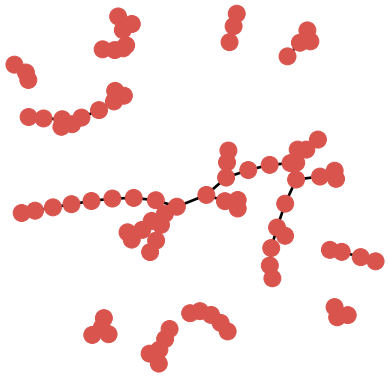
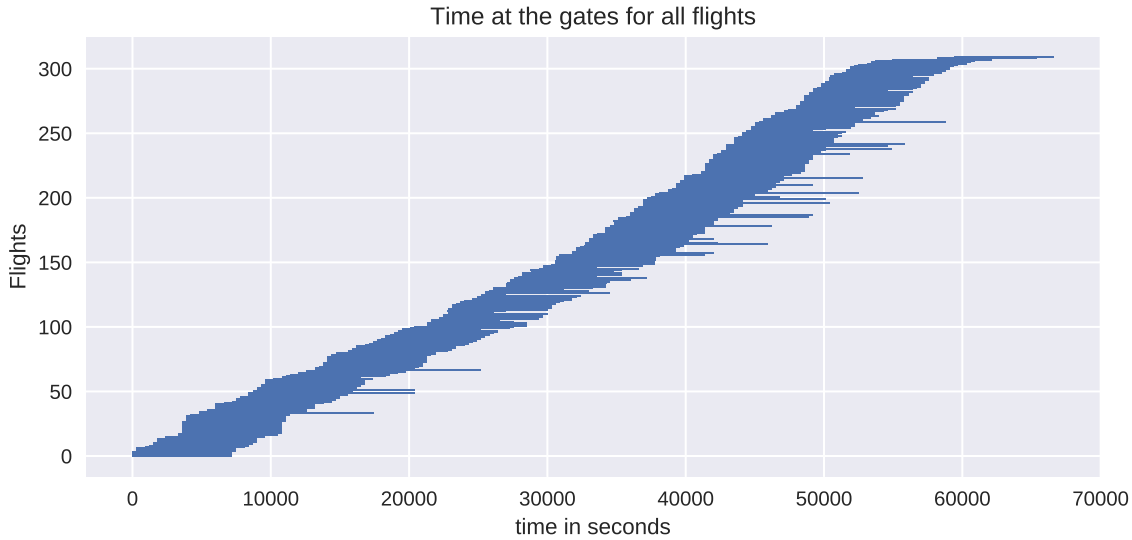
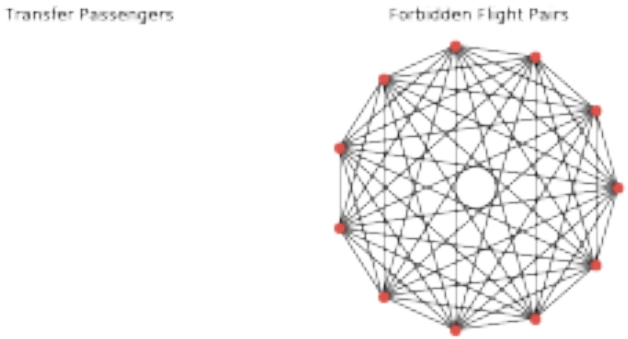
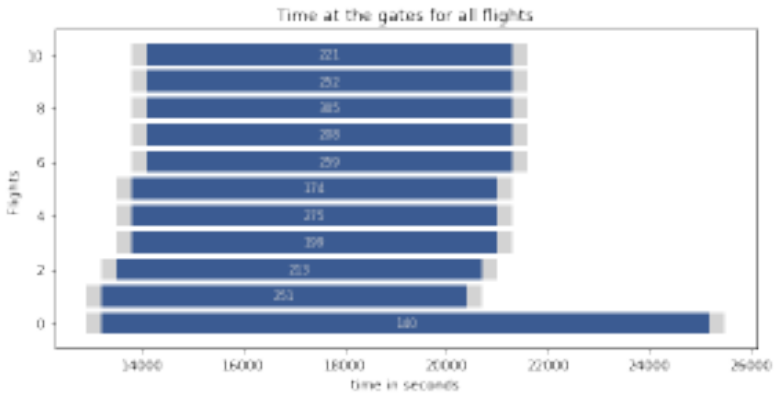




