Flow and Heat Transfer over an Exponential Stretching Sheet under the Effects of a Temperature Gradient Dependent Heat Sink and Thermal Radiation.

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Abstract: The problem of MHD flow and heat transfer in a Newtonian viscous incompressible fluid over a stretching sheet with temperature gradient dependent heat sink/source and radiation is investigated. The governing partial differential equations are converted into ordinary differential equations by similarity transformation technique. The effects of viscous dissipation, work due to deformation, thermal radiation are considered in the energy equation and the variations of dimensionless surface temperature as well as the heat transfer characteristics with various values of non-dimensional parameters like Prandtl number, suction parameter, temperature gradient dependent heat sink parameter are graphed and tabulated. The heating process of the type i) the sheet with prescribed surface temperature (PST case) is studied.

Key words; *MHD*, *viscous fluid*, *stretching sheet*, *radiation parameter*, *temperature* gradient dependent heat sink,

I. Introduction

Boundary layer behavior over a continuous moving flat sheet finds wide applications in engineering and manufacturing process in industry. These include aerodynamics extrusion of plastic sheets, rolling and extrusion in manufacturing process, the cooling of an infinite metallic plate in a cool path boundary layer along a liquid film in condensation process, the boundary layer along the material handling conveyers etc.

Since the pioneering work of Sakiadis[19] presented studies on boundary layer flow adjacent to containing moving surface in the context of chemical engineering systems. McComack & Crane[14] have provided comprehensive analysis on boundary layer flow including the flow caused by stretching of flat surface and between two surfaces under different physical situations. Since then many authors including Rajgopal et al [18], Wang [26], Thakar and Soundalgekar [24], Surmadevi et al [22], have studied the boundary layer flow caused by a stretching / continuously moving sheet for different thermo physical situation using variety of fluid models and boundary conditions.

Thermal radiation effect might play a significant role in controlling heat transfer phenomena in polymer processing industry. The quality of the final product depends to an extent on the heat controlling factors. In view of this Raptis and Perdikis [1] analysed visco-elastic flow and heat transfer past a semi-infinite porous plate having constant suction of the fluid in presence of thermal radiation. Viscous dissipation which must be taken into account in the heat transfer analysis of non-Newtonian fluid flow is excluded from this stuffy. Raptis [2] studied boundary layer flow and heat transfer of micro polar fluid past a continuously moving plate with viscous dissipation in the presence of radiation. Raptis [3] also investigated the visco-elastic fluid flow past a semi infinite plate taking into consideration of radiation using Rosseland approximation Brewester [12] when the free stream velocity and the temperature of the plate are not constant. However, this work does not deal with the situation when there would be a temperature dependent heat source/sink, viscous dissipation and suction/blowing through the porous boundary surface.

Kumari and Nath [11] studied radiation effect in a non-Darcy mixed convection flow over a solid surface immersed in a saturated porous medium using Rosseland approximation. Rafael [17] studied about visco-elastic fluid flow and heat transfer over a stretching sheet under the effects of a non uniform heat source, viscous dissipation and thermal radiation. Another effect which bears great importance on heat transfer in the viscous dissipation the determination of the temperature distribution when the internal friction is not negligible is of utmost significance in different industrial fields, such as chemical and food processing , oil exploitation and bio-engineering. Consequently, the effect of viscous dissipation is also included in the energy equation.

On the other hand, the effect of radiation on visco-elastic boundary layer flow and heat transfer problems can be quite significant at high operating temperature. In view of this, visco-elastic flow and heat

transfer over a flat plate with constant suction, thermal radiation and without viscous dissipation were studied by Raptis and Perdikis [1]. Viscous dissipation and radiation were considered by Raptis [2] and the effect of radiation was also included in reference [3] and [4]. Very recently, researches in this field have been conducted by many investigators[21], however, the effects of work due to information on visco-elastic flows and heat transfer in the presence of radiation, viscous dissipation and uniform heat source/sink have not been studied in recent years.

