



Research Article

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Folded surface elements coupled with planar scissor linkages: A novel hybrid type of deployable structures**

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Abstract: Origami folding structures can find significant applications in the general area of building design, as they can be lightweight and deployable. An inherent property of folded surfaces, which is related to the degrees of freedom of each origami crease pattern, is form flexibility. Therefore, when building-scale applications are considered, in many instances, the folded surfaces, in order to become stiff and load-bearing, need to be constrained. A study of different types of deployable structures has led to the observation that in planar scissor linkages, hinges and pivots follow the same deployment path in space, as sets of vertices in certain origami structures. Due to the similarities in their kinematic behavior, selected origami patterns and scissor linkages can function as effective kinematic pairs, leading to structures able to transform in a controlled manner through a wide range of possible spatial configurations. A few examples of combining these two types of structures already exist [1–3]. In this paper a systematic approach for coupling origami crease patterns characterized by biaxial and rotational symmetry, with translational, polar and angulated scissor linkages, towards the development of novel forms of deployable structures, has been attempted. For the design and evaluation of the kinematic performance of the developed new structures, existing geometric modeling and calculation methods, parametric and simulation processes, as well as testing with physical models have been used. It is anticipated that research in this direction will lead to promising novel hybrid types of deployable structures.

Keywords: transformable structures, kinetic architecture, origami, scissors, folding mechanisms, transformable ar-

chitecture, geometry, structural origami, parametric modelling, mechanisms design

1 Introduction

The interest in deployable structures and their applications has significantly increased since the second half of the 20th century [4]. Deployable structures, defined as structures capable of large configuration changes in an autonomous way [5], can expand and/or contract due to their geometrical, material and mechanical properties [6, 7]. In the listing of the prevailing types of deployable structures, that includes inflatable structures, tensioned membranes, scissor-hinge linkages, tensegrity structures [8] etc., origami has gained the attention of engineers and architects, due to its aesthetic, geometric, structural and elastic qualities [9]. Potential applications of origami structures include rapidly erected shelters, deployable structures that respond to weather changes, foldable solar panels etc. In addition, light and sound diffusion, which can be considered as an intrinsic property of folded surfaces, makes them suitable for other challenging architectural applications that address the lighting and acoustics of spaces [10]. However, in many instances, when building-scale applications are considered, the origami surfaces, need to be constrained in order to become stiff and load-bearing. The approach taken in this study is to attempt coupling origami surfaces with other mechanisms, specifically with scissor linkages, in order to develop rigid structures able to transform in a controlled manner.

In the following sections, the main features and properties of origami surfaces and methods for coupling origami with other mechanisms are discussed.

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2 Structural origami: main features and properties

Many researchers with expertise in different disciplines have studied the kinematic behaviour of origami structures, including Tomohiro Tachi, whose research particularly focuses on architectural applications, and Robert J. Lang and Erik D. Demaine who have significantly contributed to the study of the origami underlying mathematics and its geometric properties [11–13].

An origami model is often described by the imprint of its folding process in a fully unfolded state, the so-called crease pattern (CP). An origami CP consists of fold lines of two types: mountain and valley folds, or, posed differently, convex and concave folds. Each CP surface confined by fold lines is called a facet or plate of the origami structure. A typical attribute of some origami CPs is flat-foldability. An origami model is considered as flat-foldable, if it can reach a state in which all fold lines are folded 180° , the model undergoes no collisions, and rests completely flat *i.e.* all facets are planar.

Rigid-foldability, another attribute of some origami CPs, can be very useful in various engineering applications. Rigid origami is an origami type, the facets of which neither stretch nor bend, throughout the folding process [14]. In general, rigid origami structures can be considered as scalable mechanisms of plates and hinges that produce a unified and synchronous movement of all creases, and possess less degrees of freedom (DOF) than non-rigid ones.

In a mathematical context, origami is regarded as an ideal zero-thickness surface. However, especially in large-scale structures, material thickness should not interfere with the folding motion and therefore it should be addressed appropriately. Various thickness accommodating techniques have been proposed by researchers in the field, including shifting the rotational axis to the material edges, and tapering material to the rotational axis of the creases [11, 15].

Structural origami has yet found limited applications in architecture and building design in general. One significant reason for this, is its form flexibility, resulting from facet deformation in non-rigid origami, or from multiple DOFs in rigid origami structures. It can be observed that constraining the edges of folded surfaces, increases their stability and load carrying ability [16], as shown in Figure 1. Constraining the edges in a large-scale structure, though, would be impractical, if such a structure is expected to transform frequently.

A possible solution for constraining folded surfaces could be addressed with the addition of a complementary mechanism that follows the same deployment path in space, as edge vertices in certain origami CPs. The ability to be restrained in several stages of the deployment process constitutes a basic requirement that this mechanism needs to fulfil.

In the following sections, initial studies for restraining origami folded surfaces that make use of scissor linkages, as well as methods for coupling certain types of origami CPs with scissor linkages of similar motion, are presented.