Instance Reduction

- Slicing by time intervals





Instance Reduction

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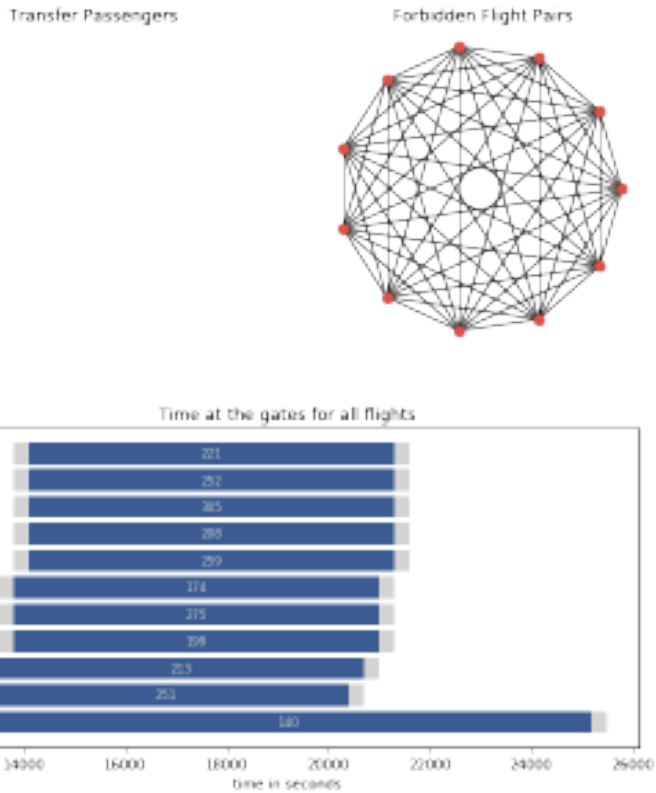
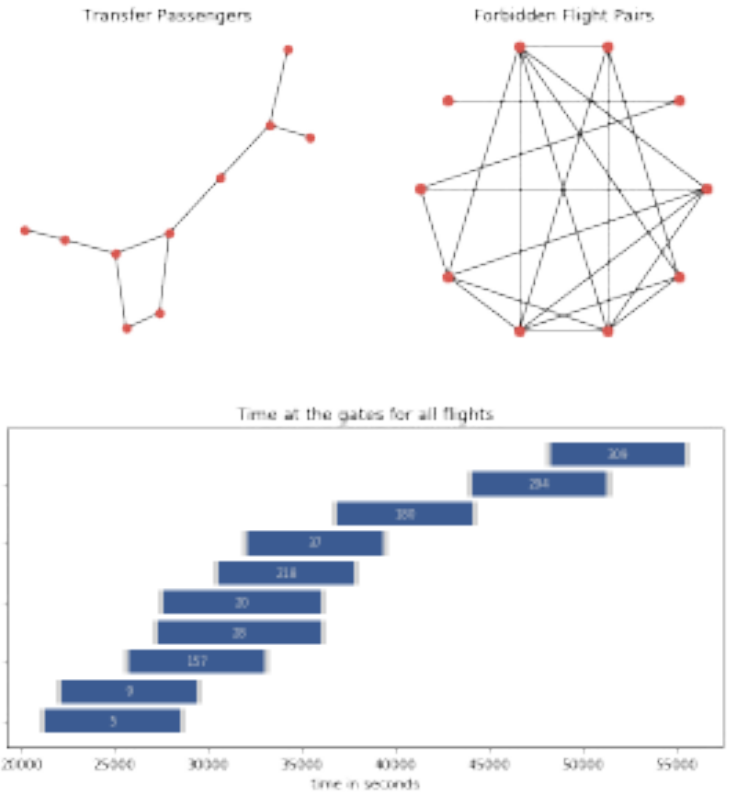
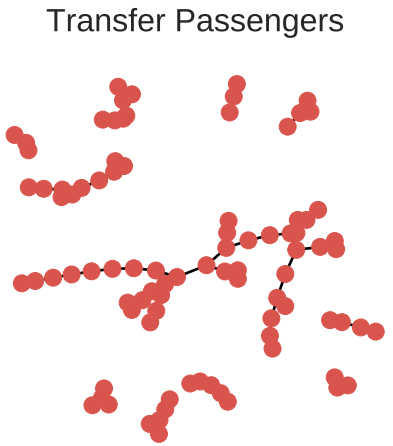