Hong and Tien [9] investigated the thermal dissipation effects on free convection heat transfer in a porous medium bounded by a vertical plate. Angirasa et al [6] provided on an analysis on heat and mass transfer effects in internally satisfied porous medium for unsteady boundary layer flow. In the present century, a century of technological advancement, exploration of industries using latest technologies in extrusions in manufacturing processes and melt-spinning processes in taking place. In the industries the extradites is stretched into a filament when it is drawn format their dye and solidifies in the desired shape through a controlled cooling system. Then for many authors including. Thakar et al [23], Anjalidevi and Thiyagrajan [7], Khan et al [10] have analysed various problems on stretching sheet for different flow model.

It is known that thermal boundary layer equation with viscous dissipation term is non-homogeneous partial differential equation involving quadratic power of the velocity gradient. To seek a similarity solution of the thermal boundary layer equation, in case of linear stretching problem, we contemplate to deal with quadratic power law thermal boundary conditions for a general case of boundary heating of the type prescribed power law surface temperature of second degree (PST). Several closed form analytical solutions for the heat transfer characteristics are obtained in the form of confluent hypergeomtric function (Kummer's functions).

II. Mathematical Formulation

In certain coordinate system (x, y), consider two dimensional free convention steady laminar boundary layer flow of an incompressible, viscous fluid caused by a moving porous sheet embedded in porous medium in presence of a temperature gradient dependent heat sink. The porous sheet is subjected to a constant suction velocity normal to the wall. The axis is taken along wall in the direction of motion of the flow and y-axis perpendicular to it. Let the components of velocity be u and v along x and y – directions respectively. It is envisaged that the sheet issues from a thin slit at the origin (0, 0) and the speed at a point on the plate is proportional to its distance from the plate but the boundary layer approximation still hold. Under the above mentioned assumptions and the following Vafai and Tien's [25] model, the steady state boundary layer equations of mass and momentum are:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} = 0 \qquad \dots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}u - \frac{\sigma B_0^2}{\rho}u \qquad \dots (2)$$

The supplementary term in the momentum equation namely the Darcian body force term $-\frac{v}{k}u$ is linear

interms of the x-direction of the stretching motion.

The appropriate boundary conditions for the problem are:

$$u = bx, \qquad v = -v_w, \qquad at \qquad y = 0 \qquad \dots (3)$$

$$u = 0, \qquad \longrightarrow \qquad as \qquad y \qquad \infty$$

III. Solution of Momentum Equation

To solve equation (2), we postulate a solution to the velocities in x and y directions as follows:

$$u = bx G'(\eta), \quad v = -\sqrt{bv} G(\eta) \text{ and } \eta = \sqrt{\frac{b}{v}} y \qquad \dots (4)$$

by using u and v, equation of continuity (1) is satisfied. Substituting equation (4) in equation (2), we obtain:

$$G''(\eta) + G(\eta)G''(\eta) - G'^{2}(\eta) - \frac{1}{(K_{2} + M)}G'(\eta) = 0 \qquad \dots (5)$$

By using (4), the boundary conditions(3) corresponding to the equation (2) reduces to:

$$\begin{array}{ccc} G'(\eta) = 1, & G(\eta) = S & at & \eta = 0 \\ G'(\eta) = 0 & & as & \eta \to \infty \end{array} \qquad ...(6)$$

Where $S = \frac{V_w}{\sqrt{v b}}$ is the suction parameter.

A

By using the boundary conditions (6) in equation (5) the solution is given us: $G(\eta) = A + B e^{-E\eta}$

Where

$$=\frac{E^{2} - (K_{2} + M)^{-1}}{E},$$

$$B = -\frac{1}{E} \& E = \frac{1}{2} \left[S + \sqrt{S^{2} + 4(1 + (K_{2} + M)^{-1})} \right]$$

Hence the exact solution shown in (7) can be expressed as :

$$G(\eta) = \frac{1}{E} \left[E^2 - (K_2 + M)^{-1} - e^{-E\eta} \right]$$
....(8)
Hence we have

$$G'(\eta) = e^{-E\eta} \qquad \dots (9)$$

IV. Heat Transfer Analysis

Energy equation by considering thermal radiation and temperature gradient dependent heat sink term is of the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{K_2}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + Q'\frac{\partial T}{\partial y} = 0 \qquad \dots (10)$$

Where k_2 is the thermal conductivity of the fluid, T is the fluid temperature, q_r is the radiation heat flux and other quantities have usual meaning [10].

