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Fluctuation of the Order Parameter and Hall Effect*),**)

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Hall conductivity σ_{xy} in the fluctuating dirty superconductors is examined microscopically above the critical temperature T_c where fluctuation effects are important.

In weak field limit, i.e., $DeH/T \ll \eta$ (D: diffusion constant, $\eta=(T-T_c)/T_c$), the Hall angle in thin films with thickness d is given by

$$\theta \equiv \frac{\sigma_{xy}}{\sigma_{xx}} = -\omega_c\tau \frac{1+(4\eta_0/(\eta-\delta))\ \ln \eta/\delta + \pi\alpha/18\cdot\eta_0/\eta^2}{1+(2\eta_0/(\eta-\delta))\ \ln \eta/\delta + \eta_0/\eta},$$

where $\omega_c=eH/mc$, $\eta_0=e^2/16d\sigma_0$, $\sigma_0=(ne^2/m)\tau$ (τ: relaxation time) and $\alpha=(2/\pi gN)$ (g: BCS coupling constant, N: state density at the Fermi energy) and δ is a pair-breaking parameter. The second term in the numerator comes from the Maki process, whereas the last is due to the AL process, which shows that the gigantic Meissner currents contribute to Hall effect in the fluctuating region. The sign of the contributions from such diamagnetic currents depends on the sign of energy derivative of the density of state function at the Fermi energy.

§ 1. Introduction

In recent years the thermodynamic fluctuations of the order parameters in superconductors near the critical temperature attracted many investigators both experimentally[1] and theoretically.[2] As regards the electrical conductivity, which has been in the center of these discussions, there exist two physically distinct processes, the one pointed out by Aslamazov and Larkin (AL)[3] and the other by Maki.[4] The former takes account of the currents carried by superconducting fluctuations, whereas the latter takes account of the additional scattering by fluctuations and thus these result in different temperature dependences of σ_{xx}. Moreover the Maki process has remarkable properties that the static conductivity is divergent in one- and two-dimensional systems. Theoretically this fact is very important and Takayama and Maki[5] have performed detailed examinations of this singularity. Practically, on the other hand, the additional pair-breaking effects first explicitly taken into account by Thompson[6] removes this difficulty and thus the theory predicts an excess conductivity σ' in a film with thickness d much smaller than the coherence length as follows:

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$$\frac{\sigma'}{\sigma_0}=\frac{2\eta_0}{\eta-\delta}\ln\frac{\eta}{\delta}+\frac{\eta_0}{\eta}, \tag{1·1}$$

where $\eta_0=e^2/16d\sigma_0(\sigma_0=(ne^2/m)\tau$, τ: relaxation time) and δ is the above-mentioned pair-breaking parameter. The first and second terms in Eq. (1·1) come from the Maki and the AL processes, respectively. The relation (1·1) is quantitatively ascertained by the experiments on Al alloys.[7)]

In order to understand more fully the physical implications of the fluctuations of the order parameter near the phase transition, other physical quantities such as orbital diamagnetism have been investigated both theoretically[8)] and experimentally.[9)]

We shall add here one more theoretical prediction. It is on the Hall effect, a phenomenon that reflects the mixed effects of electrical transport and orbital diamagnetism. The discussions are confined to the temperature region above the critical point with the perturbational treatments of the fluctuations.

The Hall effect in normal metals with a long mean free path[10)] is mainly determined by the kinematical effect of Lorentz force that curves the electron orbits. This force yields transverse currents represented by the relation $\sigma_{xy}=-\omega_c\tau\sigma_0$, where $\omega_c=eH/mc$ is the cyclotron frequency. The process that results in the above relation can be interpreted purely classically. Concerning the fluctuating superconductors the additional scattering mechanism due to fluctuations corresponding to the Maki process modifies this relation. However, as was pointed out by Kubo,[11),12)] the presence of a static magnetic field has other physical effects different from the above-mentioned kinematical ones. That is, the magnetic fields disturb the electronic distributions in energy space quantum-mechanically. This is the origin of the finite diamagnetism[13)] of the electron gas contrary to the Bohr-van Leeuwen theorem. Thus the deviations of the electronic distributions due to the magnetic field yield diamagnetic currents which contribute to the Hall effect, if the scattering mechanism of electrons is present. This contribution arises, formally speaking, from the expansion of the density matrix in terms of the magnetic fields in the Kubo formula,[14)] and not from the magnetic field dependent time evolution of the current operators. Such extra quantum mechanical contributions are, however, negligible in normal metals to the order of $(\varepsilon_F\tau)^{-1}$ (ε_F: the Fermi energy) compared with the Lorentz force effect, because these diamagnetic currents have no transverse component if the external perturbing force is absent.

In superconductors, however, the existence of the complete diamagnetism is the most essential property. Thus the quantum mechanical effects for σ_{xy} from the diamagnetic currents, neglected in normal metals, are expected to play some important roles in superconductors not only below the critical temperature but also in the fluctuating regions above T_c.

The usual approximation for the fluctuation propagator $\mathscr{D}$ is insufficient because in such treatments no contributions come from the fluctuating supercurrents represented by the AL process. This is in part obvious from the fact that negligible contributions come from the diamagnetic currents in normal metals. For this reason we must proceed one step further to the order of $(\varepsilon_F\tau)^{-1}$ or T/ε_F, which has been fully discussed elsewhere.[15] Thus we get excess Hall conductivity in a weak magnetic field due to fluctuating supercurrents,

$$\frac{\varDelta\sigma_{xy}^{AL}}{\sigma_0}=-\omega_c\tau\frac{\pi\alpha}{18}\frac{\eta_0}{\eta^2}, \tag{1·2}$$

where $\eta=(T-T_c)/T_c$ and $\alpha=2/\pi gN$ (g: BCS coupling constant). Note that, although the parameter $(\varepsilon_F\tau)^{-1}$ or T/ε_F is itself small, large currents carried by fluctuations yield important contributions for σ_{xy} in the end. Equation (1·2) is valid for nearly free electrons. For general Bloch electrons, the sign of $\varDelta\sigma_{xy}^{AL}$ changes in accordance with the sign of energy derivative of the density of states at the Fermi surface. This dependence on the surface structure is different from σ_{xy} in normal metals.

