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Flow Characteristics in the Curved Rectangular Channels

(Visualization of Secondary Flow)*

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In order to clarify the secondary flow pattern which appears in curved rectangular channels , flow visualization is made in a fully developed laminar flow. The channels have aspect ratios ranging from 0.5 to 2.5 , curvature ratios ranging from 5 to 8 and a width of 20 mm. From the experimental results the developing process of secondary flow vortices becomes clear. At a retarded layer near the outer concave wall, additional counter rotating pairs of vortices are observed. The critical Dean number takes the minimum and maximum values at the aspect ratios of about 1 and 2 , respectively. The critical Dean number increases rapidly with decreasing aspect ratio when it is smaller than 1.

Flow Mechanics, Experimental Study, Flow Visualization, Secondary Flow Vortex, Curved Rectangular Channel, Critical Dean Number,

1. Introduction

When a fluid flows through a curved channel with circular or rectangular cross section , a pressure gradient arising across the channel is required to balance the centrifugal force attributed to the curvature. As a result , a secondary flow is set up.

For the flow in the curved rectangular channel, this secondary flow is classified into two different types. One is a pair of counter rotating vortices (called the main secondary flow vortices) which are set up at a low Dean number range and extend almost all over the cross section. The other is one or more pairs of the additional counter rotating vortices (called the additional secondary flow vortices) which are set up at a retarded layer near the outer concave wall in addition to the main secondary flow vortices beyond the critical Dean number.

In a curved rectangular channel with a specified aspect ratio, the point of the maximum velocity of the main stream shifts from the center of the channel towards the outer concave wall. Therefore , a sharp velocity gradient arises in the region between the point of the maximum velocity and the outer concave wall where velocity is zero. These phenomena lead to a rapid decrease of the centrifugal force on a fluid element , and then the difference between the pressure at the outer concave wall and that at the inner convex wall becomes maximum. In such a region , the centrifugal force does not balance the inward force caused by the pressure difference. Consequently, additional secondary vortices are set up due to the unstable flow or the instability near the outer concave wall.

The first study on the secondary flow was made by Dean[(1)] and later , by Barua[(2)] and Mori et al.[(3)(4)].

A numerical analysis on the flow passing through the curved rectangular channel was made by Cheng et al.[(5)(6)] and Ghia[(7)]. They demonstrated the occurrence of the secondary flow vortices theoretically.

An experimental investigation on flow visualization was reported by Cheng et al.[(8)]. They visualized the secondary flow patterns in curved rectangular channels with a channel width of 25 mm, a curvature ratio of 5 and aspect ratios ranging from 1 to 12. They also showed that additional secondary flow vortices are set up in addition to the main secondary flow vortices. Furthermore , the relation between the aspect ratio and the critical Dean number was also found, but this relation in case when the aspect ratios less than 1 and between 2 and 3 was not found. Akiyama et al.[(9)] also reported on it.

The purpose of the present work is to clarify the phenomena of a secondary flow which appears in a fully developed laminar flow in the curved rectangular channel by means of visualizing the flow pattern. The channels used for the experiments have a width of 20 mm, curvature ratios ranging

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from 5 to 8 and aspect ratios ranging from 0.5 to 2.5 . It was found from the experiments that in the case of the aspect ratio 2 the developing process of the secondary flow vortices depends on the curvature ratio and that the critical Dean number depends on the aspect ratio.

2. Nomenclature

Often used symbols in this paper are defined as follows ;

- a : channel width of a curved rectangular channel
- b : channel height of a curved rectangular channel
- R : radius of curvature of a channel
- K : aspect ratio $= b/a$
- D_h : equivalent hydraulic diameter $= 2ab/(a+b)$
- r : curvature ratio $= R/a$
- $\bar{w}$: mean velocity
- ν : kinematic viscosity
- Re : Reynolds number $= \bar{w}D_h/\nu$
- De : Dean number $= Re\sqrt{a/R}$

Table 1. Dimensions of curved rectangular channels (Material;acryl resin)

