Open access • Journal Article • DOI:10.1002/APJ.5500120109

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Published on: 15 May 2008 - Developments in Chemical Engineering and Mineral Processing (Wiley Subscription Services, Inc., A Wiley Company)

Topics: Pressure drop, Laminar flow, Fluid dynamics, Turbulence and Reynolds number

Fluid Flow through 90 Degree Bends

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Pressure drop measurement and prediction in curved pipes and elbow bends is reviewed for both laminar and turbulent single-phase fluid flow. For curved pipe under laminar flow, the pressure loss can be predicted both theoretically and using empirical relations. The transitional Reynolds number can be predicted from an empirical relation. Turbulent flow in curved pipes can only be theoretically predicted for large bends but there are a large number of empirical relations that have proved to be accurate. Elbow bends have proven to be difficult to both measure and represent the pressure loss. Methods of overcoming such problems are outlined. There was no reliable method of theoretically predicting pressure drop in elbow bends. Experimental measurements showed considerable scatter unless care was taken to eliminate extraneous effects. Reliable data are highlighted and an empirical method is proposed for calculation of pressure drop in elbow bends.

Introduction

Scant attention is usually given to the exact effect of fittings in the design of piping systems. Their influence on the overall pressure drop can be significant for single-phase flow. In this work attention is focussed on the determination of pressure drop in bends with single-phase flow. An extension to multiphase flows will be presented in a subsequent paper.

Single-phase deflected flow can be divided arbitrarily into two separate cases, that for curved pipe at high R/d values, and for sharp elbow bends at low values of R/d. The two are obviously interlinked but normally the former has fully developed flow with the virtual absence of inlet and outlet tangent effects, while the latter elbow bend has undeveloped flow with both inlet and outlet tangent effects.

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I. Curved Pipe

There have been a number of reviews on the effects of fully developed flow in curved circular tube [1-4]. Dean [5-7] used perturbation solutions to show that the central core of the fluid moved more rapidly than the average flow and was forced outwards from the centre of curvature by centrifugal action. On the other hand, the slower moving wall region of the fluid moved inwards towards the centre of curvature where the pressure was lower. Thus a two helical vortex secondary flow was established radially to the main axial flow, as shown in Figure 1, as mirror imaged rotating spirals in the upper and lower semi-circular cross section of the pipe. Such steady secondary flow caused an increase in frictional resistance over that in a straight pipe. Additional secondary vortices occur above some definite Dean number [8-13], as defined by:

$$De = Re\left(\frac{d}{D}\right)^{0.5} \tag{1}$$

The dynamic similarity depended mainly on the Dean number, De, although the effect of curvature or helical coil pitch could not be ignored with tubes of large d/D ratios [14] or large coil pitch p/D [15-20]. Some alternative definitions of De have been used mainly for convenience in the analytical treatment.

(i) Laminar Flow

Analysis of curved pipe laminar flow originally used the method of successive approximations [5-7] employing the extended Stokes series [21-24] that proved successful only for $De < 200$. Boundary layer techniques gave valid solutions for $De > 800$ [25-29]. Finite difference methods were shownto apply for $De > 400$ [10, 30-38] except when Fourier series expansions were used [10, 12, 14, 39]. Recently computational fluid dynamics (CFD) [40-43] has shown agreement with reliable data [25, 34, 44-48] over the whole laminar flow region and helped resolve the debate over the analytical methods [10, 36, 49-51]. The increased frictional resistance caused by secondary motion in curved pipe has often been presented as the ratio of the friction factors for curved and straight pipe under the same conditions based on the actual centre line length of the curvature. Such an approach presents only the consistent developed pressure drop within the main body of the curved length excluding entrance [35, 52-77] and exit [78] effects. As the R/d value of the curved pipe falls, these latter effects become increasingly important. Despite the success of CFD in predicting pressure drop in curved pipe with laminar flow, it often is of practical value to have either a theoretical relation [5-7, 10, 12, 21, 22, 24-28, 33, 34, 36, 39, 79] or empirical relation [16, 33, 40, 45, 47, 48, 80-87] to use in calculations of pressure drop. The most reliable relations proved to be those of White [45] for the range $11.6 < De < 2000$:

$$\frac{\phi_s}{\phi_c} = 1 - \left[1 - \left[\frac{11.6}{De}\right]^{0.45}\right]^{\frac{1}{0.45}} \tag{2}$$

