



## UvA-DARE (Digital Academic Repository)

### Focused electron emission from planar quantum point contacts

de Raedt, H.; Garcia, N.; Saenz, J.J.

**DOI**

[10.1103/PhysRevLett.63.2260](https://doi.org/10.1103/PhysRevLett.63.2260)

**Publication date**

1989

**Published in**

Physical Review Letters

[Link to publication](#)

**Citation for published version (APA):**

de Raedt, H., Garcia, N., & Saenz, J. J. (1989). Focused electron emission from planar quantum point contacts. *Physical Review Letters*, 63(20), 2260-2263. <https://doi.org/10.1103/PhysRevLett.63.2260>

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

## Focused Electron Emission from Planar Quantum Point Contacts

H. De Raedt,<sup>(1,2)</sup> N. García,<sup>(3,4),(a)</sup> and J. J. Sáenz<sup>(5)</sup>

<sup>(1)</sup>Physics Department University of Antwerp, Universiteitsplein 1, B-2610 Wilrijk, Belgium

<sup>(2)</sup>Natuurkundig Laboratorium, University of Amsterdam, Valckenierstraat 65,  
1018 XE Amsterdam, The Netherlands

<sup>(3)</sup>Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

<sup>(4)</sup>IBM Research Division, Zurich Research Laboratory, CH-8803 Rüschlikon, Switzerland

<sup>(5)</sup>Departamento Materia Condensada, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

(Received 21 June 1989)

A planar quantum-point-contact device is proposed capable of producing focused electron beams. This solid-state device consists of a tunnel barrier attached to a quantum point contact. By solving the time-independent and time-dependent Schrödinger equation of the 2D system, it is shown that the angular spread of the emitted beam can be reduced to less than  $10^\circ$ , is coherent and collimated in energy, and is able to produce interferences. By applying a magnetic field for which electron orbits have a radius of  $2 \mu\text{m}$ , it should then be possible to focus the beam down to  $0.15 \mu\text{m}$ .

PACS numbers: 73.40.Gk, 73.50.Jt

Progress in technology to fabricate devices of a size comparable with the electron wavelength has made it possible to study physical phenomena on microscopic and mesoscopic length scales. These devices provide a unique laboratory to investigate all kinds of quantum interference effects. Experimental work on point sources, ultrasharp<sup>1</sup> and teton tips,<sup>2</sup> has demonstrated that the properties of the field-emitted electron beam differ markedly from those of beams emitted by conventional sources. This may prove to be of a great importance for electron holography and interferometry.<sup>3-7</sup> In GaAs heterostructures such electron sources have been realized by connecting two parts of a high-mobility two-dimensional electron gas (2DEG) through a narrow constriction.<sup>8,9</sup> Experimentally it has been demonstrated that the conductance is quantized, the quantization being a function of the width of the constriction. Theoretical calculations of the conductance of electrons moving from one reservoir via a constriction to another reservoir are in agreement with experiment.<sup>10-15</sup>

All these experiments have in common that the electrons are emerging from a narrow (few electron wavelengths) constriction. A fundamental difference between the atomic tips and the GaAs devices is that in the latter case the electron has to move through a constriction only whereas in the former it also has to tunnel through the metal-vacuum potential. Theoretical calculations<sup>16</sup> have shown that the presence of a planar tunnel barrier is essential to focus the field-emitted electron beam and to explain the salient features of the experiments<sup>1,2</sup> on a qualitative level.

The aim of the present paper is to propose a solid-state device which is capable of producing collimated electron beams. A schematic picture of the geometry of the device, together with an energy diagram, is shown in Fig. 1. The two reservoirs, having a Fermi wavelength  $\lambda_1$  and  $\lambda_2$ , are connected by a constriction, created by applying a

potential  $V$  in shaded area I.<sup>8,9</sup> The constriction, characterized by its width  $W$  and its length  $L$ , quantizes the conductance as a function of  $W$  (or  $V$ ). From analogy with the field-emission model<sup>16</sup> we want to introduce into this device a *tunnel barrier* of a thickness  $s$  and a strength  $\phi$  in order to focus the current. This might be accomplished by depositing a layer (region II, see Fig. 1) of GaAlAs of a thickness  $s$  or by inducing a potential barrier.<sup>17</sup> In contrast to the field-emission case where, to a good approximation, the barrier has a triangular shape, in the GaAs device it will more closely resemble a rectangular potential well. Inducing a triangular tunnel barrier is perhaps possible by linearly changing the Ga-As composition as a function of the distance in the layer. Although the focusing power of a rectangular barrier is