Using Rossel and approximation for radiation [12] we can write

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y} \qquad \dots \dots (11)$$

Here σ is the Stefan - Boltzmann constant and k^{*} is the absorption coefficient. Further we assume that the temperature difference within the flow is such that T⁴ may be expanded in a Taylor series about T_{∞} and neglecting higher order terms we get

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4.$$

The supplementary term in the energy equation namely the temperature gradient dependent heat sink $Q' \frac{\partial T}{\partial y}$ is

a linear function of the temperature field.

Prescribed Surface Temperature (PST) case : In this case, the boundary conditions are

where l is the characteristic length.

To solve heat equation (10), we introduce the following non-dimensional quantity for the temperature variable T as

$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}} \qquad \dots \dots (13)$$

using (4), (12) and (13) equation (10) reduce to

where

.....(7)

$$P_r = \frac{\mu C_p}{k_2}$$
 - Prandtl number ; $N = \frac{\mu C_p K^*}{4\sigma T_{\infty}^3}$ - Radiation parameter

for temperature as $\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}$ and corresponding dimensionless boundary conditions are $\theta(\eta) = 1$ at $\eta = 0$

$$\theta(\eta) \to 0$$
 as $\eta \to \infty$...(15)
To obtain the solution of equation (14), we introduce a new change of variable ζ defined as

$$\zeta = \frac{-P_r \left(1+Q^*\right)}{E^2} \left(\frac{3N}{3N+4}\right) e^{-E\eta} \qquad \dots \dots (16)$$

Hence with the help of (12) and (16), equation (14) transforms to

$$\zeta \theta'' + [1 - a_0 - \zeta] \theta' + \frac{2}{1 + Q^*} \theta = 0 \qquad \dots (17)$$

where $a_0 = \left[\frac{P_r (1 + Q^*)}{\alpha^2} - \frac{-P_r (1 + Q^*)}{\alpha} \frac{V_w}{\sqrt{bv}} \right] \left(\frac{3N}{3N + 4} \right) e^{-E\eta}$

and the corresponding boundary conditions converts to

The solution of equation (17) under the boundary conditions (18) interms of confluent hypergeomtric function is obtained in terms of η as

$$\theta(\eta) = e^{-Ea_0\eta} \frac{M[a_0 - S; 1 + a_0; -K_3 e^{-E\eta}]}{M[a_0 - S; 1 + a_0; -K_3]} \qquad \dots (19)$$

where $S = \frac{2}{1 + Q^*}$ and $K_3 = \left[\frac{-P_r(1 + Q^*)}{E}\right] \left(\frac{3N}{3N + 4}\right)$

Skin Friction

The non-dimensional skin-friction (τ) at the wall $\eta = 0$ is derived as

$$\tau = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = bxG''(\eta) \qquad \dots (20)$$

and obtained for different sets of values of permeability parameter k_2 , suction parameter S and stretching rate bx are tabulated in the following table 1.

k ₂	S	bx	τ
10	0.5	1.0	-1.32819
20	0.5	1.0	-1.30475
40	0.5	1.0	-1.29682
30	1.0	1.0	-1.56193
30	1.5	1.0	-2.13875
30	0.5	1.5	-1.89329
30	0.5	2.0	-2.76563
Table 1. Skin-friction.			

V. Results and Discussion

Results for the velocity profiles are depicted fig(1). By studying the velocity for various positions. From Fig(1a) and Fig (1b) it is seen that velocity decreases for increasing values of permeability parameter k_2 and magnetic parameter M. It is also observed that physically k_2 expresses the pressure of porous matrix and quantities the hydraulic conductivity of the porous medium. The permeability acts as Darcy force term

 $\frac{-1}{k_2}G'(\eta) = 0$ for $k_2 \to \infty$ in the transformed equation and serves to retard the momentum in the positive x-

direction.