- Connected components of transfer passenger graph



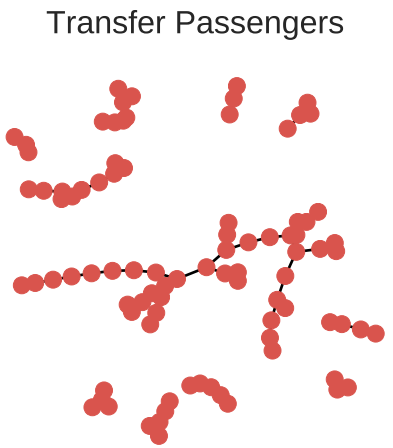
Instance Reduction

- Use connected components of transfer passenger graph to find small hard instances



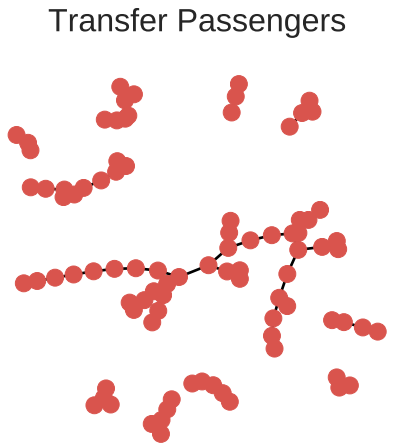
Instance Reduction

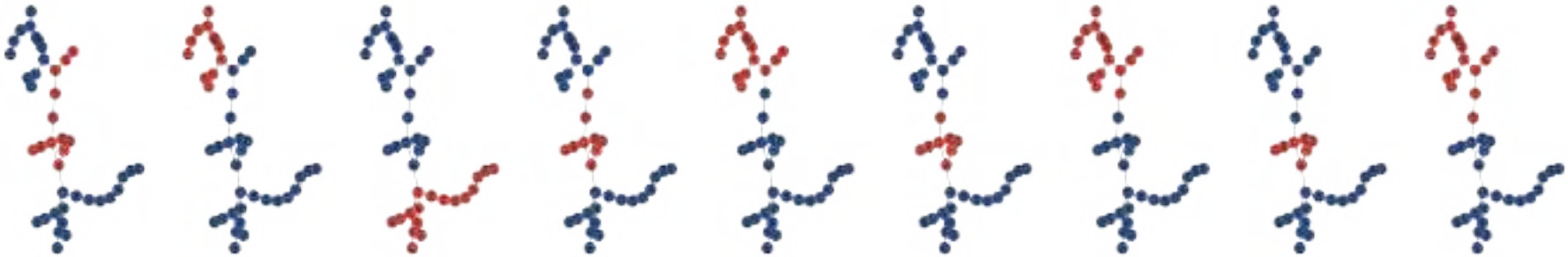
- Use connected components of transfer passenger graph to find small hard instances
- Choose number of gates close number of maximal clique



