

This file has been cleaned of potential threats.

If you confirm that the file is coming from a trusted source, you can send the following SHA-256 hash value to your admin for the original file.

a15cc51ddf43ac1c17447c99cb437a80f6d973d28ea45b41266afa9ca707a099

To view the reconstructed contents, please SCROLL DOWN to next page.

FLUID PHENOMENA IN SCRAMJET COMBUSTION SYSTEMS¹

E. T. Curran

Director, Aero Propulsion and Power Directorate, Wright Laboratory,
Wright-Patterson Air Force Base, Ohio 45433

W. H. Heiser

Department of Aeronautics, United States Air Force Academy, USAF
Academy, Colorado 80840

D. T. Pratt

Department of Mechanical Engineering, University of Washington, Seattle,
Washington 98195

KEY WORDS: hypersonic air-breathing propulsion, dual-mode combustors, combustor-isolator interactions, supersonic mixing

ABSTRACT

This paper reviews important advances in understanding and predicting the behavior of scramjet combustion systems since the classic article published in this series by Antonio Ferri (1973). The review focuses on basic fluid phenomena and is divided into three distinct sections. The first briefly describes progress that has been made in the design and demonstration of practical scramjets around the world, especially in the US, the FSU, France, and Germany. The second provides a contemporary exposition of the aerothermodynamics of the dual-mode ramjet/scramjet combustion system, accounting for the role of the isolator in preventing unstart from either thermal choking or flow separation due to heat release. The third part summarizes the current state of knowledge regarding fuel-air mixing in dual-mode ramjet/scramjet combustors, especially the potential of axial vorticity to increase mixing effectiveness over that of lateral vorticity.

¹The US Government has the right to retain a nonexclusive, royalty-free license in and to any copyright covering this paper.

INTRODUCTION

A classic article reviewing several important aspects of supersonic combustion was published in this series by Antonio Ferri (1973) under the title "Mixing-Controlled Supersonic Combustion." The principal issues considered in the article were the basic concept and performance potential of hypersonic air-breathing supersonic combustion ramjets (called, briefly, *scramjets*), the behavior of supersonic combustion flames, chemical reaction processes of hydrogen-air combustion, fuel-air mixing processes, the multidimensional interaction between combustion and fluid dynamics known as thermal compression, and modeling suitable for numerical analysis of the complex combustor flow fields.

In the intervening time period, major national programs significantly advanced both the understanding and technology of scramjet engines. To his credit, many of those efforts centered on the issues previously identified by Ferri. Consequently, this article reviews the progress made in these critical areas over the past two decades, with two notable exceptions. First, the science of chemical reactions has, with the help of modern computers, grown into a field so large that it must be considered separately. Fortunately, this is a sign of success, and much of what is required for scramjet design is readily available. Second, Computational Fluid Dynamics (CFD) has been harnessed to the maximum extent possible to analyze scramjets, from freestream to exhaust. This was necessary because the flow conditions are so far removed from previous test experience that no other means were available to predict flow behavior. This was the greatest challenge to date for CFD techniques, and their growth was exponential and the progress made has been quite impressive. Therefore, CFD techniques are deserving of a review of their own.

PRACTICAL PROGRESS

The past two decades have also brought significant progress towards the realization of practical scramjets. The analytical and experimental efforts concerning scramjet engines and their components have been an international endeavor with major programs proceeding primarily in the US, the former USSR (FSU), France, and Japan. Several ground test engines have been fabricated and tested at simulated flight conditions, primarily at speeds less than Mach 10. The only successful flight tests have been those of the FSU as reported by Roudakov et al (1993), with demonstration of supersonic combustion operation at a flight speed of approximately Mach 5. The approximate performance of hydrogen-fueled engine systems is compared in Figure 1. As noted by Ferri, the main attraction of the scramjet lies in its potential to operate efficiently to very high Mach numbers. It should also be noted that any air-breathing hypersonic flight

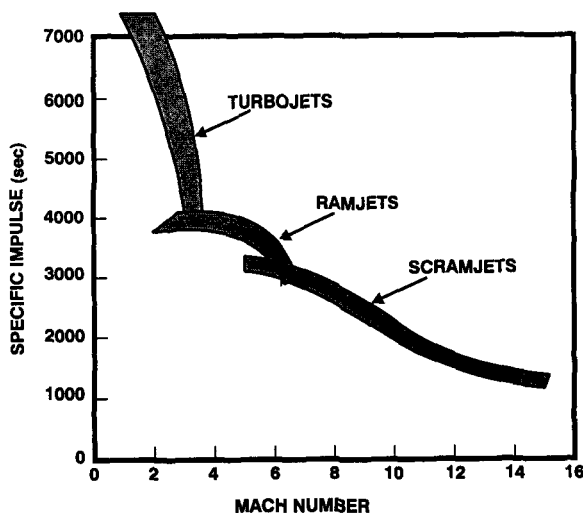


Figure 1 Approximate performance of hydrogen-fueled engines.

vehicle operating at maximum speeds in excess of about Mach 8 will require a combined cycle engine, which operates efficiently at lower speeds as a turbo-accelerator, later transitioning to a conventional subsonic-burning ramjet mode, and finally operating with scramjet propulsion. The ramjet and scramjet engine layouts are shown in Figures 2 and 3, where one-dimensional flow is assumed. Obviously it is desirable to extend the performance of a scramjet over the widest possible range of Mach numbers to reduce the complexity of the lower-speed propulsion system. It is also strongly desirable to avoid the complexity of variable-geometry operation. One approach proposed by Ferri to permit fixed geometry scramjet engine operation at speeds as low as Mach 4 was the use of three-dimensional engine flow fields combined with "thermal compression" by

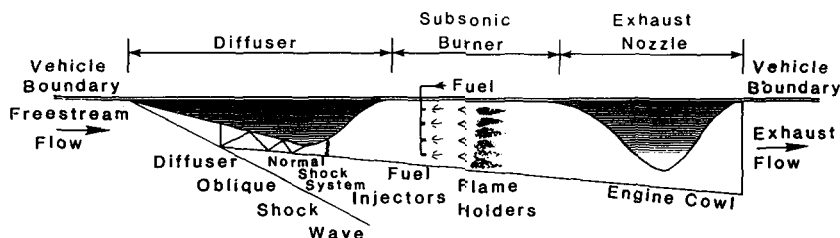


Figure 2 Schematic diagram of a two-dimensional or planar geometry ramjet engine.

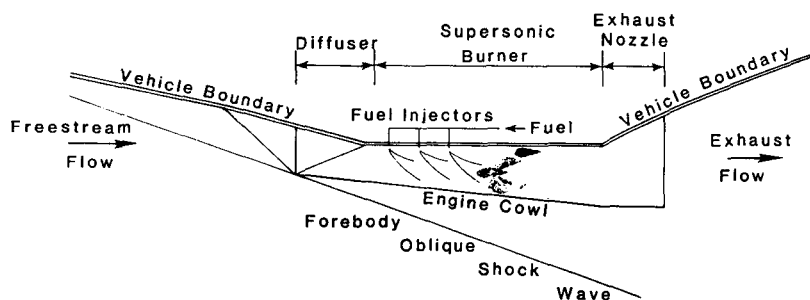


Figure 3 Schematic diagram of a two-dimensional or planar geometry scramjet engine.

which the supersonic heat addition in one part of the flow was allowed to compress the flow outside the flame region. This approach is discussed at length in the Ferri review paper and was critically assessed by Billig et al (1968). Billig et al concluded that considerable gains were theoretically possible at the lower flight Mach numbers with high fuel-air ratios. However, the practical difficulties of tailoring the fuel injection to produce the required regions of thermal compression are formidable and this approach was overtaken by the engineering development of the dual-mode engine concept proposed by Curran & Stull (1964) and was experimentally verified in the US and elsewhere (Waltrup et al 1976). The dual-mode engine, discussed later in this review, is able to operate initially as a subsonic-burning conventional ramjet and subsequently to transition to supersonic combustion operation at higher speeds.

The common rudimentary processes of the scramjet engine are compression of the freestream air to a lower supersonic Mach number, fuel injection, mixing and combustion in a supersonic stream, and finally expansion of the burned stream to discharge conditions. The actual components corresponding to these supposedly sequential processes are, of course, the inlet, fuel-injection system, combustor, and exhaust nozzle. In practice, one additional component is needed, particularly for lower-speed operation, namely the inlet isolator. This device is typically a constant-area supersonic diffuser serving to contain the complex wave structure commonly known as a pseudo-shock, which can accompany the deceleration of the supersonic flow from the inlet to the lower speed at the combustor entry. This shock structure, usually referred to as the precombustion shock structure, provides the flow boundary condition upstream of the region of heat addition. The shock structure itself will change as the combustion heat release is varied. The key engineering function of the isolator in an engine is to prevent the static pressure rise associated with combustor operation from unstating the inlet. The isolator may be a distinct component of the

engine, or its function may be fulfilled by a segment of the overall combustion system.

Heat Addition in Ducts

The combustion system is the heart of any class of ramjet engine, and the gas dynamics of such combustors are the key to understanding engine operation. In the 1940s, when gas turbine combustors, afterburners, and subsonic combustion ramjets were being intensively developed, considerable attention was directed to the study of heat addition to subsonic flows in constant-area combustion chambers.

After some early confusion, an expository paper by Foa & Rudinger (1949a) clarified the phenomenological behavior of such flows with particular emphasis on the role of flow boundary conditions. The interested reader will find in this paper an extremely useful historical discussion. The same authors also considered heat addition to supersonic flows in a constant-area duct, publishing a less well-known report (Foa & Rudinger 1949b). Those early studies of heat addition triggered interest in the unsteady flows arising from such heat addition (e.g. Stocker 1952) and also in nonuniform heat addition in ducts (Marsh & Horlock 1961).

Regarding heat addition to supersonic flows in constant-area ducts, the most significant fact is that only a limited amount of heat can be added to the flow before the steady flow breaks down. This amount is given by the expression

$$\left(\frac{q}{C_p T} \right)_{\text{limit}} = \frac{(M^2 - 1)^2}{2(\gamma + 1)M^2},$$

where q is the heat addition per unit mass, C_p is the specific heat at constant pressure, T is the static temperature, M is the axial Mach number, γ is the ratio of specific heats, and all are initial conditions except q . Because of this relationship, very little heat can be added at low supersonic combustor entry Mach numbers (and correspondingly low supersonic flight speeds) in constant-area combustors. Consequently, most practical scramjet engine designs utilize combustors incorporating area relief, either by step increases in area or by use of diverging duct segments. It is also apparent that the longitudinal heat release must be carefully tailored in a given duct geometry to avoid local or global unstart of the combustor. As a consequence of the limited utility of constant-area ducts for scramjet engines at lower flight speeds, the gas dynamic analysis of heat addition in variable area combustors has been of interest. A noteworthy monograph by Zierep (1974) gives an excellent treatment and survey of such analyses to that date.

The equations governing heat addition in these circumstances are well known, particularly in the simplest case excluding wall friction and momentum and

mass addition:

$$\frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{V} \frac{dV}{dx} = -\frac{1}{A} \frac{dA}{dx}$$

$$\frac{1}{\gamma M^2} \frac{1}{p} \frac{dp}{dx} + \frac{1}{V} \frac{dV}{dx} = 0$$

$$-\frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{\gamma p} \frac{dp}{dx} = \frac{1}{C_p T} \frac{dq}{dx}.$$

Here ρ is the density, x is the axial coordinate, V is the axial velocity, A is the throughflow area, and p is the static pressure. For prescribed $q(x)$ and $A(x)$, the above equations can be solved to yield the following derivatives,

$$\frac{1}{\rho} \frac{d\rho}{dx} = \frac{1}{1 - M^2} \left(\frac{M^2}{A} \frac{dA}{dx} - \frac{1}{C_p T} \frac{dq}{dx} \right)$$

$$\frac{1}{p} \frac{dp}{dx} = \frac{\gamma M^2}{1 - M^2} \left(\frac{1}{A} \frac{dA}{dx} - \frac{1}{C_p T} \frac{dq}{dx} \right) \quad (1)$$

$$\frac{1}{V} \frac{dV}{dx} = \frac{-1}{1 - M^2} \left(\frac{1}{A} \frac{dA}{dx} - \frac{1}{C_p T} \frac{dq}{dx} \right) \quad (2)$$

$$\frac{1}{T} \frac{dT}{dx} = \frac{1}{1 - M^2} \left[(\gamma - 1) M^2 \frac{1}{A} \frac{dA}{dx} + (1 - \gamma M^2) \frac{1}{C_p T} \frac{dq}{dx} \right] \quad (3)$$

$$\begin{aligned} \frac{1}{M} \frac{dM}{dx} = \frac{1}{1 - M^2} \left[- \left(1 + \frac{\gamma - 1}{2} M^2 \right) \frac{1}{A} \frac{dA}{dx} \right. \\ \left. + \frac{1 + \gamma M^2}{2} \frac{1}{C_p T} \frac{dq}{dx} \right]. \end{aligned} \quad (4)$$

The above equations have been applied to diverse technological processes ranging from the problem of condensation shocks in Laval type nozzles (Wegener & Mack 1958, Pouring 1961) to combustion processes in MHD power generators (Woodley 1967). For scramjet engines, early interest was shown in evaluating combustor losses for the simple flow processes associated with equating the left-hand side of Equations (1), (3), and (4) to zero. In addition to constant-area heat addition, these basic processes correspond respectively to heat addition at constant static pressure (and correspondingly constant velocity), constant static temperature, and constant Mach number. Useful deductions on the relative losses and geometries of ideal supersonic combustion systems can be made by examining these basic flows or sequential combinations of these basic flows, e.g. constant-area heat addition to Mach 1, followed by continuing heat addition at constant Mach number.