3 Initial studies: coupling cases

Considering the geometry of origami structures, we note that most flat-foldable CPs include at least two boundary edges consisting of a sequence of alternating mountain and valley folds, that locally fold and unfold resembling an accordion-pleated surface. A review of various types of deployable structures has led to the observation that the hinges and pivots of planar scissor linkages follow a similar path in space as selected sets of edge vertices in certain origami CPs. Scissor linkages are scalable single-DOF mechanisms that consist of multiple scissor-hinged bar pairs. All scissor-pair components are linked, so that the rotation of one local assembly affects the behaviour of the entire mechanism. This principle of propagation reduces the actuation and control mechanism to one variable, specifically

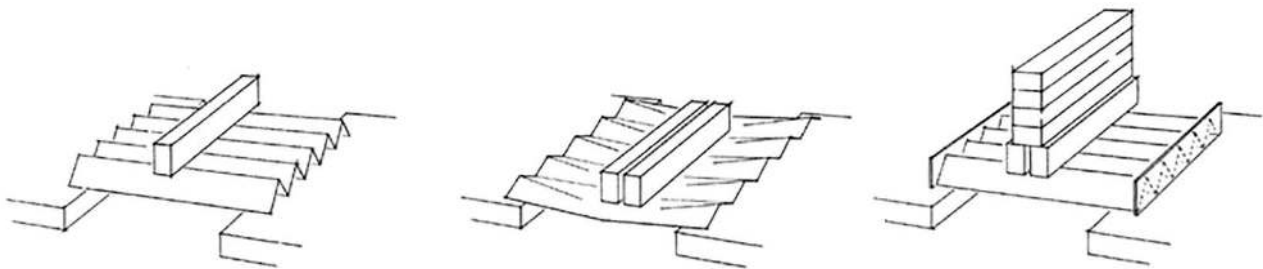


Figure 1: Constraining the edges of folded surfaces increases their stability and load carrying ability

to the rotation of only one component. It also determines the synchronized and smooth transformation between fully retracted and deployed positions [17].

The most common types of planar scissor linkages, namely translational, polar and angulated, have been initially considered, for coupling surfaces with alternating mountain and valley folds, usually referred to as ‘accordion’ CP. Three general coupling cases are categorized and examined (Figure 2).

The translational scissor unit consists of two straight bars of length l , linked to each other at their midpoints by a revolute joint (pivot). Each unit connects to the next one at the upper and lower hinges by revolute joints as well. In the translational scissor linkages, the lines connecting each pair of upper and lower hinges, called unit lines, remain parallel during all phases of deployment.

The polar scissor unit consists of two straight bars of length l , linked to each other at an intermediate point other than the midpoint, thus dividing the bar into vary-

ing lengths l_1 and l_2 . In this way, the upper hinges, pivots and lower hinges lie on concentric circles. The unit lines intersect at the centre of these circles, forming angle φ , which varies during deployment and, as a result, the intersection point moves closer to the unit as the curvature increases [18].

In an angulated scissor unit consisting of two angulated bars with a central kink of amplitude ω , each bar is divided by the pivot in equal parts of length l_1 . Unlike polar scissors, the segment angle φ between two angulated scissor unit lines is constant during deployment. Therefore, angulated scissors can be used for radially deploying closed-loop structures [19]. The relationship between the φ and ω angles is given by equations (1) and (2), where ψ is the angle between one part of the angulated bar and the line connecting the upper and lower hinge of the same bar. The length l of this line can be calculated by equation (3).