Instance Reduction

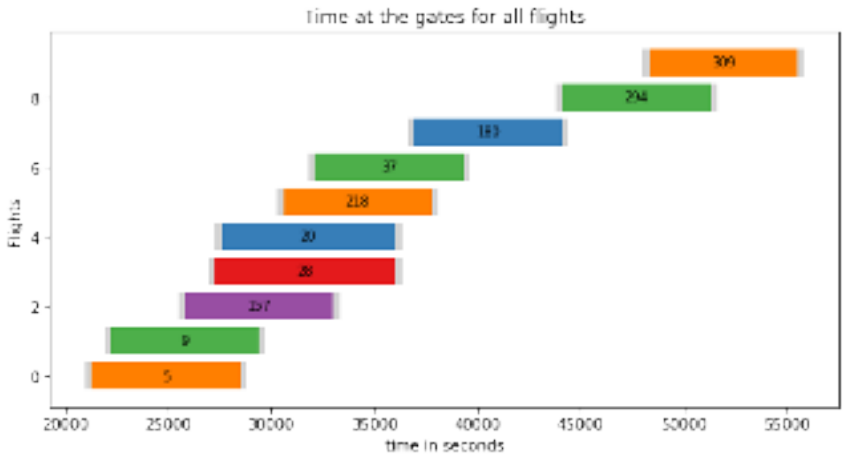
- Use connected components of transfer passenger graph to find small hard instances
- Choose number of gates close number of maximal clique
- Split largest connected component of the transfer passenger graph

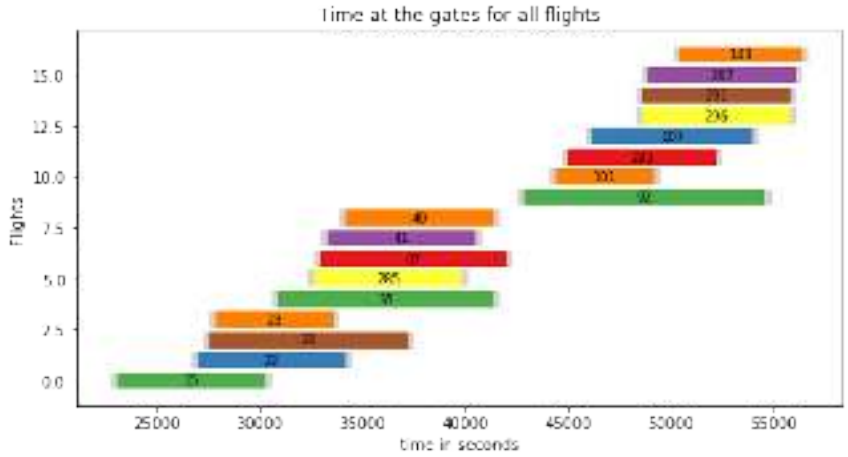




Classical Solution

- Getting exact solutions using classical solver SCIP





Embedding

- Standard D-Wave API

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- 5 embeddings for each instance

Embedding

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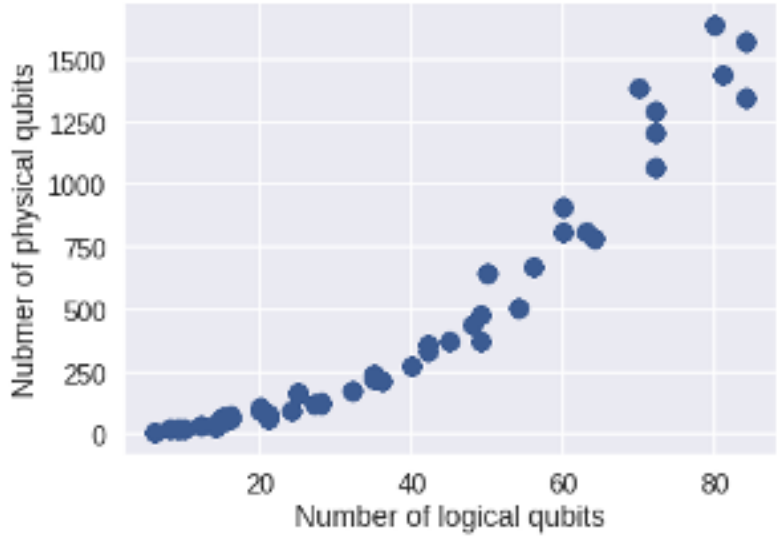