For the range 13.5 < De < 2000, then the relation by Ito [28, 52]:

$$\frac{\phi_C}{\phi_S} = \frac{21.5\text{De}}{[1.56 + \log \text{De}]^{5.73}} \tag{3}$$

In both cases, $\phi_C = \phi_S$ below the lower De value. There was a tendency for Equation (2) to under predict (-1%) at low De values and over predict (+2.5%) at high De numbers. The Ito correlation [28, 52] was marginally better but still possessed the same under and over prediction tendency at low and high De numbers respectively. The Mishra and Gupta [16] relations of Equations (4) to (6) are given below:

$$\frac{\phi_C}{\phi_S} = 1 + 0.033(\log \text{De}_m)^{4.0} \tag{4}$$

$$\text{De}_m = \text{Re}\left(\frac{d}{2R_{co}}\right)^{0.5} \tag{5}$$

$$R_{co} = \frac{D}{2}\left[1 + \left(\frac{p}{\pi D}\right)^2\right] \tag{6}$$

They gave a consistently high result (+2.7%) but did allow the effect of coil pitch to be correlated by the inclusion of R_{co}, the radius of coil curvature.

Secondary flow in curved geometries were shown to be influenced by the actual cross sectional area shape. Laminar flow in curved channels of rectangular [11, 88-98], square [61, 62, 99-108], triangular [109-113], elliptical [30, 114-117], semi circular [37, 118, 119] and annular [120] cross sections have been examined. Turbulent flow studies have been carried out for curved channels of rectangular [121, 122] and square [102, 123, 124] cross sections. Helical coils were the preferred geometry used but other designs have been employed [27, 32, 39, 46, 53, 54, 57, 61, 64, 66-69, 71, 72, 78, 88, 94, 99, 101, 107, 108, 110, 114, 115, 121-123, 125]. Additional work has been reported on laminar pulsating flow in curved pipes [2, 58, 126-153], on rotating curved pipes [51, 154-164], twisted pipe with torsion effects [38, 76, 165-175], and for non-Newtonian flow [51, 176-182].

(ii) Transitional Flow

Taylor [183] among others [1, 16, 25, 45, 46, 54, 81-83, 85, 149, 184-190,192] have demonstrated that the secondary flow formed in a curved pipe was a steady helical swirling motion that stabilised the axial flow such that the transitional Reynolds number to turbulent flow (Re_{cr}) exceeded that for straight pipe (Re_{sc}). Spedding and Chen [191] have shown that the elimination of disturbances in the flow or vibration then allowed flow in straight pipe to remain laminar up until $\text{Re}_{scr} \cong 40{,}000$ in contrast to that normally registered, i.e. $\text{Re}_{scr} = 2100\text{-}2700$. The same effect has been shown to apply to curved pipe for d/D < 0.01 [1, 46, 52]. This aspect of the laminar to