The associated losses in total pressure can be found from

$$\frac{1}{p_t} \frac{dp_t}{dx} = \frac{-\gamma M^2}{2\left(1 + \frac{\gamma-1}{2} M^2\right)} \frac{1}{C_p T} \frac{dq}{dx}.$$

With heat addition in a diverging combustor, special attention has to be paid to the occurrence of sonic flow conditions in the duct. Such a *choked thermal throat* is technologically important to enable a dual-mode combustor to operate in the ramjet mode. As shown by Zierp (1974) and Heiser & Pratt (1994), one requires that

$$\frac{1}{A} \frac{dA}{dx} = \frac{1}{C_p T} \frac{dq}{dx}$$

hold at Mach 1. For additional discussion, see Möhring (1979), Guha (1994), and particularly Delale et al (1993). As already noted, the simple one-dimensional equation set can be solved for a given $A(x)$ and prescribed heat release $q(x)$. However, it was soon discovered in early scramjet engine development that such models were not useful for interpreting engine test data because of the impossibility of prescribing $q(x)$ a priori and the difficulty of accounting for practical engine effects such as wall shear and precombustion shock structure. Furthermore, the complexity of the fuel injection, mixing, and combustion processes prevented accurate modeling of heat release along the divergent combustor, and alternative methods of predicting combustor performance were sought. One such method was to approximate the integral appearing in the one-dimensional momentum equation relating combustor exit (e) and inlet (i) conditions:

$$\int_i^e p dA = (m_e V_e + p_e A_e) - (m_i V_i + p_i A_i). \quad (5)$$

In one of the early studies of scramjet potential, Weber & MacKay (1958) utilized Equation (5) to study the effect of various postulated pressure profiles in the combustor on engine performance. However, later studies, primarily in the US, focused attention on the so-called Crocco relationship, which relates pressure and duct flow area as follows

$$pA^{\varepsilon/(\varepsilon-1)} = \text{constant}. \quad (6)$$

Equation (6) is used not only to evaluate the overall momentum equation from duct inlet to exit,

$$\begin{aligned} F_W &= \int_i^e p dA = (m_e V_e + p_e A_e) - (m_i V_i + p_i A_i) \\ &= (1 - \varepsilon)(p_e A_e - p_i A_i), \end{aligned} \quad (7)$$

but is also used to relate the pressure and area at every plane in the combustor. An extensive treatment of this relationship has been given by Crocco (1958), who also applied Equation (6) to pseudo-shocks in variable-area ducts, nonuniform flows, and mixing. Examination of Equation (6) will also show that $\varepsilon = 1$ corresponds to constant-area flow, $\varepsilon = 0$ to constant-pressure flow, and constant-Mach number flow results when $\varepsilon = -\gamma M^2$. In addition to the basic expression for the wall pressure integral given in Equation (7), relatively simple formulae relating pressure, area, temperature, and stream thrust ratios can be derived. The classic treatment, given by Billig (1967), included some early experimental validation of the Crocco approach. In subsequent years the appropriateness of assuming a Crocco relationship to describe the supersonic combustion process in a duct was discussed by Cookson (1975), Ramaty et al (1982), and more recently by Kremer (1993). The two principal concerns expressed early were, first, that the Crocco assumption implied a monotonic pressure variation along the divergent duct (which was not generally observed in experimental work) and, second, that the use of an "entropy limit" criterion to select an appropriate ε value to match an experimental pressure distribution implied that the flow at the exit plane of the combustor approached an isentropic flow. The third, and most recently expressed concern, was that the implied heat-release distribution along the duct was unrepresentative of real engine processes.

Despite these objections to the Crocco relationship, this power law has been most successfully applied to the modeling of the scramjet combustion process, including wall shear and heat transfer, and the overall optimization of scramjet engines, particularly the dual-mode engine (Billig & Sullins 1993).

The key to successful application of the Crocco model was the recognition of the existence of a shock structure (pseudo-shock) at the combustor entry, evidence of which is seen in typical engine pressure distributions. Billig & Dugger (1968) gave an early treatment of this shock-structure upstream boundary condition and the combustion process; the evolution of the "Crocco" approach from these early beginnings is well documented by Billig (1993).

As noted earlier, the prediction of heat release along the combustor is a difficult task, which has been circumvented by the use of integral methods. However, a dedicated approach to this prediction was initiated by Anderson (1974) and other research workers at NASA Langley. The fundamental thrust of this approach was to determine the heat release distribution from the basic processes of fuel injection, fuel-air mixing, and chemical reaction. In principle, this method should enable the synthesis of successful combustor design from the component processes of fuel injection (perpendicular, parallel, or angled directions, strut or wall location), flame stabilization and mixing devices (wall recesses, in-stream systems), and duct contour. Such a synthesis must of course

avoid the pitfalls of thermal choking at lower flight speeds, excessive loss mechanisms, and excessive heat transfer. Anderson (1986) and Sullins & Waltrup (1985, 1986) provide interesting discussions on the differing approaches to engine analysis, and a perspective on the applicability of both approaches to the design and analysis of scramjet combustion systems is given by Pratt & Heiser (1993). A considerable data base on scramjet component processes and overall engine performance has been established at NASA Langley and is well documented by Northam & Anderson (1986), an extensive paper with over two hundred references.

Anderson et al (1990) have given a more recent progress report on very high speed scramjets, focusing attention on a major thrust in NASA research, namely the evaluation of scramjet performance at speeds between Mach 10 and Mach 25. The testing of such engines and their components demands pulse facilities of the shock-tube and expansion tube class with unique instrumentation for determining component performance. Furthermore, the complex and unsteady flows encountered in such facilities require extensive CFD capability. The merging of CFD analysis with pulse facility data has enabled the creation of parametric data bases for the preliminary design of engine components. In addition to work in the US, NASA has supported scramjet testing in the unique impulse facilities of the University of Queensland, Australia (Gai 1992).

A major accomplishment by NASA was the design and testing of the Hypersonic Research Engine (HRE). This was a dual-mode, axisymmetric, hydrogen-fueled engine ground tested at conditions corresponding to flight Mach numbers of 5, 6, and 7. These tests provided definitive engine performance data and demonstration of dual-mode performance. Andrews & Mackley (1993) have reviewed the achievements of this program, and further engine data analysis has recently been given by Hoose (1994).

Substantial scramjet research has been conducted in Europe. In France, the potential of the supersonic combustion ramjet had been foreseen and experimental studies were soon undertaken. This work culminated in the study of a hydrogen-fueled, dual-mode, axisymmetric engine known as ESOPE, which was similar to the HRE but unique in many aspects. The combustion system of this engine was tested at Mach 5.6 and 6.0 in the Modane test facility in the early 1970s. These tests confirmed the feasibility of a dual-mode Mach 3.5/7.0 engine and are discussed by Contensou et al (1972). Unfortunately, scramjet research was not continued in the later 1970s and lay relatively dormant until initiation of a new research program, termed PREPHA. The PREPHA program was launched in 1991 and focused on scramjet engine technology. It is primarily aimed at ground testing a scramjet at simulated flight Mach numbers of 6 (at Aerospatiale-Subdray) and 8 (at ONERA Modane). Interestingly, France has

been able to support the FSU in flight testing scramjet engines. In November 1992 an axisymmetric, dual-mode, hydrogen-fueled, scramjet engine was flight tested in the range Mach 3.5 to 5.35 in the subsonic and supersonic combustion modes. Test results from this hypersonic flying laboratory vehicle named Kholod are discussed by Roudakov et al (1993).

It should be noted that this joint FSU-French test was the second reported flight test of the Kholod vehicle. The first test of the same type of axisymmetric engine took place in November 1991, marking a significant milestone in the impressive program of scramjet development carried out by the FSU over approximately the last 35 years. The FSU program has been notable for detailed aerodynamic studies (particularly in the area of pseudo-shocks), for extensive testing of combustion systems and the exposition of their gasdynamics, and for a wide variety of ground tests on specific engine configurations. It is impossible to do justice to the scope of the FSU work in this short review. The interested reader is referred to an AGARD Lecture Series volume, organized by Sosounov (1993), which is dedicated to FSU research and development efforts.

Other countries are working in collaboration with the FSU. Walther et al (1993) reports on some interesting joint work with Germany. Such work is, of course, in addition to the existing German Hypersonic Propulsion Technology program, which embraces hydrogen-fueled ramjet development to speeds of Mach 7 (Kramer et al 1993).

Additional hypersonic propulsion work is being pursued elsewhere in Europe; Broadbent (1991) has compiled a valuable survey of some British contributions. Also, a substantial analytical and experimental effort on scramjet engines is emerging in Japan (Nouse 1993). Additional studies have also been published by Chinese researchers.

AEROTHERMODYNAMICS OF THE DUAL-MODE COMBUSTION SYSTEM

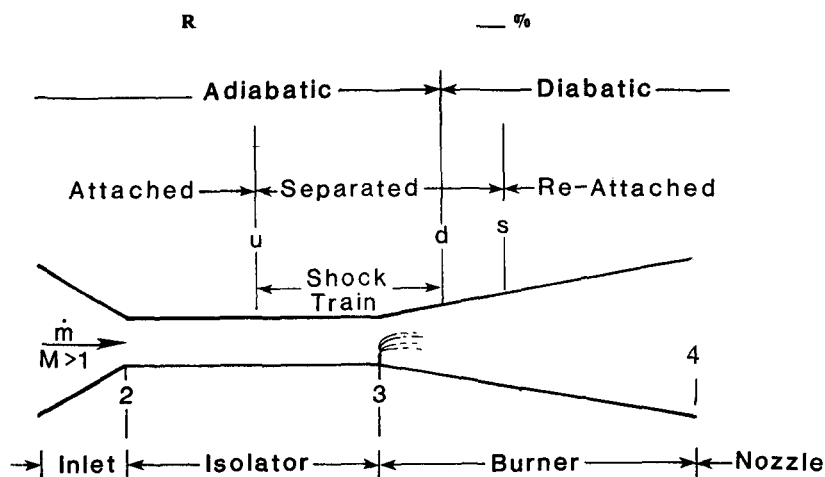
A critical requirement of ram compression air-breathing engines is that they have good performance over an extremely wide spectrum of operating conditions, including flight Mach numbers that range from subsonic to hypersonic. Consequently, designers have focused their attention for the past two decades on largely one-dimensional concepts that provide great thermodynamic cycle flexibility within relatively simple geometries, rather than on the multi-dimensional issues of "thermal compression" as proposed by Ferri (1973). The following discussions of advances in the aerothermodynamics of the new concepts are excerpted from the extensive material appearing in Heiser & Pratt (1994).

A schematic diagram of the dual-mode ramjet/scramjet combustor is presented in Figure 4. The principal features of this device are: 1. a constant-area diffuser called an inlet *isolator*, which can provide burner entry Mach number and static pressure conditions anywhere between those at station 2 and those corresponding to a normal shock wave at station 2; 2. no local physical throats; and 3. a fuel distribution system that allows considerable control over the axial distribution of the release of combustion energy (hereinafter regarded as heat addition without mass addition).