Microscopic treatments of the Maki process, on the other hand, show

$$\frac{\varDelta\sigma_{xy}^{M}}{\sigma_0}=-\omega_c\tau\frac{4\eta_0}{\eta-\delta}\ln\frac{\eta}{\delta}, \tag{1·3}$$

which is less singular than the contributions from the AL process.

In § 2 Hall angle in weak field limit $DeH/T\ll\eta$ ($D=vl/3$; v is Fermi velocity, l is mean free path) is calculated, whereas in § 3 the contributions from the fluctuating supercurrents are discussed in detail for all values of the magnetic field.

§ 2. Hall conductivity and Hall angles

The model we discuss in this section is the BCS effective Hamiltonian for nearly free electrons.

$$\mathcal{H}=\int\psi^\dagger\Big(\frac{1}{2m}\boldsymbol{p}^2+\sum_j U(\boldsymbol{r}-\boldsymbol{R}_j)\Big)\psi d\boldsymbol{r}-g\int\psi_\uparrow{}^\dagger\psi_\downarrow{}^\dagger\psi_\downarrow\psi_\uparrow d\boldsymbol{r}, \tag{2·1}$$

where U is the impurity potential with short force-range. Cases for arbitrary Bloch electrons are discussed in § 3.

Hall conductivity σ_{xy} in the limit of weak magnetic fields is given by the general formula[14] for σ_{xy} in terms of the vector potential $\boldsymbol{A_q}$ to its first order. As will be shown in § 3, the expansion parameter in this case is $DeH/\eta T$, and "the weak magnetic field" means that $DeH/T<\eta$.

$$\sigma_{\mu\nu}=\frac{1}{i\omega}K_{\mu\nu}^{\alpha}A_{q\alpha}\Big|_{\omega\to0}, \tag{2·2}$$

$$K^{\alpha}_{\mu\nu}(i\omega_\lambda)=\frac{e^2}{mc}\delta_{\nu\alpha}[\mathcal{L}_\mu(\boldsymbol{q},i\omega_\lambda)-\mathcal{L}_\mu(\boldsymbol{q},0)]+\frac{1}{c}[\mathcal{L}^{\alpha}_{\mu\nu}(\boldsymbol{q},i\omega_\lambda)-\mathcal{L}^{\alpha}_{\mu\nu}(\boldsymbol{q},0)],\tag{2·3}$$

$$\mathcal{L}_\mu(\boldsymbol{q},i\omega_\lambda)=-\frac{1}{\beta}\int_0^\beta d\tau\int_0^\beta d\tau'\exp[i\omega_\lambda(\tau-\tau')]\langle T\hat{J}_\mu(\boldsymbol{q},\tau)\hat{\rho}(-\boldsymbol{q},\tau')\rangle,\tag{2·4}$$

$$\mathcal{L}^{\alpha}_{\mu\nu}(\boldsymbol{q},i\omega_\lambda)=\frac{1}{\beta}\int_0^\beta d\tau\int_0^\beta d\tau'\int_0^\beta d\tau''$$
$$\times\exp[i\omega_\lambda(\tau-\tau'')]\langle T\hat{J}_\mu(\boldsymbol{q},\tau)\hat{J}_\alpha(-\boldsymbol{q},\tau')\hat{J}_\nu(\boldsymbol{0},\tau'')\rangle,\tag{2·5}$$

where

$$\hat{\boldsymbol{J}}(\boldsymbol{k})=\frac{ie}{2m}\int d\boldsymbol{r}e^{-i\boldsymbol{k}\boldsymbol{r}}\{\psi^\dagger(\boldsymbol{r})\boldsymbol{\nabla}\psi\boldsymbol{\nabla}(\boldsymbol{r})-\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\},\tag{2·6}$$

$$\hat{\rho}(\boldsymbol{k})=\int d\boldsymbol{r}e^{-i\boldsymbol{k}\boldsymbol{r}}\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}).\tag{2·7}$$

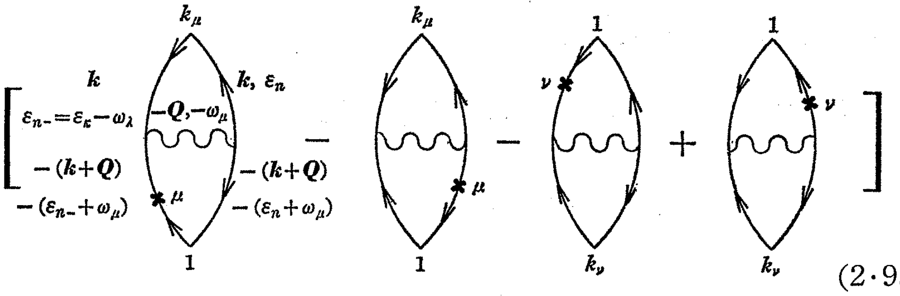
We take electronic charge as $-e$ $(e>0)$ in this paper.

As we are concerned with the uniform magnetic fields, only the linear parts of $\boldsymbol{q}$ in $K^{\alpha}_{\mu\nu}$ are necessary. Applying similar procedures developed in Ref. 16), we get, after straightforward manipulations, the following expressions:

$$K^{\alpha}_{\mu\nu}(\boldsymbol{q},i\omega_\lambda)\equiv K^{\alpha(\mathrm{M})}_{\mu\nu}(\boldsymbol{q},i\omega_\lambda)+K^{\alpha(\mathrm{AL})}_{\mu\nu}(\boldsymbol{q},i\omega_\lambda),\tag{2·8}$$

$$K^{\alpha(\mathrm{M})}_{\mu\nu}=-\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})$$