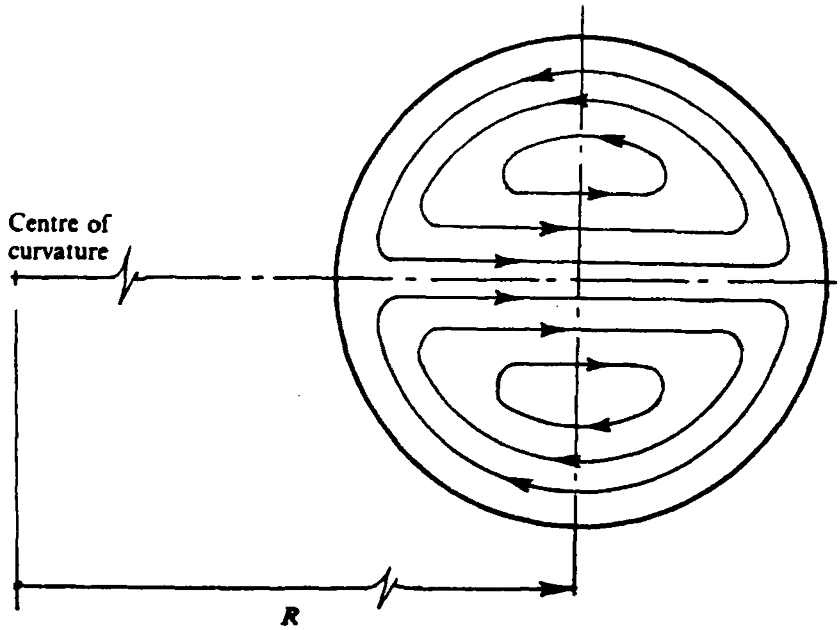

Figure 1. Secondary flow patterns in the pipe cross-section down stream of a bend.

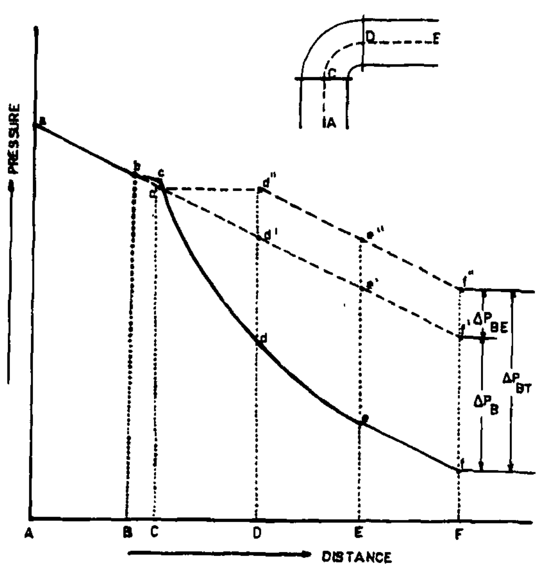

Figure 2. Schematic diagram of the single phase frictional pressure loss in a horizontal 90° elbow bend.

turbulent transition is only of academic interest since the required tranquil conditions would not apply in an operating plant. White [45] suggested the critical transition for curved pipe occurred at $\phi_{cr} = 0.0045$ and could be calculated using Equations (1) and (2). This method gave a result significantly greater (+3%) than experiment [25, 45, 46, 52, 82, 149, 183-187, 190], but performed badly when $d/D > 0.03$. Ito [49] suggested the relation:

$$Re_{cr} = 2x10^4 \left(\frac{d}{D}\right)^{0.32} \tag{7}$$

The average result was within +17% of the data, being poor in the region $d/D < 0.01$. Other suggested transitional relations for curved pipe [1, 16, 81, 83, 188] gave even worse performances. In addition only two of the relations [1, 16] approached the straight pipe value Re_{sc} in the limit when $D \rightarrow \infty$. A better prediction of data can be obtained with the relation:

$$\frac{Re_{cr}}{Re_{scr}} = 7.5 \left(\frac{d}{D}\right)^{0.25} \tag{8}$$

(iii) Turbulent Flow

Turbulent flow in curved pipes has proved to be more complex than for laminar flow. The measured velocity distribution [102, 123, 126, 192, 193] showed that the maximum axial velocity was near the pipe outside wall with the slower moving fluid chiefly restricted to a relatively thin shedding layer near the walls. The flow pattern was more complex than in the laminar flow case [78] but has been qualitatively predicted using a two-equation turbulence model [57]. The radial pressure gradient created in the bend propagated upstream into the inlet pipe through the wall boundary layer. The loss coefficient was affected by the upstream and downstream tangents [52, 78]. The maximum pressure loss was achieved when the fluid had passed through a rotational angle of about 120° [193, 194]. Ward-Smith [195] showed that the gross pressure loss for square and circular cross-sectional curved pipe were about the same. Using the shedding layer concept of Weske [123], Ito [52] and Mori and Nakayama [27] developed predictive models for circular pipe which gave results below the available experimental data. The same was true for the model of Pantakar et al. [57] and the application of corrections for elliptical effects being transmitted through the pressure field [75, 125].

