Flux Flow Noise Power Spectra in the Presence of Local Pinning Interactions*

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We derive expressions for the dc flux flow voltage and for the flux flow noise power spectrum in type II superconductors using a model in which flux bundles travel a distance l *less than the sample width* L *before being stopped for finite times by local pinning interactions. The frequency dependence of the power spectrum is shown to be identical to our earlier derivation, where we assumed that pinning and release of flux bundles occurred in zero times, an assumption we now show to be incorrect. This frequency dependence has been shown to lead to good agreement for experimentally measured transit times, which have repeatedly been shown to be too short when obtained from models of uninterrupted flux transit across the sample width. The concept of a pinned fraction arises naturally in our model, and occurs because of two factors; first, in certain regions the Lorentz force is not sufficient to overcome pinning, and second, in regions where flux flow occurs, fluxoids will be held up for brief periods because of interaction with local pinning centers. The expressions for the noise power at zero frequency and the dc voltage are modified by factors that depend on averages of* l.

1. INTRODUCTION

Several studies of the flux flow state in type II superconductors have been made using measurements of the noise power spectra as an investigative technique.[1–5] These spectra arise because fluxoids traversing a sample, either singly or in bundles, give rise to individual voltage pulses with a height proportional to the product of the flux bundle size and the velocity of the bundle, and a duration equal to the transit time of the bundle across the sample width. Clem has predicted that these spectra should depend on the geometry of the sample and the arrangement of the measuring circuit.[6,7] For a foil sample with width L much less than the probe separation d, the pulse

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shape as observed by the measuring circuit can be approximated to be rectangular in shape. For such a pulse the power spectrum would be of the form

$$W(f)\,df = 2\Phi V_{\mathrm{av}}\left(\frac{\sin \pi f \tau_c}{\pi f \tau_c}\right)^2 df \qquad (1)$$

where Φ is the magnitude of the moving flux entities, V_{av} is the dc voltage, df is the measuring bandwidth, and τ_c is the transit time of fluxoids across the sample width. The noise spectra should thus yield a critical frequency $f_c = 1/\tau_c = v/L$, where v is the fluxoid velocity. The fluxoid velocity, however, can be calculated directly from the dc voltage V_{av} and the magnetic field B:

$$v = \frac{V_{\mathrm{av}}}{Bd} \qquad (2)$$

Critical frequencies obtained from the noise spectra and from the velocities should therefore be comparable.

Measurements made on polycrystal foil samples that do not have low defect densities have repeatedly led to noise spectra characterized by critical frequencies higher than predicted by Eq. (2). To overcome this difficulty, van Gurp[1] and Jarvis and Park[4] have used the concept of a pinned fraction of vortices p such that the velocity of the unpinned vortices is enhanced:

$$v = \frac{V_{\mathrm{av}}}{Bd(1-p)} \qquad (3)$$

We have previously presented results in which we also observed noise spectra yielding higher critical frequencies than predicted by Eq. (2), although these critical frequencies increased linearly with V_{av}/B as required by Eq. (2).[8] Our noise spectra also fell off at high frequencies as $1/f$ rather than the more rapid decrease predicted by Eq. (1). To overcome these difficulties, we applied a suggestion originally made by van Gurp[9] with respect to flux motion in type I superconductors. Thus we assumed that the flux did not directly transit the sample producing a pulse of length τ_c. Instead, we assumed that the fluxoid motion is interrupted momentarily as fluxoids are temporarily held up at pinning centers. The fluxoid motion thus gives rise to subpulses, each of duration τ_i, and the total spectrum represents the total contribution from the various subpulses:

$$W(f) = \int_{\beta\tau_c}^{\alpha\tau_c} g(\tau_i)\, W(f\tau_i)\, d\tau_i \qquad (4)$$

where

$$W(f\tau_i) = 2\Phi V_{av}\left(\frac{\sin \pi f\tau_i}{\pi f\tau_i}\right)^2 \qquad (5)$$