K	r	R(mm)	a(mm)	b(mm)
0.5	5	100	20	10
	6	120		
	7	140		
	8	160		
0.75	5	100	20	15
	6	120		
	7	140		
	8	160		
1.0	5	100	20	20
	6	120		
	7	140		
	8	160		
1.25	5	100	20	25
	6	120		
	7	140		
	8	160		
1.5	5	100	20	30
	6	120		
	7	140		
	8	160		
2.0	5	100	20	40
	6	120		
	7	140		
	8	160		
2.5	5	100	20	50
	6	120		
	7	140		
	8	160		

3. Experiments

The dimensions of the curved rectangular channels used in this experiment are listed in Table 1. These channels were made of acryl resin. A schematic drawing of the experimental apparatus is shown in Fig.1. Air from blower was drawn into an air tank to reduce the pulsation and was brought to a laminar flow meter which was connected to a Betz-type manometer. The volume flow rate was evaluated from reading the Betz-type manometer. The air temperature was measured at the exit of laminar flow meter. Then the air was led separately to a smoke generator and to the main channel by adjusting the valve. Air was mixed with the incense smoke in the smoke generator and passed through a water strainer and a filter of absorbent cotton. Then , the air was again made confluent with the flow in the main channel. The air which was mixed with smoke passed through the entrance channel and was conducted to the curved rectangular channel. The secondary flow pattern was observed at the exit of the curved rectangular channel by using a slit light source which was placed perpendicularly to the main stream. The flow pattern was photographed by a camera, mounted with telephoto lens of 300 mm , which was installed in front of the exit of the curved channel.

4. Experimental Results and Discussion

4.1 Pattern of Secondary Flow

Typical photographs are shown in Fig.2 to Fig.9 when the aspect ratios K = 0.5, 0.75, 1.0, 1.25, 1.5, 2.0 and 2.5, respectively , to demonstrate the developing process of various types of the secondary flow in the curved rectangular channels.

The left side of these photographs is the inner convex wall and the right side is the outer concave wall. It was found from the results of the experiments of flow visualization that the secondary flow pattern in the curved rectangular channel changes with increasing Dean number.

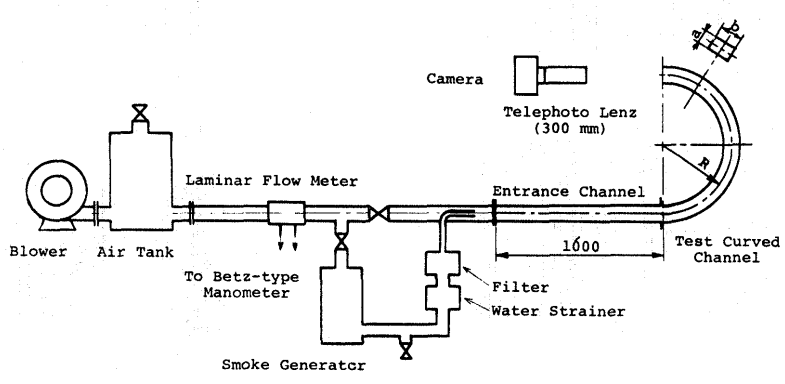


Fig.1 Schematic diagram of experimental apparatus

Changing process of flow pattern.

(a) Occurrence of the secondary flow vortices only [e.g. Fig.2(a)].

(b) Occurrence of retarded layer near the outer wall in addition to the main secondary flow vortices [e.g. Fig.2(b)].

(c) Occurrence of the additional secondary flow vortices developed from the retarded layer [e.g. Fig.2(c)].

(d) Impossible to judge.

The flow patterns in the cases of aspect ratios K = 0.5 and 0.75 are shown in Figs.2 and 3, respectively, they change as mentioned above. One pair of the additional secondary flow vortices is set up [e.g. Fig.2(c) and Fig.3(b)]. The situations shown in Figs.2(a) and (c) are similar to the results obtained from the numerical analysis by Cheng et al.[Fig. 3(b) in Ref.6].

