

Flows around Two Tandem Circular Cylinders
at Very High Reynolds Numbers*

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Flows around two tandem circular cylinders in a stream up to the range of much higher Reynolds numbers than the critical one were investigated in a low speed wind tunnel. By making the surface of the upstream circular cylinder rough with two types of surface roughness, the critical Reynolds number of the cylinders was reduced.

Drag coefficients and Strouhal numbers of the two tandem circular cylinders were measured and the flow patterns on the cylinders were visualized by the surface oil-flow technique, in the subcritical flow regime, in the supercritical one where there were laminar bubbles followed by turbulent separation and in the transcritical one where purely turbulent separation occurred without laminar bubbles but with an extremely regular vortex shedding behind the cylinders.

The flow characteristics of the two tandem circular cylinders were discussed and the effects of Reynolds numbers and the gap spacing between the two cylinders were made clear.

1. Introduction

Recently, the aerodynamical and the aeroelastic characteristics of two circular cylinders placed in a stream have received wide attention, for their practical applications are seen in various areas of engineering: heat exchangers, transmission lines, chimney stacks, jetties, off-shore structures, and so on. So the author studies about flows around two circular cylinders in a tandem arrangement as the most basic and fundamental research.

Succeeding to the measurements of drag forces by Biermann and Herrstein⁽¹⁾, numerous works^(2~12) have been done on the flows around two tandem circular cylinders. All Reynolds numbers which are treated in their experiments, however, are comparatively low and the flows around the upstream cylinder of their two tandem cylinders are subcritical. There are few experimental works in the range of very high Reynolds numbers of the flow regime which represent most of the practical applications in civil, mechanical or electrical engineering.

Even in the case of a single circular cylinder, higher Reynolds number than the critical one can not be easily achieved in wind tunnels. So, in the reference (13), the author used a cylinder with surface roughness in order to promote transition to turbulence in the boundary layers around a circular cylinder and consequently to reduce the actual Reynolds number at which a purely turbulent separation occurs.

And the author could draw some interesting conclusions concerning the roughened cylinder.

Flows around the roughened cylinder become supercritical ones where there are observed laminar bubbles at relatively low Reynolds number and further the supercritical flow regime is followed by the flow regime in which separation is purely turbulent without laminar bubbles. This flow regime is called transcritical, according to Roshko's definition⁽¹⁴⁾, where the author found a vortex shedding with an extremely high periodicity. Flow around a single circular cylinder at very high Reynolds numbers is very complicated, and hence flows around two tandem cylinders at such Reynolds numbers seem to be still more complicated.

In the present study the flow characteristics of two circular cylinders in a tandem arrangement are clarified by measurements of drag forces and Strouhal numbers and also by visualization tests of the surface flows on the cylinders. A roughened cylinder is used as an upstream one to give insight into flow characteristics of two tandem cylinders at much higher Reynolds numbers.

Nomenclature

C_D :	drag coefficient ($=D/1/2\rho U^2 d l$)
D :	drag force
d :	diameter of circular cylinder (=30cm)
f :	frequency of velocity fluctuation
l :	span-length of circular cylinder (=200cm)
Re :	Reynolds number ($=Ud/\nu$)
S :	gap spacing between two tandem cylinders
St :	Strouhal number ($=df/U$)
U :	velocity of main stream
δ :	diameter of particle roughness

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- θ : angular position from stagnation point
 ν : kinematic viscosity of air
 ρ : density of air

Subscripts

- 1: upstream
 2: downstream

2. Experimental apparatus and method

Experiments were conducted in a low speed wind tunnel. Test section of the wind tunnel was rectangular in form, 4m high by 2m wide, where longitudinal turbulence was, no more than 0.1% of the main stream.

Cylinder models were $d=30\text{cm}$ in diameter and $l=200\text{cm}$ in span-length, placed horizontally in a tandem arrangement as shown in Fig. 1, and clamped outside the side walls of the wind tunnel. The blockage ratio of this cylinder model to the test section was so small, 0.075 that the blockage corrections were omitted.

The Reynolds number was varied in the range from 0.4×10^5 to 6.2×10^5 and the gap spacing between the two cylinders was varied in the interval $0.07 < S/d < 5.3$.

The surface of a smooth cylinder model was covered with a smooth thin aluminum sheet. As a roughened cylinder two different types of cylinders were prepared: one cylinder with a pair of roughness strips of 1 cm wide of small polystyrene of 1.5 mm in diameter locally attached at ± 50 deg. from the stagnation point, and the other one with polystyrene of 2.7 mm in diameter glued over its whole surface in such a manner as to give a fairly uniform random distribution.

The drag forces of two cylinders were measured by the strain-gauges cemented on the supporting arms of the cylinder models and the frequency measurement of the wake velocity was made with constant temperature hot-wire anemometers. Each of two hot-wire probes was placed at its own position behind the upstream and the downstream cylinders as shown in Fig. 1.