A number of empirical relations of various forms have been suggested to predict turbulent pressure drop in curved tubes [16, 27, 48, 52, 81, 83-85, 87, 186, 196-202]. The more realistic models are listed in Table 1. For smooth pipes, the Ito [52] relation of Equation (10) gave predictions within ±1% of the reliable data. Equations (11), (13)-(15) and (19) performed well but with a somewhat wider spread of the error. The relations suggested by Kubair and Varrier [81], and another second model given by Koutsky and Adler [186], did not exhibit the correct variation with Re and d/D respectively. The model by Rao and Sadasivudu [84] with secondary relations by

Table 1. *Prediction of turbulent pressure drop in curved pipes.*

$\phi_c = \phi_s + 0.005\left(\frac{d}{D}\right)^{0.5}$	(9)	[196]
$\phi_c\left(\frac{D}{d}\right)^{0.5} = 0.003625 + 0.038\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{-0.25}$	(10)	[52]
$\phi_c = \phi_s\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{0.05}$	(11)	[52]
$\phi c = \phi_s \exp\left[2\pi\frac{d}{D}\right]$	(12)	[197]
$\phi_c = 0.0395\left(\frac{d}{D}\right)^{0.5}\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{-0.2}$	(13)	[186]
$\phi_c = \phi_s\left[1 + 0.0823\left(1 + \frac{d}{D}\right)\left(\frac{d}{D}\right)^{0.53}\mathrm{Re}^{0.25}\right]$	(14)	[87]
$\phi_c\left(\frac{D}{d}\right)^{0.5} = 0.0395\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{-0.2}\left[1 + 0.112\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{-0.2}\right]$	(15)	[27]
$\phi_c\left(\frac{D}{d}\right)^{0.5} = 0.024\left[\mathrm{Re}\left(\frac{d}{D}\right)^{2.5}\right]^{-0.167}\left[1 + 0.068\left[\mathrm{Re}\left(\frac{d}{D}\right)^{2.5}\right]^{-0.167}\right]$	(16)	[27]
$\phi_c = 0.042\left[\mathrm{Re}\left(\frac{D}{d}\right)^{0.5}\right]^{-0.2}$	(17)	[83]
$\phi_c = \phi_s\left[1.83\left(\frac{d}{D}\right)^{0.1}\right]$	(18)	[198]
$\phi_c\left(\frac{D}{d}\right)^{0.5} = 0.00206 + 0.0394\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{-0.227}$	(19)	[199]
$\phi_c = 0.001875 + 0.3165\left(\frac{d}{D}\right)^{0.275}\mathrm{Re}^{-0.4}$	(20)	[201]
$\phi_c = \phi_s + 0.00375\left(\frac{d}{D}\right)^{0.5}$	(21)	[16]
$\phi_c = \phi_s + 8.7891\left(\frac{d}{D}\right)^{0.3621}\mathrm{Re}^{-0.3137}\left(\frac{\varepsilon}{d}\right)^{0.6885}$	(22)	[202]

Schmidt [87], Ito [52], Mori and Nakayama [27] and Ruffell [201], showed generally poor performances. Some workers have highlighted an effect due to pipe roughness [201-203], and Das [202] has presented a model incorporating ε/d. Equations (9), (12), (17), (18), (20) and (21) all gave over-predictions which were attributable to being derived from work on non-smooth tubes of various ε/d values. Equations (16) and (22) tended to give low predictions. Equation (21) allowed for the effect of coil pitch and according to Singh and Mishra [200] also for the effects of certain non-Newtonian flows.

II. Elbow Bends

Steady single-phase flow has been reported for pipe elbow bends generally with $R/d < 5$ for various deflection angles up to 360°. Considerable interest has been shown in the 180° U bend [53, 57, 64, 66-68, 78, 114, 123, 193, 194, 204-206] but far less attention has been directed to the more widely used 90° elbow bend. Generally the pressure drop through elbow bends is considerably larger than for the straight pipe equivalent and adds significantly to the losses sustained in piping systems. Designers usually apply the general rule that a 90° elbow bend has a pressure drop equivalent of 30 to 50 pipe-diameters length of straight pipe [207]. However, when the estimation of pressure drop can have a critical effect on operation or plant safety, such as on the downstream side of a relief value, a more exact method is desirable.