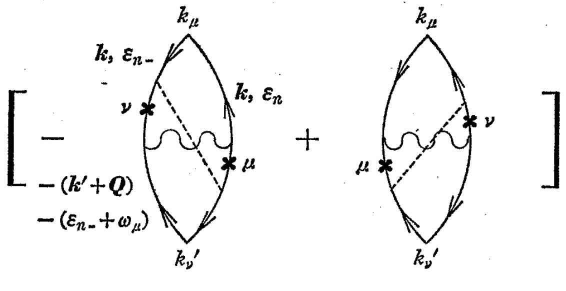
$$\times T^2\sum_n\sum_\mu\sum_{K,Q}$$



(2·9a)

$$-2\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})$$

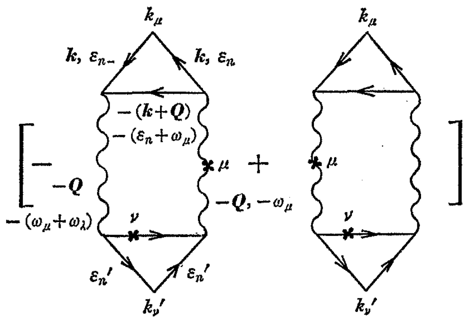
$$\times T^2\sum_n\sum_\mu\sum_{K,Q}$$



, (2·9b)

$$K_{\mu\nu}^{\alpha(\mathrm{AL})}=-4\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})$$

$$\times T^2\sum_n\sum_{n'}T\sum_\mu\sum_{\boldsymbol{k},\boldsymbol{Q}}\Bigg[-$$



$$\Bigg]. \qquad (2\cdot10)$$

In these expressions, the wavy lines and dotted lines represent the fluctuation propagators and impurity potentials, respectively. The vertex μ or ν on a propagator means that this propagator should be partially differentiated with respect to the μ-th or the ν-th component of the wave vector. $K_{\mu\nu}^{\alpha(\mathrm{M})}$ and $K_{\mu\nu}^{\alpha(\mathrm{AL})}$ come from the Maki process and the AL process, respectively. The fluctuation propagator $\mathscr{D}(\boldsymbol{Q}, i\omega_\mu)$ that carries the momentum $-\boldsymbol{Q}$ and the energy $-i\omega_\mu$ is given by Maki[4] as follows:

$$\mathscr{D}(\boldsymbol{Q}, i\omega_\mu)^{-1}=-N[\eta+\lambda_0|\omega_\mu|+\lambda Q^2], \qquad (2\cdot11)$$

where N is the density of states at the Fermi energy and $\eta=(T-T_c)/T_c$, $\lambda_0=\pi/8T$, $\lambda=\pi D/8T$.

The calculations of $K_{\mu\nu}^{\alpha(\mathrm{M})}$ are performed as follows. In $K_{\mu\nu}^{\alpha(\mathrm{M})}$ we can safely set $\omega_\mu=0$ from the first as far as we are concerned with the dominant contributions near T_c. As the third term in the curly bracket in Eq. (2·9a) is equal to the first of the linear order of ω_λ, we get

$$\begin{aligned}\text{Eq. }(2\cdot9\text{a}) &= -2\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})T^2\sum_{\boldsymbol{Q}}\mathscr{D}(\boldsymbol{Q},0)\\ &\times\sum_k\sum_{\boldsymbol{k}}k_\mu G(\boldsymbol{k}, i\varepsilon_n)G(\boldsymbol{k}, i\varepsilon_{n-})\frac{k_\mu+Q_\mu}{m}[G^2(\boldsymbol{k}+\boldsymbol{Q}, -i\varepsilon_n)G(\boldsymbol{k}+\boldsymbol{Q}, -i\varepsilon_{n-})\\ &\quad -G(\boldsymbol{k}+\boldsymbol{Q}, -i\varepsilon_n)G^2(\boldsymbol{k}+\boldsymbol{Q}, -i\varepsilon_{n-})]\Lambda(i\varepsilon_n, i\varepsilon_n;\boldsymbol{Q})\Lambda(i\varepsilon_{n-}, i\varepsilon_{n-};\boldsymbol{Q})\\ &= -\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})\frac{4\varepsilon_{\mathrm{F}}}{3}T^2\sum_{\boldsymbol{Q}}\mathscr{D}(\boldsymbol{Q},0)\\ &\times\sum_n\sum_{\boldsymbol{k}}\Lambda(i\varepsilon_n, i\varepsilon_n;\boldsymbol{Q})\Lambda(i\varepsilon_{n-}, i\varepsilon_{n-};\boldsymbol{Q})G(\boldsymbol{k}, i\varepsilon_n)G(\boldsymbol{k}, i\varepsilon_{n-})\\ &\times[G^2(\boldsymbol{k}, -i\varepsilon_n)G(\boldsymbol{k}, -i\varepsilon_{n-})-G(\boldsymbol{k}, -i\varepsilon_n)G^2(\boldsymbol{k}, -i\varepsilon_{n-})],\end{aligned} \qquad (2\cdot12)$$

where $\varepsilon_{\mathrm{F}}=k_{\mathrm{F}}^2/2m$ is the Fermi energy,

$$\Lambda(i\varepsilon_n, i\varepsilon_{n+\nu};\boldsymbol{Q})=\begin{cases}\dfrac{|\tilde{\varepsilon}_n+\tilde{\varepsilon}_{n+\nu}|}{|2\varepsilon_n+\omega_\nu|+DQ^2}, & \varepsilon_n\varepsilon_{n+\nu}>0,\\ 1, & \varepsilon_n\varepsilon_{n+\nu}<0,\end{cases} \qquad (2\cdot13)$$

$$\tilde{\varepsilon}_n=\varepsilon_n\left(1+\frac{1}{2\tau|\varepsilon_n|}\right),$$

$$\tau^{-1}=2\pi n_i u^2 N,$$

where n_i and u are the number density of impurities and the Fourier transform of U, respectively.