$$\psi = \frac{\varphi}{2} \tag{1}$$

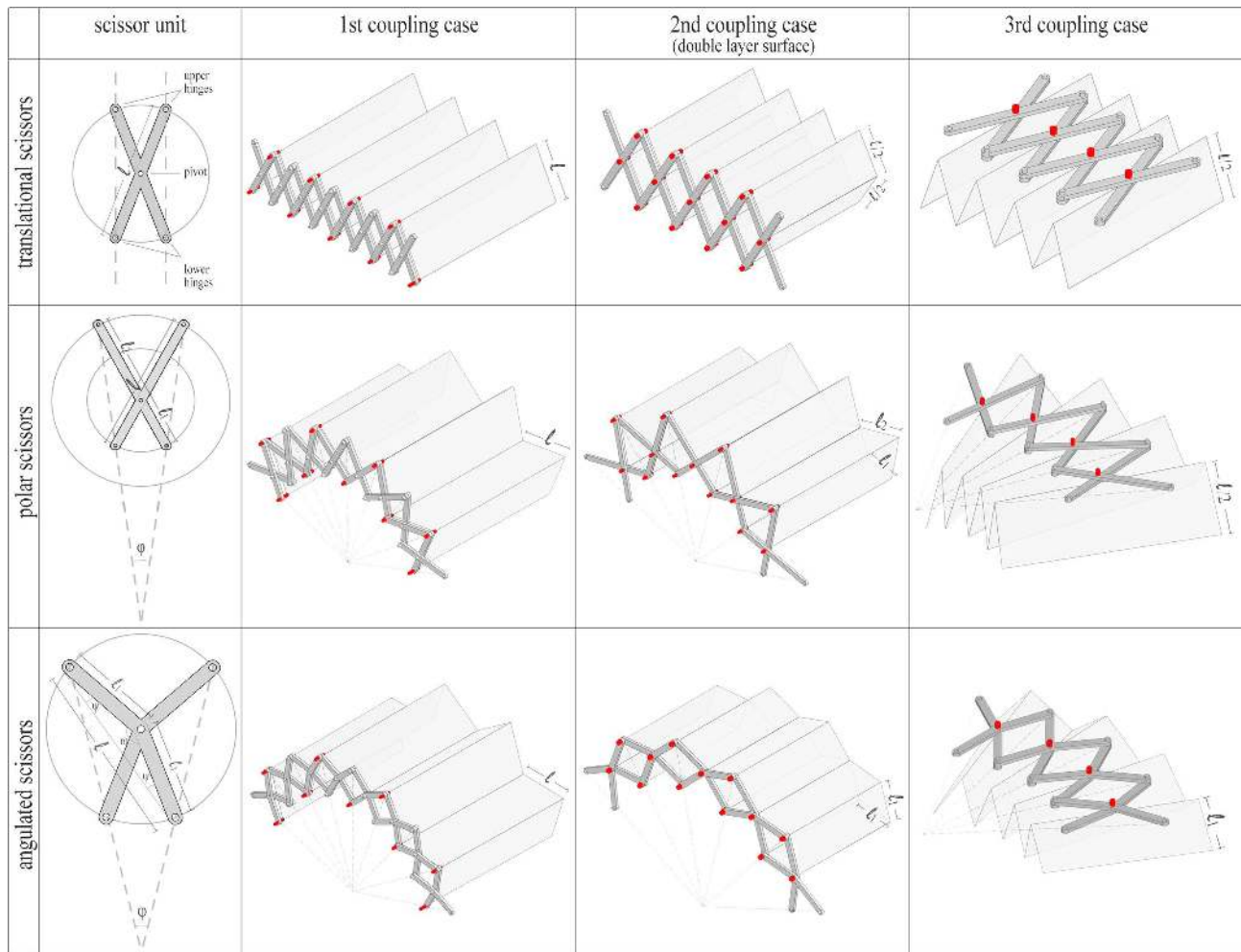


Figure 2: Scissor units and three general cases of coupling translational, polar and angulated linkages with accordion pleated surfaces

$$\omega = 180 - 2\psi \quad (2)$$

$$l = 2l_1 \cos \psi \quad (3)$$

In the first coupling case that we studied, the folded surface's width is a multiple of l , depending on the scissor type. The surface edge vertices are anchored on every alternate upper and lower scissor hinge. In Figure 2 the anchors are marked with red colour. The second coupling case could find applications in architecture when a double layer surface is needed, such as in spaces that need heat insulation or sound isolation. The upper folded surface is anchored on scissor pivots and upper hinges, whereas the lower surface on pivots and lower hinges, respectively.

The resulting mechanisms of the first and second coupling cases possess one degree of freedom, assuming that the accordion folded surfaces consist of rigid plates. In the case of non-rigid origami, folds tend to 'open up', as they diverge from the scissor linkage.

The third coupling case could be utilized in situations where a large span of folded surface needs to be controlled and/or stiffened at intermediate points rather than only at the edges, or when the plane of the scissors linkage is required to remain parallel to the surface's deployment plane. The third case of coupling polar and angulated scissor linkages with accordion folded surfaces is feasible only if a critical bending of the facets is allowed (non-rigid origami) or, if the facets are triangulated with complementary fold lines.

4 Methodology

The methods used to design and analyse the kinematics of the studied hybrid deployable structures that occur from the coupling of folded surfaces with scissor linkages include parametric modelling and kinematic simulation tools, as well as testing with small-scale physical models. The parametric definition of the scissors and origami structures is developed in the Grasshopper[®] visual programming environment of Rhinoceros[®] 3D software, whereas for the simulation of the origami folding process, the interactive live-physics plug-in Kangaroo has been utilized. Specifically, the developed algorithm involves three basic steps. Initially, scissor linkages are defined geometrically in Grasshopper according to their type (translational, polar, angulated, or combinations) and input parameters, such as bar length, pivot shift, kink angle, unit count, etc. are entered. The second step involves the creation of a mesh of the origami CP, according to the variables of the scissors definition. At the third step, the constraints such as facet planarization and

anchors are defined in the Kangaroo engine, which is used for the simulation of the folding process of the surface.

Physical models of the mechanisms were also constructed to test the foldability and stiffness of certain systems.

5 Hybrid deployable structures: typological and morphological studies

Several types of hybrid deployable structures have been developed and studied. They are classified in the following sections according to their spatial deployment method, the origami CP used for each structure, and the configurations of the structure before, during and after deployment. The studies have been focused on the geometrical construction and possible variations of each hybrid structure, as well as the analysis of the motion and degrees of freedom of the mechanisms.

5.1 Uniaxial transformation surface structures

Uniaxial transformation structures can be constructed using two planar scissor linkages mounted on the upper and lower edges of a double layer accordion pleated surface, as shown in Figure 3a. The first and the last scissor pair of each linkage is trimmed on their pivots, so that the two folded surfaces can be joined together on the right and left edge of the structures. Vertical bars, placed at these two extreme scissor pivots, force the two linkages to maintain a parallel position to each other while deploying. Vertical bars could also be placed at any intermediate pivot pair of the linkages, if needed, in order to stiffen larger spans. The single-DOF resulting mechanism can be mounted on a vertical axis, at any of the scissor pivot points with, or without the ability to rotate around it. These structures, can be arranged from a fully retracted (left) to expanded (right) configurations and can be restrained at any of their intermediate states, by constraining the rotation of only one of the scissor pairs.

In a similar manner, curvilinear panels can be constructed using polar (Figure 3b) or angulated scissor linkages (Figure 3c). Although these structures present identical configurations at certain points during their deployment, their kinematic behaviour is different, considering the increasing curvature of polar scissor linkages in con-

trast to the constant curvature of angulated ones [8], as seen in their expanded configurations.