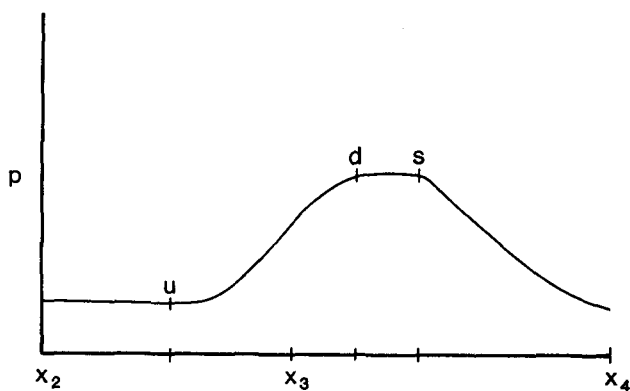
When operating in the *ramjet* mode, the flow is choked ($M = 1$) somewhere downstream of the isolator, causing a large back pressure at the burner entry and a *normal shock train* with a subsonic core flow to form inside the isolator and upstream of the burner entry station. As long as the back pressure does not exceed the ability of the inlet isolator to contain the normal shock train, the isolator performs the same functions as the conventional ramjet trans-section. If the isolator cannot contain the required shock train, the inlet unstarts as it would in any ramjet. Choking in the divergent combustor passage is brought about by tailoring the axial distributions of heat release and throughflow area so that the former drives the local Mach number up to 1 and the latter continues the acceleration to supersonic velocities. This process is known as *thermal choking*, and the location of the sonic flow is known as the choked thermal throat.

When operating in the *scramjet* mode, the flow is everywhere supersonic, and it would appear that there is no need for an inlet isolator. However, frequently the rate of area increase is insufficient to relieve the *thermal blockage* or *occlusion* resulting from heat addition to a supersonic stream, and an adverse pressure gradient arises. If the pressure increases too abruptly within the burner, the wall boundary layer will separate and the pressure rise will propagate freely upstream through the separated wall boundary layer, even though the confined core flow remains supersonic. In this case an *oblique shock train* with a supersonic core flow forms inside the isolator and prevents unstart of the engine inlet.

Visualizing the complex sequence of thermodynamic processes involved in dual-mode ramjet/scramjet operation can be greatly facilitated by 1. modeling them as steady, one-dimensional compressible flows of calorically perfect gases (Shapiro 1953) and 2. representing them on dimensionless, static enthalpy-kinetic energy coordinates (or, for brevity, the *H-K diagram*) as shown in Figure 5. The local static enthalpy and kinetic energy are most conveniently made dimensionless by dividing by the total enthalpy of the relative freestream flow. An attractive feature of the *H-K diagram* is that constant values of many basic flow properties appear as straight lines, as seen in Figure 5. The single important exception is the isoline of constant impulse function, which is parabolic in form in the region of interest.



(a)



(b)

Figure 4 (a) Axial location designations for engine geometry (stations 2, 3, and 4) and aerothermodynamic processes (stations u, d, and s). (b) Typical axial distribution of wall pressure for scramjet mode operation.

A virtue of the H - K diagram is that the familiar "simple types" of compressible flows (Shapiro 1953) are easily pictured, as noted in Figure 5. The H - K diagram is not a thermodynamic state diagram, because kinetic energy is a mechanical property of the flow. The special value of the H - K diagram to air-breathing propulsion applications is illustrated in Figures 6 and 7, where it is employed to visualize the sequences of internal flow processes that occur within ramjet and scramjet engines, respectively.

Combustor Design Considerations

For the case of one-dimensional flow of a calorically perfect gas with negligible friction or mass addition, the ordinary differential equation for the axial variation of Mach number $M(x)$ is given by Shapiro (1953) as

$$\frac{dM}{dx} = M \left(\frac{1 + \frac{\gamma-1}{2} M^2}{1 - M^2} \right) \left[-\frac{1}{A} \frac{dA}{dx} + \frac{(1 + \gamma M^2)}{2} \frac{1}{T_t} \frac{dT_t}{dx} \right], \quad (8)$$

where T_t is the total temperature. Although $A(x)$ may, of course, be prescribed by the designer, $T_t(x)$ is not precisely known in advance because it depends on

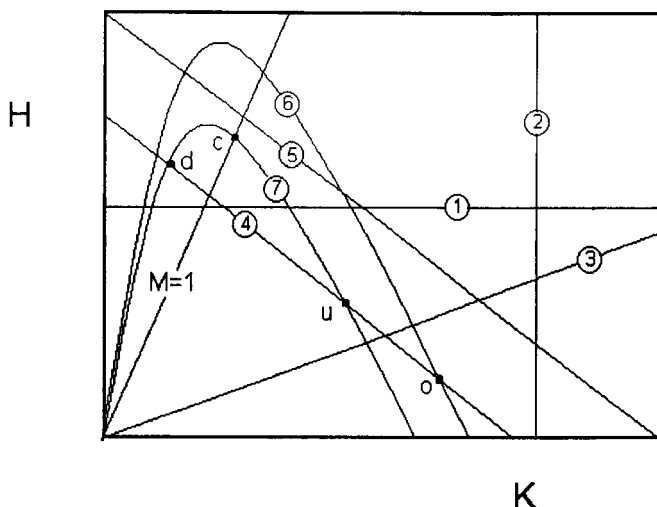


Figure 5 The H - K diagram, depicting constant property isolines and "simple types" of compressible flows. Point 0 = freestream reference state. Point c = choked condition at constant impulse. Points u and d denote end states of normal shock. Circled numbers denote isolines of constant: 1 static enthalpy, static temperature; 2 kinetic energy, velocity, pressure (for frictionless flow); 3 Mach number; 4 total enthalpy, total temperature, $T_t/T_0 = 1$; 5 post heat release adiabat, $T_t/T_0 > 1$; 6 impulse function/stream thrust, area (frictionless flow), $I = I_0$; 7 impulse function, $I < I_0$. 4 and 5 represent Fanno flow. 6 and 7 represent Rayleigh flow.

where χ is the nondimensional distance along the burner axis downstream of the point at which combustion begins, θ is a constant that depends on the method of fuel injection and of fuel-air mixing, and τ_b is the nondimensional total temperature rise ratio T_{14}/T_{12} within the burner.

The procedure for calculating the axial distribution of properties in entirely supersonic combustion is straightforward. Given the burner entry Mach number M_2 , the area distribution $A(x)$, and the heat release distribution $T_t(x)$, Equation (8) is numerically solved for $M(x)$ as a discrete point set along the burner axis. As long as $M(x)$ remains above 1, Equation (8) never becomes singular, and no difficulties are encountered. All other flow properties of interest for which entry values are known—such as static temperature T , static pressure p , total pressure p_t , and velocity V —can be determined from $M(x)$ at each point by means of the collection of “useful integral relations” (Shapiro 1953).

A process path for an example case of entirely supersonic combustion is shown as line A on the H - K diagram of Figure 8. At burner entry, where the heat addition rate dT_t/dx is greatest, the process path is close to the desirable constant- p /constant- K trajectory because the pressure rise due to heat addition is largely counteracted by the pressure relief due to area increase. However, as dT_t/dx tapers off along the burner axis the process path approaches, successively, a path of constant M , T , and, near the burner exit, T_t . Note that, for the example of line A, the minimum burner Mach number is 1.33. The location of the minimum Mach number is called the *thermal throat*, although its location is not known in advance.

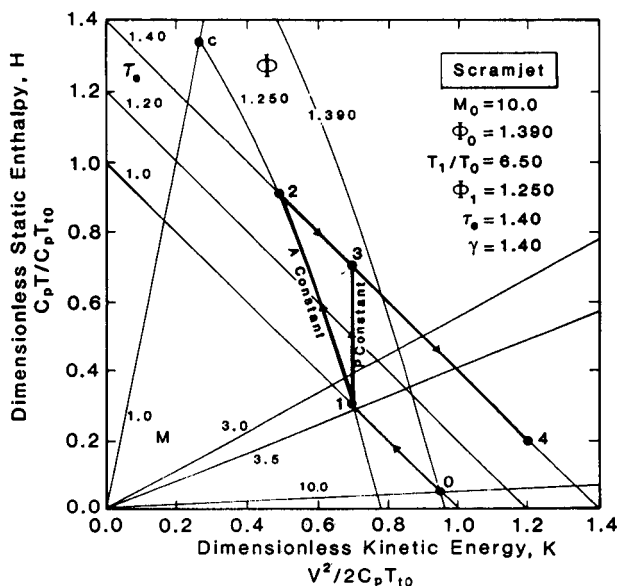


Figure 7 The H - K diagram for a scramjet. The same process numbering as in Figure 6 is used.

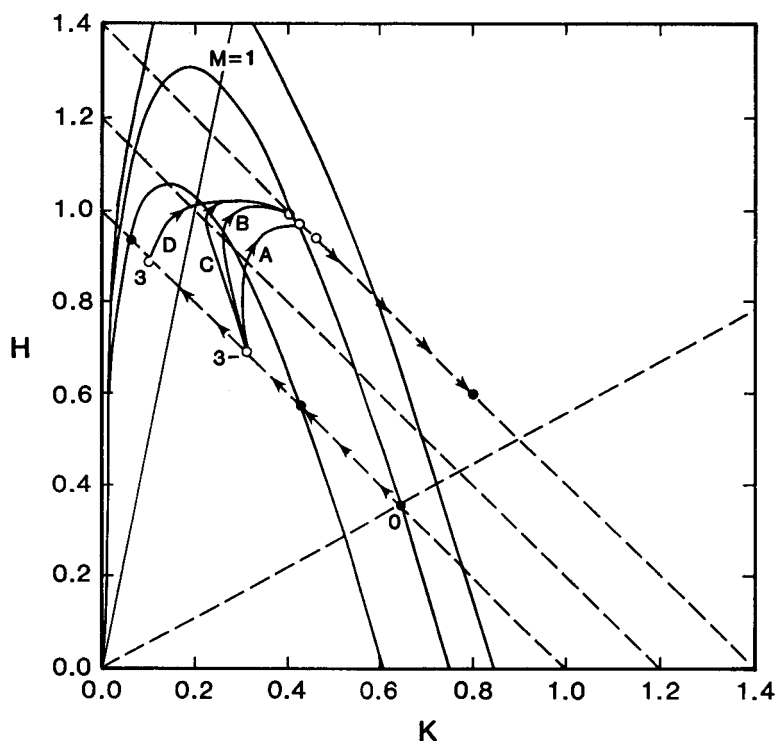


Figure 8 Scramjet-to-ramjet mode transition by reducing linear $A(x)$ burner area ratio A_4/A_3 for the same $T_t(x)$.

The burner process path of line A of Figure 8 produces an exit static pressure that is considerably less than the entry value, which can reduce cycle thermal efficiency, supersonic mixing, and chemical kinetic reaction rates (Meshcheryakov & Sabelnikov 1988). To improve the situation, the burner exit area can be reduced, such as in lines B and C of Figure 8, which have thermal throat Mach numbers of 1.19 and 1.03, respectively. These changes increase the static pressure throughout the combustor, although the combustor remains in the scramjet mode of operation.

Finally, a further slight reduction of the combustor exit area causes the flow to reach Mach 1 and choke at the thermal throat. The flow is now controlled at the choking location and must be subsonic at burner entry, which also causes the entry Mach number to be lower there and thus allows more "room" for the specified heat addition. This is shown as line D of Figure 8. The combustor is now in the ramjet mode of operation.

The axial location of the choked thermal throat, should choking occur, must occur at a unique location where both the numerator and denominator of Equation (8) are zero, i.e.

$$\frac{1}{A} \frac{dA}{dx} - \frac{(1+\gamma)}{2} \left(\frac{1}{T_t} \frac{dT_t}{dx} \right) = 0. \quad (9)$$

Equation (9) gives sufficient conditions for choked flow to exist, but it does not give the necessary conditions. If Equation (9) is satisfied, it is possible for choked flow to occur within the burner. If Equation (9) is not satisfied, the burner is not capable of ramjet mode operation. The Mach number positive and negative axial derivatives dM/dx at the choking point can also be obtained from algebraic relationships given in "special conditions at the sonic point" of Shapiro (1953). If $A(x)$ and $T_t(x)$ do admit a choking point, Equation (8) is numerically integrated *upstream* from there to the burner entry, starting with the two calculated dM/dx values, resulting in process lines D and one only slightly to the left of line C of Figure 8. These limiting processes determine the burner entry conditions outside of which either all-subsonic ($M_3 < M_{3+}$) or all supersonic ($M_3 > M_{3-}$) combustion is possible. Burner entry Mach numbers between M_{3+} and M_{3-} are not possible for the particular burner specified by $A(x)$ and $T_t(x)$.