Embedding

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- 5 embeddings for each instance
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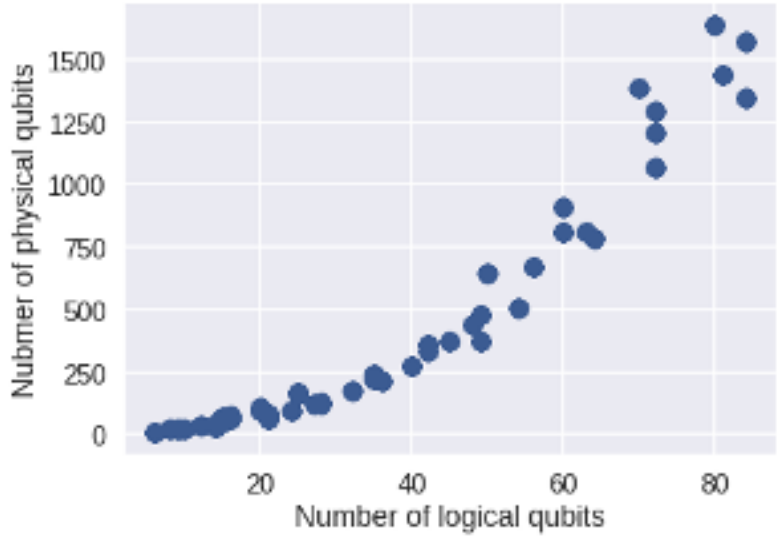
- Maximum 90 logical qubits



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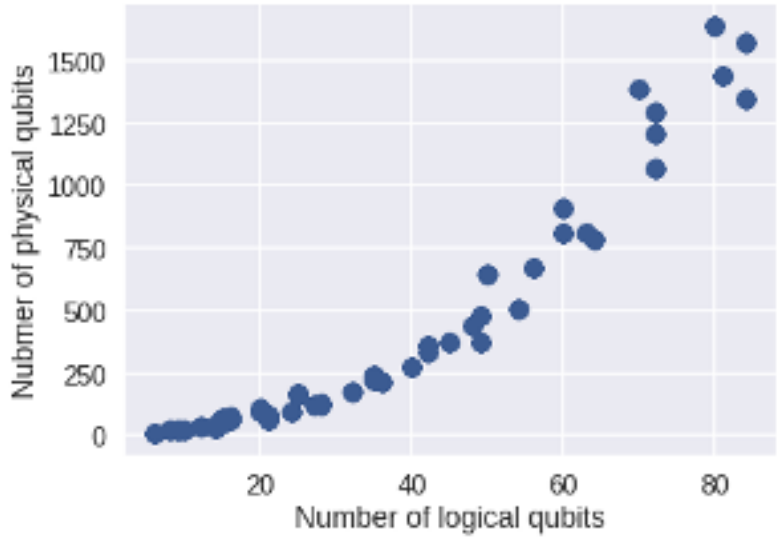
  ($\#Variables = \#Flights \cdot \#Gates$)



Annealing Results

- 10000 runs with different parameter settings

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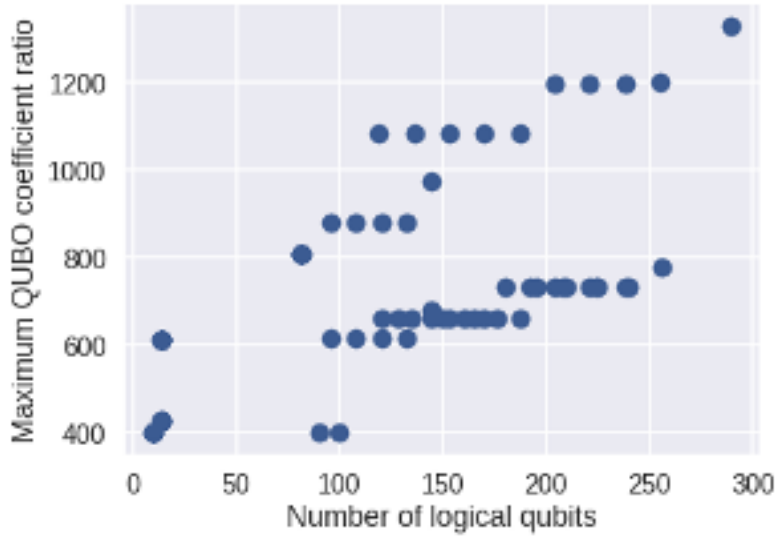
  Ratio of maximal to minimal coefficient too large!