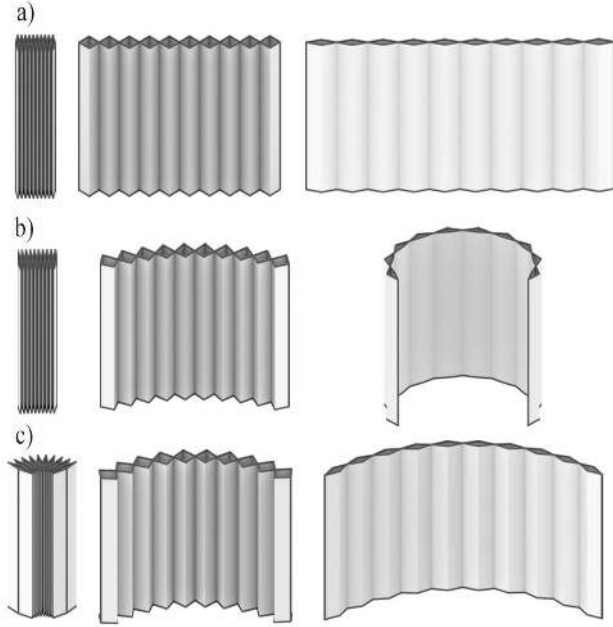


Figure 3: Deployment sequences of uniaxial transformation structures consisting of a) translational, b) polar and c) angulated scissor linkages

5.2 Biaxial transformation surface structures

Biaxial transformation surface structures can be constructed using the miura-ori origami CP, primarily selected due to its single-DOF property. The miura-ori unit, as shown in Figure 4a, consists of four identical parallelogram plates, and can be defined by three parameters: side lengths α and b , and sector angle φ . We note that in a rigid miura-ori unit (Figure 4c), edge vertices A, B, C remain on xy plane and vertices C, D, E on xz plane, throughout the folding process.

In order to create a linkage that transforms in accordance to these miura-ori edge vertices (A, B, C, D, E), two scissors that lie on perpendicular planes are coupled, connected on the edge of their intersecting planes. The resulting mechanism (Figure 5) consists of two pairs of linkages: the horizontal pair (bottom and top) formed of translational scissors of bar length equal to 2α , and the vertical pair (left and right) formed of translational scissors of bar length equal to $2b$. Although in this case the two pairs of scissor linkages follow a parallel motion, before coupling the linkages with the folded surfaces, the motion of the horizontal linkages pair is independent of the motion of the

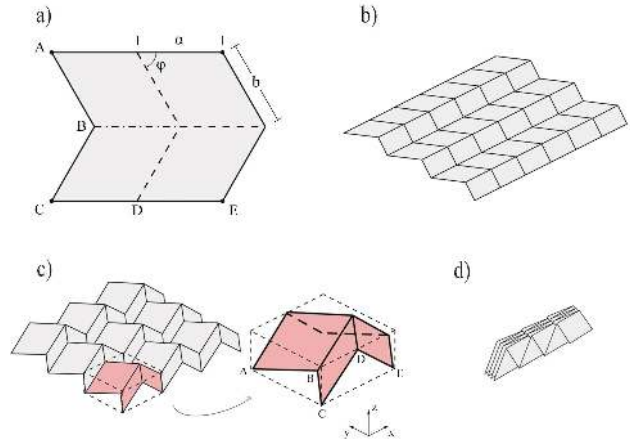


Figure 4: Miura-ori CP: a) unit, b) tessellation, c) half-way folded tessellation and unit detail, d) fully folded configuration

vertical linkages pair, forming a 2-DOF mechanism. When the scissor linkages are coupled with a double layer miura-ori folded surface, the mechanism becomes single-DOF. Modifying the sector angle φ of the miura-ori CP, the deployment path of the structure changes. In this case, the structure is able to retract in a more compact configuration for a maximum φ of almost 90° (Figure 5b).

By altering the alignment of zig-zag creases on of a miura-ori CP, an arc-miura pattern is generated (Figure 6). The edge vertices of a fully folded arc-miura tessellation lie along arc profiles, as seen in Figure 6c.

The arc-miura unit (Figure 6a) is constructed from four identical trapezoidal surfaces defined by four distinct side lengths α_1 , α_2 , b_1 and b_2 as well as sector angles φ_1 and φ_2 , with the stipulation that $\varphi_1 > \varphi_2$, $\alpha_1 < \alpha_2$, and $b_1 < b_2$. Among these six parameters only four are independent, as they have to meet the conditions:

$$b_1 \sin \varphi_1 = b_2 \sin \varphi_2 \quad (4)$$

$$\alpha_2 + b_1 \cos \varphi_1 = \alpha_1 + b_2 \cos \varphi_2 \quad (5)$$

In a half-folded configuration (Figure 6b), three dihedral angles θ_A , θ_{MZ} , and θ_{VZ} , and four edge angles η_{MZ} , η_{VZ} , η_{MA} , and η_{VA} can be defined [20]. As shown in a fully folded configuration (Figure 6c) each unit is subtended by fold angle ξ . This angle is null when the surface is unfolded and increases while folding till the maximum ξ angle of the fully folded configuration. Dihedral and edge angles as well as the unit angle are given by equations (6)–(12).

$$(1 + \cos \eta_{MZ}) (1 - \cos \eta_{MA}) = 4 \cos^2 \varphi_1 \quad (6)$$

$$\cos \eta_{MA} = \sin^2 \varphi_1 \cos \theta_{MZ} - \cos^2 \varphi_1 \quad (7)$$