$g(\tau_i)$ is a distribution function for the τ_i and the limits of integration on the subpulse duration are expressed in terms of the unimpeded transit time τ_c. Our assumption, following van Gurp, was that the fluxoids were held up for sufficiently short times at pinning centers so that the average transit time τ_c was not altered. By using as a distribution function

$$g(\tau_i) = 1/(\alpha-\beta)\tau_c, \qquad \alpha\tau_c > \tau_i > \beta\tau_c$$

we fit the experimental data very well by using α and β as adjustable parameters. In general, best fits are obtained for $\beta = 0$, whereas $\alpha = 1$ for well-annealed samples, but decreases as grain sizes are reduced, thus producing shorter average subpulse times. See Fig. 1.

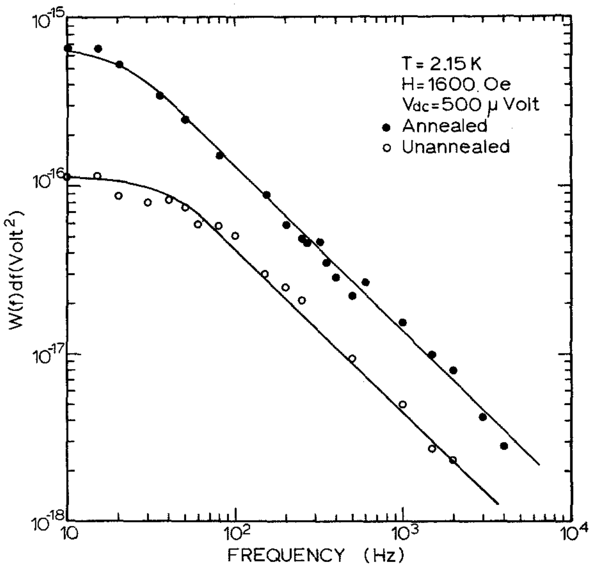

Fig. 1. Noise power spectra for $Pb_{80}In_{20}$ foil. Measurements are taken at $H = 1600$ Oe, $V = 500\ \mu V$, and $T = 2.15$ K. Sample dimensions are: width $L = 0.41$ cm; probe separation $d = 2.3$ cm; thickness 0.024 cm. Open points were taken before sample was annealed, and solid points after annealing eight days at 260 C. The solid curves represent best fits and yield $\alpha = 0.40$ (unannealed) and $\alpha = 1.00$ (annealed). β is approximately zero in both cases.

Although the model has been successful in helping us interpret noise spectra, we now understand that it has been derived assuming an incorrect value for the height of the subpulses. Further, the idea of fluxoids held up for vanishingly small times at pinning centers is incorrect. The purpose of this paper is to present a more comprehensive model for fluxoid motion interrupted by pinning. The principal result of this model is that the form for the frequency dependence of the noise spectrum is as given by Eqs. (4) and (5), although the value of the spectrum at zero frequency $W(0)$ is altered. We are led to an explicit expression for the pinned fraction, although our interpretation of this concept is somewhat altered. An expression for the dc voltage as a function of current is also obtained which yields a nonlinear region at low currents as previously derived by Baixeras and Fournet.[10]

2. DIFFICULTIES WITH PREVIOUS MODEL

As noted, the model of fluxoids being held at pinning centers for vanishingly small times is incorrect. This is because in the limit of zero stopping time, the end of one subpulse is exactly correlated with the beginning of the succeeding subpulse. The subpulses are therefore not statistically independent and interference terms would appear in the correlation function from which the noise spectrum is derived. Since the subpulses are not independent, the summation of their individual power spectra cannot be carried out as is done in Eq. (4).