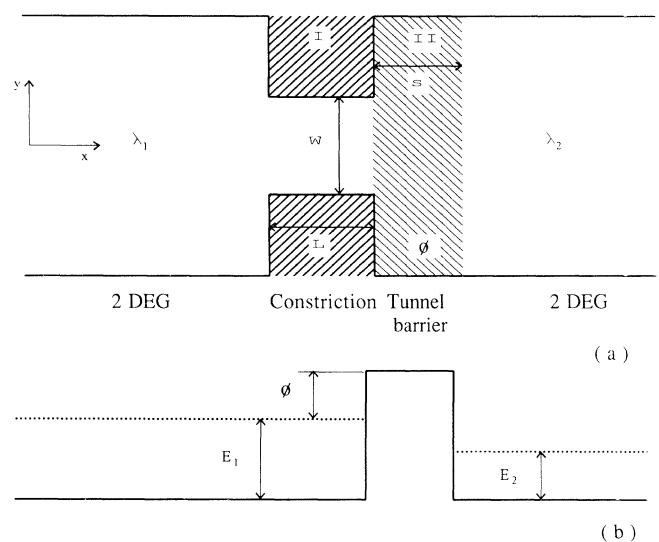


FIG. 1. Schematic layout and corresponding energy diagram of the proposed GaAs device.

less than that of a triangular barrier,<sup>16</sup> in both cases it is substantial. By increasing the doping in the reservoir 2, i.e., by increasing the Fermi energy (or reducing  $\lambda_2$ ), the electrons would be accelerated just outside the constriction, leading to an extra electron focusing similar to the field focusing in field emission. This could be mimicked also by applying a large potential between reservoirs 1 and 2.

To examine the focusing properties of this device theoretically we have solved the appropriate stationary Schrödinger equation (SSE) and time-dependent Schrödinger equation (TDSE). Assuming that the electrons inside the metal can be modeled by a free-electron gas, the total intensity of emitted electrons can be written as

$$\frac{I}{V} = \frac{e^2}{\pi\hbar} \sum_f \sum_i T_{f,i}(E_F) = \frac{e^2}{\pi\hbar} \sum_f T(\theta_f), \quad (1)$$

where  $E_F$  is the Fermi energy and the summation runs over all angles of incidence  $\theta_i$  and over all possible outgoing angles  $\theta_f$ . To solve the SSE and calculate  $T_{f,i}(E_F)$  a "matching scattering technique" is employed which has proven itself in scanning tunneling microscopy,<sup>18</sup> quantum resistances of interfaces,<sup>19</sup> and quantum point contacts.<sup>10</sup> This technique consists of matching the wave function in the metal, i.e., a linear superposition of plane waves (incident plus reflected waves), with the appropriate linear combination of waves in the tunnel barrier and in the vacuum region.

In Fig. 2 we present the outcome of some model calculations for the intensity as a function of the outgoing angle  $\theta_f$ ,  $I(\theta_f) \propto \sum_i T_{f,i}(E_F) = T(\theta_f)$ . For curve *a* we have chosen  $\lambda_1 = \lambda_2 = 335 \text{ \AA}$ , a Fermi energy  $E_F = 20$

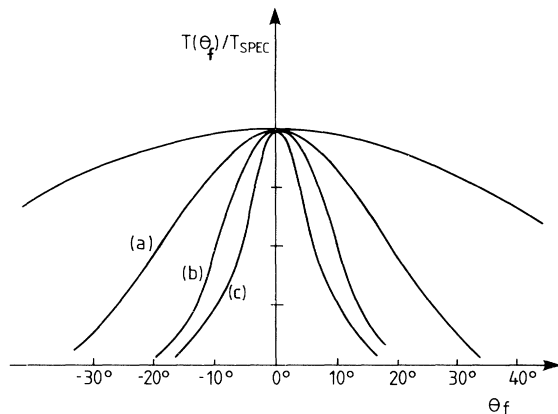


FIG. 2. Angular distribution of electrons emitted by a quantum point contact, modeling the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  device depicted in Fig. 1, for the case without barrier. The curves labeled *a*, *b*, and *c* are with barrier and for different parameters: *a*,  $\lambda_1 = \lambda_2 = 335 \text{ \AA}$ ; *b*,  $\lambda_1 = 335 \text{ \AA}$  and  $\lambda_2 = 167 \text{ \AA}$ ; and *c*, for the same wavelengths as in *b* but for a thicker barrier.  $T_{\text{SPEC}}$  denotes the total transmitted intensity in the direction normal to the point contact. SSE calculation.

meV,  $\phi = 20 \text{ meV}$  corresponding to a tunnel decay length of  $53 \text{ \AA}$ , a  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer of thickness  $s = 371 \text{ \AA}$  and  $x = 0.06$ . We have taken into account that the effective mass of the electron is that of GaAs, equal to 0.067 the bare electron mass. Curve *b* is obtained from a similar calculation but instead of  $\lambda_2 = 335 \text{ \AA}$ , we have taken  $\lambda_2 = 167 \text{ \AA}$ . In all the cases we take  $W = 2\lambda_F$ . For comparison the results without tunnel barrier are also shown.