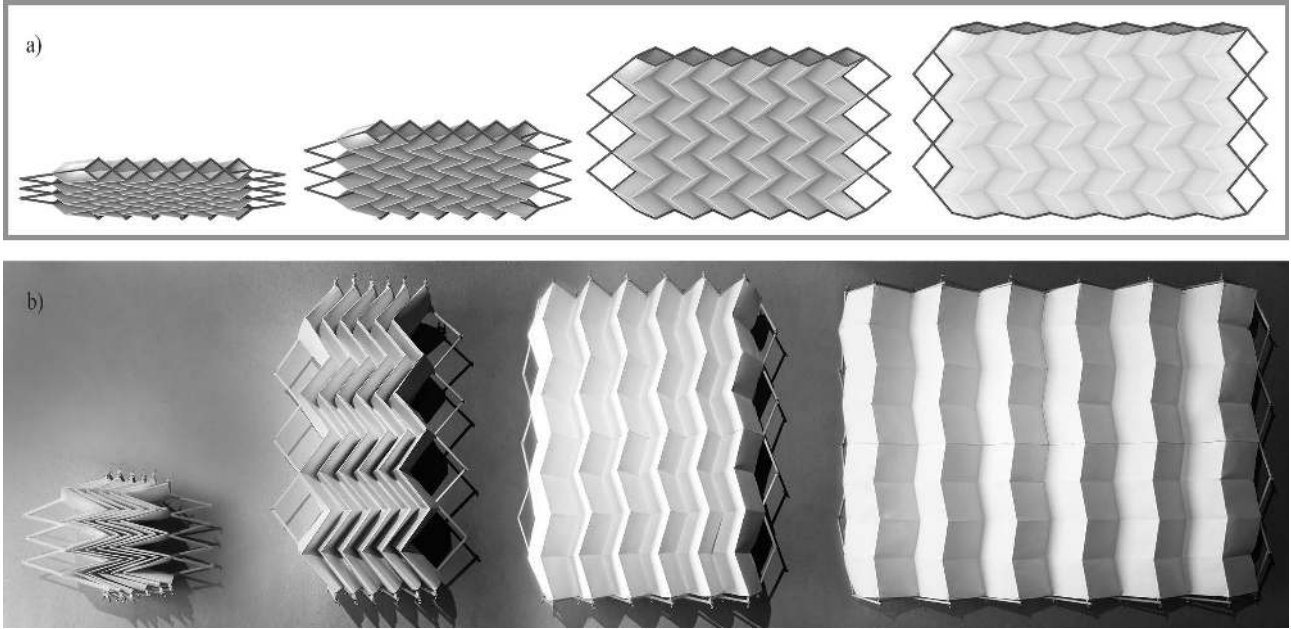


Figure 5: Deployment sequences of biaxial transformation surface structures: a) Parametric model with sector angle $\varphi = 50^\circ$ and b) physical model with $\varphi = 80^\circ$

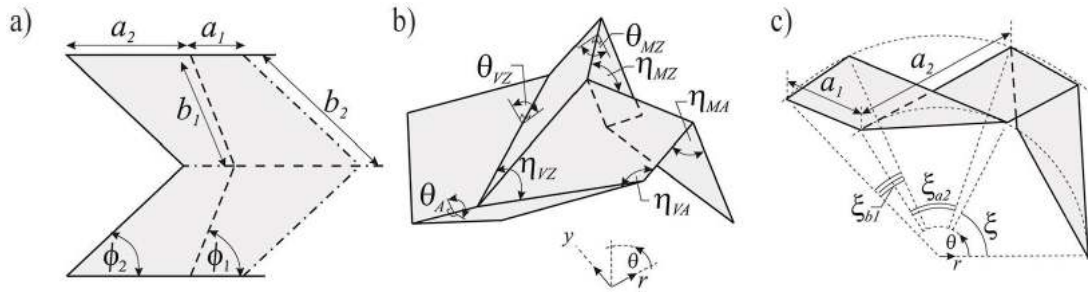


Figure 6: Arc-miura CP unit: a) unfolded, b) half-folded and c) fully folded [20]

$$\cos \eta_{MZ} = \sin^2 \varphi_1 \cos \theta_A + \cos^2 \varphi_1 \quad (8)$$

$$\xi = \eta_{VA} - \eta_{MA} \quad (12)$$

$$(1 + \cos \eta_{VZ})(1 - \cos \eta_{VA}) = 4 \cos^2 \varphi_2 \quad (9)$$

$$\cos \eta_{VA} = \sin^2 \varphi_2 \cos \theta_{VZ} - \cos^2 \varphi_2 \quad (10)$$

$$\cos \eta_{VZ} = \sin^2 \varphi_2 \cos \theta_A + \cos^2 \varphi_2 \quad (11)$$

Given a rigid arc-miura tessellation of a base unit meeting the conditions (4) and (5), and a θ_A angle between 0° and 180° , angle ξ can be defined from equations (6)–(12). In order to transform relatively, unit angle ξ must coincide with segment angle φ of the scissor linkage. Angle ξ is not constant, therefore angulated scissor linkages with constant φ angle cannot be used. On the other hand, polar linkages acquire a maximum curvature when fully expanded, unlike the arc-miura CP where curvature is minimum at its fully expanded configuration. For the coupling of arc-miura folded surfaces with scissor linkages of similar deployment path, a modified scissor linkage is proposed. The modified scissor unit consists of two bars joined at the center (pivot) with a sliding revolute joint (Figure 7b). This modification increases the linkage's degrees of freedom, as each unit can shift from translational (Figure 8c) to angulated con-