Accommodating the supersonic combustor entry flow to the conditions imposed by thermal choking is accomplished by the inlet isolator. If we assume frictionless flow without mass or heat addition and allow the exit flow to be confined to a separated core area A_c less than the total throughflow area A_2 , a one-dimensional control volume analysis of the constant-area isolator (illustrated in Figure 9) leads to the result

$$\frac{A_c}{A_2} = \left[(1 + \gamma M_2^2) \frac{M_3}{M_2} \sqrt{\frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\gamma-1}{2} M_2^2}} - \gamma M_3^2 \right]^{-1}$$

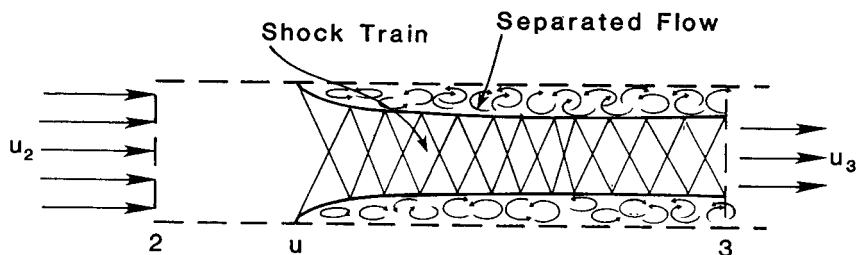


Figure 9 Control volume for isolator analysis.

As before, the remaining flow properties can be obtained from the “useful integral relations,” provided the separated core area A_c is used in the calculations rather than A_2 . The maximum and minimum possible isolator exit values of M_3 correspond both to $A_c = A_2$ and to the jump conditions across a normal shock wave. Thus, the isolator can provide a continuum of burner entry Mach numbers over the range of an upstream normal shock wave. If the combustor requires an entry Mach number less than that available to the isolator, the separation will proceed further upstream and unstart the inlet. This is an especially delicate point for constant- A combustors because transition from scramjet to ramjet mode places the isolator exactly at its normal shock limit and hence with no operating margin at all. Indeed, if a constant- A burner were specified for a dual-mode combustion system, either the inlet isolator would have to be replaced with a trans-section or enough air would have to be bled from the isolator to achieve the increasing area effect of a trans-section. Closer examination reveals that the dissipative mechanisms at work in the isolator are different from the more familiar one-dimensional Fanno and oblique shock wave flows, generating less total pressure loss for the same static temperature increase than the former, and more than the latter.

Finally, if the pressure rise due to heat addition in a supersonic flow, such as path C of Figure 8, is sufficiently large to cause wall boundary layer separation—as it frequently is in practice—the higher pressure is communicated to the isolator exit through the separated flow region, and the isolator responds by providing the necessary burner entry conditions. This situation can be identified by plateaus of constant static pressure in experimental data, which translate into vertical lines of constant K on the H - K diagram.

The importance of the interaction of the precombustion shock structure, or pseudo-shocks, and the combustion energy release process was not well recognized in early scramjet development. However, experimental and analytical work by Waltrup & Billig (1973) gave useful descriptions of the pseudo-shock geometry and characteristics, and this eventually led to optimal dual-mode engine designs (Billig & Sullins 1993). Important contributions to the understanding of isolator behavior were made through CFD (Hunter & Couch 1990; Lin et al 1991a,b; Sullins & McLafferty 1992; Elmquist 1993). Significant advances in understanding pseudo-shocks were made by FSU authors, particularly the pioneering works of Ostras & Penzin (1977) and Penzin (1989). In these and other studies, cold flow tests have been widely used, with mechanical throttling of the exit flow used to simulate the occlusion of heat addition. This technique does not explore the effect of pseudo-shock flow structure on the ensuing mixing and combustion, as has more recently been done in tests involving combustion (Baranovsky & Levin 1991, Voloshchenko

et al 1992, Vinogradov et al 1995, Sabelnikov et al 1993; see also Zimont et al 1978).

The above descriptions of isolator operation have been simplified for the sake of discussion. In practical engine configurations having wall or strut injection of fuel, the pseudo-shock behavior may be modified (Penzin 1983). The significant departures from one-dimensionality observed there may be handled by conventional nonuniformity or distortion analysis, as discussed by Ortwerth (1977) and Tretyakov (1993).

Interpretation of Experimental Data

The simplified models of dual-mode combustor behavior can be productively applied, and tested, by using them to interpret existing experimental data. The throughflow area distribution $A(x)$ is usually reported, as is the static pressure distribution $p(x)$ —the most easily measured flow property—and the entrance conditions. The goal is to determine the axial distribution of all flow properties of interest, especially $M(x)$ and $T_t(x)$.

The first step is to smooth the experimental $p(x)$ data and use it to determine the axial distribution of the impulse function $I(x)$. Whereas any smoothly decreasing function could be used to fit the $p(x)$ data in the attached region, Billig (1988, 1993) recommends the function proposed by Crocco (1958) of Equation (6). This will match any arbitrary set of decreasing $p(x)$ data, although care must be taken to obtain the best fit to the intermediate $p(x)$ data. Because it is assumed that only pressure forces act on the duct walls, the differential change in the impulse function is given by $dI = pdA$. For constant- A ducts, I is also constant.

With $I(x)$ determined, the Mach number distribution $M(x)$ is obtained from the definition of the impulse function by

$$M(x) = \sqrt{\frac{1}{\gamma} \left[\frac{I(x)}{p(x)A(x)} - 1 \right]}. \quad (10)$$

The $M(x)$ obtained from Equation (10) must be interpreted as the Mach number of the separated core flow within any separated flow range.

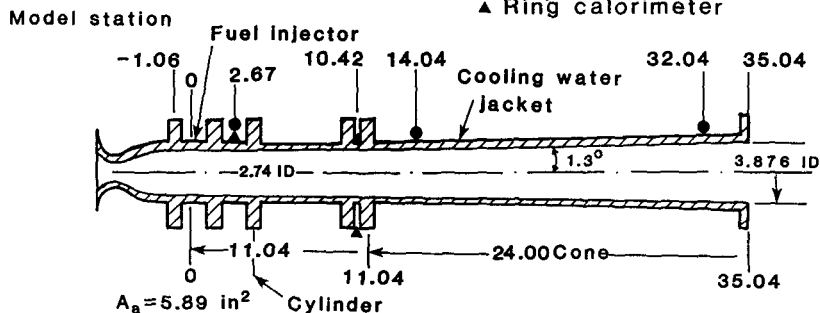
All other properties of interest may be obtained from the useful integral relations (Shapiro 1953) except where the flow is assumed to be separated and heat is being added, in which case any simple, smooth function can be used to patch T_t across the interval and then the remaining properties can be approximately determined.

Figures 10 and 11 present experimental static pressure measurements from hydrogen-air combustion in scramjet burners, reported by Billig (1967) and Billig et al (1972), as well as H - K diagrams representing the combustion process as determined by the method described above.

T_{T_a} (°R)	p_{T_a} (psia)	M_a	T_a (°R)	p_a (psia)	T_{T_f} (°R)	ER	η_c
• 4110	454	3.23	1570	7.47	1270	0.50	0.94
• 4100	454	3.23	1565	7.47	-	0	-

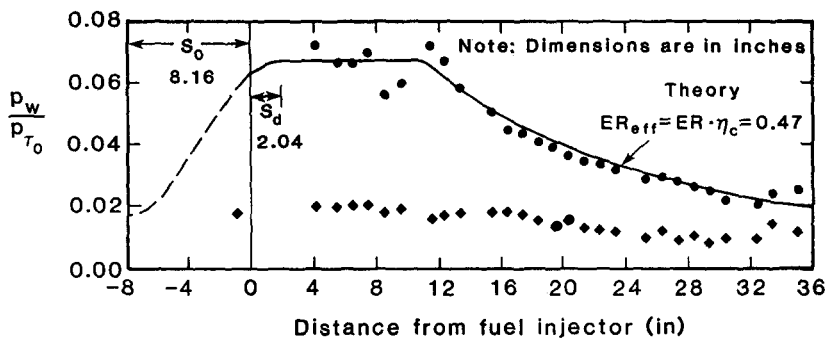
• Disc calorimeter

▲ Ring calorimeter



Note: Dimensions are in inches $A_b = 11.80 \text{ in}^2$

(a)



(b)

Figure 10 Experimental wall pressure data in a hydrogen-air combustor operating in scramjet mode. (a) Schematic diagram. (b) Pressure data. (c) $H-K$ diagram. (From Billig et al 1972.)

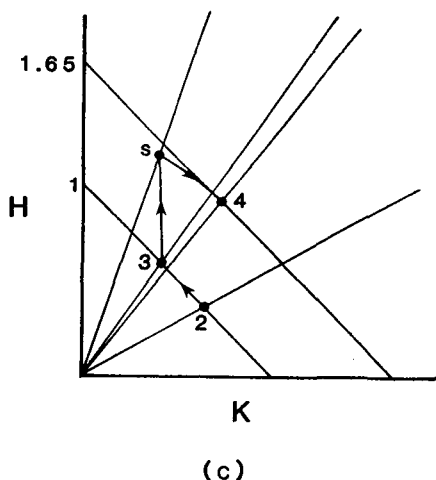


Figure 10 (continued.)

In Figure 10, the characteristic constant-pressure plateau resulting from separated flow with heat addition in a supersonic flow in a constant-area burner is apparent. Analysis of the data gives the results listed in Table 1.

Note that the static temperature rises rapidly in the precombustion shock train at burner entry and that the Mach number is clearly supersonic throughout. The H - K diagram of the analyzed process path asymptotes rapidly to the terminal adiabat $T_{t4}/T_{t2} = 1.65$ primarily along a vertical line of constant p and K . While the burner satisfies the design requirement that p_4 is approximately equal to p_2 , 95% of the heat addition occurs at a pressure more than three times greater than p_2 .

Figure 11 is the test data from Billig (1967), which is believed to be the first experimental demonstration of ramjet mode combustion in dual-mode combustor

Table 1 Analysis of wall pressure data of Figure 10¹

Station	$T_t(^{\circ}\text{R})$	$T(^{\circ}\text{R})$	$u(\text{ft s}^{-1})$	M
2	4110	1570	6081	3.23
3	4110	2510	4825	2.03
s	6449	4901	4747	1.42
4	6764	3918	6438	2.16

¹Overall H_2 -air equivalence ratio $\phi_0 = 0.50$, assumed $\gamma = 1.31$, gas constant $R = 53.56 \text{ ft-lbf/lbm-}^{\circ}\text{R}$. Station numbers refer to Figure 4.

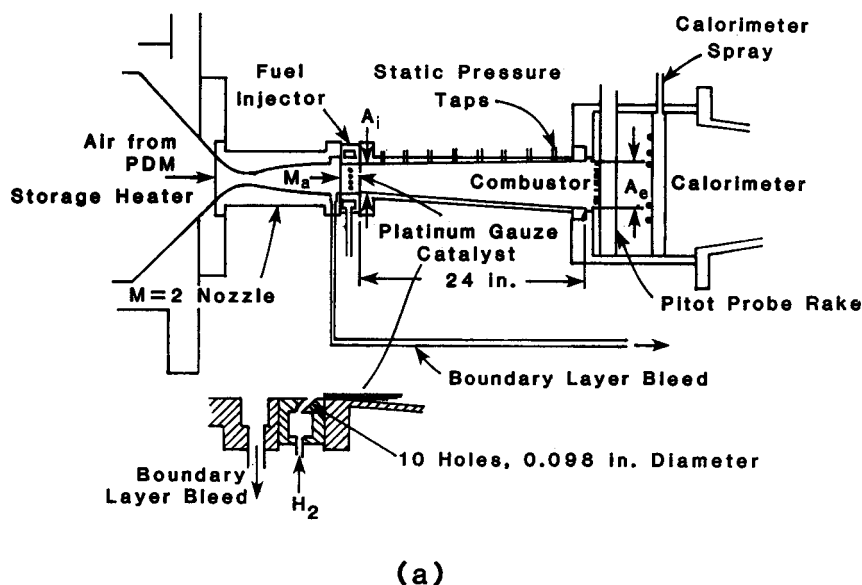


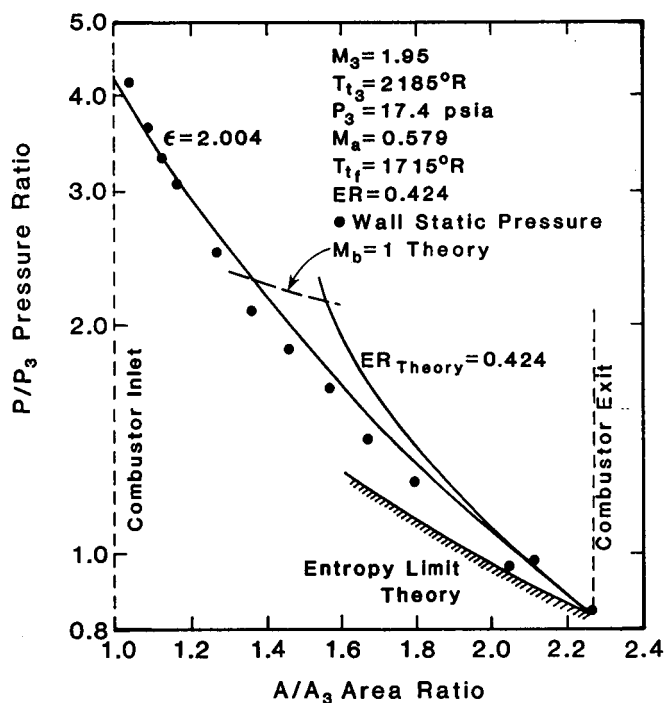
Figure 11 Experimental wall pressure data in a hydrogen-air combustor operating in ramjet mode. (a) Schematic diagram. (b) Pressure data. (c) $H-K$ diagram. (From Billig 1967.)

geometry (Curran & Stull 1964). Analysis of the data yields the results shown in Table 2.