Fig(1b), M express the pressure of magnetic field and the magnetic conductivity of the magnetic field. The magnetic parameter acts as magnetic force term and $\frac{-1}{M}G'(\eta) \rightarrow 0$ for $M \rightarrow \infty$ in the equation and

serves to retard the momentum in the positive x-direction. Thus our results related to magnetic effect are correlated with the general conclusion arrived at by the studies of Thakar et al[23].

Fig(1c) shows the variation of velocity u i.e. x-direction longitudinal velocity versus η for S= 0 (non porous wall i.e. zero section). S = 0.5, S = 1.0 and S = 1.5. It is observed that a steady decrease in u and rise in suction parameter S, with all profiles tending asymptotically to the horizontal axis. The longitudinal velocity u is observed to be a maximum in all cases at the wall i.e. $\eta = 0$. Schlichting[20] has shown that suction acts physically to increase the adherence to the wall of the hydrodynamic boundary layer, which in turn retards the flow in the longitudinal direction.

Fig(2a) shows the effect of permeability (k_2) on transverse velocity $G(\eta)$ versus η . During the initial stage when the leading edge effect is not felt, the transverse velocity vis minimum and its maximum values always occur at the end of η range and steadily rise from $\eta = 0$ to $\eta = 5$ and ultimately flattered towards the end of the range. The permeability of the porous structure k_2 and suction S in the momentum equation (5) are inherently tied to the porous matrix and generally act in accordance with experimental finding (see Vafai & Tien[25]). These results correlate well with the general conclusion arrived at by the studies of Ingham[18], Thakar et al[23].

Fig(2b) shows the effect of magnetic parameter M on transverse velocity $G(\eta)$ versus η physically M express the pressure of uniform magnetic field. Initially when magnetic effect is not felt at the edge, the velocity distribution is minimum and after wards it steadily increases and at the end of range of values of η distribution is maximum and ultimately remains same till the last. The magnetic conductivity of the magnetic parameter M acts as magnetic force and as M increases which intern decreases the transverse velocity.

In fig (2c), shows the variation of transverse velocity i.e. $G(\eta)$ versus η for various values of suction parameter S = 0.5, 1.0, 1.5. It is observed from the figure that there is steady decrease in velocity accompanies a rise in S with all profiles tending asymptotically to the horizontal axis. The velocity is observed to be a maximum in all cases at the wall u, when

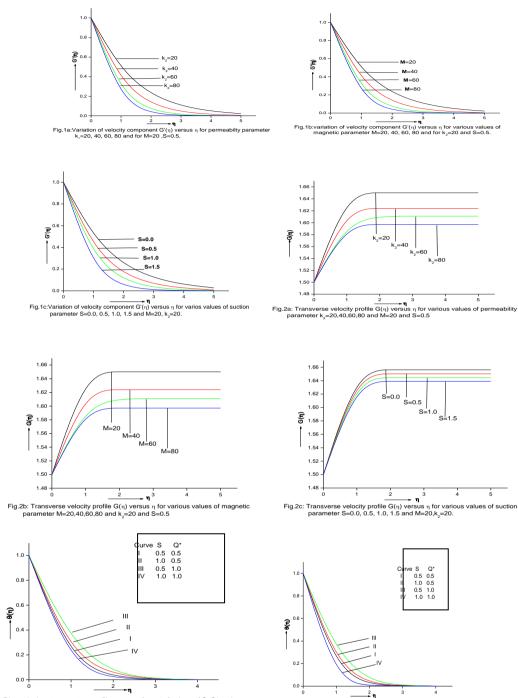
 $\eta = 0$. Ajaykumar[5] has shown that suction acts physically to increase the adherence to the wall of the hydrodynamic boundary layer which intern retards flow longitudinal direction and velocity profiles are monotonically decreasing in nature.