Retaining only the singular terms that result from the interval $\omega_\lambda>\varepsilon_n>0$ in the summation in Eq. (2·12), we have

$$\text{Eq. (2·12)}=-\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})\frac{4\varepsilon_F}{3}T^2\sum_{\boldsymbol{Q}}\mathcal{D}(\boldsymbol{Q},0)$$

$$\times\sum_{\omega_\lambda>\varepsilon_n>0}\frac{2\tilde{\varepsilon}_n}{2\varepsilon_n+D\boldsymbol{Q}^2}\cdot\frac{|2\tilde{\varepsilon}_{n-}|}{|2\varepsilon_{n-}|+D\boldsymbol{Q}^2}(4\pi iN)\left\{\frac{(\tilde{\varepsilon}_n+\tilde{\varepsilon}_{n-})^2+3(\tilde{\varepsilon}_n-\tilde{\varepsilon}_{n-})^2}{(\tilde{\varepsilon}_n-\tilde{\varepsilon}_{n-})^2(2\tilde{\varepsilon}_n)^2(2\tilde{\varepsilon}_{n-})^2}\right\}$$

$$=-2\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})2\varepsilon_F T^2 4\pi iN\tau^2$$

$$\times\sum_{\boldsymbol{Q}}\mathcal{D}(\boldsymbol{Q},0)\frac{1}{D\boldsymbol{Q}^2}\frac{1}{4\pi T}\left\{\psi\left(\frac{1}{2}+\frac{D\boldsymbol{Q}^2}{4\pi T}+\frac{\omega_\lambda}{2\pi T}\right)-\psi\left(\frac{1}{2}+\frac{D\boldsymbol{Q}^2}{4\pi T}\right)\right\}$$

$$=-\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})\frac{3n}{4}\pi i\omega_\lambda\tau^2\sum_{\boldsymbol{Q}}\frac{\mathcal{D}(\boldsymbol{Q},0)}{D\boldsymbol{Q}^2}. \qquad (2·14)$$

That is

$$\Delta\sigma_{xy}^{\mathrm{M},a}\equiv\frac{K_{\mu\nu}^{\alpha(\mathrm{M},a)}}{i\omega}A_{q\alpha}=-\sigma_{xy}^0\frac{3\pi}{4}\sum_{\boldsymbol{Q}}\frac{\mathcal{D}(\boldsymbol{Q},0)}{D\boldsymbol{Q}^2}, \qquad (2·15)$$

where $\sigma_{xy}^0=-\omega_c\tau\sigma_0$, $\omega_c=eH/mc$.

Next we calculate Eq. (2·9b).

$$\text{Eq. (2·9b)}=-2\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})n_iu^2\sum_{\boldsymbol{Q}}\mathcal{D}(\boldsymbol{Q},0)$$

$$\times T^2\sum_n\Lambda(i\varepsilon_n,i\varepsilon_n)\Lambda(i\varepsilon_{n-},i\varepsilon_{n-})\left[-\sum_{\boldsymbol{k}}k_\mu\frac{k_\mu+Q_\mu}{m}G(\boldsymbol{k},i\varepsilon_n)\right.$$

$$\times G(\boldsymbol{k},i\varepsilon_{n-})G^2(\boldsymbol{k}+\boldsymbol{Q},-i\varepsilon_n)\sum_{\boldsymbol{k}'}k_\nu'\frac{k_\nu'+Q_\nu}{m}G(\boldsymbol{k}'+\boldsymbol{Q},-i\varepsilon_n)$$

$$\times G(\boldsymbol{k}'+\boldsymbol{Q},-i\varepsilon_{n-})G^2(\boldsymbol{k}',i\varepsilon_{n-})+\sum_{\boldsymbol{k}}k_\mu\frac{k_\mu+Q_\mu}{m}G(\boldsymbol{k},i\varepsilon_n)$$

$$\times G(\boldsymbol{k},i\varepsilon_{n-})G^2(\boldsymbol{k}+\boldsymbol{Q},-i\varepsilon_{n-})\sum_{\boldsymbol{k}'}k_\nu'\frac{k_\nu'+Q_\nu}{m}G(\boldsymbol{k}'+\boldsymbol{Q},-i\varepsilon_n)$$

$$\left.\times G(\boldsymbol{k}'+\boldsymbol{Q},-i\varepsilon_{n-})G^2(\boldsymbol{k}',i\varepsilon_n)\right]$$

$$=-\frac{4i}{9}\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})n_iu^2T^2\sum_{\boldsymbol{Q}}\mathcal{D}(\boldsymbol{Q},0)$$

$$\times \mathrm{Im} \sum_n I(i\varepsilon_n, i\varepsilon_{n-}) I(-i\varepsilon_{n-}, -i\varepsilon_n) \Lambda(i\varepsilon_n, i\varepsilon_n) \Lambda(i\varepsilon_{n-}, i\varepsilon_{n-}), \tag{2·16}$$

where

$$I(i\varepsilon_n, i\varepsilon_{n-}) = \sum_k \frac{k^2}{m} G(\boldsymbol{k}, i\varepsilon_n) G(\boldsymbol{k}, i\varepsilon_{n-}) G^2(\boldsymbol{k}, -i\varepsilon_{n-}). \tag{2·17}$$

As is evident from Eq. (2·16), we need the imaginary part of Eq. (2·17) for ε_n that satisfies $\omega_\lambda > \varepsilon_n > 0$. Integrating over $\boldsymbol{k}$ in Eq. (2·17), and expanding the results in terms of $(\varepsilon_F \tau)^{-1}$, we have

$$I(i\varepsilon_n, i\varepsilon_{n-}) = -\frac{k_F^3}{4\pi} \frac{1}{\omega_\lambda + 1/\tau} \frac{1}{\tilde{\varepsilon}_n} \left[\frac{1}{\tilde{\varepsilon}_n} - \frac{3i}{2\varepsilon_F} \right]. \tag{2·18}$$