Since our observed spectra follow quite closely the functional form obtained with Eq. (4), we assume that the summation process is correct, but that the assumption of zero stopping time is incorrect. We therefore modify this assumption by associating two times with a given subpulse. For the ith pulse we let τ_i be the actual time the flux is in motion and θ_i be the time between the initiation of flux motion giving rise to the ith pulse and the initiation of the $(i+1)$th pulse. Assuming that the fluxoids are held for random times upon pinning is equivalent to assuming that the pulses are statistically independent, or that the distribution function for the θ's can be written

$$u(\theta)=(1/\theta_0)\exp(-\theta/\theta_0) \tag{6}$$

A second problem in the van Gurp model was that in the case of a flux bundle transiting the distance l_i between pinning centers, the expression for the voltage pulse for a flux bundle whose motion is uninterrupted was simply altered by replacing τ_c by τ_i (i.e., it was assumed $V_i=\Phi/\tau_i$). This is inappropriate because τ_i is determined *both* by the velocity of the moving fluxoids *and* by the distance between pinning centers. The proper form for the voltage generated by the ith pulse is therefore

$$V_i = \Phi l_i / L\tau_i \qquad \text{for} \quad \sum_{j=1}^{i} \theta_{j-1} \le t \le \sum_{j=1}^{i} \theta_{j-1} + \tau_i \tag{7}$$

We note that this expression for V_i satisfies the necessary condition

$$\sum V_i \tau_i = \Phi$$

3. CALCULATION OF THE PINNED FRACTION

We have characterized a given subpulse associated with the motion of a group of fluxoids between two pinning centers by the parameters θ_i, τ_i, and l_i. We are assuming that l_i and τ_i are independent variables, l_i being determined by the physical distribution of pinning centers in the material, whereas for a given l_i, the value of τ_i will be determined by the driving force on the flux. For generality we should also assume that the size of the flux bundle Φ_i can vary from one subpulse to the next.

We note that the concept of a time of motion τ_i and a stopping time $\theta_i - \tau_i$ allows us to define two velocities for each subpulse. While the flux is in motion the velocity is given by

$$v_i = l_i / \tau_i = \Phi_i J / \eta \tag{8}$$

where η is the viscosity coefficient and J is the transport current density. One can also obtain the average velocity over the entire time associated with the ith subpulse:

$$\bar{v}_i = l_i / \theta_i = \Phi_i (J - J_i) / \eta \tag{9}$$

J_i represents a critical current density for the ith pinning event, which Yamafuji and Irie[11] have related to the relationship between v_i and $\bar{v}_i$. The distribution in times θ_i for a given applied current is therefore related to a distribution in l_i, Φ_i, and J_i. For the distribution function of J_i we use the form

$$\begin{aligned} f(J_i) &= (1/J_0) \exp\left[-(J_i - J_c)/J_0\right], \qquad && J_i \ge J_c \\ &= 0 && J_i < J_c \end{aligned}$$

used by Baixeras and Fournet[10] in deriving the shape of the flux flow current–voltage characteristic. J_c is the minimum value of the critical current density required to produce flux motion, and J_0 determines the spread in values of J_i.

One can, at this point, introduce the concept of a pinned fraction. For the combination of parameters giving rise to the ith type of subpulse, and for a current in excess of J_i, the pinned fraction will be the fraction of the time

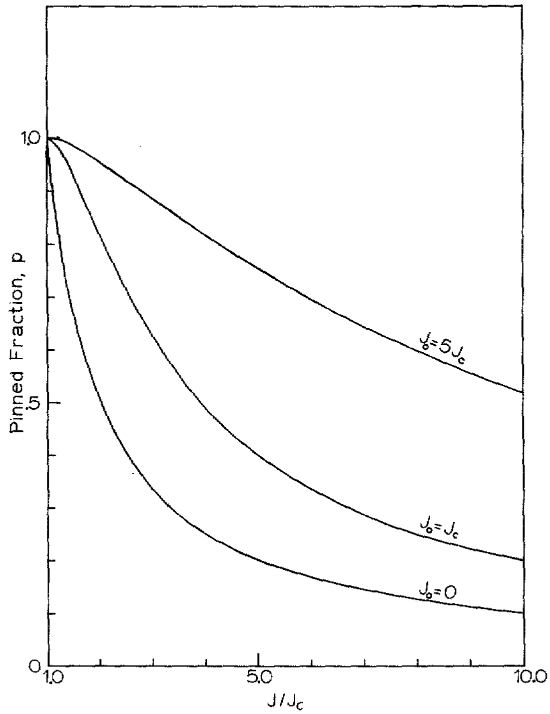

Fig. 2. Curves showing pinned fraction p from Eq. (11) as a function of the normalized current J/J_c. The parameter J_0, which gives rise to the difference between the curves, represents the spread in local pinning strengths from the minimum value J_c.