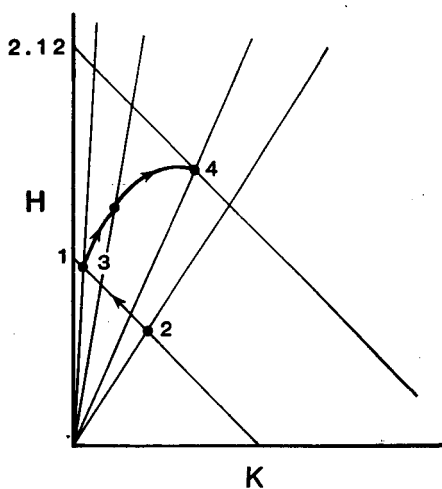
Two points are worthy of special note here. First, the burner entry state corresponds to point 3+ of Figure 8, the normal shock limit for inlet unstart. These data were collected as part of a series of experiments during which Billig first recognized the need for an inlet isolator (Billig 1993). Lacking an isolator, Billig used boundary layer bleed to stabilize the normal shock at burner entry. Consequently, the pressure begins to drop immediately downstream of the stabilized normal shock wave at burner entry, so that the flow is attached throughout the burner, and $T_t(x)$ may be inferred accurately from integral analysis. Second, the flow passes smoothly through the sonic plane—critical point as it accelerates from subsonic flow at burner entry to supersonic flow at burner exit.

FUEL-AIR MIXING PROCESSES

Ferri's (1973) review suggests that parallel-flow, shear-layer mixing, or possibly lateral fuel jet injection into the supersonic air stream, would be adequate to accomplish efficient supersonic combustion. However, Ferri did not acknowledge contemporary warnings that these two mixing schemes may not be



(b)



(c)

Figure 11 (continued)

Table 2 Analysis of wall pressure data of Figure 11²

Station	$T_t(^{\circ}\text{R})$	$T(^{\circ}\text{R})$	$u(\text{ft s}^{-1})$	M
2	2185	1320	3410	1.95
3	2185	2068	1252	0.57
s	2170	2053	1252	0.57
*	3895	3321	2777	1.00
4	4627	3180	4411	1.62

²Overall H_2 -air equivalence ratio $\phi_0 = 0.424$, assumed $\gamma = 1.31$, gas constant $R = 53.56 \text{ ft-lbf/lbm-}^{\circ}\text{R}$. Station numbers refer to Figure 4.

adequate and that it may be necessary to enhance the rate of fuel-air mixing by introducing axial vorticity (Swithenbank & Chigier 1969) or *premixing*—that is, to initiate the fuel-air mixing process upstream of the combustor (Rubins & Bauer 1966).

Measures of Local "Goodness" of Mixing

Regrettably, there is at present no theoretical framework to describe how the flow transitions from the near-field processes of stirring or macromixing (Brodkey 1967) to the far-field, turbulence-dominated micromixing, to (given sufficient stay time) the overall equivalence ratio resulting from the relative mass flow rates of the fuel and air streams. Consequently, it is necessary to rely on empirical, global or "engineering" measures of mixing effectiveness. Anderson (1974) defined such an empirical, one-dimensional measure of the degree of mixing completeness that accounted for both near- and far-field mixing while accounting for stoichiometry. This measure is termed the *mixing efficiency* $\eta_M(x)$ and is defined at any axial station within the combustor as "the amount of fuel that would react if complete reaction occurred without further mixing divided by the amount of fuel that would react if the mixture were uniform." Beach (1992) defines $\eta_M(x)$ simply as "the fraction of fuel that is mixed so it could react." In the lexicon of mixing (Brodkey 1967), $\eta_M(x)$ is the fraction of the *minor mixant* (defined as either fuel or air, depending on whether the overall equivalence ratio is less than or greater than unity, respectively) that is micromixed with the other mixant to flammable (near-stoichiometric) proportion at any axial location. Swithenbank et al (1991) point out that the term "mixing efficiency" ought to be reserved to describe some measure of a mixing "cost/benefit" ratio, where the "cost" is the conversion into mixing power of thermal and/or mechanical energy from the fuel or air stream, or loss of thrust (Papamoschou 1994) or *thrust potential* (Riggins & McClinton 1991) due to mixing power expended, and the "benefit" is either increased rate of growth of the near-field mixing layer

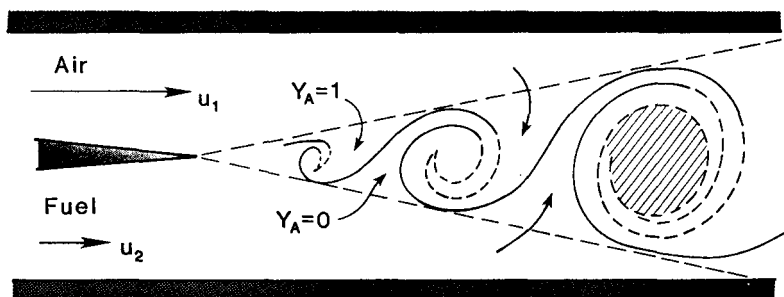


Figure 12 Formation of vortex structures in a transitional shear layer, for $u_1 > u_2$. Dashed curves at mixant boundaries indicate molecular diffusion. Cross-hatched area represents micromixed region.

and/or reduction of the axial distance required for near-complete micromixing, both of which contribute to improved completeness of combustion.

Mixing in a Turbulent Shear Layer

By far the greatest progress in supersonic combustion state of the art has occurred in mixing and reaction in shear layers arising from parallel flow of air and fuel streams. This work was initiated in the late 1960s by Anatol Roshko, together with colleagues and a succession of distinguished doctoral students, at CalTech. The physics of shear-layer mixing has been treated exhaustively in prior volumes of this series and elsewhere. Dimotakis & Hall (1987) presented a quasi-one-dimensional model for combined mixing and chemistry in reacting shear layers, based on a Lagrangian transient stirred-reactor representation of the engulfment-stretching-micromixing model of Broadwell & Breidenthal (1982), as illustrated in Figure 12. Pulsonetti et al (1988) proposed a time-mean Eulerian mixing-plus-chemistry stirred reactor model for the same configuration, as shown in Figure 13.

An excellent summary of two decade's work on reacting shear layers, especially as it pertains to scramjet combustion, is given by Dimotakis (1991) and summarized in a tutorial manner in Heiser & Pratt (1994). The principal results having implications for supersonic combustion are

1. At low convective Mach numbers, the time-mean growth rate of the turbulent shear layer is linear;
2. following mixing transition, the mixing layer grows at about one-half the growth rate of the shear layer;
3. due to the difference in molecular weight, the micromixture of hydrogen

and air within the mixed layer is too fuel rich to initiate combustion in the near field; and

4. at convective Mach numbers as low as $M_c = 0.2$, marked suppression of the shear-layer growth rate occurs, and by $M_c = 0.8$ and greater, the shear-layer growth rate is suppressed to only 0.2 of its incompressible value.

This suppression apparently remains constant with increasing convective Mach number, at least up to $M_c = 2$ (Papamoschou 1989). While the reasons for this behavior are not fully understood, it is speculated that the principal suppression mechanism is the stabilization of the modes of fluid instability that are responsible for the formation of the large vortex structures. Breidenthal (1992) has proposed a "sonic eddy" model to explain this phenomenon.

These findings lead to the conclusion that shear-layer mixing is not suitable for supersonic combustion of hydrogen and air at any but the lowest flight Mach numbers.

Data from experimental measurements of supersonic mixing between parallel coflowing streams of hydrogen and air (Beach 1972) can be represented approximately by the empirical relations

$$\eta_{M_0} \cong \begin{cases} x/L_{m,F}, & \phi_0 \leq 1 \\ x/L_{m,A}, & \phi_0 > 1 \end{cases} \quad (11)$$

where $L_{m,F}$ and $L_{m,A}$ are the axial distances required for all of the fuel or air, as appropriate, to be mixed to stoichiometric proportion.

Since the micromixed portion of fuel and air is assumed to be in near-stoichiometric proportion as long as $\eta_M < 1$, the distance L_m required to achieve $\eta_M = 1$ is greatest when the overall equivalence ratio ϕ_0 is unity. The

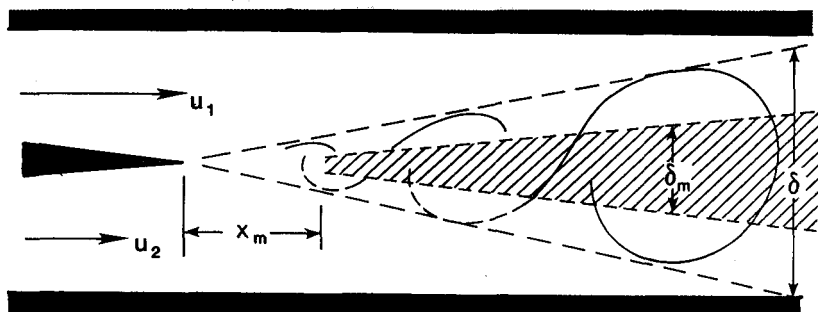


Figure 13 Growth of time-averaged shear layer thickness δ and of mixing layer thickness δ_m , following mixing transition.

axial distance at which the minor mixant is depleted is given approximately by the empirical relations (Pulsonetti et al 1988, Northam et al 1991)

$$\frac{L_{m,F}}{b} \cong 0.179 C_m e^{1.72\phi_0}, \quad \phi_0 \leq 1 \quad (12a)$$

$$\frac{L_{m,A}}{b} \cong 3.333 C_m e^{-1.204\phi_0}, \quad \phi_0 > 1, \quad (12b)$$

where b is the sum of the entry scales of segregation for both streams, and where the mixing constant C_m in Equation (12) is reported as varying from 25 to 60.

The formal definition of $\eta_M(x)$ does not state how the mixing rate of the major mixant is to be represented following depletion of the minor mixant. Pulsonetti et al (1988) suggest "the major constituent stream is assumed to continue to mix at the rate it was mixing when the minor constituent ran out, until it too is depleted." This assumption can be represented by

$$L_{m,0} = \begin{cases} L_{m,F} + \frac{1 - \phi_0}{\phi_0 (d\eta_M/dx)_{x=L_{m,F}}}, & \phi_0 \leq 1 \\ L_{m,A} + \frac{\phi_0 - 1}{(d\eta_M/dx)_{x=L_{m,A}}}, & \phi_0 > 1 \end{cases} \quad (13)$$

where $L_{m,0}$ is the axial distance required to fully micromix *both* streams. Even though Equations (12) show that the required axial distance $L_{m,F}$ or $L_{m,A}$ to deplete the minor mixant is decreased for off-stoichiometric mixtures, Equation (13) shows that the length required to fully mix both streams $L_{m,0}$ remains approximately equal to C_m , irrespective of ϕ_0 .