Thus we get

$$\begin{aligned}\text{Eq. (2·16)} &= \frac{4mk_F^4 T^2 \tau^3 i}{3\pi^2} \frac{e^3}{m^2 c} (q_\mu \delta_{\nu\alpha} - q_\nu \delta_{\mu\alpha}) n_i u^2 \\ &\quad \times \sum_Q \mathcal{D}(\boldsymbol{Q}, 0) \sum_{\omega_\lambda > \varepsilon_n > 0} \frac{1}{\tilde{\varepsilon}_n (2\varepsilon_n + D\boldsymbol{Q}^2)(-2\varepsilon_n + D\boldsymbol{Q}^2)} \\ &= \frac{i\omega_\lambda}{4} \pi \tau^2 \frac{e^3}{m^2 c} (q_\mu \delta_{\nu\alpha} - q_\nu \delta_{\mu\alpha}) \sum_Q \frac{\mathcal{D}(\boldsymbol{Q}, 0)}{D\boldsymbol{Q}^2},\end{aligned}$$

or in other words,

$$\Delta\sigma_{xy}^{\mathrm{M}, b} \equiv \frac{1}{i\omega} K_{\mu\nu}^{\alpha(\mathrm{M}, b)} A_{q\alpha} = \sigma_{xy}^0 \frac{\pi}{4} \sum_Q \frac{\mathcal{D}(\boldsymbol{Q}, 0)}{D\boldsymbol{Q}^2}. \tag{2·19}$$

Combining Eqs. (2·15) and (2·19), we have the total contribution to σ_{xy} from the Maki process,

$$\Delta\sigma_{xy}^{\mathrm{M}} \equiv \Delta\sigma_{xy}^{\mathrm{M}, a} + \Delta\sigma_{xy}^{\mathrm{M}, b} = -\frac{\pi}{2} \sigma_{xy}^0 \sum_Q \frac{\mathcal{D}(\boldsymbol{Q}, 0)}{D\boldsymbol{Q}^2}. \tag{2·20}$$

Next we will discuss AL process, Eq. (2·10).

$$\begin{aligned}K_{\mu\nu}^{\alpha(\mathrm{AL})} &= -4 \frac{e^3}{m^2 c} (q_\mu \delta_{\nu\alpha} - q_\nu \delta_{\mu\alpha}) C^2 \sum_Q Q_\mu T \sum_{\omega_\mu} \\ &\quad \times \left[\frac{\partial}{\partial Q_\mu} \mathcal{D}(\boldsymbol{Q}, i\omega_\mu + i\omega_\lambda) \mathcal{D}(\boldsymbol{Q}, i\omega_\mu) - \mathcal{D}(\boldsymbol{Q}, i\omega_\mu + i\omega_\lambda) \frac{\partial}{\partial Q_\mu} \mathcal{D}(\boldsymbol{Q}, i\omega_\mu) \right],\end{aligned} \tag{2·21}$$

where

$$C = T \sum_n \frac{k_\nu^2}{m} G(\boldsymbol{k}, i\varepsilon_n) G(\boldsymbol{k}, i\varepsilon_n) G^2(\boldsymbol{k}, -i\varepsilon_n),$$

and it is equal to $-2Nm\lambda$. Summations over ω_μ in Eq. (2·21) are performed as follows to the first order of $\omega (= i\omega_\lambda)$:

$$T\sum_{\omega_\mu}\left[\frac{\partial}{\partial Q_\mu}\mathscr{D}(\boldsymbol{Q}, i\omega_\mu+i\omega_\lambda)\mathscr{D}(\boldsymbol{Q}, i\omega_\mu)-\mathscr{D}(\boldsymbol{Q}, i\omega_\mu+i\omega_\lambda)\frac{\partial}{\partial Q_\mu}\mathscr{D}(\boldsymbol{Q}, i\omega_\mu)\right]$$

$$=\frac{\omega}{\pi}\,\mathrm{Im}\,\mathscr{P}\int_{-\infty}^{\infty}dxN(x)\left[\frac{\partial}{\partial x}\frac{\partial}{\partial Q_\mu}\mathscr{D}^R\mathscr{D}^R-\frac{\partial}{\partial x}\mathscr{D}^R\frac{\partial}{\partial Q_\mu}\mathscr{D}^R\right.$$

$$\left.-\mathscr{D}^A\frac{\partial^2}{\partial x\partial Q_\mu}\mathscr{D}^R+\frac{\partial}{\partial x}\mathscr{D}^R\frac{\partial}{\partial Q_\mu}\mathscr{D}^A\right], \qquad (2\cdot22)$$

where the principal part of the integration is to be taken and

$$N(x)=[e^{\beta x}-1]^{-1},$$

$$\mathscr{D}^R(\boldsymbol{Q}, x)\equiv\mathscr{D}(\boldsymbol{Q}, x+i\delta).$$

Writing

$$[\mathscr{D}^R(\boldsymbol{Q}. x)]^{-1}=-N[\eta+\lambda Q^2-i\lambda_0 x],$$

$$[\mathscr{D}^A(\boldsymbol{Q}, x)]^{-1}=-N[\eta+\lambda Q^2+i\lambda_0{}^* x],$$

we can calculate Eq. (2·22), retaining the most singular terms in η as follows:

$$\text{Eq. (2·22)}=-\omega\,\mathrm{Im}\,\frac{T\lambda Q_\mu}{N^2}\frac{\lambda_0}{[\eta+\lambda Q^2]^4}. \qquad (2\cdot23)$$

We see from Eq. (2·23) that the simple form of the fluctuation propagator, Eq. (2·11), which gives real λ_0 results in no contributions from the AL process. This result corresponds to the fact that the diamagnetic currents in normal metals are negligible for the Hall effect. Then we need $\mathscr{D}$ exactly up to this order, which is given as follows[15] for any Bloch electrons with density of states N.

$$[\mathscr{D}^R(\boldsymbol{Q}, \omega]^{-1}=-N\left\{\ln\frac{T}{T_c}+\psi\left(\frac{1}{2}+\zeta\right)-\psi\left(\frac{1}{2}\right)\right.$$

$$\left.+\frac{\omega}{2}\frac{N'}{N}\left[1+\frac{1}{gN}-\psi\left(\frac{1}{2}+\zeta\right)+\psi\left(\frac{1}{2}\right)-\ln\frac{T}{T_c}\right]\right\}, \qquad (2\cdot24)$$

where $\psi(z)$ is the di-gamma function and

$$\left.\begin{aligned}&\zeta=\frac{1}{4\pi T}(-i\omega+DQ^2)\left(1-\frac{\omega}{2}\frac{N'}{N}\right),\\&D=\frac{a}{N}\tau,\\&a=\sum_{\boldsymbol{k}}\delta(\varepsilon_F-\varepsilon(\boldsymbol{k}))\left(\frac{\partial}{\partial k_\mu}\varepsilon(\boldsymbol{k})\right)^2.\end{aligned}\right\} \qquad (2\cdot25)$$