3. Experimental results and discussion

3.1 Two smooth cylinders in a tandem arrangement

3.1.1 Reynolds number effects

The drag coefficients (C_{D1} , C_{D2}) and Strouhal numbers (St_1 , St_2) of two smooth cylinders with the gap spacing $S/d=2$ and 4 are shown against Reynolds number in Fig. 2.

The values of C_{D1} and St_1 of the upstream cylinder remain almost constant up to Reynolds number of 3×10^5 , i.e. $C_{D1}=1.0$ ($S/d=2$) and 1.2 ($S/d=4$), and $St_1=0.14$ ($S/d=2$; the amplitude of velocity fluctuation is very slight.) and 0.19 ($S/d=4$). At the critical Reynolds number of 3.8×10^5 , the value of C_{D1} abruptly drops accompanied with a remarkable increase of Strouhal number. For Reynolds numbers greater than 4.4×10^5 , the value of C_{D1} of the upstream cylinder remains small, i.e. $C_{D1}=0.3$ ($S/d=2$) and 0.4 ($S/d=4$).

On the other hand, the values of C_{D2} and St_2 of the downstream cylinder change against Reynolds numbers subject to the above-mentioned variations for the upstream cylinder; up to Reynolds number of 3.0×10^5 the downstream cylinder receives a thrust force for small spacings and a small drag force for large spacings; beyond the critical Reynolds number where an abrupt drop of C_{D1} occurs, the value of C_{D2} increases to become greater than that of C_{D1} , at both values of the spacing, $S/d=2, 4$.

The results of the visualization tests by the surface oil-flow technique are presented, for example at the spacing $S/d=2$, in Fig. 3. At Reynolds number of 2.5×10^5 the boundary layer on the upstream cylinder

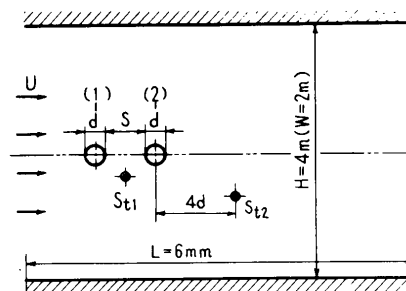
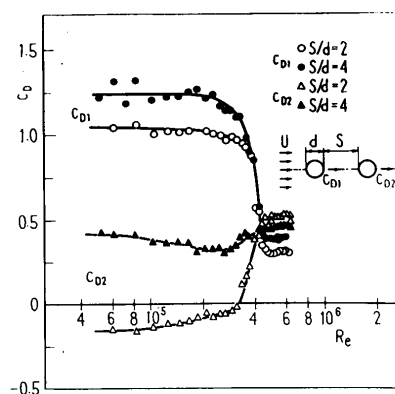
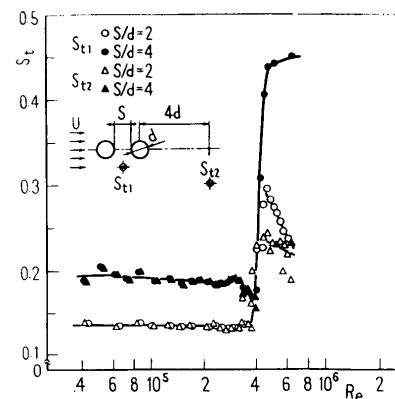


Fig. 1 Arrangement of two tandem cylinders



(a)



(b)

Fig. 2 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (St_1 , St_2) of two smooth cylinders in a tandem arrangement

separates around 70 deg. but the behavior of a surface oil film on the downstream cylinder is obscure. At a Reynolds number of 4.8×10^5 which is considerably greater than the critical one, there can be clearly observed laminar bubbles on the surface of the upstream cylinder, where the flow separates around 90 deg., reattaches at 105 deg. and finally separates at 120 deg. Then on the downstream cylinder, the flow is visualized to reattach around 30 deg. and to separate at 125 deg., without laminar bubbles.

Fig.4(a) shows typical spectra of the velocity fluctuations behind two tandem cylinders with the spacing of 2, at the above Reynolds number of 4.8×10^5 . The power spectral densities of both velocities are considerably broad band, and their center Strouhal numbers are not equal to each other. Further at the spacing of 4, in Fig.4(b), the peaked Strouhal number for the upstream cylinder is 0.45 and equal to

the corresponding value of a single smooth cylinder, but this component decays too fast to be detected behind the downstream cylinder. So the Strouhal number of the highest peak for the downstream one is 0.23 and not equal to St_1 of the upstream.