The flow patterns in the case of aspect ratio K=1.0 are shown in Fig.4. Figs.4(a), (b) and (c) correspond respectively to the changing process of the flow patterns (a), (b) and (c) mentioned above. One pair of the additional secondary flow vortices and its development can be seen in Ref.8 (fig.4). These flow patterns seem to indicate almost the same tendency as those pointed out by the authors. The secondary flow pattern, Fig.4(a),is similar to the secondary flow streamlines shown in Ref.6 [Fig.2(a)] and Ref.7 [Fig.6(c)] . Fig.4(c) is similar to the streamlines given in Ref.6 [Fig.2(b) to (d)] and Ref.7[Fig.7(a)] which show one pair of the additional secondary flow vortices.

The additional secondary flow vortices disappear, as shown in Fig.4(d), as Dean number increases over that at the situation as shown in Fig.4(c) in which the vortices are developing. Cheng et al. pointed out these phenomena and showed the secondary flow streamlines at the situation in which the additional secondary flow vortices disappeared in Ref.6[Fig.2(e)].

With increasing Dean number two pairs of the additional secondary flow vortices are set up from each corner of outer wall as shown in Fig.4(e). It seems that these secondary flow patterns have not be reported in any other paper.

Flow patterns in Fig.4 indicate the same tendency in the range of the curvature ratios from 5 to 8.

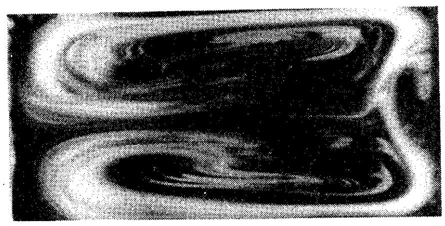
The secondary flow patterns in the case of aspect ratios K = 1.25 and K = 1.5 are shown in Fig.5 and Fig.6, respectively. These flow patterns changed in the same manner as in the case when aspect ratio K = 0.5 until a certain Dean number and one pair of the additional secondary flow vortices was set up. With further increase of Dean number, two pairs of the additional secondary flow vortices were set up in some cases. However, when these vortices were set up the flow pattern fluctuates considerably.

The flow patterns in the case of the aspect ratio K = 2.0 are shown in Figs.7 and 8. It is of particular interest to note that the occurrence of the additional secondary flow vortices depends on the curvature ratio.

In the case of the curvature ratio r = 5 , one pair of the additional secondary flow vortices was set up at a certain Dean number as shown in Fig.7(c) and two pairs of the additional secondary flow vortices appeared with increasing Dean number as shown in Fig.7(d).

On the other hand, in the case of the curvature ratio r = 8, two pairs of the additional secondary flow vortices were

(a) De=140, Re=396

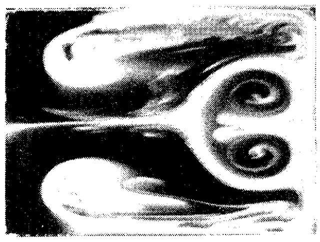


(b) De=261, Re=738

(c) De=275, Re=777

Fig.2 K = 0.5, r = 8

(a) De=203, Re=574



(b) De=282, Re=797

Fig.3 K = 0.75, r = 8

set up first at a certain Dean number as shown in Fig.8(c). These vortices developed further as Dean number increased and thus one pair of the additional secondary flow vortices appeared as shown in Fig.8(d). These phenomena are reverse to those at curvature ratio r = 5.

Dean number at which the additional secondary flow vortices begin to be set up at r=8 is about 30 lower than that at r=5.

In the curved rectangular channel with K=2.0, r=6, one pair of the additional secondary flow vortices changes into two pairs with an increasing Dean number similarly to the case of K=2 , r=5 . When K=2.0 and r=7, two pairs of the additional secondary flow vortices change into one pair similarly to the case of K = 2.0 and r = 8. It seems that the value of the critical curvature ratio is in the range of r = 6 to 7. Fig.7 is similar to Fig.5 in Ref.8.