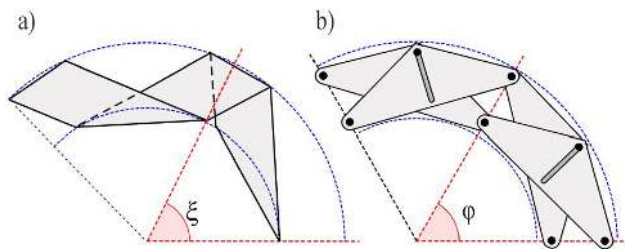


Figure 7: a) Arc-miura CP fully folded configuration b) Proposed modified scissor with sliding revolute joint

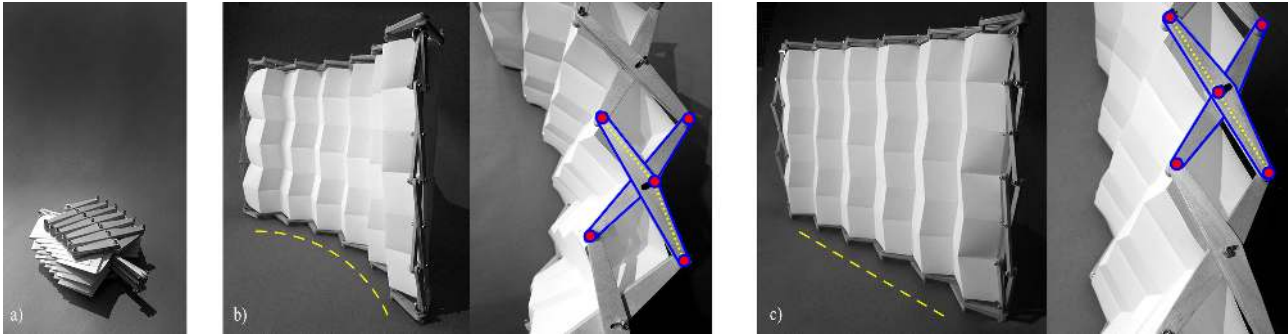


Figure 8: Physical model of a biaxial transformation structure consisting of a non-rigid arc-miura CP: a) retracted, b) expanded along arc and modified scissor detail, c) expanded linear and modified scissor detail

figurations (Figure 8b), with a central kink angle varying between minimum ω and 180° . Minimum kink angle ω can be calculated from equations (1) and (2), given that φ is equals to the maximum unit angle ξ of the arc-miura CP.

A non-rigid physical model of the proposed mechanism was constructed (Figure 8). In the expanded configurations, the panel can shift from linear (Figure 8c) to curvilinear (Figure 8b) shapes, due to facet deformation. On the contrary, a rigid arc-miura CP, which possesses only one DOF, would constrain the scissor linkages to a specific amplitude angle ω for every unique θ_A angle, forming a single-DOF structure of increasing global curvature along one axis, when expanding.

5.3 Yoshimura CP deployable vault

The so-called yoshimura or diamond CP consists of a series of mirrored isosceles triangles of side length α , and base angle φ , forming a rhombuses tessellation, as shown in Figure 9a. The yoshimura CP generates curvature while folding, forming arch-like elements in fully folded configurations, as shown in Figure 9b. The generated curvature is related to angle φ , as larger angles form structures with less global curvature [21].

The proposed yoshimura CP deployable vault (Figure 10a) is coupled with three translational scissor link-

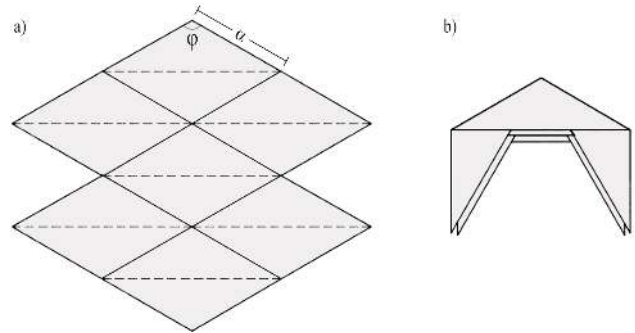


Figure 9: Yoshimura CP: a) unfolded and b) fully folded configurations

ages (blue) of bar length equals to side length α , anchored on edge and intermediate CP vertices (red). The linkages are connected to each other with bars at every alternate hinge, following the geometry of the CP, as the bars lie on rhombuses diagonals. In this way, the scissor linkages are constrained to deploy parallel to each other, and the resulting linkage acts as a supporting frame for the folding surface. However, depending on the use, facet rigidity, and the scale of the structure, a varying number of connecting bars could be used, or alternatively, only one central scissor linkage without any bars. The resulting kinematically paired mechanism folds into a vaulted surface of increasing height and decreasing width, as shown in a sequence

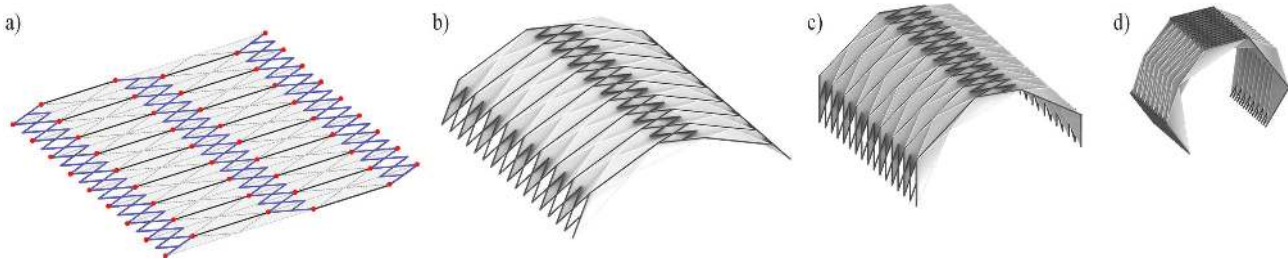


Figure 10: Yoshimura CP deployable vault: a) expanded, b) and c) intermediate, d) retracted configuration

of spatial configurations in Figure 10. The entire structure can be ideally controlled by the motion of any of the three translational linkages.