In Eqs. (2·24) and (2·25), N' is the derivative of N with respect to the Fermi energy. In Eq. (2·25), $\varepsilon(\boldsymbol{k})$ is the band energy of the Bloch band. It is assumed to be of cubic symmetry and then a is independent of the vector component μ. For weak magnetic fields, i.e., $DeH/T\ll\eta$, we can expand $\psi(z)$ and get

$$[\mathscr{D}^R(\boldsymbol{Q},\omega)]^{-1}=-N\left[\ln\frac{T}{T_c}-i\lambda_0\omega\left(1+\alpha\frac{T}{\varepsilon_F}i\right)+\lambda Q^2\right],\tag{2·26}$$

where

$$\alpha=\frac{4\varepsilon_F}{\pi}\frac{N'}{N}\frac{1}{gN}.\tag{2·27}$$

For nearly free electron systems under consideration, $D=vl/3$ and

$$\alpha=\frac{2}{\pi}\frac{1}{gN}.\tag{2·28}$$

From Eqs. (2·23) and (2·26), we obtain

$$K_{\mu\nu}^{\alpha(\mathrm{AL})}=4\omega\frac{e^3}{m^2c}(q_\mu\delta_{\nu\alpha}-q_\nu\delta_{\mu\alpha})C^2\frac{T}{N^2}\lambda\lambda_0\alpha\frac{T}{\varepsilon_F}\sum_{\boldsymbol{Q}}\frac{Q_\mu{}^2}{(\eta+\lambda\boldsymbol{Q}^2)^4}\tag{2·29}$$

or

$$\varDelta\sigma_{xy}^{\mathrm{AL}}=\sigma_{xy}^0\frac{\pi^2}{12}\alpha\sum_{\boldsymbol{Q}}\frac{\lambda^2Q_\mu{}^2}{ND(\eta+\lambda\boldsymbol{Q}^2)^4}.\tag{2·30}$$

Consequently, the excess Hall conductivity $\varDelta\sigma_{xy}$ due to fluctuations is given by

$$\varDelta\sigma_{xy}=\sigma_{xy}^0\left[-\frac{\pi}{2}\sum_{\boldsymbol{Q}}\frac{\mathscr{D}(\boldsymbol{Q},0)}{D\boldsymbol{Q}^2}+\frac{\pi^2\alpha}{12}\sum_{\boldsymbol{Q}}\frac{\lambda^2Q_\mu{}^2}{ND(\eta+\lambda\boldsymbol{Q}^2)^4}\right].\tag{2·31}$$

For bulk samples,

$$\varDelta\sigma_{xy}^{(3)}=\frac{3\sqrt{3}}{2}\pi^{3/2}\left(\frac{T}{\varepsilon_F}\right)^{1/2}\frac{1}{(p_Fl)^{3/2}}\sigma_{xy}^0\left[\frac{1}{\eta^{1/2}}+\frac{7}{144}\alpha\pi\frac{1}{\eta^{3/2}}\right],\tag{2·32}$$

and for a thin film with thickness d much smaller than coherence length

$$\varDelta\sigma_{xy}^{(2)}=\eta_0\sigma_{xy}^0\left[\frac{4}{\eta-\delta}\ln\frac{\eta}{\delta}+\frac{\pi\alpha}{18}\frac{1}{\eta^2}\right],\tag{2·33}$$

where $\eta_0=e^2/16d\sigma_0$.

In (2·33), we introduce a pair-breaking parameter first adopted by Thompson[6] to avoid the difficulty of divergence in the static conductivity from the Maki process.

By use of Eq. (2·32) or (2·33) and the excess conductivity $\varDelta\sigma_{xx}$ obtained formerly,

$$\varDelta\sigma_{xx}^{(3)}=\sigma_0\frac{3\sqrt{3}}{16}\pi^{3/2}\left(\frac{T}{\varepsilon_F}\right)^{1/2}\frac{1}{(p_Fl)^{3/2}}\frac{1}{\eta^{1/2}},\tag{2·34}$$

$$\varDelta\sigma_{xx}^{(2)}=\sigma_0\eta_0\left[\frac{2}{\eta-\delta}\ln\frac{\eta}{\delta}+\frac{1}{\eta}\right],\tag{2·35}$$

we finally get Hall angles as follows:

$$\theta \equiv \frac{\sigma_{xy}}{\sigma_{xx}} = -\omega_c\tau\left\{1+\frac{3\sqrt{3}}{2}\left(\frac{T}{\varepsilon_F}\right)^{1/2}\frac{1}{(p_F l)^{3/2}}\left(\frac{1}{\eta^{1/2}}+\frac{7}{144}\alpha\pi\frac{1}{\eta^{3/2}}\right)\right\}$$
$$\times\left\{1+\frac{3\sqrt{3}}{16}\pi^{3/2}\left(\frac{T}{\varepsilon_F}\right)^{1/2}\frac{1}{(p_F l)^{3/2}}\frac{1}{\eta^{1/2}}\right\}^{-1} \tag{2·36}$$

for bulk cases and

$$\theta = -\omega_c\tau\left\{1+\frac{4\eta_0}{\eta-\delta}\ln\frac{\eta}{\delta}+\frac{\pi\alpha}{18}\frac{\eta_0}{\eta^2}\right\}\left\{1+\frac{2\eta_0}{\eta-\delta}\ln\frac{\eta}{\delta}+\frac{\eta_0}{\eta}\right\}^{-1} \tag{2·37}$$

for films.

Note that, although the Maki process and the AL process yield contributions to σ_{xx} with the same temperature dependence (for bulk samples), they give different temperature dependence for σ_{xy}. Even for bulk samples, the Hall effect due to superconducting fluctuations is to be observed if we get $\eta\sim10^{-2}$, $\varepsilon_F\tau\sim10$, $\tau T\sim10^{-1}$ and $\omega_c\tau\sim10^{-4}$. In thin films if $\varepsilon_F\tau\sim10^2$, $\tau T\sim10^{-1}$ and $\omega_c\tau\sim10^{-4}$, then the condition of weak field limit is satisfied for $\eta\sim10^{-1}$ and we expect deviation of the Hall angle due to fluctuations.