From the above results, the variations of drag coefficients and Strouhal numbers of the upstream cylinder with Reynolds numbers are found to be almost similar to those of a single smooth cylinder, and the drag coefficients and Strouhal numbers of the downstream cylinder change with the variations of flows around the upstream cylinder. So in the following sections, the four regimes of the flow around only the upstream cylinder, that is, the subcritical, the critical, the supercritical and the transcritical regimes, will be applied for two tandem cylinders.

3.1.2 Effects of the gap spacing

Fig.5 shows typical values of drag coefficients and Strouhal numbers of two tandem cylinders at Reynolds numbers of $(1.7, 2.5) \times 10^5$ as the subcritical flow regime, 3.8×10^5 as the critical flow regime and $(5.6, 6.3) \times 10^5$ as the supercritical flow regime, compared with the values obtained by various authors for different Reynolds numbers. The Reynolds number ranges which are measured by Biermann⁽¹⁾ ($Re=10^5$), Imaichi⁽²⁾ ($Re=2.1 \times 10^4$), and Ishigai⁽³⁾ ($Re=1.5 \times 10^3 - 1.5 \times 10^4$), all are

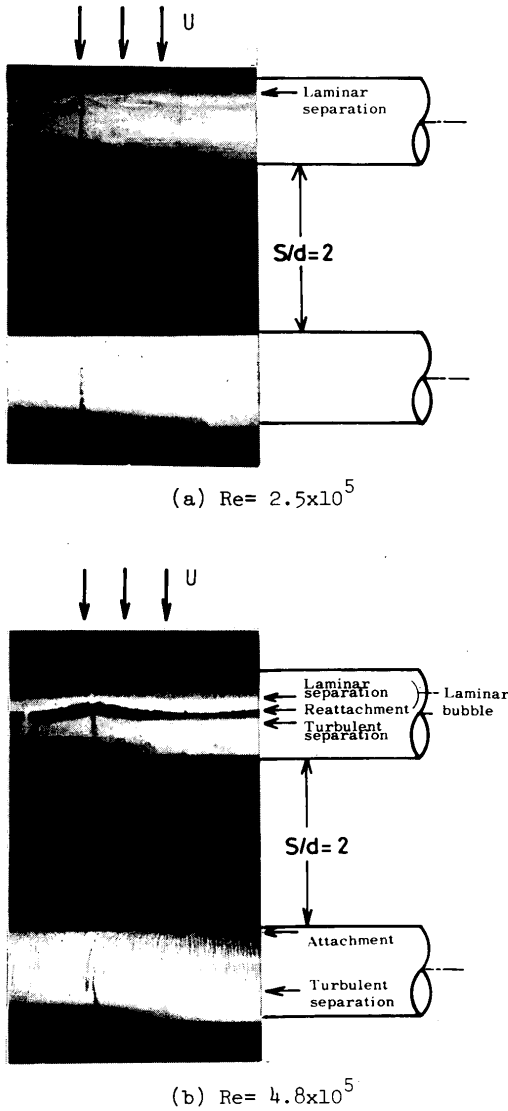


Fig.3 Flow patterns on surface visualized by oil film technique, $S/d=2$

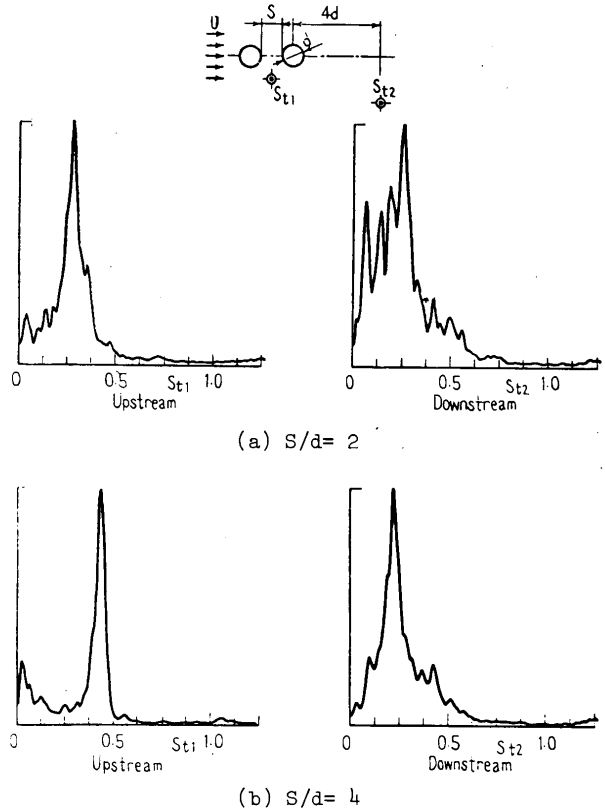


Fig.4 Typical fluctuating velocity spectra of two smooth cylinders in a tandem arrangement, $Re=4.8 \times 10^5$

less than the critical Reynolds number and hence between the results of the present author's subcritical Reynolds numbers of $(1.7, 2.5) \times 10^5$ and the other authors' ones there is a good agreement.