which such pulses on the average spend pinned. Thus, using Eqs. (8) and (9),

$$p_i = (\theta_i - \tau_i)/\theta_i = J_i/J, \qquad J \geq J_i \tag{10a}$$

$$= 1, \qquad J < J_i \tag{10b}$$

For the sample as a whole, the pinned fraction will be, for a given driving current J, determined by the fraction of time each group spends pinned, as well as on the distribution of the J_i. Thus for $J \geq J_c$, $p = \int_{J_c}^{\infty} p_i f(J_i)\, dJ_i$, and

$$p = \int_{J_c}^{J} \frac{J_i}{JJ_0} \exp\left(-\frac{J_i - J_c}{J_0}\right) dJ_i + \int_{J}^{\infty} \frac{1}{J_0} \exp\left(-\frac{J_i - J_c}{J_0}\right) dJ_i$$

$$p = \frac{J_c + J_0\{1 - \exp[-(J - J_c)/J_0]\}}{J} \tag{11}$$

Although we have used the distribution function for $f(J_i)$ as used by Baixeras and Fournet, the pinned fraction we obtain at a given J is much larger than that which they find (see Fig. 2). The physical reason for this difference is their assumption that if $J > J_i$, the ith group of fluxoids will definitely be in motion. In our model, such fluxoids will spend part of their time in motion and part of their time pinned. As a result, even in the linear flux flow region, we find that for a sample with J_0 comparable to J_c a sizable fraction of the total flux is at any given time not in motion.

4. CALCULATION OF THE DC VOLTAGE AND THE NOISE SPECTRUM

For identical pulses occurring at a rate n, the noise spectrum can be calculated (Carson's theorem)[12] from

$$W(f) = 2n|F(f)|^2 \tag{12}$$

where

$$|F(f)|^2 = \left|\int_{-\infty}^{\infty} y(t) \exp(-2\pi i f t)\, dt\right|^2 \tag{13}$$

If we use for $y(t)$ the voltage subpulses given by Eq. (7), then for a given type of subpulse characterized by specific values for the variables θ_i, τ_i, l_i, and Φ_i,

$$|F_i(f)|^2 = \frac{\Phi_i^2 l_i^2}{L^2}\left(\frac{\sin \pi f \tau_i}{\pi f \tau_i}\right)^2 \tag{14}$$

and the associated power spectrum will be

$$W_i(f) = 2n_i \frac{\Phi_i^2 l_i^2}{L^2}\left(\frac{\sin \pi f \tau_i}{\pi f \tau_i}\right)^2 \tag{15}$$

With the assumption that the events are random, which is equivalent to assuming that the times θ_i are statistically independent, we have for the total spectrum containing all subpulses

$$W(f) = \sum_i W_i(f) \tag{16}$$

Assuming that values of θ_i, τ_i, l_i, and Φ_i are continuously distributed, the summation can be converted to an integral by using a distribution function $\gamma(\theta_i, \tau_i, l_i, \Phi_i)$ such that

$$\sum n_i = \int\!\!\int\!\!\int\!\!\int \gamma(\theta_i, \tau_i, l_i, \Phi_i)\, d\theta_i\, d\tau_i\, dl_i\, d\Phi_i = N \tag{17}$$

where N is the total repetition rate for all subpulses. Assuming further that there is no coupling between θ_i, τ_i, l_i, and Φ_i,

$$\gamma(\theta_i, \tau_i, l_i, \Phi) = Ng(\tau_i)h(l_i)k(\Phi_i)u(\theta_i) \tag{18}$$