There have been a number of different ways proposed to calculate the elbow pressure drop from raw experimental data. A future paper on multiphase flow in bends will show that the most consistently reliable method of calculation is that detailed by Ito [204] and others [208]. Figure 2 is the pressure drop through a horizontal 90° elbow bend. AC and DF are the inlet and outlet pipe tangent lengths, CD is the total elbow bend centreline length, while BC and DE are the inlet upstream and outlet downstream transitional regions. The actual pressure distribution is a-b-c-d-e-f, while a-b-c'-d'-e'-f' is the straight pipe distribution for the equivalent centre line pipe length. The pressure drop f-f' is that due to the elbow bend itself ΔP_B. The corrected pressure distribution a-b-c'-d''-e''-f'' includes a straight pipe loss equivalent to the length CD of the elbow bend centre line which results in f'-f'' the equivalent pressure drop ΔP_{PE} due to the length of the elbow bend. The straight pipe pressure drops in the upstream and downstream tangent legs have been extended to C and D, the points of physical commencement of the elbow legs. Thus the true pressure loss due to the bend is ΔP_{BT}. This approach of treating the elbow bend as a complete entity has the advantage of making the bend resistance easier to formulate and the actual results possess a more consistent pattern.

Perry [207] has detailed other methods used to calculate elbow bend pressure drops from raw measurement data. In general they gave smaller absolute values of resistance, required more detailed calculation and resulted in scatter and inconsistencies in subsequent handling. In some cases the distance CD was reduced to that of the actual bend in the elbow, i.e. without any length for its legs to flanges, etc. In other cases compensation was made for the centre line distance, or the centre line intersectional distance, of the bend in order to arrive at a value for ΔP_B stripped of any element due to length within the system. The differences between approaches can be significant, particularly as R/d increases as shown in Figure 3. There remains the additional complication that some workers have not unambiguously stated the exact method used to calculate either the elbow or bend pressure drop. Characterisation of the bend is usually made by the R/d parameter but in some cases [209, 210] it is unclear how the bend was described. What is certain is that the inner wall of the bend must coincide with that of the connecting straight pipe if excessive pressure drop is to be avoided. For example, a screwed elbow bend of $R/d = 0.814$ and $R/d_b = 0.63$ had about double the total bend pressure drop ΔP_{BT} of a smooth inner wall equivalent [204]. In addition Pigott [210] showed that crinkling on the inner bend wall, due to faulty manufacture, appreciably increased the bend pressure drop.

Often the pressure drop has been presented in terms of velocity heads instead of the more useful equivalent pipe length expressed as internal pipe diameters, i.e. l_e/d. The use of K values results in a pressure drop parameter that varies with pipe roughness, where:

$$K = \left[\frac{\Delta P / \rho g}{V^2 / 2g} \right] \tag{23}$$

The equivalence between the two methods of presentation is given by:

$$\ell_e / d = K / (8\phi) \tag{24}$$

For elbow bends the excess pressure drop due to both the separation at the inner wall and that in the tangent legs BC and DE was proportionally much greater than that observed for curved pipes at larger R/d values. Eustice [114] showed by dye visualisation studies that even for laminar flow separation, reversal of flow and greater turbulence was observed when $R/d < 3$. Analytical evaluation using both inviscid potential and rotational flow has given general qualitative agreement with the observed pressure distribution through elbow bends [1]. Ito [204] reported a significant circumferential pressure variation within the actual elbow bend, much greater in fact than for curved pipe. On the outer wall of the elbow bend the observed pressure was high due to fluid impaction. However at the inner bend wall, separation of the boundary layer with subsequent formation of secondary circulation currents took place resulting in a low value of the recorded pressure in this region. Stagnation points have been observed in both of these regions [211]. The growth and separation of the boundary layer, particularly at the inner wall of the bend, grossly changed the velocity distribution within the bend for both laminar [51, 55, 61, 69, 101, 102, 110, 125, 194] and turbulent [57, 78, 102, 125, 195] conditions, inducing cyclic elements downstream of sharp bends [53, 75, 212-214]. This latter switching of downstream secondary flow patterns can explain the inability of analytical methods (including CFD) to effectively handle elbow bends [204, 215]. Interactive effects have been observed with closely spaced elbow bends [1, 3, 216].