The flow pattern at the aspect ratio K=2.5, as shown in Fig.9, takes the same process as that at K = 2.0, r = 8 and two pairs of the additional secondary flow vortices are set up independently of the aspect ratio r.

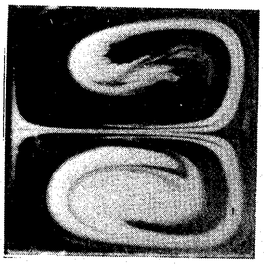
4.2 Critical Dean Number (Dec)

As was mentioned in Sec.4-1 regarding to the changing process of flow patterns, a Dean number at which the flow pattern changes from the situation (b) to (c) is named the critical Dean number (Dec).

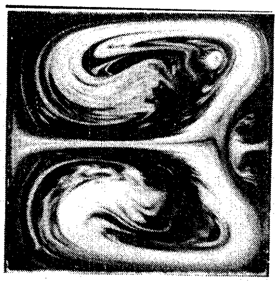
The critical Dean number is determined from the results of the experiments in flow visualization for the curved rectangular channels with various aspect ratios K and curvature ratios r. Fig.10 shows the relation between K and r . Figs.2(b), 3(a),4(b),5(a),6(a), 7(b), 8(b) and 9(a) show the flow patterns at which Dean numbers are very close to the critical values at each given aspect ratio.

The broken line in Fig.10 shows the experimental results by Cheng et al. for the curved rectangular channels with a = 25 mm, r=5 and K≧1.

In Fig.10, the region where the Dean number is lower than the solid line is designated as the stable region and the region where the Dean number is higher than



(a) De=93, Re=265



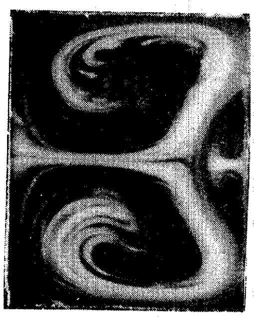
(b) De=139, Re=393

(c) De=183, Re=518

(d) De=383, Re=1083

(e) De=527, Re=1489

Fig.4 K = 1.0, r = 8



(a) De=133
Re=376

(b) De=249
Re=704

Fig.5 K = 1.25, r = 8

(a) De=151
Re=426

(b) De=216
Re=612

Fig.6 K = 1.5, r = 8

this is designated as the unstable region.

The following facts will be found from Fig.10.

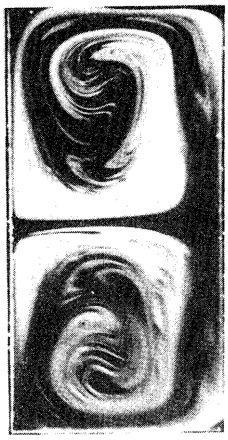
(1) The critical Dean number Dec becomes minimum in case when the aspect ratio K is about 1 and increases in case when both $K>1$ and $K<1$, but the rate of increase in case when $K<1$ is remarkably higher than that when $K>1$. The critical Dean number takes the minimun value in case when the aspect ratio K is from 2.0 to 2.5. Cheng et al. showed streamlines obtained from the numerical analysis in cases when the aspect ratios K=0.5 and K=1.0 in Ref.6 (Figs.2 and 3). They did not discuss in regard to the critical Dean number in Ref.6. However, after transforming the Dean number De and the curvature ratio r defined by them into the present De and r, respectively, the critical Dean number Dec will become as follows.

When K=0.5, r=2/3X100, $217<Dec<245$,
when K=1.0, r=100, $151<Dec<202$.

The numerical results obtained by Cheng et al. show that Dec for K=0.5 is higher than that for K=1.0 . This indicates a similar tendency to that of the present experimental results as shown in Fig.10.