5.4 Polygon flasher CP umbrella

Based on the “radial flasher” CP, designed by Jeremy Shafer [22], we developed the polygon flasher CP (Figure 11a), which can be geometrically constructed from any regular polygon. It is a pattern of radial symmetry generating umbrella-shaped folded surfaces, which retract to their own perimeters while folding. Given any regular n -gon, the CP is constructed by extending every edge of the polygon, forming mountain folds, and bisecting the interior angles with valley folds. The n -gon flasher CP produces a regular $2n$ -gon perimeter which retracts towards the center like a vortex, while folding (Figure 11b). This CP can be fully retractable, only if a critical bending of the triangular plates is allowed, therefore, it is not rigid-foldable, nor flat-foldable, as it cannot be folded into a fully flat configuration. The central polygon plate though, remains planar throughout the folding process, therefore this part may be constructed out of a lightweight rigid material.

The n -gon flasher CP is coupled with an angulated closed-loop linkage of n scissor pairs. The folded surface can be anchored at the boundary vertices of the valley folds,

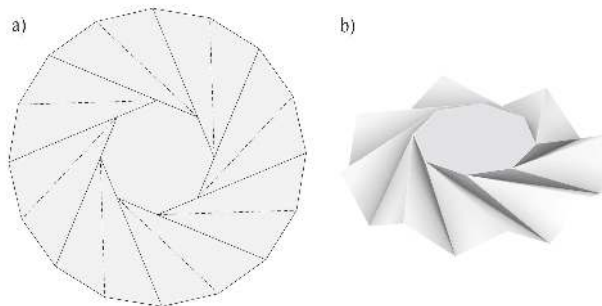


Figure 11: Octagon flasher: a) crease pattern, b) half folded configuration

on either of the inner (Figure 12) or outer hinges, or the scissor pivots (Figure 13). The coupled mechanism can ideally be controlled by changing the distance between two opposite linkage points, or the rotation of one local scissor pair assembly. A physical model of an icosagon flasher was constructed, and then mounted on inner scissor hinges. The mechanism expands (Figure 12b) and retracts (Figure 12a), with the central rigid polygonal plate remaining on the same plane throughout the folding process.

The parametric model of the proposed mechanism, though, retracts in a slightly different manner, as shown in Figure 13. Due to plate minimum bending constraints in Kangaroo, the central polygon plate moves vertically through deployment, approaching a cylinder surface in

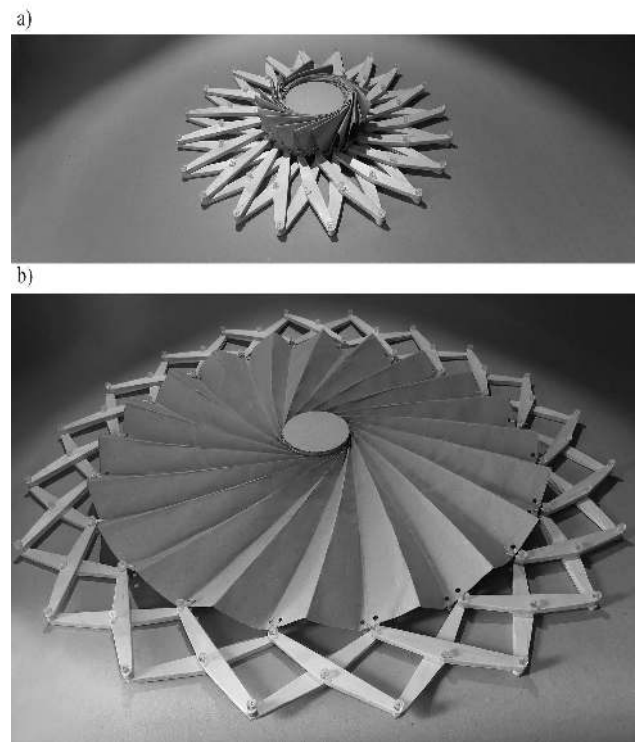


Figure 12: Physical model of an icosagon flasher in a) retracted and b) expanded configurations

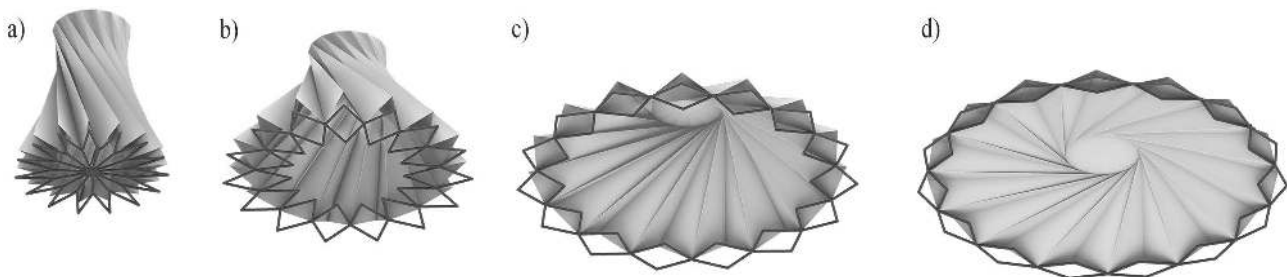


Figure 13: Deployment sequence of a 15-gon flasher CP, anchored on the pivots of an angulated scissors closed-loop linkage