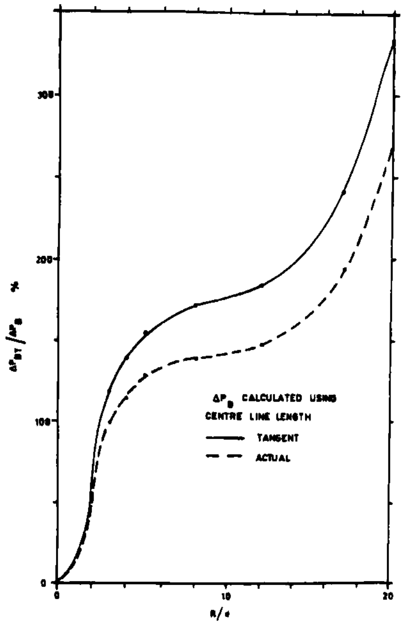

Figure 3. *The total elbow pressure drop over that for the bend pressure loss corrected for the centre line length and the centre line tangent length.*

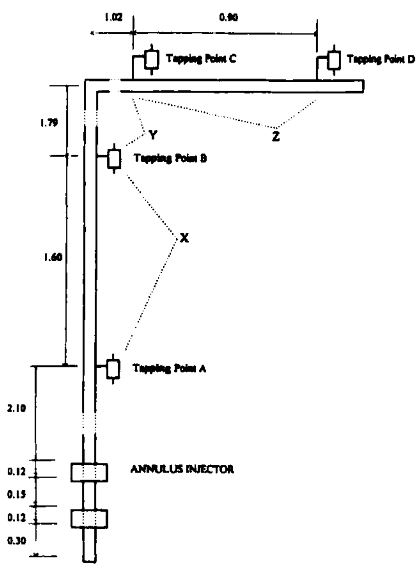

Figure 4. *Schematic diagram of the test section.*

Experimental Details

The experimental measurements were designed and conducted in the light of the foregoing review. The apparatus is shown schematically in Figure 4, and the actual details of the commercial 90° elbow bend are given in Figure 5. The system was designed as a multiphase rig but was operated here using air up to a flowrate of 0.02m^3 s^{-1} at atmospheric conditions. Measurement of flow was by calibrated rotameters. The apparatus was made from clear perpex piping (0.026 m i.d.). The elbow bend was standard PVC 0.026 m i.d. fitting with an R/d of 0.0654. The bend and piping matched exactly so as to present a smooth inner surface to the flow. Tapping points of 0.002 m i.d. were used to measure pressure drop across three separate sections of the apparatus X, Y and Z. Measurement was by a calibrated Zephyr micromanometer (Air-Neotronics). Further details of the apparatus and its operation and verification are given elsewhere [217].

Results and Discussion

Single-phase air flow through the tangent lengths X and Z were in agreement with the Moody diagram pressure loss for smooth pipes, when the former was corrected for gravity head. The fluctuation range in the inlet tangent X approximately doubled in size around the transitional Re_{cr} range.

Figure 6 shows the results of this work together with data from a number of sources [204, 208-210, 218-221] some of which exhibit a wide variation. There was even a wide divergence between quoted data and the original source in some instances. For example, Fitzsimmons [218] reported data by Beji [219] that were much higher in actual values than the original, no doubt caused by attempting to bring data to a consistent basis. Some results in Figure 6 can be expected to be rather low since they were calculated as ΔP_B only [213, 222], or had insufficient elbow tangent lengths. The most reliable data were those by Ito [204, 220, 221] to which other workers [208, 222] have shown reasonable agreement, as also did some models [223, 224]. The scatter of unreliable data evidenced in Figure 6 was magnified when used to handle multiphase flow in elbow bends.

