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FLUID DYNAMICS WITH A COMPUTATIONAL PERSPECTIVE

Modern fluid dynamics is a combination of traditional methods of theory and analysis and newer methods of computation and numerical simulation. Underlying both are the principles of fluid flow. *Fluid Dynamics with a Computational Perspective* synthesizes traditional theory and modern computation. It is neither a book on methods of computation, nor a book on analysis; it is about fluid dynamics – consistent with the state of the art in that field. The book is ideal for a course on fluid dynamics. The early chapters review the laws of fluid mechanics and survey computational methodology and the subsequent chapters study flows where the Reynolds number increases from creeping flow to turbulence, followed by a thorough discussion of compressible flow and interfaces. Although all significant equations and their solutions are presented, their derivations are informal. References for detailed derivations are provided. A chapter on intermediate Reynolds number flows provides illustrative case studies by pure computation. Elsewhere, computations and theory are interwoven.

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To Cinian, Seth, and Sanja, who were patient and supportive while we wrote, and to Professor Enoch Durbin, for a lifetime of encouragement

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Preface

This is a book on fluid dynamics. It is not a book on computation. Many excellent books on fluid dynamics are available: why is another needed?

In recent decades, numerical algorithms and computer power have advanced to the point that computer simulations of the Navier–Stokes equations have become routine. This vastly expands our ability to solve these equations, further extending our understanding of fluid flow and providing a tool for engineering analysis. Computer simulations are solutions of a different nature from classical exact and approximate solutions. They are numerical data rather than formulas. One of our objectives in this text is to relate computer solutions to theoretical fluid dynamics. Indeed, it is this goal, rather than computation as a tool for complex engineering analysis, that provides the guideline for this text. Computer solutions can reproduce closed-form and approximate solutions; they can illuminate the merits and limits of simple analyses; and they can provide entirely new solutions of varying degrees of complexity. The time is ripe to integrate computer solutions into fluid dynamics education.

From a pedagogical perspective, readily available, commercial computational fluid dynamics (CFD) software provides a new resource for teaching fluid dynamics. This software converts CFD from a technique used by researchers and engineers in industry into a readily accessible facility. It is a challenge to integrate such software packages into the educational structure. Most of the examples in this book have been computed with commercial software, and exercises to be solved with such software have been suggested. How far to go in this direction was a true quandary. We started out ambitiously, intending an intimate use of commercial tools, but backed off, deciding on a text that provides the reader with illustrative computations. Grids and specifications needed to effect computer simulation are described, but far short of the level of detail found in tutorials. The endeavor was to make the book useable either on its own or as an adjunct to an integrated course on fluid dynamics and computation. A lecturer might supplement this text with a computer laboratory, instructing students via tutorials.

This certainly is not a book on methods of computational fluid dynamics. We assume that a student who uses this text in conjunction with computational analysis has access to CFD software, including both flow solver and mesh generator. Methods and algorithms are mentioned only to the extent that they bear on the fidelity of computations. In other words, the perspective is that of a code user, not of a

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developer. One impetus for this book was the observation by many of our colleagues that students are increasingly proficient at using software, but often without the understanding of fluid dynamics needed to use it effectively. We hope this text will be a resource for educating prospective software users.

Despite the power of computer simulation, it has not supplanted theory and analysis in fluid dynamics teaching, even at advanced levels; nor have theoretical efforts diminished within the research community. No wholesale displacement of theory by computation is remotely on the horizon. The complexity and intricacy of flow phenomena are too great, and the range of applications are too vast, for any single approach to be sufficient. Hence, in this book, we endeavor both to present some basics of the theory of fluid flow and to explore them by computer simulation. The intent is a marriage between classical theory and modern computer-aided analysis.

Deciding how much mathematics to include was another quandary. A great many texts detailing analyses and solutions to governing equations are available. It is not our intent to reproduce such material. At the same time, there is a need to establish a framework for computer-aided analysis. This means that equations and solutions must be cited. To a large extent, we have quoted results with informal derivations, providing references where detailed developments can be found. In some cases, formulas might appear out of the blue, but in almost all cases, some basis has been provided, without rigor.

The prerequisite to this book is a course on basic fluid dynamics, including elementary viscous flow. The book is self-contained, but the pace might seem fast without the prerequisite. It is directed to students at advanced undergraduate level or to graduate students at master's level. It is also meant for scientists and engineers who want background in viscous flow phenomena.

After two chapters that provide background on the laws of fluid mechanics and survey computational methodology, the next four chapters increase the Reynolds number from creeping flow to turbulence. They are followed by a chapter on compressible flow and a final chapter on interfaces. We have been guided by the content of standard curricula in fluid mechanics.

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