§ 3. Discussion of contributions from diamagnetic currents

We examined the Hall conductivity σ_{xy} microscopically not only from the Maki process but also from the AL process, indicating large contributions from the fluctuating supercurrent (AL process). The latter contribution comes from small modifications of the fluctuation propagators of the order of T/ε_F.

The sign of this contribution depends on that of (N'/N). In normal metals the sign of the Hall angle is determined by that of

$$\sum_{\boldsymbol{k}}\delta(\varepsilon_F-\varepsilon(\boldsymbol{k}))\left[\left(\frac{\partial\varepsilon}{\partial k_x}\right)^2\frac{\partial^2\varepsilon}{\partial k_y{}^2}-\frac{\partial\varepsilon}{\partial k_x}\frac{\partial\varepsilon}{\partial k_y}\frac{\partial^2\varepsilon}{\partial k_x\partial k_y}\right], \tag{3·1}$$

which is not necessarily equal to that of (N'/N). Thus it can occur that the fluctuations decrease the Hall angle in normal state.

Let us now examine in this section the properties of the effect of fluctuating diamagnetic currents on σ_{xy} including the cases of high magnetic fields, i.e., $DeH/T\gtrsim\eta$. Moreover the electrons are assumed to be any Bloch electrons. By use of the formulations developed by Mikeska and Schmidt[17] and Usadel,[18] we have for a film with thickness d,

$$\Delta\sigma_{xy} = -4\frac{e^2}{\omega}\frac{2\pi}{d}(2ND)^2\sum_{n=0}^{\infty}\frac{(2eH)^2}{(2\pi)^2}\frac{n+1}{2}A^2_{n,n+1}$$
$$\times T\sum_{\mu}[\mathcal{D}_{n+1}(i\omega_\mu)\mathcal{D}_n(i\omega_\mu+i\omega_\lambda)-\mathcal{D}_n(i\omega_\mu)\mathcal{D}_{n+1}(i\omega_\mu+i\omega_\lambda)]_{i\omega_\mu\to\omega+i\delta}$$
$$= -\frac{18e^2}{\pi^2 d}N^2D^2(eH)^2\sum_{n=0}^{\infty}(n+1)A^2_{n,n+1}$$

$$\times \mathrm{Im}\,\mathcal{P}\int dx N(x)\Big[\mathcal{D}^R_{n+1}\frac{\partial}{\partial x}\mathcal{D}_n{}^R-\mathcal{D}_n{}^R\frac{\partial}{\partial x}\mathcal{D}^R_{n+1}$$

$$+\mathcal{D}_n{}^A\frac{\partial}{\partial x}\mathcal{D}^R_{n+1}-\mathcal{D}^A_{n+1}\frac{\partial}{\partial x}\mathcal{D}_n{}^R\Big], \qquad (3\cdot 2)$$

where $\mathcal{D}_n{}^R$ is equal to $\mathcal{D}^R(\boldsymbol{Q},\omega)$, Eq. (2·26), with replacement of $\boldsymbol{Q}^2=4eH(n+\frac{1}{2})$, and $A_{n,n+1}$ is given by Eq. (A·13) of Ref. 18),

$$A_{n,n+1}=\frac{1}{4eHD}\Big[\psi\Big(\frac{1}{2}+\frac{DeH}{\pi T}\Big(n+\frac{3}{2}\Big)\Big)-\psi\Big(\frac{1}{2}+\frac{DeH}{\pi T}\Big(n+\frac{1}{2}\Big)\Big)\Big]. \qquad (3\cdot 3)$$

Here D is given by Eq. (2·25). In order to evaluate Eq. (3·3), the expansion of $[\mathcal{D}_n{}^R(x)]^{-1}$ in terms of x is sufficient,

$$[\mathcal{D}_n{}^R]^{-1}=-N\Big[t_n-\frac{ix}{T}a_n\Big], \qquad (3\cdot 4)$$

where

$$t_n=\ln\frac{T}{T_c}+\psi\Big(\frac{1}{2}+e_n\Big)-\psi\Big(\frac{1}{2}\Big),$$

$$a_n=\frac{1}{4\pi}\psi^{(1)}\Big(\frac{1}{2}+e_n\Big)\Big(1+i\frac{T}{\varepsilon_F}\alpha_n\Big),$$

$$\alpha_n=2\pi\varepsilon_F\frac{N'}{N}\Big\{-e_n+\frac{1}{\psi^{(1)}(\frac{1}{2}+e_n)}\Big[\frac{1}{gN}-\psi\Big(\frac{1}{2}+e_n\Big)+\psi\Big(\frac{1}{2}\Big)-\ln\frac{T}{T_c}\Big]\Big\}, \qquad (3\cdot 5)$$

$$e_n=\frac{DeH}{\pi T}\Big(n+\frac{1}{2}\Big).$$

By use of t_n and a_n, we get

$$\Delta\sigma_{xy}=-\frac{32e^2}{\pi d}(DeH)^2\,\mathrm{Im}\sum_{n=0}^{\infty}(n+1)A^2_{n,n+1}\frac{a_n{}^*a_{n+1}}{t_nt_{n+1}(a_n{}^*t_{n+1}+a_{n+1}t_n)}. \qquad (3\cdot 6)$$

In the weak field limit, we can replace

$$A_{n,n+1}=\frac{\pi}{8T},$$

$$t_n=\eta+\frac{\pi DeH}{2T}\Big(n+\frac{1}{2}\Big), \qquad (3\cdot 7)$$

$$a_n=\frac{\pi}{8}\Big(1+\frac{T}{\varepsilon_F}\alpha i\Big),$$

and then

$$\mathrm{Im}\frac{a_n{}^*a_{n+1}}{a_n{}^*t_{n+1}+a_{n+1}t_n}=\frac{\pi^2 DeH}{64T}\frac{T}{\varepsilon_F}\alpha\Big[\eta+\frac{\pi DeH}{T}(n+1)\Big]^{-2}. \qquad (3\cdot 8)$$

If H is weak enough, $\eta \gg DeH/T$, we can transform the summation over n into integral in Eq. (3·6) and get the same result as Eq. (2·30).

