Focal switch: a new effect in low-Fresnel-number systems

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It is shown for the first time we believe, that when a spherical wave illuminates a certain type of diffracting screen, in addition to the expected focal-shift effect, depending on the value of the Fresnel number of the focusing system, a focal switch effect can appear, i.e., an increase in the height of the lateral lobe of the axial-intensity distribution over that of the central lobe. © 1996 Optical Society of America

It is well known that when a monochromatic converging spherical wave is diffracted by a circular aperture the point of maximum intensity in the diffracted field is not at the geometric focus but displaced toward the aperture, resulting in the so-called focal-shift effect.[1–3] More recently, it has been recognized that the focal-shift effect is also present in obscured systems,[4,5] in focused Gaussian beams,[6–8] or in general in any type of diffracting screen.[9] Moreover this effect has been shown to appear not only on the optical axis but on any line directed toward the geometric focus of the spherical wave front.[10]

The goal of this research is to recognize the existence of a certain kind of diffracting screen where the expected focal-shift effect can be accompanied by another interesting effect: an increase in the height of a secondary lobe of the axial-intensity distribution over the height of the central lobe, resulting in an effective permutation of the focal point.

We start by considering a rotationally symmetric diffracting screen whose amplitude transmittance is $t(r)$ and that is illuminated by a monochromatic converging spherical wave. Then the amplitude distribution along the optical axis in the vicinity of the focal point, within the paraxial approximation, is[9]

$$u(z) = \exp(ikz)\,\frac{2\pi}{i\lambda f(f+z)}\int_0^{r_o} t(r) \times \exp\left[-i2\pi\,\frac{z}{2\lambda f(f+z)}\,r^2\right] r\,\mathrm{d}r, \tag{1}$$

where r_o is the external radius of the screen, z is the axial coordinate as measured from the paraxial focal point, and f is the focal length of the system, as shown in Fig. 1.

Next it is convenient to employ the next geometric mapping:

$$\zeta = \left(\frac{r}{r_o}\right)^2 - 0.5, \qquad q(\zeta) = t(r), \tag{2}$$

which converts the integral of Eq. (1) into a one-dimensional (1-D) Fourier transform. Then Eq. (1) can be rewritten, apart from an irrelevant phase factor, as

$$u(z) = \frac{\pi N}{(f+z)}\int_{-0.5}^{0.5} q(\zeta)\exp\left[-i2\pi\,\frac{Nz}{2(f+z)}\,\zeta\right]\mathrm{d}\zeta = \frac{\pi N}{(f+z)}\,u'(z), \tag{3}$$

where $N = r_o^2/\lambda f$ represents the Fresnel number of the aperture, i.e., the number of Fresnel zones that are covered by the aperture as viewed from the geometric focus, $z = 0$. Function $u'(z)$ is the axial-

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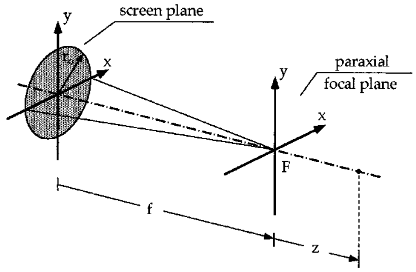

Fig. 1. Geometry of the diffraction problem.

amplitude distribution when the factors external to the integral are not taken into account.

Finally, the intensity distribution along the optical axis is given by the squared modulus of Eq. (3), that is,

$$I(z) = \left[\frac{\pi N}{(f+z)}\right]^2 \left| \int_{-0.5}^{0.5} q(\zeta) \exp\left[-i2\pi \frac{Nz}{2(f+z)}\zeta\right] \mathrm{d}\zeta \right|^2 = \left(\frac{\pi N}{(f+z)}\right)^2 I'(z). \quad (4)$$

From Eq. (4) it follows that the axial-intensity distribution is governed by the product of two terms that can be interpreted from the point of view of the Huygens–Fresnel principle in the following way. The first term, $I'(z)$, involves the squared modulus of the 1-D Fourier transform, with a scale factor $Nz/2(f+z)$, of the mapped version of the amplitude transmittance of the diffracting screen, $q(\zeta)$. This term describes at any axial point the effect on intensity from interference by the Huygens spherical wavelets proceeding from all points of the diffracting screen and whose amplitude depends on the transmittance of the screen. In particular, in the case of a purely absorbing screen the Huygens wavelets arrive in phase at the geometric focus and maximum intensity is achieved. However, as the secondary wavelets propagate, their amplitude suffers an attenuation that is proportional to the inverse covered distance. This attenuation is described in Eq. (4) by the term $1/(f+z)^2$. This term, whose value increases with negative values of z, is responsible, in the case of Fresnel numbers with low values, for the displacement toward the screen of the maximum of the axial-intensity distribution, as we discuss below.