The reliable data in Figure 6 had a minimum in the pressure drop at R/d = 2.5. Ito [204] showed that similar minimum values occurred for the 45° and 180° bends that were respectively below and above the 90° elbow bend data shown in Figure 6. This minimum highlights the optimum design of an elbow bend in order to minimise pressure drop. The minimum in Figure 6 proved to be insensitive to Re_G. Of course the pressure drop does not go on increasing with R/d indefinitely, but eventually returns to the straight pipe value as R/d $\rightarrow \infty$. With large values of R/d the dominant influence on excessive pressure loss was outer wall friction. As R/d was reduced the length of the bend decreased, causing a steady reduction in the excess pressure drop. At R/d > 10 the elbow bend pressure drop can be predicted within ±1% by Equations (10), (11), (13)-(15) and (19) for turbulent flow conditions. Below this value of R/d, the pressure loss in the entry and exit tangent lengths begins to be of importance and also when separation of flow on the inner bend wall starts to have an effect. Below R/d $\cong$ 5, the latter separation becomes the predominant cause of pressure loss.

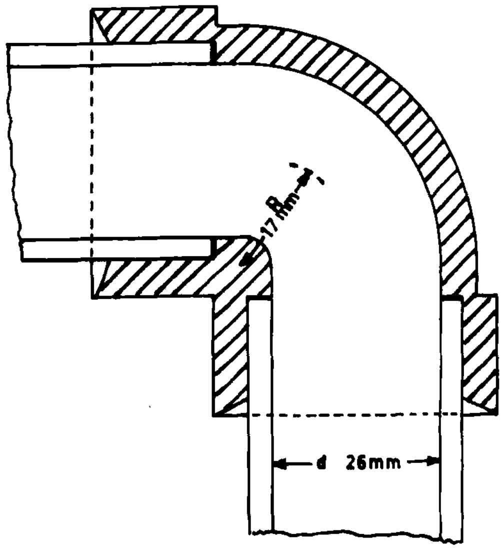

Figure 5. *The elbow bend, which is the total physical size of the bend shown cross hatched and not just the bend element.*

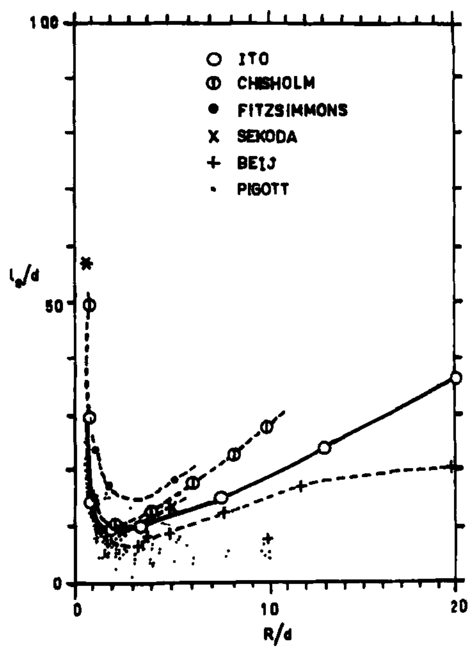

Figure 6. *Single phase pressure loss in horizontal 90° elbow bends at Re = 1.5×10^4.*

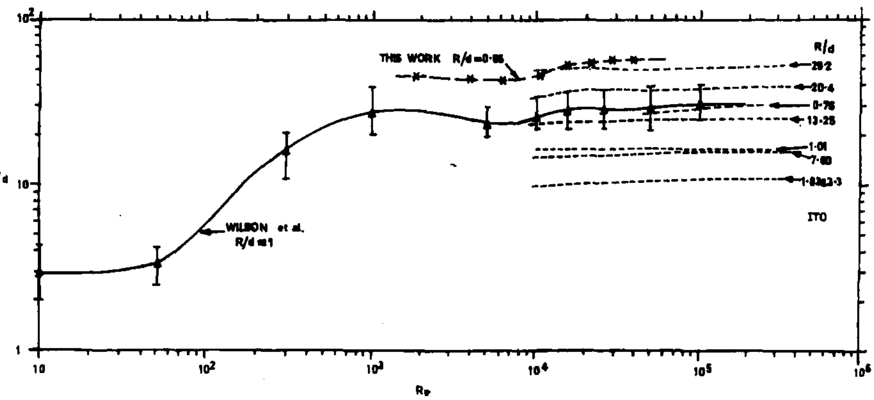

Figure 7. *The effect of Reynolds number on the single-phase pressure loss in horizontal 90° elbow bends.*