At the critical spacing the value of which is 2.8 there is found a distinct step-like jump of drag coefficients or Strouhal numbers of the subcritical Reynolds numbers of $(1.7, 2.5) \times 10^5$. The feature of this discontinuous jump is investigated in detail by Ishigai, et al. (3), Imaichi, et al. (2), Tanida, et al. (6) and so on. This discontinuity is caused by a sudden change

from one flow pattern to another at this critical spacing. The first flow pattern is observed up to the critical spacing and it hardly produces a vortex shedding behind the upstream cylinder, for the flow separating from the upstream cylinder reattaches to the downstream one. Therefore strong velocity fluctuations are detected only behind the downstream cylinder, and this Strouhal number is small, 0.14. The drag coefficients C_{D1} and C_{D2} gradually change, respectively in the opposite directions.

Beyond the critical spacing, the second flow pattern appears and it produces periodic vortex sheddings behind both cylinders, the Strouhal number of which is 0.19 and almost equal to the case of a single circular cylinder. C_{D1} of the upstream cylinder is 1.2 and C_{D2} of the downstream one remains very small, 0.3. The author already reported in reference (6) that C_{D2} of the downstream cylinder approaches C_{D1} of the upstream one with an increasing spacing at low Reynolds numbers. So, it is inferred that C_{D2} shows a fairly strong dependence on Reynolds number.

At the critical Reynolds number of 3.8×10^5 , the drag coefficients and Strouhal numbers exhibit an intermediate tendency between the subcritical and the supercritical flow regimes, being almost independent of the spacing. There is a considerable scatter in data of the Strouhal numbers in the range of small spacings.

In the supercritical flow regime where the laminar bubbles occur on the surface of the upstream cylinder, $Re = (5.0, 6.3) \times 10^5$, the drag coefficients and Strouhal numbers of both cylinders are seen to be slightly affected by the spacing. Therefore the

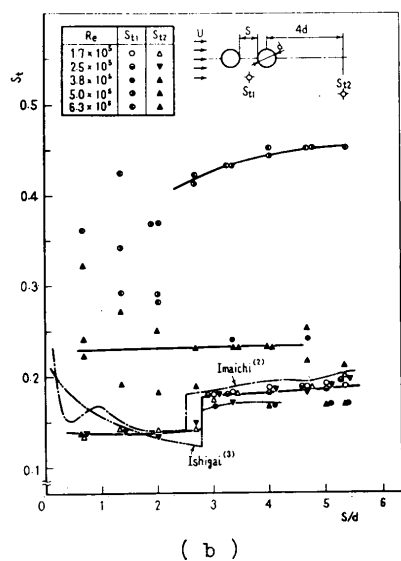
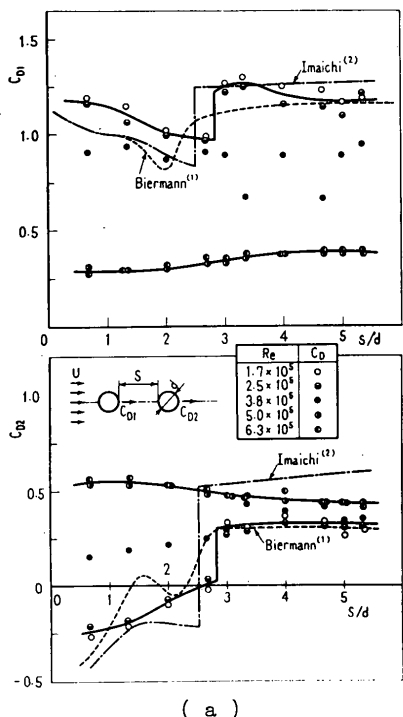


Fig.5 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (S_{t1} , S_{t2}) of two smooth cylinders in a tandem arrangement

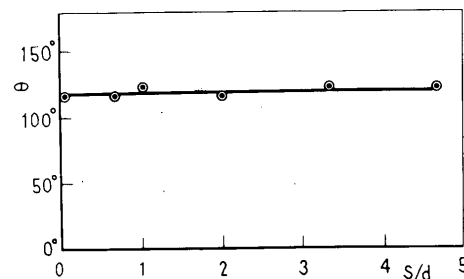
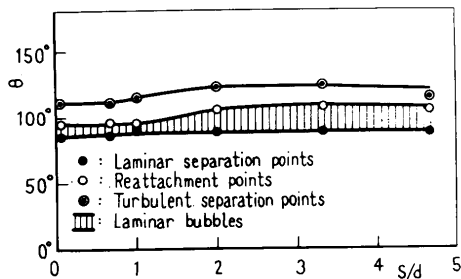
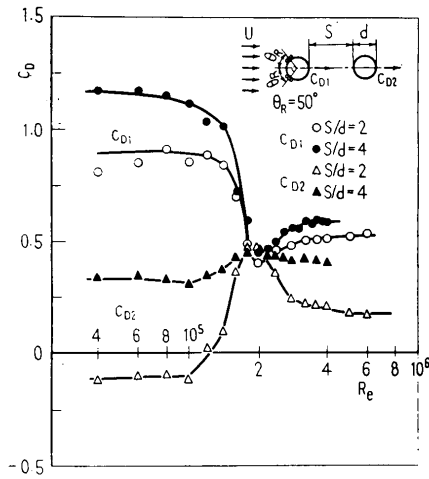