From Fig. 2 it is seen that the electron beam, emerging from a device consisting of only a constriction, has a very large angular spread ( $\sim 60^\circ$ ), defined by the angle at which  $I(\theta_f)$  drops a factor  $1/e$  (i.e., half of the total angular width). It has been suggested<sup>20,21</sup> that a constriction of smoothly, adiabatically changing shape<sup>22</sup> can also focus the electron beam. However, the adiabatic approximation only holds if the width of the constriction changes in a region of length  $S \gg \lambda_F$ .<sup>22,23</sup>

Switching on a tunnel barrier greatly reduces the angular spread. As shown in Fig. 2, the angular spread can be reduced up to  $\lesssim 10^\circ$ .<sup>16</sup> To illustrate the effect of the thickness of the tunnel barrier, Fig. 2 curve *c* depicts the results of a calculation for the same parameters as for curve *b* but for a thicker barrier ( $530 \text{ \AA}$ ) which is approximately 1.5 times the wavelength of the electrons in reservoir 1. Our calculations clearly demonstrate that the method we propose is quite efficient in focusing the beam. This focusing effect should be stronger the lower the tunnel barrier is (for  $\phi > 0.1E_1$ ), the thicker the GaAlAs deposit (i.e., the thicker the tunnel barrier) is, and the smaller  $\lambda_2$  is. For a given current, the optimal focusing effect is obtained for a thick and shallow barrier. The ideal is a triangular barrier having a low tunnel barrier and a small slope.

Another complementary method to analyze the focusing properties of the constriction-tunnel-barrier system is to follow the time evolution of the electron wave packet by solving the TDSE for a quantum particle that moves towards the constriction and tunnels through the potential barrier  $V(\mathbf{r})$ . In our case,  $V(\mathbf{r}) = \infty$  in region I and at the boundary of the simulation box,  $V(\mathbf{r}) = \phi$  in region II (the tunnel barrier), and  $V(\mathbf{r}) = 0$  otherwise. The computer simulation consists of preparing a wave packet and letting the wave packet evolve in time according to the TDSE. The TDSE is solved numerically by means of an algorithm based on a fourth-order Trotter formula.<sup>24</sup>

A convenient choice for the initial wave packet is a Gaussian of width  $\kappa$  moving towards the constriction with wave vector  $\mathbf{q} = (q_x, q_y)$ . By changing the center of the initial wave packet or the angle  $\theta_i = \arctan(q_y/q_x)$  the way in which the packet approaches the constriction can be altered. The energy of the incident packet is chosen to be very close  $E_F$  and  $|\mathbf{q}| = q = \kappa_F$ . After (part of) the wave packet has traveled through the constriction and has tunneled through the potential barrier (if present), a Fourier transform with respect to the spatial coordinates of the transmitted wave packet  $\tilde{\psi}(\mathbf{r}, t)$  is carried out.

Constriction  $L=5\lambda_F, W=2\lambda_F$   
 Transmitted wave: Fourier space  
 $\Theta_i = 14^\circ, T(\Theta_i) = 6010^{-2}$

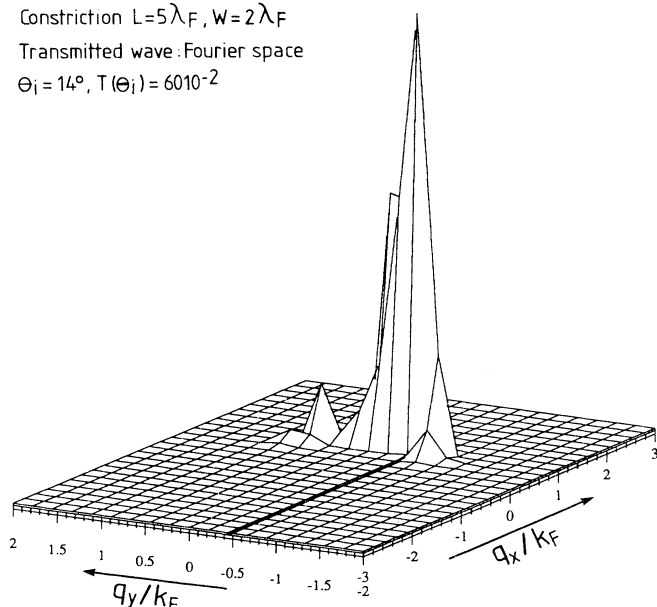


FIG. 3. Fourier transform of the wave packet emitted by a constriction. The direction of propagation corresponds to  $q_x$ . TDSE simulation.