On the other hand, if the magnetic field is strong enough, $DeH/T \gtrsim \eta$, only the term corresponding $n=0$ is relevant to the singularity. In this case

$$A_{0,1}=\frac{1}{4DeH}\left\{\psi\left(\frac{1}{2}+\frac{3DeH}{2\pi T}\right)-\psi\left(\frac{1}{2}+\frac{DeH}{2\pi T}\right)\right\},$$

$$t_0=\frac{T-T_c(H)}{T_c(H)}\equiv\eta_H,$$

$$t_1=\psi\left(\frac{1}{2}+\frac{3DeH}{2\pi T}\right)-\psi\left(\frac{1}{2}+\frac{DeH}{2\pi T}\right),$$

$$a_0=\frac{1}{4\pi}\psi'\left(\frac{1}{2}+e_0\right)\left(1+i\frac{T}{\varepsilon_F}\alpha_0\right),$$

$$a_1=\frac{1}{4\pi}\psi'\left(\frac{1}{2}+e_1\right)\left(1+i\frac{T}{\varepsilon_F}\alpha_1\right),$$

$$\alpha_0=2\pi\varepsilon_F\frac{N'}{N}\left\{-e_0+\frac{1}{\psi'(\frac{1}{2}+e_0)\,gN}\right\},$$

$$\alpha_1=2\pi\varepsilon_F\frac{N'}{N}\left\{-e_1+\frac{1}{\psi^{(1)}(\frac{1}{2}+e_1)}\left[\frac{1}{gN}-\psi\left(\frac{1}{2}+\frac{3DeH}{2\pi T}\right)+\psi\left(\frac{1}{2}+\frac{DeH}{2\pi T}\right)\right]\right\}, \tag{3·9}$$

where $T_c(H)$ is the root of the equation

$$\psi\left(\frac{1}{2}+\frac{DeH}{2\pi T}\right)-\psi\left(\frac{1}{2}\right)+\ln\frac{T}{T_c}=0.$$

We, in this paper, fix the strength of the magnetic field and vary the temperature. Substitution of Eq. (3·9) into Eq. (3·6) yields

$$\Delta\sigma_{xy}=-\sigma_0\eta_0\frac{16T}{\pi}\frac{N'}{N}\frac{1}{\eta_H}\left\{-e_1\psi'\left(\frac{1}{2}+e_1\right)+\left[\frac{1}{gN}-\psi\left(\frac{1}{2}+e_1\right)+\psi\left(\frac{1}{2}+e_0\right)\right]\right\}$$

$$\equiv-16\frac{T}{\pi}\frac{N'}{N}\frac{\sigma_0\eta_0}{\eta_H}B. \tag{3·10}$$

In this limit, singularity with respect to the temperature deviation differs from that in the weak field limit.

In the similar situation as in Eq. (3·10), σ_{xx} is given as[17]

$$\Delta\sigma_{xx}=\frac{4\sigma_0\eta_0}{\eta_H}, \tag{3·11}$$

and then

$$\Delta\sigma_{xy}=-\frac{4B}{\pi}T\frac{N'}{N}\sigma_{xx}. \tag{3·12}$$

The singularity in this case with respect to η_H is similar for σ_{xx} and σ_{xy}. Moreover the Maki process and other processes besides the AL process yield contributions with the same singularity. But, for nearly free electrons and in such a temperature range as

$$\eta_H \lesssim \frac{DeH}{T} \ll 1 , \tag{3·13}$$

the contribution from the AL process (3·10) is

$$\varDelta\sigma_{xy}^{\rm AL} = -\sigma_0 \frac{4\eta_0}{\eta_H} \frac{T}{\varepsilon_{\rm F}} \alpha , \tag{3·14}$$

whereas the one from the Maki process is,[19]

$$\varDelta\sigma_{xy}^{\rm M} = -\omega_c\tau \frac{8\eta_0}{\eta_H} \sigma_0 . \tag{3·15}$$

Thus Eq. (3·14) is a major contribution to the nearly free electron systems, but this relation may hold for any Bloch electrons. Thus, using[19],[20]

$$\varDelta\sigma_{xx}^{\rm AL} = \varDelta\sigma_{xx}^{\rm M} = -\frac{4\eta_0}{\eta_H}\sigma_0 , \tag{3·16}$$

in this region, we obtain

$$\varDelta\sigma_{xy} = -2T\frac{N'}{N}\alpha \varDelta\sigma_{xx} . \tag{3·17}$$

§ 4. Summary of results

The Hall angle for a film with thickness d in the weak field limit of magnetic fields, $\eta \gg DeH/T$, is given

$$\theta = -\omega_c\tau \left\{1 + \frac{4\eta_0}{\eta - \delta} \ln \frac{\eta}{\delta} + \frac{\pi\alpha}{18} \frac{\eta_0}{\eta^2}\right\} \left\{1 + \frac{2\eta_0}{\eta - \delta} \ln \frac{\eta}{\delta} + \frac{\eta_0}{\eta}\right\}^{-1} ,$$

where $\eta_0 = e^2/16d\sigma_0$, $\eta = (T - T_c)/T_c$, $\alpha = 2\pi^{-1}(1/gN)$ (g: coupling constant, N: density of states at the Fermi energy) and δ is the pair-breaking parameter. The second and third terms in the numerator come from the Maki and the AL processes, respectively. The latter contribution due to fluctuating supercurrents has very strong singularity with respect to η and emerges from the small corrections of the fluctuation propagators of the order of $T/\varepsilon_{\rm F}$. The sign of this contribution is dependent on the sign of the curvature of N near the Fermi energy, and then the fluctuations can either increase or decrease the Hall angle near T_c. In this respect the experiments on the Hall effect in fluctuating superconductors are expected to afford detailed information on the Fermi surface.

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