Mixing with Normal Fuel Injection

Figure 14 illustrates the significant fluid mechanical features of fuel injected from a normal jet into a supersonic crossflow (Billig et al 1971). The most significant feature is that the supersonic crossflow is displaced by the fuel jet much as if a cylindrical rod were inserted into the freestream from the wall at a right angle. As a result, a detached normal shock wave forms just upstream of the jet, causing the upstream wall boundary layer to separate. In addition, a bluff-body wake region is formed immediately downstream of the jet core. The recirculating flow in the wake acts exactly as the subsonic flame stabilization zone in a gas turbine combustor primary zone or as the wake of the flameholding gutter in ramjet combustors and turbojet afterburners.

The overall effect of the normal jet on the flow field is to anchor the mixing layer firmly to the jet core, reducing the mixing distance to not only zero, but actually to a small negative distance, as the mixing and flameholding zone extends slightly upstream of the jet via the separated boundary layer. Unfortunately,

Underexpanded jet injected normally into a supersonic freestream

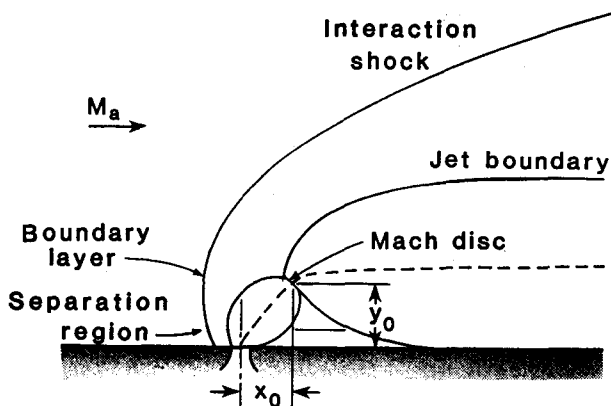


Figure 14 Schematic of flow field in normal fuel injection (Billig et al 1971, Rogers 1971).

the significant gain in near-field mixing completeness is not without cost. The detached normal shock wave causes a severe local loss of total pressure, which together with the total pressure loss due to boundary layer separation, leads to a significant decrease in overall cycle efficiency.

Experimental measurements of the axial variation of the mixing parameter $\eta_M(x)$ for normal injection of hydrogen into a crossflowing air stream (Rogers 1971) can be represented approximately by an empirical relation (Heiser & Pratt 1994),

$$\eta_{M_{90^\circ}} = \left[\frac{x}{L_m} + \frac{1}{(50 + 1000\alpha)} \right]^\alpha, \quad (14)$$

where α is a fit parameter that varies from $\alpha = 0.17$ for widely spaced injectors to $\alpha = 0.25$ for closely spaced injectors, and where L_m is the axial distance required for the minor mixant to be depleted while mixing to stoichiometric proportion, as given by Equation (12).

If one compares values of $\eta_M(x)$ for parallel and normal injection at the same value of x/L_m , determined from Equations (11) and (14), the normal jet mixing layer can be seen to grow initially (small x) at a significantly greater rate than the parallel jet mixing layer. It should be especially noted that the presence of the recirculating wake region behind the jet results in η_M being significantly greater than zero at $x = 0$: Specifically, Equation (14) gives values of $\eta_M(0) = 0.24$ and 0.40 for closely and widely spaced injectors, respectively.

However, it is of even greater significance that the value of L_m that is to be used in Equation (14) is the same value given by Equation (12a). While normal injection "jump-starts" the mixing and causes a greater initial growth rate of η_M , the far-field mixing is apparently somewhat weakened in such a way that approximately the same axial distance is required for the minor mixant to be fully ingested into the mixing layer as with parallel fuel injection! Apparently, the near-field mixing is dominated by the stirring or macromixing driven by the large-scale vortices generated by the jet-freestream interaction, whereas the far-field mixing depends as before only on the small-scale turbulence within the plume-mixing layer, such that the far-field mixing essentially has no memory of the near-field stirring. The transition from near-field to far-field mixing takes place about 10 to 20 jet diameters downstream (Hollo et al 1992).

It is of course possible to inject the fuel into the cross stream at angles between 0 and 90 degrees, in order to enhance the near-field mixing, while holding down the severity of shock losses from the impingement of the fuel jet on the freestream. To reasonably good approximation, so-called vectored jet injection can be modeled by linearly varying the fit parameter α in Equation (14) between $\alpha = 0.17$ to 0.25 for 90° injection and $\alpha = 1$ for parallel fuel injection (Pulsonetti et al 1988).

The Need for Axial Vorticity

Dimotakis' (1991) summary of more than 20 years of research on mixing in turbulent shear-mixing layers shows that the most significant finding for supersonic combustion is that shear between coflowing fuel and air streams cannot achieve near-field mixing to near-stoichiometric proportions of air with hydrogen, the fuel of choice for hypersonic propulsion. It appears that lateral or spanwise vorticity, whether generated by shear layers, lateral fuel injection, or separated base-wake flows, is unlikely to produce the required H_2 /air mixing at a reasonable cost in mixing power.

It is now recognized that the flow instabilities induced in both parallel fuel injection and normal fuel injection give rise principally to lateral or spanwise vorticity and only secondarily to axial or streamwise vorticity (vorticity vectors aligned in the x direction). Given the general agreement that mixing from vectored fuel injection is costly in terms of total pressure loss, and that the dominant sense of the vorticity responsible for the near-field stirring is predominantly lateral vorticity, the logical next step is to try stirring with axial vortices.

Axial Vorticity in the Fuel Stream

In 1969, Swithenbank & Chigier postulated that "substantial increase in mixing rates can be obtained by applying a swirling motion to the fuel jet." It was well known at that time that this was true for *subsonic* mixing and combustion, but

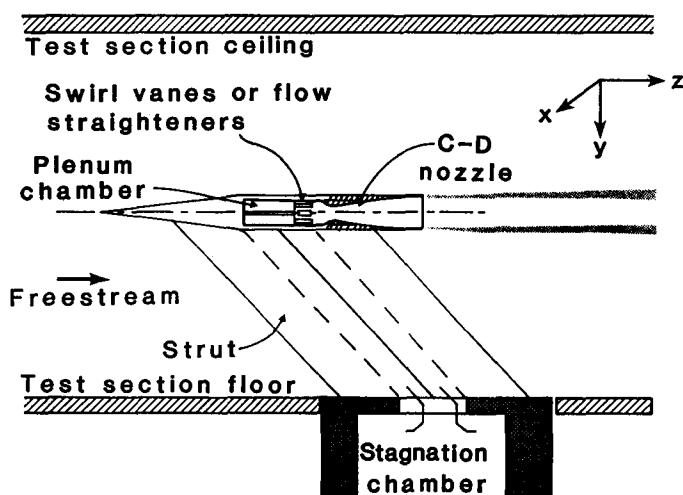


Figure 15 Vortex injector for imparting swirl to fuel jet in supersonic mixing. (After Naughton et al 1992.)

there was very little experimental verification to support their assertion. Two experiments reported in 1972–1973 revealed little or no mixing enhancement by swirling the fuel jet as it issued parallel to a supersonic air stream. However, more recent investigations reported in 1989–1990 supported Swithenbank & Chigier's assertion. In a review of this historical research, Naughton et al (1992) state that "increases in mixing rate of up to 60% are possible through the addition of streamwise vorticity" using the supersonic, swirled injection fuel nozzle shown in Figure 15. Although this improvement in mixing rate over parallel injection is no greater than for normal fuel injection, it is anticipated that, based on theoretical considerations as yet unsubstantiated, the same degree of mixing can be achieved with less mixing power expended than with normal fuel injection.

Another method of studying axial vorticity in the parallel-flow fuel jet involves inducing the vorticity through secondary flows. These flows can arise when supersonic fuel flows through a converging tapered slot jet that features an elliptic-to-conical duct transition just before sonic injection of the fuel into the parallel air stream (Gutmark et al 1989).

Axial Vorticity in the Air Stream

A variety of mixing devices have been proposed recently, all with the basic objective of converting a fraction of the flow energy in the air stream into

tangential kinetic energy, in the hope that the resulting axial vortex will sweep through and entrain an unswirled, parallel-injected fuel jet. These devices have been termed *hypermixers*.

One class of such devices that has been studied intensively in recent years is an array of wall-mounted ramps of various configurations, as illustrated in Figures 16 and 17.

In configuration (a) of Figure 16, an oblique shock stands at the base of the ramp where it rises from the wall. The compressed air above the ramp spills over the sides into the lower pressure along the ramp sidewall, forming a counter-rotating pair of axial vortices in the sense shown in the end view of Figure 16. The design goal of this configuration is for the axial vortices to entrain the central fuel jet, ultimately leading to downstream mixing.

In configuration (b) of Figure 16, the wall is turned away from the flow, while the top surface of the ramp remains in the plane of the upstream wall. When the wall has relieved far enough to expose the fuel jet in the downstream end of the ramp, the wall turns back into the flow until it is again parallel with the plane of the upstream wall. In this case, a Prandtl-Meyer expansion fan is anchored at the upper edge of the inclined plane, causing a pressure difference between the flow

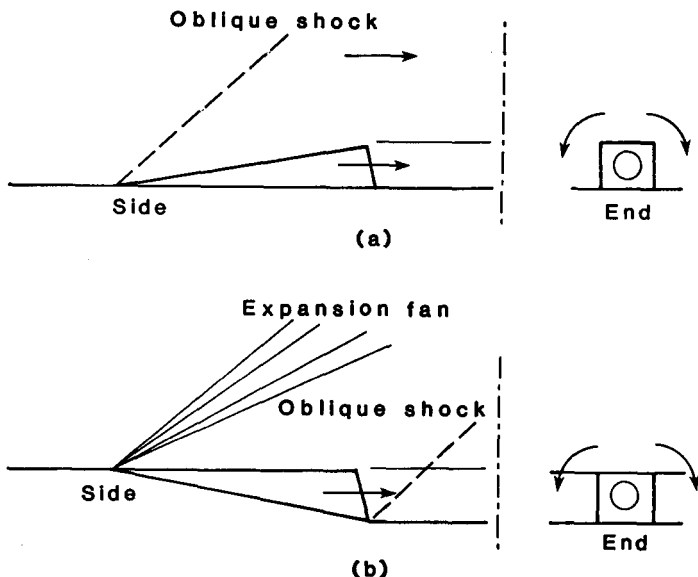


Figure 16 Geometry of wall-mounted, unswept ramp fuel injectors. (a) Raised ramp. (b) Relieved or exposed ramp.

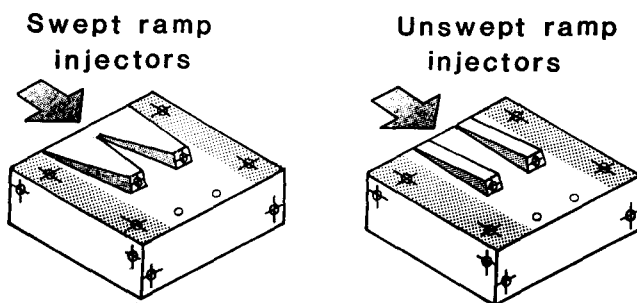


Figure 17 Perspective view of swept and unswept ramp fuel injectors. (After Northam et al 1991.)

on the upper ramp surface and the expanded flow along the sidewalls of the ramp. The design goal is also to form counter-rotating pairs of axial vortices having the same sense of rotation as in configuration (a). However, configuration (b) is significantly different from (a) in that the fuel jet now has to pass through a planar oblique shock anchored at the lower edge of the inclined ramp, where the wall is turned back into the freestream. If the fuel jet is gaseous H_2 , its mass density is much less than that of the adjacent air, due to the $\sim 28 : 1$ difference in molecular weight of fuel and air. This fact, which is responsible for the poor mixing in parallel fuel injection, may possibly be used to our advantage. In addition to the axial vorticity generated by the flow spilling over the shoulders of the relieved ramp, additional axial vorticity is generated due to *baroclinic torque* at the mixant interface between the fuel jet and the air stream. This occurs because a strong spatial gradient of mass density exists at the mixant interface, with the gradient direction radially outward from the surface of the cylindrical mixant interface. At the same time, a strong pressure gradient exists, with direction normal to the surface of the planar oblique shock, through which the fuel jet must pass. Wherever the density and pressure gradients are not collinear, baroclinic torque is generated. The pointwise rate of generation of baroclinic torque in the vicinity of the fuel-air interface is given by (Waitz et al 1992)

$$\rho \frac{D}{Dt} \left(\frac{\omega}{\rho} \right) = \frac{1}{\rho^2} \nabla \rho \times \nabla p,$$

where ω is the vorticity.