Fig.6 Positions of laminar bubbles, flow separation and reattachment, $Re = 5.0 \times 10^5$

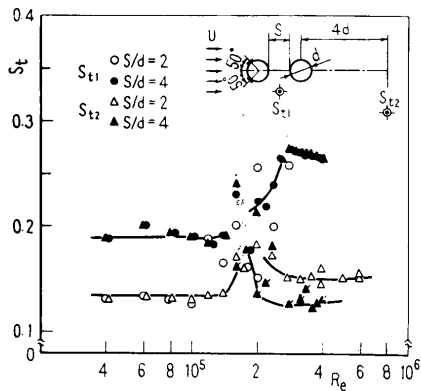
critical regime shows a disappearance of the discontinuous jump of the drag coefficients and Strouhal numbers, and with an increasing spacing, C_{D1} gradually increases and C_{D2} is reduced at a slow rate, but always a positive drag force; C_{D1} approaches C_{D2} . It is interesting, however, that the values of C_{D1} are always less than those of C_{D2} in the whole range of spacings.

Beyond the critical Reynolds number, the occurrence of laminar bubbles on the upstream cylinder makes the value of C_{D1} less than at subcritical Reynolds numbers in a similar way to a single circular cylinder, followed by a contraction of the reversed flow regime and a narrow wake, resulting in an increase of C_{D2} of the downstream cylinder.

Fig.6. shows the points of flow separation and reattachment at the supercritical Reynolds number of 5.0×10^5 which were gained from the motion of a surface oil-film in visualization tests. The positions of laminar bubbles and flow separations hardly change with the spacing, which corresponds to the fact that C_{D1} and C_{D2} remain constant and independent of the spacing in the supercritical flow regime.



(a)



(b)

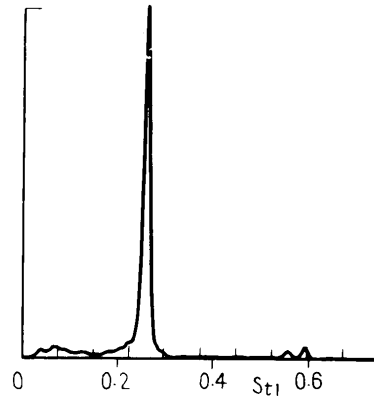
Fig.7 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (S_{t1} , S_{t2}) of a roughened cylinder and a smooth one in a tandem arrangement

3.2 A roughened cylinder and a smooth cylinder in a tandem arrangement

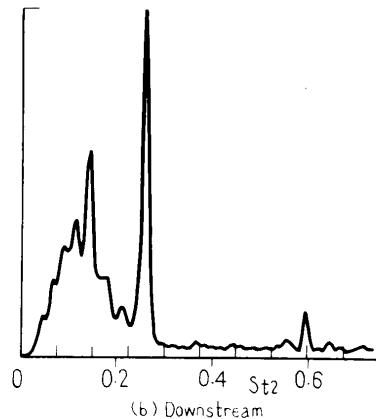
Two types of surface roughness are used as roughness elements on the surface of an upstream cylinder in a tandem arrangement. A downstream cylinder remains smooth.

3.2.1 A cylinder locally attached with roughness strips

Drag coefficients (C_{D1} , C_{D2}) and Strouhal numbers (S_{t1} , S_{t2}) of two tandem cylinders which consist of an upstream cylinder locally attached with roughness strips and a downstream smooth cylinder, are plotted against Reynolds numbers in Fig.7. This figure shows the results of the spacing of 2 and 4 as typical examples of the gap spacing between two cylinders. The measurements of the change of the drag coefficients and Strouhal numbers of both cylinders show that the critical Reynolds number is reduced to 1.5×10^5 , which is considerably lower than the critical one of a smooth cylinder. The motion of a surface oil film indicates that there are formed laminar bubbles on the surface of this upstream cylinder at very low Reynolds number of 2.0×10^5 and at this Reynolds number, while C_{D1} of the upstream cylinder reaches a minimum, C_{D2} of the downstream one is maximum, being almost equal to $(S/d=4)$ or greater than $(S/d=2)$ C_{D1} .