Because the scale factor of the 1-D Fourier transformation is proportional to the Fresnel number of the screen, for high values of N the function $I'(z)$ is so sharp about the geometric focus, $z = 0$, that its value is negligible unless z is small enough that it can be ignored when it appears in $1/(f+z)^2$. In this case the axial-intensity distribution is governed only by the interference term, $I'(z)$. However, when the Fresnel number is small, the Fourier transformation provides a function that smoothly decreases in the vicinity of the focus. Now z cannot be ignored, and then the term $1/(f+z)^2$ shifts the intensity peak to negative values of z, resulting in the focal-shift effect. Then it is clear that for a given diffracting screen the lower the Fresnel number is, the greater the amount of focal shift.

Now we address the following question: For a fixed value of geometric parameters, r_0 and f, of the optical setup, i.e., for a given value of the Fresnel number, is it possible to predict when a diffracting screen is more inclined to suffer a focal shift than other screens are? To answer this question, we use the reasoning described above in which it is stated that the smoother the slope of function $I'(z)$ in the neighborhood of the paraxial focal point, the greater the influence of factor $1/(f+z)^2$ in the axial-intensity distribution, and then the greater the amount of the focal shift. Therefore it follows that a diffracting screen that produces axial superresolution, i.e., an axial-intensity distribution in which the central lobe becomes narrower compared with that of the circular aperture, is less sensitive to the focal-shift effect than a diffracting screen that produces axial apodization. In other words, the greater the capacity of a screen for producing axial apodization, the greater its sensitivity to the focal shift.

To illustrate this reasoning, in Fig. 2 we have

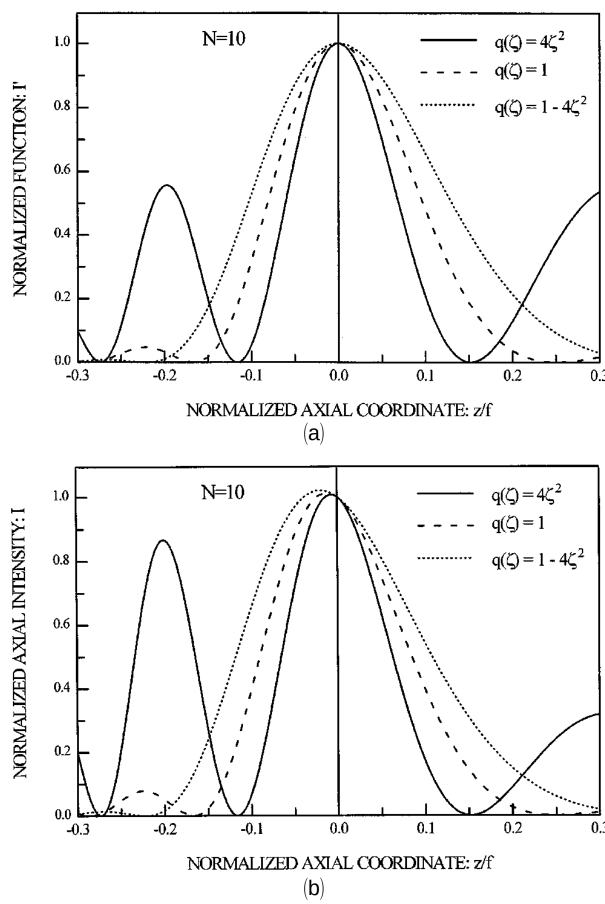

Fig. 2. For the $N = 10$ case (a) the normalized version of the function $I'(z) = |u'(z)|^2$ corresponding to the diffracting screens with transmittance $q(\zeta) = 4\zeta^2$ (solid curve) and $q(\zeta) = 1 - 4\zeta^2$ (dashed curve), and to the corresponding circular aperture (dashed curve); (b) normalized axial-intensity distribution for the same screens as in (a).

depicted for a relatively low value of the Fresnel number, $N = 10$, the normalized axial-intensity distribution, $I(z) = |u(z)|^2$, compared with the normalized version of the function $I'(z) = |u'(z)|^2$, for the case of two well-known diffracting profiles: $q(\zeta) = 4\zeta^2$, which produces axial superresolution, and $q(\zeta) = 1 - 4\zeta^2$, which produces axial apodization. The normalization is such that $I(z = 0) = I'(z = 0) = 1$. It is apparent from this figure that, as we predicted, the amount of focal shift is clearly greater in the case of the axially apodizing profile.