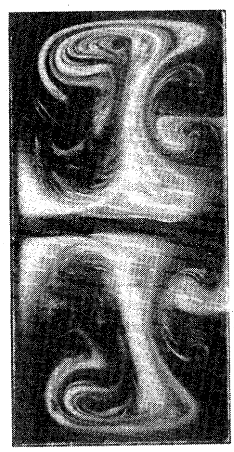
The relation between De and K is similar to the results obtained by Cheng et al.[8] in the range of K = 1.0 to 2.0 .

(2) Dec decreases as the curvature ratio r increases from 5 to 8 in the range of the aspect ratios K = 0.5 to 2.5 . The

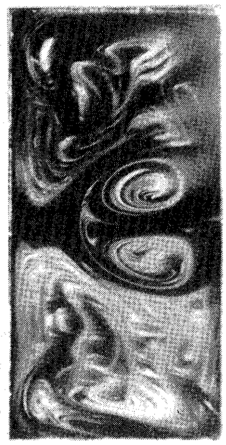


(a) De=127 Re=360

(b) De=180 Re=508

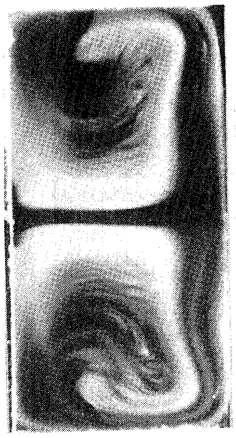


(c) De=224 Re=634



(d) De=391 Re=1106

Fig.8 K = 2.0, r = 8



(a) De=131 Re=293

(b) De=222 Re=496

(c) De=448 Re=1001

(d) De=596 Re=1332

Fig.7 K = 2.0, r = 5

(a) De=174 Re=491

(b) De=279 Re=789

Fig.9 K = 2.5, r = 8

rate of decrease of Dec is higher for a larger K , for example when K = 2.5 Dec for r=5.0 is about 40 higher than that for r = 8.0.

In case when two pairs of the additional secondary flow vortices are set up at K=2.0 or 2.5, the flow pattern shown in Fig.8(b) appears in a considerably wide range of Dean number . It is difficult to judge from the situation shown in Fig.8(b) whether the fluid in the retarded layer near the outer concave wall flows in vortical state or not. Therefore, for determining the value of Dec it is likely that some error may be involved in the experimental results of visualization.

Cheng et al.[8] did not give Dec in the range of 2<K<3. Difference in Dec between the present results and those by Cheng et al.[8] is about 30 for K=1.0 . It seems that this difference comes from the judgement about the occurrence of the secondary flow pattern when Dec is to be determined.

5. Conclusions

From the visualization of a secondary flow pattern in a fully developed laminar flow region in curved rectangular channels with aspect ratios K = 0.5 to 2.5 and curvature ratios r = 5 to 8 , the following conclusions were obtained.

(1) In the laminar stable region at a low Dean number, only main secondary flow vortices appear, and as the Dean number increases due to instability of flow in the retarded layer near the outer concave wall, one or two pairs of the additional secondary flow vortices are set up . In case when aspect ratio K=2.0, the process of occurrence of the additional secondary flow vortices depends on the curvature ratio r.

(2) From the relation between the critical Dean number Dec and aspect ratio K , it is likely that Dec becomes minimum when K is nearly 1.0 and becomes maximum when K is nearly in the range 2.0 to 2.5 . On the other hand, Dec decreases with increasing curvature ratio and this tendency becomes prominent with increasing aspect ratio.

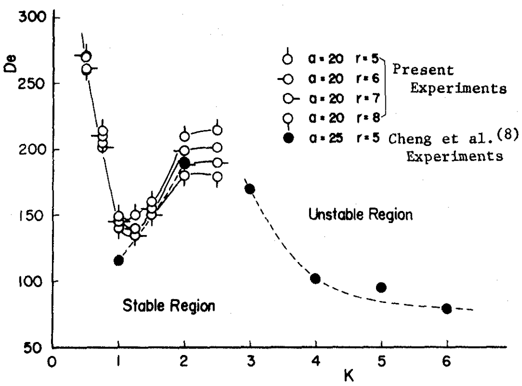


Fig.10 Relation between De and K

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