(a) Upstream



(b) Downstream

Fig.8 Typical fluctuating velocity spectra of a roughened cylinder and a smooth one in a tandem arrangement, $Re=4.0 \times 10^5$, $S/d=4$

In the range of much higher Reynolds numbers, the laminar bubbles disappear and the separation of the boundary layers on the surface of the upstream cylinder becomes purely turbulent, which is called the transcritical flow regime after Roshko⁽¹⁴⁾. The disappearance of the laminar bubbles is followed by an increase of C_{D1} , a reduction of C_{D2} and the resumption of vortex shedding, and then beyond Reynolds number of 2.8×10^5 both C_{D1} and C_{D2} keep constant, as seen in Fig.7.

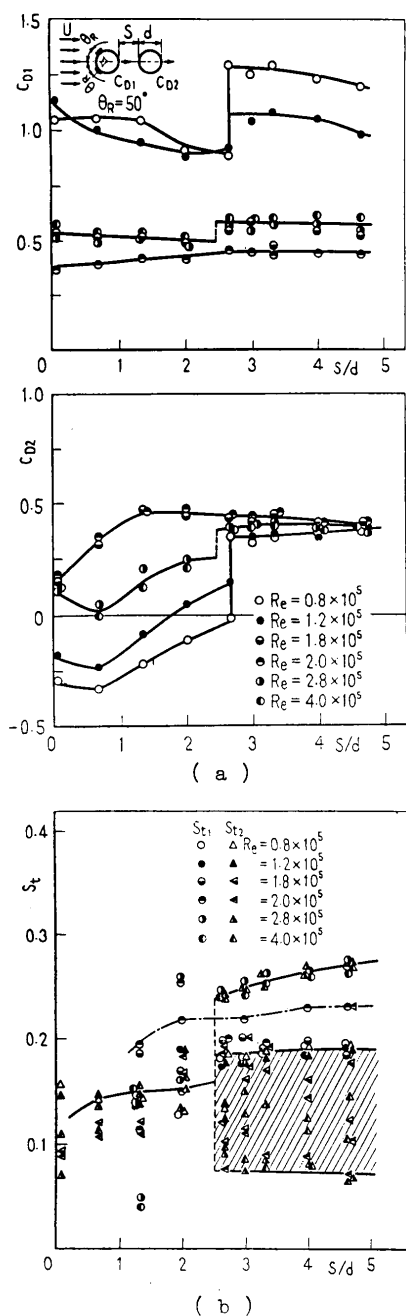


Fig.9 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (St_1 , St_2) of a roughened cylinder and a smooth one in a tandem arrangement

Fig.8 shows typical examples of the power spectral density of the wake velocities at the transcritical Reynolds number ($Re = 4.0 \times 10^5$) plotted against Strouhal number. The spectrum of velocity fluctuations behind the upstream cylinder indicates the very sharpness of the peak in Fig.8(a), which implies that the vortex shedding from the upstream cylinder resumes in this flow regime. On the other hand in the spectrum of the downstream cylinder there are exhibited two distinct peaks at the Strouhal numbers of 0.14 and 0.26, the latter being due to the components of the vortex shedding synchronized with that behind the upstream cylinder and the former due to the vortex shedding directly from the downstream cylinder, which is in the same way as Fig.4. The Strouhal numbers corresponding to two peaks are plotted in Fig.7(b), where the values St_2 of the second peak is denoted by the symbol \blacktriangle .

In the transcritical flow regime where the laminar bubbles disappear from the upstream cylinder, the Strouhal number St_2 is approximately constant and equal to 0.15 within smaller spacing, and at greater spacing both St_1 and St_2 become 0.27 to 0.26 which agrees with the case of a single cylinder with the roughness strips⁽¹³⁾.

Next, in Fig.9 typical examples of the drag coefficients and Strouhal numbers of the roughened and the smooth cylinders are indicated against the spacing at the subcritical Reynolds number ($Re = 0.8 \times 10^5$, 1.2×10^5), the critical Reynolds number ($Re = 1.8 \times 10^5$), the supercritical Reynolds number ($Re = 2.0 \times 10^5$) and the transcritical Reynolds number ($Re = 2.8 \times 10^5$, 4.0×10^5). From this figure, the curves of the subcritical and the critical Reynolds numbers are seen analogous to the preceding case of two smooth cylinders. A bistable nature of the flow phenomenon is seen in Fig.9, i.e. two different values of the drag coefficients and the Strouhal numbers of $Re = 0.8 \times 10^5$ may intermittently occur at critical spacing ($S/d = 2.7$), which suggests that two different flow patterns are intermittently changing for some period of time at this spacing. This bistable nature of flow phenomenon was already confirmed by different authors^(2,3). And it is the same as the case of the two smooth cylinders that at supercritical Reynolds number there can not be found a discontinuous jump of the drag coefficients or the Strouhal numbers.