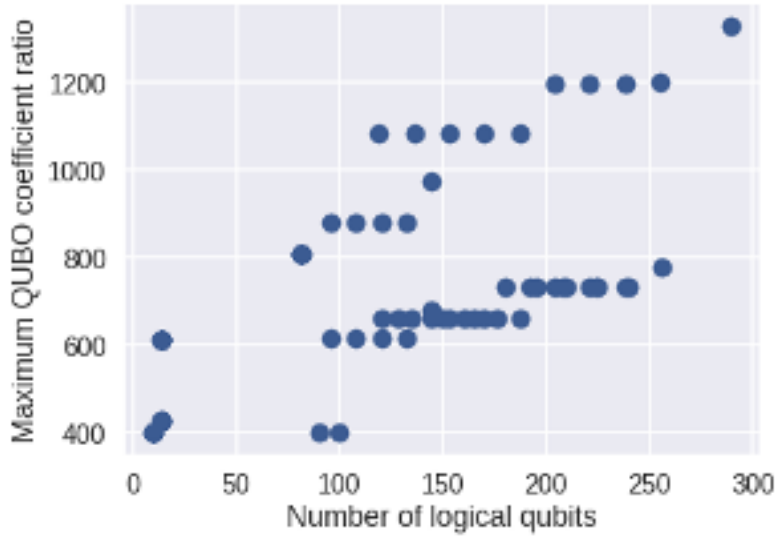


Annealing Results

- 10000 runs with different parameter settings
- Even for the small instances:

  Ratio of maximal to minimal coefficient too large!

⇒ Success probability close to 0



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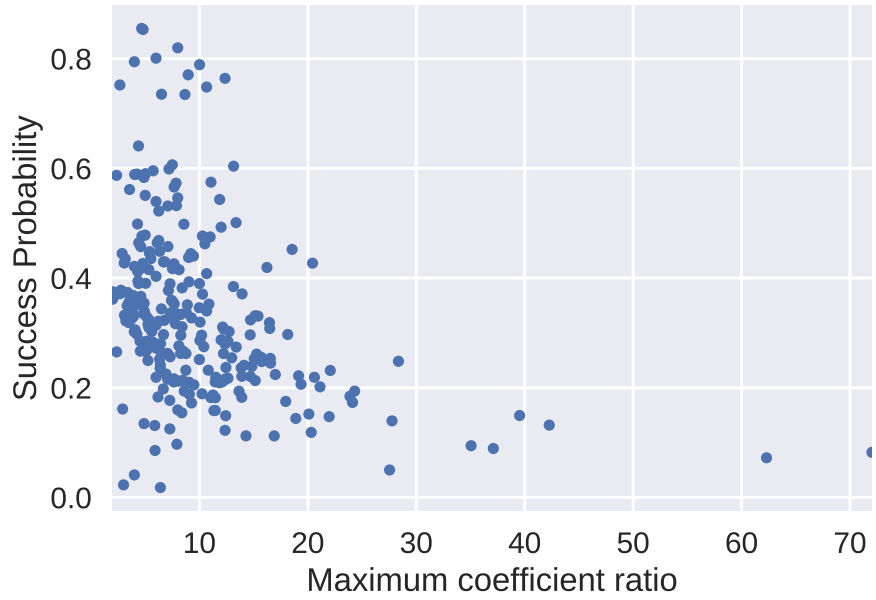