where $u(\theta_i) = (1/\theta_0)\exp(-\theta_i/\theta_0)$ is required for statistical independence and the distribution functions $g(\tau_i)$, $h(l_i)$, and $k(\Phi_i)$ are separately normalized to unity, i.e.,

$$\int_{\beta\tau_c}^{\alpha\tau_c} g(\tau_i)\,d\tau_i = 1$$

where here the limits of integration have been expressed in terms of the transit time calculated from Eq. (2), $\tau_c = B\,dL/V_{av}$. Using the fact that the integration over θ_i yields unity, we obtain for the power spectrum

$$W(f) = \frac{2N}{L^2}\int_{\Phi_{min}}^{\Phi_{max}} \Phi_i^2 k(\Phi_i)\,d\Phi_i \int_{l_{min}}^{l_{max}} l_i^2 h(l_i)\,dl_i$$
$$\times\int_{\beta\tau c}^{\alpha\tau_c} g(\tau_i)\left(\frac{\sin \pi f\tau_i}{\pi f\tau_i}\right)^2 d\tau_i$$
$$W(f) = \frac{2N}{L^2}\langle\Phi^2\rangle\langle l^2\rangle\int_{\beta\tau_c}^{\alpha\tau_c} g(\tau_i)\left(\frac{\sin \pi f\tau_i}{\pi f\tau_i}\right)^2 d\tau_i \tag{19}$$

where N is the total pulse rate for all events.

The average voltage, on the other hand, is

$$\begin{aligned} V_{av} &= \sum_i n_i V_i \tau_i \\ &= (N/L)\iiiint \Phi_i l_i g(\tau_i)h(l_i)k(\Phi_i)u(\theta_i)\,d\tau_i\,dl_i\,d\Phi_i\,d\theta_i \\ &= N\langle\Phi\rangle\langle l\rangle/L \end{aligned} \tag{20}$$

The total pulse rate N is given by the product of the total number of flux bundles that exist within the sample M and $\langle 1/\theta_i\rangle$. Using Eq. (9) for θ_i,

$$\left\langle\frac{1}{\theta_i}\right\rangle = \frac{\langle\Phi\rangle}{\langle l\rangle\eta}\int_{J_c}^{J} (J-J_i)f(J_i)\,dJ_i$$

$$\left\langle\frac{1}{\theta_i}\right\rangle = \frac{\langle\Phi\rangle}{\langle l\rangle\eta}\left\{J - J_c - J_0\left[1-\exp\left(-\frac{J-J_c}{J_0}\right)\right]\right\}$$

and

$$V_{av} = \frac{M\langle\Phi\rangle^2}{\eta L}\left\{J - J_c - J_0\left[1 - \exp\left(-\frac{J - J_c}{J_0}\right)\right]\right\} \tag{21}$$

or

$$V_{av} = \frac{B\langle\Phi\rangle R_n \sigma_n}{\eta}\left\{I - I_c - I_0\left[1 - \exp\left(-\frac{I - I_c}{I_0}\right)\right]\right\} \tag{22}$$

where R_n and σ_n are the normal-state resistance and conductivity. This is the same expression obtained by Baixeras and Fournet, even though, as we have noted, we have a different interpretation and a higher value for the pinned fraction than that which they calculate. Representative V–I curves are shown in Fig. 3.

Finally we can write

$$W(f) = \frac{2}{L} V_{av} \frac{\langle\Phi^2\rangle\langle l^2\rangle}{\langle\Phi\rangle\langle l\rangle} \int_{\beta\tau_c}^{\alpha\tau_c} \left(\frac{\sin \pi f \tau_i}{\pi f \tau_i}\right)^2 g(\tau_i)\, d\tau_i \tag{23}$$