In the transcritical flow regime that can be realized by the surface roughness, there occurs a considerable regular vortex shedding in wake and a step-like jump is displayed in the curves of the drag coefficients and the Strouhal numbers, again. It may be inferred from Fig.9 that the jumps of C_{D1} and C_{D2} are smaller than those for the subcritical flow regime, i.e. C_{D1} jumps from 0.6 to 0.5 and C_{D2} from 0.25 to 0.4 at the critical spacing the value of which is about 2.5. The critical spacing is also less than that for the subcritical Reynolds numbers, which may imply that the formation region for vortex shedding shrinks and becomes smaller. Further the curve of Strouhal number shows resemblance

to that for the subcritical Reynolds numbers, i.e. within the spacing of 2.5 a distinct vortex shedding, S_{t2} of which is about 0.15, is detected behind the downstream cylinder, and beyond the spacing of 2.5 a vortex shedding suddenly appears behind both the upstream and downstream cylinders, S_{t1} and S_{t2} of which are equal, being 0.24 to 0.27. The Strouhal numbers of the second peak shown in Fig.8 are plotted with hatching in Fig.9.

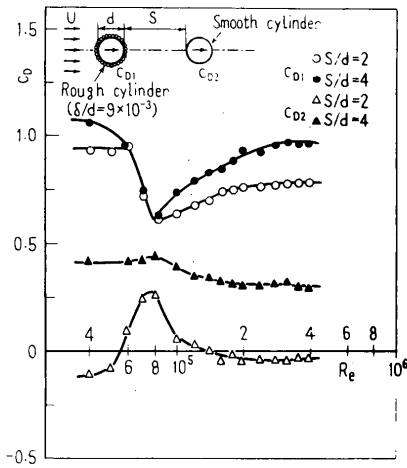
3.2.2 A cylinder covered densely with roughness particles

Fig.10 indicates the drag coefficients and Strouhal numbers of two tandem cylinders, an upstream cylinder being covered densely with roughness and a downstream one being smooth. The critical Reynolds number is found to be 0.65×10^5 , the value of which is remarkably reduced by the surface roughness. The curves of the drag coefficients and the Strouhal numbers for lower Reynolds

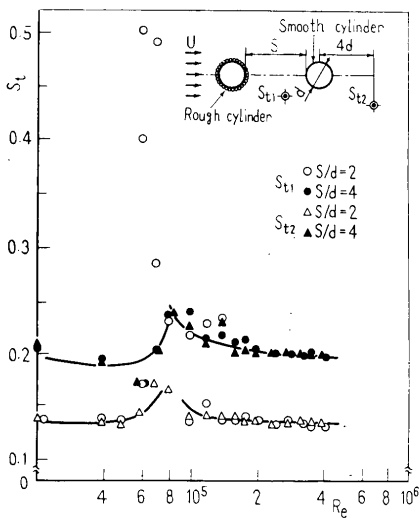
numbers than the critical one are in agreement with the corresponding curves for the smooth cylinders and at Reynolds number of 0.8×10^5 , C_{D1} reaches the minimum whereas C_{D2} is the maximum.

Further with an increasing Reynolds number there occur gradual changes of C_{D1} and C_{D2} , respectively upward and downward, and beyond Reynolds number of 1.5×10^5 the downstream cylinder comes to receive a thrust force at a small spacing ($S/d=2$) or small drag force at a large spacing ($S/d=4$), in a similar way to the subcritical flow regime. The motion of an oil film flow in visualization tests reveals the disappearance of laminar bubbles, beyond Reynolds number of 2.0×10^5 and hence the flow around the upstream cylinder is confirmed to be transcritical. Fig.11 shows spectra of wake velocities of this flow regime i.e. $Re=4.0 \times 10^5$ and $S/d=4$. The very extreme sharpness of the peak at the Strouhal number of 0.2 indicates that the vortex is shed with a very high periodicity.