Fig (3a) shows the variations in temperature $\theta(\eta)$ with η for the effects of various combinations of S and Q^{*}. Maximum temperature distribution is occurred at the curve 1 for which S=0.5 and Q=0.5. This physically means that an union of low suction with low wall temperature and weak heat sink. Further temperature distribution is decreasing for rise in values S and Q^{*} values. However the value of suction parameter S for curve II is higher than for the curve III and curve IV. This implies that S impinges relatively less effects on lowering the temperature in comparison to the values of P and Q^{*}. Such a scenario implies that a stronger heat sink or greater wall temperature, parameter plays more dominant role for lowering the temperature.

Fig(3b) depicts the graph of variations in heat transfer with permeability parameter k_2 illustrating the effects of S and Q^{*} values. As the heat sink parameter Q^{*} rises and for increasing values of permeability parameter k_2 from curve I to curve IV, the temperature distribution is lowered. However for curve III, the P value is higher and for the curve II, the suction parameter S is higher which explains the fact that suction plays a more powerful role in reducing the temperature.

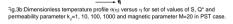
Fig(4a) is drawn for temperature profile for various values of Prandtl number P_r and radiation parameter N when $V_w = -0.04$. This fig depicts the graphical representation of temperature profile for various values of Prandtl number P_r and radiation parameter. The effect of increasing values of Prandtl number P_r is to decrease temperature at a point in the flow field, as there would be a thinning of the thermal boundary layer, as a result of reduced thermal conductivity. The combined effect of increasing values of P_r and N is to reduce temperature largely in the boundary layer flow region.

Fig(4b) is plotted for the same data sets as those of fig(4a) respectively except for $V_w = 0.04$. In these cases temperature profiles follow similar pattern as in the case of fig (4a). However, the comparative study of fig(4a) and (4b) reveals that the effect of blowing parameter $V_w = 0.04$ is to increase the temperature throughout the boundary layer except on the wall where it attains unity in PST case. This is due to the fact that thermal boundary layer increases in the case of blowing.



.3a:Dimensionless temperature profile versus $_{\rm N}$ for set of values of S, Q* and magnetic parameter M=20, 40, 60, 80 permeability parameter k_2=20 in PST case.

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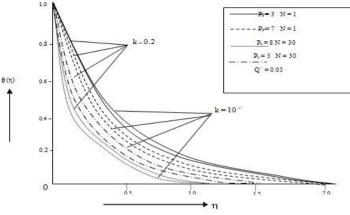
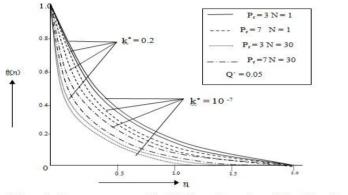


Fig. (4b) Dimensionless temperature profiles $\theta(\eta)$ for various values of Prandtl number $~P_r$ and radiation parameter N when y_{ss} = 0.04



 $r_{Mg_c}(4a)$ Dimensionless temperature profiles $\theta(\eta)$ for various values of Prandtl number P_r and radiation parameter N when V_{ge} = -0.04

VI. Conclusions:

The important conclusions of the study are as follows:

- 1. The horizontal velocity is maximum at $\eta=0$ for all values of k_2 , M and S. and decreases rapidly with increasing values of η .
- 2. The longitudinal velocity decreases with rise in the magnitude in the suction parameter S.
- 3. The transverse velocity is minimum at $\eta=0$ for all values of S, k_2 and M and increases slowly with increasing values of S, k_2 and M.
- 4. An increase in suction parameter S, wall temperature parameter P or heat sink parameter Q results in lowering the temperature field steadily.
- 5. An increase in the permeability parameter and magnetic parameter increases the skin-friction while an increase in suction parameter decreases the skin-friction.
- 6. The combined effect of increasing values of Prandtl number P_r and radiation parameter N is to reduce the temperature largely in the boundary layer flow region.
- 7. The effect of the suction parameter v_w is to increase the numerical value of wall temperature gradient and that of blowing is to decrease the same for all values of small visco-elastic parameter k_2 , Prandtl number P_r and radiation parameter N in absence of frictional heating.

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