Annealing Results

- Running structurally equivalent but random instances with much smaller ratios
- 10000 runs with different parameter settings
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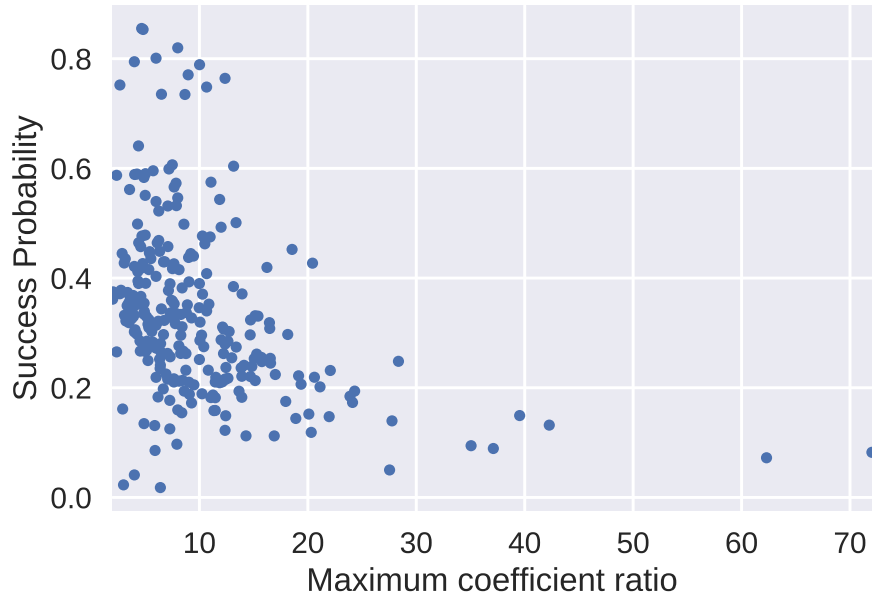
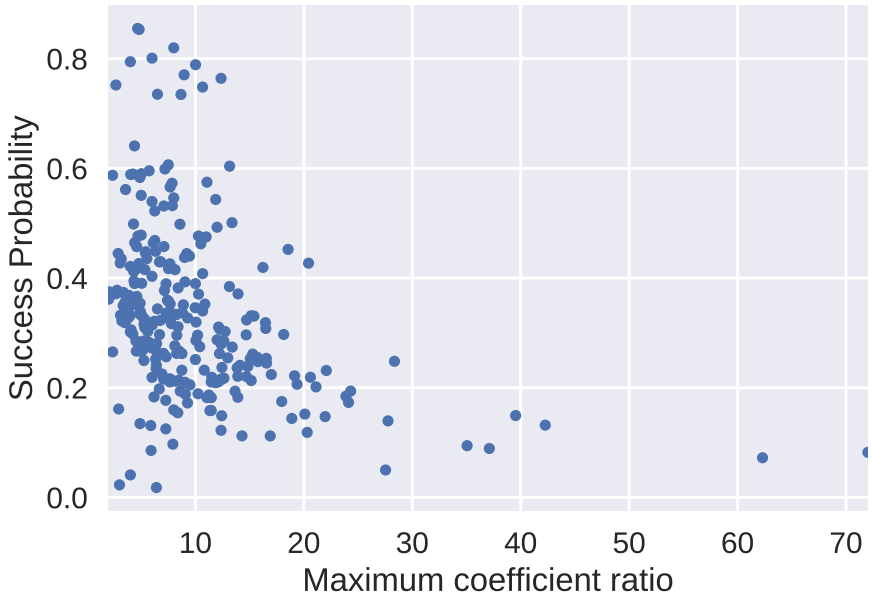
  - ratio needs to be <30
  - wide distribution



Annealing Results

- Running structurally equivalent but random instances with much smaller ratios
- 10000 runs with different parameter settings
- Differentiated results
  - ratio needs to be <30
  - wide distribution
  - due to other parameters like intra chain coupling of embedding



Outlook

- Adjust embedding/annealing parameters for more clear results

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- Investigate new D-Wave features
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- Using QAOA algorithm (related gate based approach)

Thank You