From a CFD modeling study comparing near-field mixing for a round, parallel-injected jet with and without passage of the jet through an oblique shock (but without a ramp), Drummond (1992) concluded that there was a significant increase in the near-field fuel-air mixing and subsequent combustion in the case of the shocked jet compared with the unshocked jet. However, it is not clear at

this time whether the gain in mixing enhancement due to the vorticity generated by baroclinic torque is sufficient to justify the additional loss of total pressure caused by the oblique shock, through which *all* of the air stream must pass.

Furthermore, the effectiveness of unswept ramps as axial vorticity mixers is questionable, as the mixing that does occur may be due principally to stirring by *lateral* vortices generated in the wake flow, downstream of the ramp face in the vicinity of the jet exit plane as shown in the end view of Figure 16, and only secondarily by the weak axial vortices generated by spillage over the ramp sidewalls (JT Martin, SL Mausshardt & RE Breidenthal 1992, unpublished manuscript).

To strengthen the formation of axial vorticity, *sweepback* has been added to both raised and relieved ramps, as illustrated in Figure 17. This configuration has been shown to be superior to the corresponding unswept ramp configurations for enhancement of near-field mixing.

Other devices for inducing axial vorticity into the air stream include inserting various configurations of vortex generators into the freestream such as "micropylons" (Avrashkov et al 1990, Baranovsky & Levin 1991), swept delta-wing tab mixers (Broadwell 1984, Zaman et al 1992), "dogtooth" tabs (Swithenbank et al 1991), and counter-swirling pairs of axial vortices (Cutler & Johnson 1995). An interesting hybrid scheme, which induces axial vorticity in both the fuel and air streams, uses serrated or fluted downstream edges on the fuel-air splitter plate, somewhat like the corrugated edge of a pie crust (McVey & Kennedy 1989). The laterally undulating surface introduces alternately raised and relieved ramps into both streams as they flow past the corrugated splitter plate. Although this fluted or lobed mixer arrangement has been found to be very effective in subsonic mixing, both for bypass air mixing and noise reduction in turbojet engine exhaust nozzles, there is as yet no experimental confirmation of its effectiveness in supersonic flows. Despite their promise for increased near-field mixing, there has been little experimental investigation of the effectiveness of any of these devices for enhanced fuel-air mixing at reasonable cost of total pressure loss.

In a modeling study of chemical kinetics in scramjet burners, Jachimowski (1988) suggests an exponential function representation for the mixing parameter. This suggestion can be represented as (Heiser & Pratt 1994)

$$\eta_M = \frac{1 - e^{-Ax/L_m}}{1 - e^{-A}}, \quad (15)$$

where A is a fit parameter that varies between 1 and 5 to represent increasingly effective near-field mixing (Riggins & McClinton 1991, Riggins et al 1992) and where L_m is the distance for minor mixant depletion as given by Equations (12). Values of $A = 1.77$ and 3.4 may be used in Equation (15) to represent the axial

mixing parameter of unswept and swept 10° -raised ramp mixers, as illustrated in Figures 16a and 17, based on results of a CFD comparison of swept ramp, unswept ramp, and 30° vectored wall injection mixers (Riggins & McClinton 1991). In a subsequent study, Riggins et al (1992) found that $A = 4.9$ fits the mixing parameter for a swept ramp mixer at a higher freestream Mach number. Both studies concluded that the swept ramp is a more effective mixer than either vectored wall injection or unswept ramps and that the unswept ramp performed so poorly that it should not be considered further as a candidate mixing device.

The mixing parameter model for normal injection, Equation (14), together with the linear variation scheme for wall injection between 0 and 90 degrees described in connection with Equation (14), did not agree well with the mixing parameter data in Avrashkov et al (1990) for an overall equivalence ratio $\phi_0 = 3.0$.

Mach Number Limitations

The empirical mixing efficiency correlations reviewed here, Equations (11)–(15), were all obtained at local Mach numbers between 1 and 3, and their applicability to higher combustor Mach numbers required for hypersonic flight is questionable.

CONCLUDING REMARKS

This article began with an acknowledgment of the role of Dr. Ferri and his team in pioneering scramjet research. In recent years some interesting recollections of his efforts have been authored by Chinitz (1993) and Peschke (1993). The reader will find these to be valuable supplements to Dr. Ferri's published works.

The ultimate challenge to the aeronautical engineer is the achievement of orbital flight by a single-stage vehicle taking off from Earth and using primarily air-breathing propulsion. This is an elegant, but so far elusive, concept for which the hydrogen-fueled scramjet is the candidate prime mover. At this point in time, the performance required from this engine for flight speeds above about Mach 12–15 has yet to be demonstrated. Much research has been done to enhance engine performance at such hypersonic speeds, and novel concepts, such as the oblique detonation wave engine (Heiser & Pratt 1994) may yet have to be harnessed. Nevertheless, for the immediate future, various aerothermodynamic and chemical means of improving vehicle performance closely related to the material of this article will continue to be explored.

Any *Annual Review* chapter, as well as any article cited in an *Annual Review* chapter, may be purchased from the Annual Reviews Preprints and Reprints service.
1-800-347-8007; 415-259-5017; email: arpr@class.org

Literature Cited

- Anderson GY. 1974. An examination of injector/comburntor design effects on scramjet performance. *Proc. 2nd Int. Symp. Air Breathing Engines, Sheffield, Engl.* London: R. Aeronaut. Soc.
- Anderson GY. 1986. Comment on: a comparison of scramjet integral analysis techniques. *J. Propuls. Power* 2(1):94
- Anderson GY, Kumar A, Erds J. 1990. Progress in hypersonic combustion technology with computation and experiment. *AIAA Pap. No. 90-5254*. Presented at AIAA Int. Aerosp. Planes Conf., 2nd, Orlando, FL
- Andrews EH, Mackley EA. 1993. Review of NASA's hypersonic research engine project. *AIAA Pap. No. 93-2323*. Presented at AIAA/ASME/SAE/ASEE Joint Propuls. Conf. Exhib., 29th, Monterey, CA
- Avrashkov V, Baranovsky S, Levin V. 1990. Gasdynamic features of supersonic kerosene combustion in a model combustion chamber. *AIAA Pap. No. 90-4568*. Presented at AIAA Int. Aerosp. Planes Conf., 2nd, Orlando, FL
- Baranovsky S, Levin V. 1991. Wide-range combustion chamber of ram jet. *AIAA Pap. No. 91-5094*. Presented at AIAA Int. Aerosp. Planes Conf., 3rd, Orlando, FL
- Beach HL Jr. 1972. Supersonic mixing and combustion of a hydrogen jet in a coaxial high-temperature test gas. *AIAA Pap. No. 72-1179*. Presented at AIAA/SAE Joint Propuls. Specialist Conf., 8th, New Orleans, LA
- Beach HL Jr. 1992. Supersonic combustion status and issues. In *Major Research Topics in Combustion*, ed. MY Hussaini, A Kumar, RG Voight, pp. 1-20. New York: Springer-Verlag. 650 pp.
- Billig FS. 1967. Design of supersonic combustors based on pressure-area fields. *11th Int. Symp. Combustion*, pp. 755-69. Pittsburgh, PA: Combustion Inst. 1200 pp.
- Billig FS. 1988. Combustion processes in supersonic flow. *J. Propuls. Power* 4(3):209-16
- Billig FS. 1993. Research on supersonic combustion. *J. Propuls. Power* 9(4):499-514
- Billig FS, Dugger GL. 1968. The interaction of shock waves and heat addition in the design of supersonic combustors. *12th Int. Symp. Combustion*, pp. 1125-39. Pittsburgh, PA: Combustion Inst. 1292 pp.
- Billig FS, Dugger GL, Waltrup PJ. 1972. *Proc. 1st Int. Symp. Air Breathing Engines, Marseilles, France*. Sheffield, Engl.: Sheffield Univ.
- Billig FS, Orth RC, Lasky M. 1968. Effects of thermal compression on the performance estimates of hypersonic ramjets. *J. Spacecr. Rockets* 5(9):1076-81
- Billig FS, Orth RC, Lasky M. 1971. A unified analysis of gaseous jet penetration. *AIAA J.* 9(6):1048-58
- Billig FS, Sullins GA. 1993. Optimization of combustor-isolator in dual-mode scramjets. *AIAA Pap. No. 93-5154*. Presented at AIAA/DGLR Int. Aerosp. Planes and Hypersonic Technol. Conf., 5th, Munich, Ger.
- Breidenthal RE. 1992. Sonic eddy—a model for compressible turbulence. *AIAA J.* 30(1):101-4
- Broadbent EG. 1991. Hypersonic aerodynamics. *Phil. Trans. Phys. Sci. Eng.* 335(1637):87-224
- Broadwell JE. 1984. Delta wing nozzle assembly for chemical lasers. *US Patent No. 4466100*
- Broadwell JE, Breidenthal RE. 1982. A simple model of mixing and chemical reaction in a turbulent shear layer. *J. Fluid Mech.* 125:397-410
- Brodkey RS. 1967. *The Phenomena of Fluid Motions*, pp. 327-51. Reading/Menlo Park/London: Addison-Wesley. 737 pp.
- Chinitz W. 1993. Some supersonic and hypersonic research at GASL in the 1960s and 70s. *AIAA Pap. No. 93-2327*. Presented at AIAA/ASME/SAE/ASEE Joint Propuls. Conf. and Exhib., 29th, Monterey, CA
- Contensou P, Marguet R, Huet C. 1972. Etude theoretique et experimental d'un statoreacteur a combustion mixte. *ICAS Pap. No. 72-24*. Presented at Congr. Int. Council Aeronaut. Sci., 8th, Amsterdam, The Netherlands
- Cookson RA. 1975. An analysis of non-constant area heat addition due to combustion in a supersonic air-stream. *TR-75-1483*. Washington, DC: USAF Office Sci. Res.
- Crococo L. 1958. One-dimensional treatment of steady gas dynamics. In *Fundamentals of Gas Dynamics*, ed. HW Emmons, pp. 64-349. Princeton, NJ: Princeton Univ. Press. 749 pp.
- Curran ET, Stull FD. 1964. *USAF Tech. Rep. RTD-TDR-63-4097*. Wright-Patterson AFB, OH: USAF Aero-Propuls. Lab.
- Cutler A, Johnson C. 1995. The use of swirling jet pairs to provide rapid penetration in scramjet combustors. *AIAA Pap. No. 95-0099*. Presented at AIAA Aerosp. Sci. Meet. and Exhib., 33rd, Reno, NV
- Delale CF, Schnerr GH, Zierap J. 1993. The mathematical theory of thermal choking in nozzle flows. *ZAMP* 44:943-76
- Dimotakis PE. 1991. Turbulent free shear layer mixing and combustion. In *High-Speed Flight Propulsion Systems*, ed. SNB Murthy, ET