Next the drag coefficients and Strouhal numbers of these two tandem cylinders in this flow regime are plotted against the spacing in Fig.12, compared with the results of the subcritical flow regime. In this figure, the results for the three transcritical Reynolds numbers of $(2.0, 2.8, 4.0) \times 10^5$ are almost equal and hence hardly

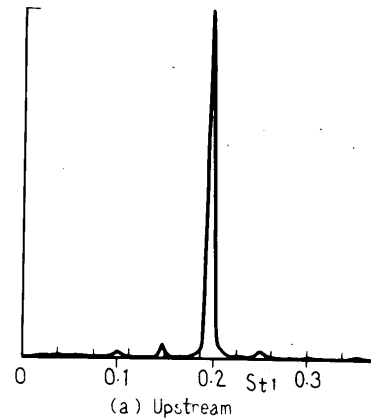


(a)

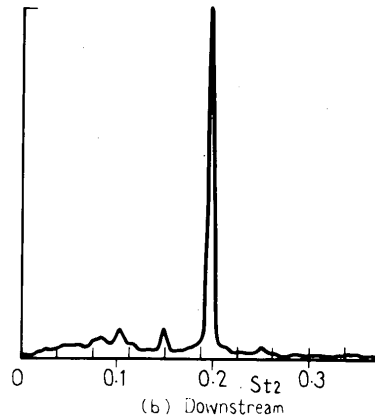


(b)

Fig.10 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (S_{t1} , S_{t2}) of a roughened cylinder and a smooth one in a tandem arrangement



(a) Upstream



(b) Downstream

Fig.11 Typical fluctuating velocity spectra of a roughened cylinder and a smooth one in a tandem arrangement, $Re=4.0 \times 10^5$, $S/d=4$

dependent on Reynolds number. For smaller spacing than the critical one, 2.7, C_{D1} is reduced gradually from 0.9 to 0.8, C_{D2} increases continuously from a negative value to 0.1 and Strouhal number is 0.15 to 0.13. At the critical spacing of 2.7, however, C_{D1} and C_{D2} jump from low values to considerably high ones, and these low and high values are intermittently displayed for some period of time. Beyond the critical spacing, the increase of spacing brings little change of both C_{D1} and C_{D2} . These behaviors of the drag coefficients for the transcritical flow regime against the spacing show much resemblance to those for the subcritical one.

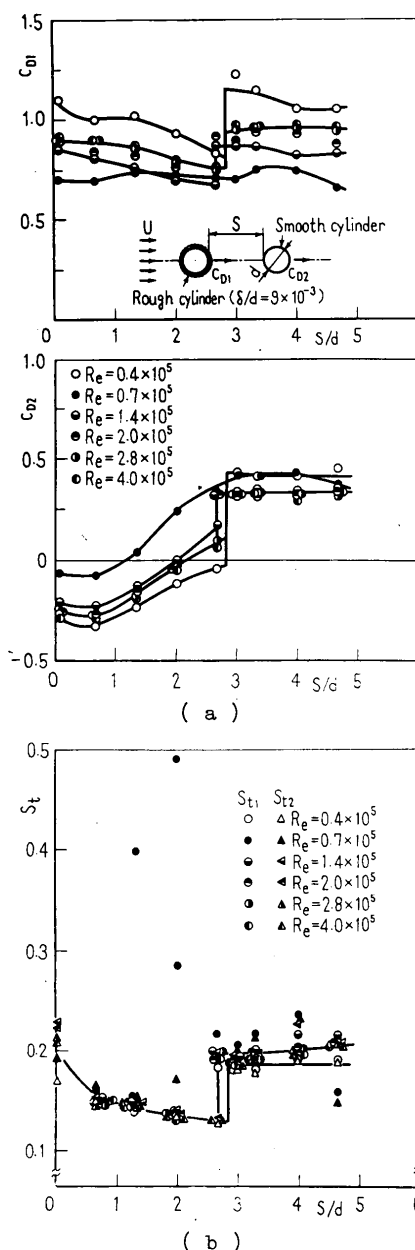


Fig.12 (a) Drag coefficients (C_{D1} , C_{D2}) and (b) Strouhal numbers (St_1 , St_2) of a roughened cylinder and a smooth one in a tandem arrangement

4. Conclusions

The flow characteristics of two cylinders in a tandem arrangement were experimentally investigated up to the range of much higher Reynolds numbers than the critical one. The following conclusions were obtained.

(1) The variations of flows around two tandem cylinders with Reynolds number are almost similar to those of a single circular cylinder.

(2) In the subcritical flow regime, there occur distinct step-like jumps of the drag coefficients and the Strouhal numbers of two tandem cylinders at the critical spacing the value of which is 2.8. And the author's results agree well with the other authors' ones.

(3) When two types of roughened cylinders are used as an upstream cylinder, each of their critical Reynolds numbers is reduced and its value is almost equal to the case of a single roughened cylinder.

(4) In the supercritical flow regime the positions of the laminar bubbles and flow separation hardly change with the spacing, which corresponds to the fact that the drag coefficients of two tandem cylinders remain constant, regardless of the spacing.

(5) In the transcritical flow regime where the laminar bubbles disappear from the upstream cylinder, a vortex shedding occurs with an extremely high periodicity, which is similar to the subcritical regime, and the drag coefficients and Strouhal numbers abruptly jump at the critical spacing the value of which is 2.5 or 2.7.

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