The analysis of the curve corresponding to the axially superresolving profile in Fig. 2(b) reveals that, simultaneously with the effect of displacement of the maximum of the central lobe toward the aperture, another quite interesting effect appears. This additional effect is due to the existence of a lateral maximum that is closer to the aperture with a relatively high value of irradiance and relatively far from the geometric focus [see Fig. 2(a)]. This fact permits the term $1/(f + z)^2$ to have a great increase in value in the axis zone where this lateral maximum is located. Then, when the product of the two terms of Eq. (4) is done to give the axial-intensity distribution, it results that the height of the lateral lobe is approximately the same as that of the central lobe. This fact implies that in practice it is rather difficult to distinguish between these two maxima.

It is clear that this effect becomes more significant as the value of the Fresnel number decreases. Therefore it follows that for Fresnel-number values lower than $N = 10$, the height of the lateral lobe is greater than that of the central lobe. The result is an effective permutation of the focal point. We refer to this as the focal-switch effect, which to the best of our knowledge has never been reported on.

To illustrate this result we represent in Fig. 3 the normalized version of function $I(z)$ for screen $q(\zeta) = 4\zeta^2$ and for $N = 5$. Note from this figure that now the maximum of the lateral lobe is ~25% higher than that of the central lobe.

To determine the range of Fresnel numbers for which the focal-switch effect takes place, in Fig. 4

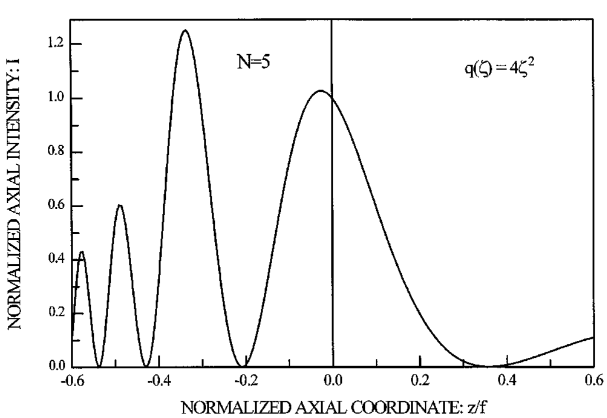

Fig. 3. Normalized intensity distribution corresponding to the axially superresolving screen, $q(\zeta) = 4\zeta^2$, and $N = 5$.

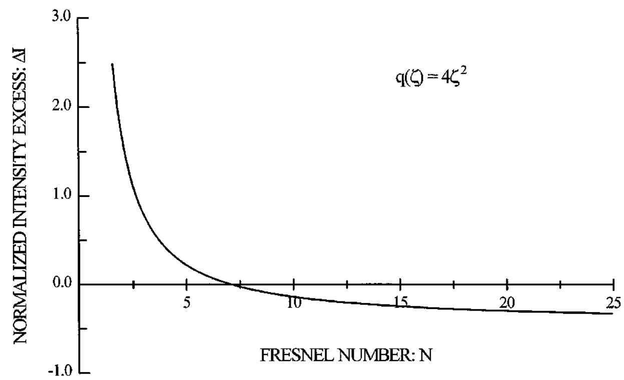

Fig. 4. Relative excess, $\Delta I = (I_L - I_C)/I(0)$, of the maximum intensity I_L of the lateral lobe over the maximum intensity I_C of the central lobe for systems with different Fresnel numbers.

we have represented the relative excess, $\Delta I = (I_L - I_C)/I(0)$, of the maximum intensity, I_L, of the lateral lobe over the maximum intensity, I_C, of the central lobe. We see that at $N < 7.1$ the focal-switch effect takes place, whereas at $N < 2.5$, ΔI increases dramatically as N decreases.

Finally it is important to point out that, if we define the focal-shift effect strictly as a variation suffered by the position of the maximum of the axial intensity when the Fresnel number is low, we find that for a certain kind of screen, which in principle has low sensitivity to the focal shift, in practice, because of the focal-switch effect, a very great displacement of the position of the maximum and then a very great focal shift results.

Summarizing, we have stated that for a fixed value of the Fresnel number, the sensitivity of a diffracting screen to the focal-shift effect is closely connected with its capacity to produce axial apodization. In this context we found, for sufficiently low values of the Fresnel number, that for certain axially superresolving screens, simultaneously with the predicted focal-shift effect, an increase in the irradiance of the lateral lobe over the irradiance in the central maximum takes place, resulting in an effective permutation of the focal point. To illustrate our result, we have shown a numerically evaluated example.

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