The variation of l_e/d against Re_G is given in Figure 7. The extensive work of Wilson et al. [225] has been restricted to lower diameters where the full effect of the tangential regions was present. Experimental spread is indicated by the bars on the data. The laminar flow region was in agreement with the data of Beck [226]. At high Re_G values in the turbulent regime l_e/d varied very little with Reynolds number. By contrast K values exhibited a steady fall with rising Re_G and therefore K was not a valid parameter for expressing bend pressure losses. In the transitional region the data showed a dip which is paralleled in this work but not with the Ito data [204, 220, 221]. The Wilson et al. [225] results were obtained on industrial screwed fittings and therefore registered values above those of Ito [204, 220, 221] for smooth bends.

The elbow bend pressure drop ΔP_{BT} obtained in this work are shown in Figures 6 and 7. In Figure 6 the result was for a practical bend with an R/d value below those used by Ito [204, 220, 221] that gave a value in agreement with the logarithmic extrapolation of their data. The data obtained in this work showed a wide divergence from that of others [209, 210, 218, 219] because of the different basis used to obtain and calculate the results. Figure 7 shows the data obtained in this work to be in good agreement with that of Ito [204, 220, 221]. Ito [204] suggested an empirical correlation for pressure loss prediction which did not agree with the data of this study, mainly because the correlation factor showed rapid change in the region of $R/d \leq 0.7$.

The theoretical development of Chisholm [209] and Matthew [215] showed little agreement with either the data of this work or that of Ito [204, 220, 221].

Crawford et al. [227] have shown that the elbow bend pressure drop can be predicted by the addition of $(l_e/d)_c$ from Equation (10) (or Equation 15) and:

$$\ell_e / d = 22.2126\left[\mathrm{Re}\left(\frac{d}{D}\right)^2\right]^{0.7888} \mathrm{Re}^{-0.71438} \tag{25}$$

where $$(\ell_e / d)_c = 19.8333\ \phi_c\ \mathrm{Re}^{0.25}\left(\frac{D}{d}\right) \tag{26}$$

Conclusions

A review of single-phase flow in curved pipes showed the Dean number to be a defining parameter. Theoretical developments have been successful with laminar flow predictions but more practical empirical relations [45, 52] allowed accurate pressure loss prediction. The critical Reynolds number Re_{cr} for transition to turbulent flow was greater than for straight pipe. A new relation is presented that allows for variation in pipe roughness. For turbulent flow in curved pipes a number of empirical relations have been developed for pressure loss prediction in smooth [27, 52, 87, 186] and rough pipes [16, 83, 196-199, 201], but a theoretical treatment has proved to be difficult except at high R/d values. Single-phase flow in elbow bends is reviewed with potential problems of measurement and data handling emphasised. The fluid dynamics in elbow bends proved to be complex, with separation and boundary layer effects causing downstream flow to exhibit cyclic characteristics that have not been amenable to theoretical treatment. An empirical correlation is presented for pressure loss prediction in elbow bends. These data will be essential for the effective handling of two- and three-phase flow in bends.

Nomenclature

d	Pipe internal diameter, m.
d_b	Internal bend diameter, m.
D	Diameter of bend, m.
De	Dean number, Re $(d/D)^{0.5}$
g	Gravitational constant, ms^{-2}
K	Pressure drop factor, eqn. (23)
ℓ_e	Pipe length equivalent to elbow bend pressure drop, m.
p	Coil pitch, m
P	Pressure, kg m^{-1} s^{-2}
R	Radius of bend element centre line, m.
R_{co}	Radius of curvature, Equation (6)
Re_{ac}	Reynold number, $dv\rho/\mu$
V	Velocity, m s^{-1}
ϵ	Roughness, m.
μ	Fluid velocity, kg m^{-1} s^{-1}
ρ	Fluid density, kg m^{-3}
σ	Surface tension, kg s^{-2}
ϕ	Friction factor

Subscripts

C	Curved pipe
CO	Coil
CR	Critical
G	Gas
B	Bend
T	Total
E	Equivalent
M	Modified
S	Straight pipe
SC	Straight pipe critical value

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