- Curran, pp. 265-340, Vol. 137 *Progress in Astronaut. Aeronaut. Ser.* Washington, DC: AIAA. 537 pp.
- Dimotakis PE, Hall JL. 1987. A simple model for finite chemical kinetics analysis of supersonic turbulent shear layer combustion. *AIAA Pap. No. 87-1879*. Presented at AIAA/SAE/ASME/ASME Joint Propuls. Conf. and Exhib., 23rd, San Diego, CA
- Drummond JP. 1992. Mixing enhancement of reacting parallel fuel jets in a supersonic combustor. *AIAA Pap. No. 91-1914*. Presented at AIAA/SAE/ASME Joint Propuls. Conf. and Exhib., 27th, Sacramento, CA
- Elmqvist AR. 1993. Evaluation of a CFD code for analysis of normal-shock trains. *AIAA Pap. No. 93-0292*. Presented at AIAA Aerosp. Sci. Meet. and Exhib., 31st, Reno, NV
- Ferri A. 1973. Mixing-controlled supersonic combustion. *Annu. Rev. Fluid Mech.* 5:301-38
- Foa JV, Rudinger G. 1949a. On the addition of heat to a gas flowing in a pipe at subsonic speed. *J. Aeronaut. Sci.* 16(2):84-94, 119
- Foa JV, Rudinger G. 1949b. On the addition of heat to a gas flowing in a pipe at supersonic speed. *USONR Rep. No. HF-534-A-2*. Ithaca, NY: Cornell Aeronaut. Lab.
- Gai SL. 1992. Free piston shock tunnels: developments and capabilities. *Prog. Aerosp. Sci.* 29:1-41
- Guha A. 1994. Thermal choking due to nonequilibrium condensation. *J. Fluids Eng.* 116:599-604
- Gutmark E, Schadow KC, Parr TP, Parr DM, Wilson KJ. 1989. Combustion enhancement by axial vortices. *AIAA J. Propuls. Power* 5(5):555-60
- Heiser WH, Pratt DT. 1994. *Hypersonic Airbreathing Propulsion*. Washington DC: AIAA. 587 pp.
- Hollo SD, McDaniel JC, Hartfield RJ Jr. 1992. Characterization of supersonic mixing in a nonreacting Mach 2 combustor. *AIAA Pap. No. 92-0093*. Presented at Aerosp. Sci. Meet. and Exhib., 30th, Reno, NV
- Hoose KV. 1994. Combustor kinetic energy efficiency analysis of the Hypersonic Research Engine data. *AIAA Pap. No. 94-2818*. Presented at AIAA/ASME/SAE/ASME Joint Propuls. Conf. and Exhib., 30th, Indianapolis, IN
- Hunter LG, Couch BD. 1990. A CFD study of precombustion shock-trains from Mach 3-6. *AIAA Pap. No. 90-2220*. Presented at AIAA/SAE/ASME/ASME Joint Propuls. Conf. and Exhib., 26th, Orlando, FL
- Jachimowski CJ. 1988. An analytical study of the hydrogen-air reaction mechanism with application to scramjet combustion. *NASA TP-2791*. Hampton, VA: NASA Langley Res. Center
- Kramer P, Heitmeir F, Bissinger N, Voss N. 1993. German Hypersonics Technology Programme propulsion technology—status 1993. *AIAA Pap. No. 93-5094*. Presented at AIAA/DGLR Int. Aerosp. Planes and Hypersonic Technol. Conf., 5th, Munich, Ger.
- Kremer F. 1993. Heat addition in a non-constant area supersonic combustor. *AIAA Pap. No. 93-5020*. Presented at AIAA/DGLR Int. Aerosp. Planes and Hypersonics Technol. Conf., 5th, Munich, Ger.
- Lin P, Rao GVR, O'Connor GM. 1991a. Numerical investigation on shock wave/boundary-layer interactions in a constant area diffuser at Mach 3. *AIAA Pap. No. 91-1766*. Presented at AIAA Fluid Dynamics, Plasma Dynamics & Laser Conf., 22nd, Honolulu, HI
- Lin P, Rao GVR, O'Connor GM. 1991b. Numerical analysis of normal shock train in a constant area isolator. *AIAA Pap. No. 91-2162*. Presented at AIAA/SAE/ASME/ASME Joint Propuls. Conf. and Exhib., 27th, Sacramento, CA
- Marsh H, Horlock JH. 1961. Diabatic gas flows. *J. Fluid Mech.* 10:513-24
- McVey J, Kennedy J. 1989. Flame propagation enhancement through streamwise vorticity stirring. *AIAA Pap. No. 89-0619*. Presented at AIAA Aerosp. Sci. Meet. and Exhib., 27th, Reno, NV
- Meshcheryakov EA, Sabelnikov VA. 1988. Reduced heat production due to mixing and kinetic factors in supersonic combustion of unmixed gases in an expanding channel. *Fiz. Goreniya Vzryva* 24(5):23-32
- Möhring W. 1979. On flows with heat addition in Laval nozzles. In *Recent Developments in Theoretical and Experimental Fluid Mechanics*, pp. 179-85. New York: Springer-Verlag
- Naughton JW, Cattafesta L, Settles GS. 1992. Experiments on the enhancement of compressible mixing via streamwise vorticity. *AIAA Pap. No. 92-3549*. Presented at AIAA/SAE/ASME/ASME Joint Propuls. Conf. and Exhib., 28th, Nashville, TN
- Northam GB, Anderson GY. 1986. Supersonic combustion ramjet research at Langley. *AIAA Pap. No. 86-0159*. Presented at AIAA Aerosp. Sci. Meet. and Exhib., 24th, Reno, NV
- Northam GB, Capriotti DP, Byington CS, Greenberg I. 1991. Mach 2 and Mach 3 mixing and combustion in scramjets. *AIAA Pap. No. 91-2394*. Presented at AIAA/ASME/SAE/ASME Joint Propuls. Conf. and Exhib., 27th, Sacramento, CA
- Nouse H. 1993. Japan's research and development program for airbreathing engine technologies. In 11th Int. Symp. Air Breathing

- Engines Symposium Papers, ed. FS Billig, 1:41-49. Washington, DC: AIAA, 716 pp.
- Ortwerth PJ. 1977. A generalized distortion theory of internal flows. *USAF Tech Rep. No. AFWL-TR-77-18*. Kirtland AFB, NM: USAF Weapons Lab.
- Ostras VN, Penzin VI. 1977. Changes in separated flow caused by throttling of a supersonic flow in a round duct. *Fluid Mech. Sov. Res.* 6(2):8-16
- Papamoschou D. 1989. Structure of the compressible turbulent shear layer. *AIAA Pap. No. 89-0216*. Presented at the AIAA Aerosp. Sci. Meet., 27th, Reno, NV
- Papamoschou D. 1994. Thrust loss due to supersonic mixing. *AIAA J. Propuls. Power* 10(6):804-9
- Penzin VI. 1983. Interaction between a pseudoshock and an obstacle. *Uch. Zap. TSAGI* 14(5)
- Penzin VI. 1989. Pseudoshock and separated flows in rectangular channels. *Fluid Mech. Sov. Res.* 18(1):1-12
- Peschke WT. 1993. An approach to in-situ analysis of scramjet combustor behavior. *AIAA Pap. No. 93-2328*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 29th, Monterey, CA
- Pouring AA. 1961. Thermal choking and condensation in nozzles. *Phys. Fluids* 8(10):1802-10
- Pratt DT, Heiser WH. 1993. Isolator-combustor interaction in a dual-mode scramjet engine. *AIAA Pap. No. 93-0358*. Presented at AIAA Aerosp. Sci. Meet. and Exhib., 31st, Reno, NV
- Pulsonetti MV, Erdos J, Early K. 1988. An engineering model for analysis of scramjet combustor performance with finite rate chemistry. *AIAA Pap. No. 88-3258*. Presented at AIAA/ASME/SAE/ASEE Joint Propuls. Conf. and Exhib., 24th, Boston, MA
- Ramaty Y, Spiegler E, Tinmat YM. 1982. The self-adjustment mechanism model of scramjet combustors. *19th Int. Symp. on Combustion*, pp. 549-56. Pittsburgh, PA: Combustion Inst. 1572 pp.
- Riggins DW, McClinton CR. 1991. Analysis of losses in supersonic mixing and reacting flows. *AIAA Pap. No. 91-2266*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 27th, Sacramento, CA
- Riggins DW, McClinton CR, Rogers RC, Bittner, RD. 1992. A comparative study of scramjet ignition strategies for high Mach number flows. *AIAA Pap. No. 92-3287*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 28th, Nashville, TN
- Rogers RC. 1971. Mixing of hydrogen injected from multiple injectors normal to a supersonic airstream. *NASA TND-6476*. Hampton, VA: NASA Langley Res. Cent.
- Roudakov AS, Schickman Y, Semenov V, Novelli P, Fourt O. 1993. Flight testing an axisymmetric scramjet—Russian recent advances. *IAF Pap. No. 93-S.4.485*. Presented at Int. Astronaut. Fed. Congr., 44th, Graz, Austria
- Rubins PM, Bauer RC. 1966. A hypersonic ramjet analysis with premixed fuel combustion. *AIAA Pap. No. 66-648*. Presented at AIAA Propuls. Joint Specialist Conf., 2nd, Colorado Springs, CO
- Sabelnikov VA, Voloschenko OV, Ostras VN, Sermanov VN. 1993. Gas dynamics of hydrogen-fueled scramjet combustors. *AIAA Pap. No. 93-2145*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 29th, Monterey, CA
- Shapiro AH. 1953. *The Dynamics and Thermodynamics of Compressible Fluid Flow*, 1:219-60. New York: Ronald Press. 647 pp.
- Sosounov V. 1993. Research and development of ram/scramjets and turbojets in Russia. *AGARD Lect. Ser. No. 194*. Paris: AGARD
- Stocker PM. 1952. The transients arising from the addition of heat to a gas flow. *Proc. Cambridge. Philos. Soc.* 48:482-98
- Sullins GA, McLafferty G. 1992. Experimental results of shock trains in rectangular ducts. *AIAA Pap. No. 92-5103*. Presented at AIAA Int. Aerosp. Conf., 4th, Orlando, FL
- Sullins GA, Waltrup PJ. 1985. A comparison of scramjet integral analysis techniques. *J. Propuls. Power* 1(2):156-58
- Sullins GA, Waltrup PJ. 1986. Reply by authors to GY Anderson. *J. Propuls. Power* 2(1):95-96
- Swithenbank J, Chigier NA. 1969. Vortex mixing for supersonic combustion. *Proc. Int. Symp. on Combustion*, 12th, pp. 1153-62. Pittsburgh: Combustion Inst.
- Swithenbank J, Eames IW, Chin SB, Ewan BCR, Yang Z, et al. 1991. Turbulent mixing in supersonic combustion systems. In *High-Speed Flight Propulsion Systems*, ed. SNB Murthy, ET Curran, pp. 341-81, Vol. 137 *Progress in Astronaut. Aeronaut. Ser.* Washington, DC: AIAA. 537 pp.
- Tretyakov PK. 1993. Determination of heat input to duct flow with pseudoshock. *Fiz. Goreniya Vzryva* 29(3):71-77
- Vinogradov VA, Kobigsky, Petrov MD. 1995. Experimental investigation of kerosene fuel combustion in supersonic flow. *J. Propuls. Power* 11(1):130-34
- Voloshchenko OV, Kolesnikov OM, Meshcheryakov YA, Ostras VN, Sabelnikov VA, Sermanov VN. 1992. *Proc. Int. Counc. Aero. Sci., 18th, Beijing*, pp. 693-702. Washington, DC: AIAA

360 CURRAN, HEISER & PRATT

- Waitz IA, Marble FE, Zukoski EE. 1992. Vorticity generation by contoured wall injectors. *AIAA Pap. No. 92-3550*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 28th, Nashville, TN
- Walther R, Sabelnikov VA, Voloschenko OV, Sermanov VN. 1993. New partnerships to meet hypersonic propulsion challenges: joint MTU-TsAGI cooperation in scramjet engine development. *AIAA Pap. No. 93-5017*. Presented at AIAA/DGLR Int. Aerosp. Planes and Hypersonics Technol. Conf., 5th, Munich, Ger.
- Waltrup PJ, Anderson GY, Stull FD. 1976. Supersonic combustion ramjet (scramjet) engine development in the United States. *Proc. Int. Symp. Air Breathing Engines, 3rd, Munich*, pp. 835-61. Munich, Ger.: DGLR. 955 pp.
- Waltrup PJ, Billig FS. 1973. Structure of shock waves in cylindrical ducts. *J. Propuls. Power* 11(10):1404-8
- Weber RJ, MacKay JS. 1958. Analysis of ramjet engines using supersonic combustion. *Tech. Note TN-4386*. Cleveland, OH: NASA Lewis Flight Propuls. Lab.
- Wegener PP, Mack LM. 1958. Condensation in supersonic and hypersonic wind tunnels. *Adv. Appl. Mech.* 5:307-442
- Woodley JG. 1967. A simplified study of open cycle magnetohydrodynamic power generation and an examination of the possible role of supersonic combustion in the cycle. *RAE Tech. Rep. TR. 67117*. Farnborough, Engl.: RAE
- Zaman KBMQ, Reeder MF, Samimy M. 1992. Supersonic jet mixing enhancement by "Delta-Tabs." *AIAA Pap. No. 92-3548*. Presented at AIAA/SAE/ASME/ASEE Joint Propuls. Conf. and Exhib., 28th, Nashville, TN
- Zierep J. 1974. Theory of flows in compressible media with heat addition. *AGARDograph No. 191*. Paris: AGARD. 61 pp.
- Zimont VL, Levin VM, Meshcheryakov. 1978. Combustion of hydrogen in a supersonic flow in a channel in the presence of a pseudodiscontinuity. *Fiz. Goreniya Vzryva* 14(4):23-26