Relevant for the focusing capacity of the device is the degree in which it redirects waves of non-normal incidence. In Fig. 3 the  $|\tilde{\psi}(\mathbf{q}, t)|^2$  is shown of a packet approaching a constriction of width  $W=2\lambda_F$  and length  $L=5\lambda_F$  but without tunnel barrier. Clearly the angle of incidence ( $14^\circ$  in this case) shows up in the transmitted packet as in shift of the maximum with respect to  $q_y=0$ . Also  $|\tilde{\psi}(\mathbf{q}, t)|^2$  is fairly broad in  $q_y$  and has a lot of structure indicating that there is a lot of diffraction at the exit plane of the constriction. Turning on the tunnel barrier ( $\phi=0.5E_F$  in these simulations) changes  $|\tilde{\psi}(\mathbf{q}, t)|^2$  drastically, as exemplified in Fig. 4. The shift of the peak position is zero (without numerical accuracy) and the width in  $q$  space in the transverse direction is much smaller than in the case without tunnel barrier. Simulations for other model parameters invariably lead to the same conclusion.

The origin of the focusing effect of the tunnel barrier can be understood with a simple picture. For large constriction widths, only the electrons within a small cone of angles around the normal incidence can go through the tunnel barrier and hence the emitted beam is focused. When the constriction is small ( $\sim\lambda_F$ ), there is a lot of diffraction at the end of it, but each of these diffracted waves is filtered by the tunneling barrier. The WKB approximation provides a nice picture to explain this effect. For a wave component having a transverse momentum  $q_T$  and energy  $E_1$  the tunneling probability for  $\phi \gg q_T^2$  reads

$$e^{-s(\phi+q_T^2)^{1/2}} \approx e^{-s\sqrt{\phi}} e^{-sq_T^2/2\sqrt{\phi}}, \quad (2)$$

Constriction  $L=5\lambda_F, W=2\lambda_F$   
 Rectangular barrier  
 Transmitted wave: Fourier space  
 $\Theta_i = 14^\circ, T(\Theta_i) = 6410^{-6}$

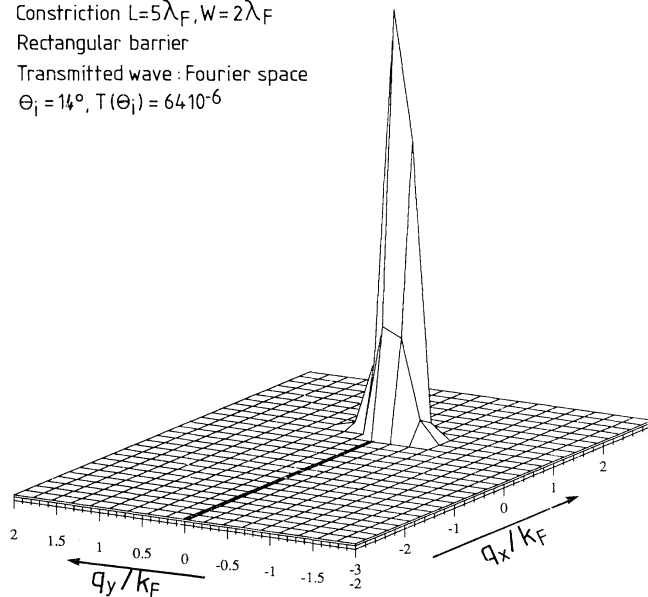


FIG. 4. Fourier transform of the wave packet emitted by a constriction followed by a rectangular barrier. TDSE simulation.

explicitly showing the attenuation (first factor) and focusing (second factor) due to the tunneling barrier. A potential of height  $\phi < 0$  will also focus the beam.<sup>20,21</sup> This mechanism is effective if  $\phi > -0.1E_1$ , for all  $W$  if the barrier is outside the constriction and for  $W \gtrsim 10\lambda_1$  if the barrier is inside the constriction. Note that the limit  $\phi \rightarrow 0$  is subtle.