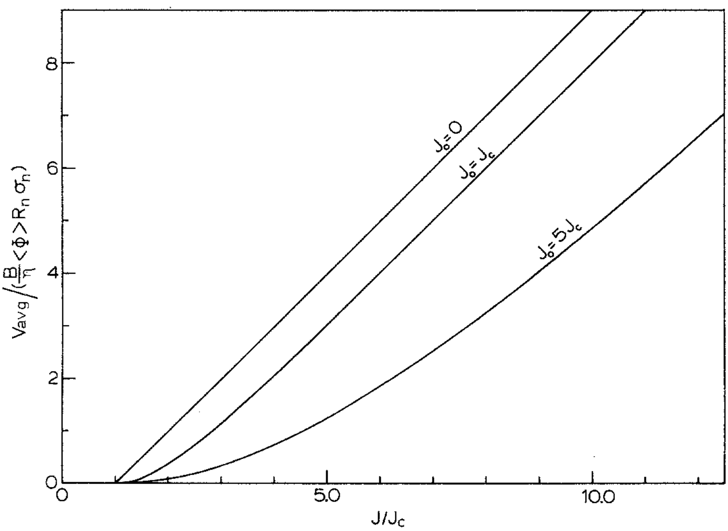

Fig. 3. Normalized flux flow voltage–current characteristics as calculated from Eq. (22). The three curves differ because of the difference in the value of J_0. Comparison with Fig. 2 shows that the value for the fraction of flux not in motion at any instant can be significant even well into the linear flux flow region.

or

$$W(f) = \frac{2BR_n\sigma_n}{L\eta}\frac{\langle\Phi^2\rangle\langle l^2\rangle}{\langle\Phi\rangle\langle l\rangle}\left\{I - I_c - I_0\left[1-\exp\left(-\frac{I-I_c}{I_0}\right)\right]\right\}$$
$$\times\int_{\beta\tau_c}^{\alpha\tau_c}\left(\frac{\sin\pi f\tau_i}{\pi f\tau_i}\right)^2 g(\tau_i)\,d\tau_i \qquad (24)$$

5. DISCUSSION

The model we have presented of the flux flow state in which flux bundles transit a sample with their motion interrupted by pinning interactions is physically realistic. Although we have no specific a priori justification for assuming the subpulses are statistically independent, interference terms would appear between subpulses if this assumption were not correct. Our further assumption of an absence of coupling between τ_i, l_i, and Φ_i for subpulses also has no prior justification. Experimentally, the noise spectrum as expressed by Eq. (23) could not be used to determine whether coupling between Φ_i and l_i exists. However, the good fit we obtain to the experimental data for the frequency dependence of the spectra would seem to indicate that τ_i is not coupled to l_i and Φ_i.

The magnitude of the noise spectrum extrapolated to zero frequency is no longer simply proportional to the average flux bundle size; thus in some sense the noise spectra are reduced in their usefulness for determining this quantity. On the other hand, there is now a dependence on the distribution of distances that flux bundles travel before becoming pinned. The magnitude of the spectrum is in fact reduced by the factor $\langle l^2\rangle/L\langle l\rangle$ from previous models. Therefore, there exists the possibility of extracting information about the distribution of distances flux bundles travel before becoming pinned. We note in this regard that preliminary measurements in a sample of PbSn in which the tin has precipitated so as to form a finely divided microstructure indicate that the noise level is very low so that $l \ll L$, thus verifying the noise reduction with short l which we predict.

Recent neutron diffraction measurements in NbTa samples,[13] which show little change from the static flux lattice when the fluxoids are far into the linear flux flow state, might seem to contradict our assumption of the flux motion being interrupted by pinning interactions. However, the NbTa sample studied shows very weak pinning, and the pinning interactions may be less effective than in the PbIn samples for which we have presented data. Moreover, as we will discuss in a separate paper in which our experimental results are presented in more detail, we envision the identity of a flux grouping as a "bundle" which exists only at the time of a pinning interaction.

For samples of macroscopic grain size in which grain boundaries provide the principal pinning mechanism, there is undoubtedly long-range correlation while fluxoids are moving within the grains and thus a nearly perfect lattice would exist for the large majority of the fluxoids. We note in this regard that the role of such long-range correlation in reducing the noise has not been included in our model.

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