The potential barrier focuses the current and increases the coherence and collimation of the beam such that strong interference effects can be produced. It also reduces the intensity of the current emitted by the quantum point contact. Thus, with tunnel barrier, the voltage required to get a certain current will be larger than without, but in return, the current will be much more focused because the effective value of  $\lambda_2$  will be reduced. Therefore, in practical applications, a compromise has to be found between having a small angular spread and a large current. Having obtained a focused electron beam, it should be possible to apply a magnetic field,<sup>25</sup> and direct the focused beam to specific areas in reservoir 2. In this manner one should be able to direct the electrons to several different gates, opening the possibility to generate strong interference effects. This may prove to be of interest for technological applications. On the basis of our calculations we estimate that an electron traveling  $2 \mu\text{m}$  can be focused on an area of a linear size of about  $0.15 \mu\text{m}$ .

We want to thank IBM Research Laboratories, Zurich, for its kind hospitality, and we deeply appreciate extensive interactions with Dr. H. Rohrer. We are grateful to L. Escapa, A. Jacoby, and Y. Imry for help-

ful discussions. The work of N.G. and J.J.S. is partially supported by a joint agreement between the Universidad Autónoma de Madrid and IBM Research Laboratories, Zurich. H.D.R. would like to thank Control Data Corporation (The Netherlands) for a generous grant of computer time on the CYBER 205, the University of Leuven for providing unrestricted access to their IBM 3090-300/VF, and the Belgian National Science Foundation for financial support.

<sup>(a)</sup>Permanent address: Departamento Materia Condensada, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.

<sup>1</sup>H-W. Fink, IBM J. Res. Dev. **30**, 460 (1986); Phys. Scr. **38**, 260 (1988).

<sup>2</sup>Vu Thien Binh and J. Marien, Surf. Sci. **102**, L539 (1988); Vu Thien Binh, J. Microsc. **152**, 355 (1988).

<sup>3</sup>N. Garcia and H. Rohrer, J. Phys. Condens. Matter **1**, 3737 (1989); P. Serena, L. Escapa, J. J. Saenz, N. Garcia, and H. Rohrer, J. Microsc. **152**, 43 (1989).

<sup>4</sup>D. Gabor, Proc. Roy. Soc. London A **454**, 197 (1949); B **64**, 449 (1951).

<sup>5</sup>A. Tonomura, Rev. Mod. Phys. **59**, 639 (1987).

<sup>6</sup>H. Lichte, Ultramicroscopy **20**, 293 (1986).

<sup>7</sup>F. G. Missiroli, G. Pozzi, and U. Valdre, J. Phys. E **14**, 649 (1981).

<sup>8</sup>B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).

<sup>9</sup>D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H.

Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988).

<sup>10</sup>N. Garcia and L. Escapa, Appl. Phys. Lett. **54**, 1418 (1989).

<sup>11</sup>L. Escapa and N. Garcia, J. Phys. Condens. Matter **1**, 2125 (1989).

<sup>12</sup>B. Kramer and J. Masek, J. Phys. C **21**, L1147 (1989).

<sup>13</sup>A. Szafer and D. Stone, Phys. Rev. Lett. **62**, 300 (1989).

<sup>14</sup>G. Kirkcenow, Solid State Commun. **68**, 715 (1989).

<sup>15</sup>E. G. Haanappel and D. van der Marel, Phys. Rev. B **39**, 5484 (1989).

<sup>16</sup>N. Garcia, J. J. Saenz, and H. De Raedt, J. Phys. Condens. Matter (to be published).

<sup>17</sup>A. Palevski *et al.*, Phys. Rev. Lett. **62**, 1776 (1989).

<sup>18</sup>N. Garcia, C. Ocal, and F. Flores, Phys. Rev. Lett. **50**, 2002 (1983); E. Stoll, A. Selloni, and P. Carnevali, J. Phys. C **17**, 3073 (1984).

<sup>19</sup>N. Garcia and E. Stoll, Phys. Rev. Lett. **37**, 445 (1988).

<sup>20</sup>C. W. J. Beenakker and H. van Houten, Phys. Rev. B **39**, 10445 (1989).

<sup>21</sup>A. Yacoby and Y. Imry, "On the quantization of the conductance of smooth point contacts beyond the adiabatic approximation," (to be published); (private communication).

<sup>22</sup>L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 239 (1988)].

<sup>23</sup>L. Escapa, N. Garcia, H. De Raedt, and J. J. Saenz (to be published).

<sup>24</sup>H. De Raedt, Comp. Phys. Rep. **7**, 1 (1987).

<sup>25</sup>H. van Houten, B. J. van Wees, J. E. Mooij, C. W. Beenakker, J. G. Williamson, and C. T. Foxon, Europhys. Lett. **5**, 721 (1988).