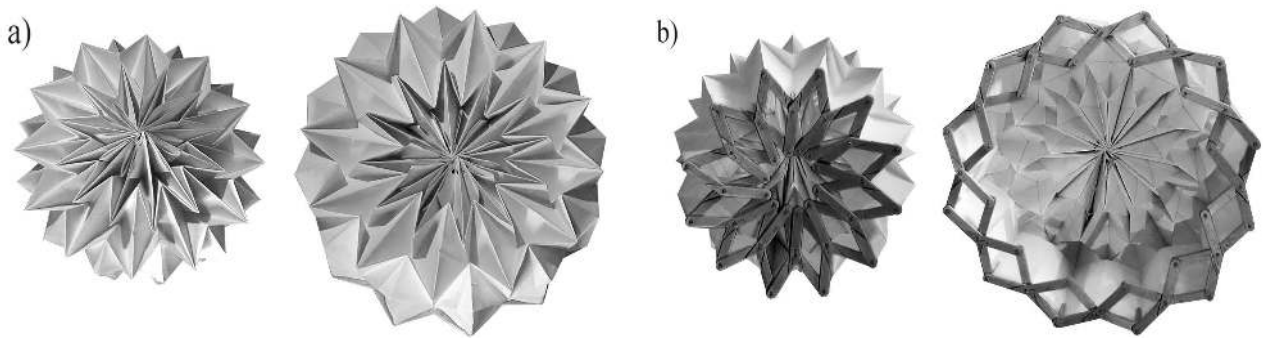


Figure 14: Waterbomb CP deployable shell in retracted and expanded configurations: a) top, b) bottom view

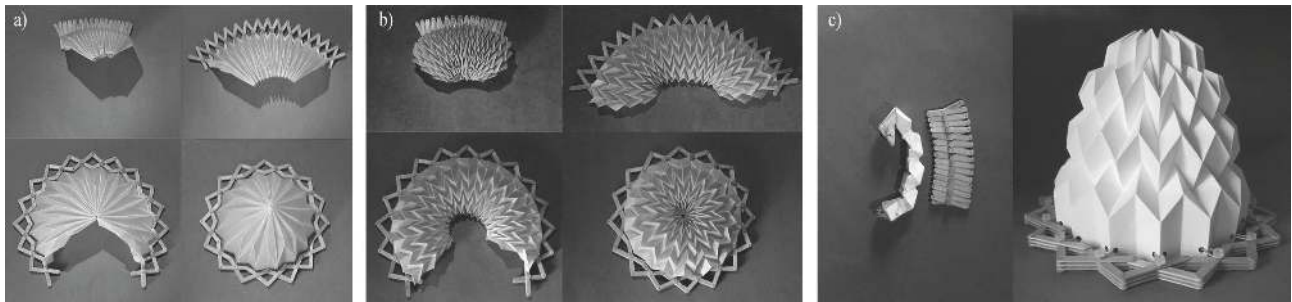


Figure 15: Deployment sequences of: a) a yoshimura CP deployable dome and b) an arc-miura CP deployable shell. c) A modified arc-miura CP deployable shell in demounted and expanded configurations

fully retracted configuration (Figure 13a). This may or may not be a desired effect of the structure, and can be additionally constrained to planar deployment, if the surface material permits critical plate bending.

5.5 Shell structures' studies with physical models

The following hybrid structures types are mainly part of our work in progress, and include some examples of non-rigid origami physical models, not yet parametrically simulated and tested. The conceptual model shown in Figure 14 consists of the so-called waterbomb CP, kinematically paired with a closed-loop of angulated scissors linkage. Initially, the surface is joined on opposite edges of the CP, forming a cylinder and then folded on pre-pleated crease lines. The edge vertices of one of the two remaining free CP edges are joined together, forming the centre 'peak' of the structure, and the opposite edge vertices are anchored on the scissor linkage's outer hinges and pivots. The resulting dome-like structure retracts and expands as shown in top (Figure 14a) and bottom (Figure 14b) view.

The three conceptual models shown in Figure 15 are all coupled with the same polar scissors linkage. This linkage deploys into a circular perimeter and can be restrained in

this state, by joining the inner and outer hinges of the first and last scissor pair. In these examples, the folded surfaces are all anchored on scissor pivots. Respectively, a double surface could be anchored on both inner and outer scissor hinges. The deployable dome shown in Figure 15a, is a modification of the yoshimura CP. The model of Figure 15b is folded out of an arc-miura CP and deploys like a torus, forming a shell of double curvature at the 'locked' configuration. A modification of the arc-miura CP generates the deployable shell shown on Figure 15c, in demounted and expanded configuration.

6 Conclusions

Although a few examples of combining origami surfaces with scissor linkages already exist, in this research a systematic approach for coupling these two types of structures has been attempted. To this end, a method for coupling various CP origami folded surfaces with common and modified types of planar scissor linkages has been developed. Specifically in the studies that were conducted, several forms of origami CPs characterized by biaxial and rotational symmetry have been coupled with translational, polar and an-

gulated scissor linkages, towards the development of novel forms of hybrid deployable structures.

These new concepts of deployable structures have been organized and presented in reference to their spatial deployment method, the origami CP used for each structure, and the configurations of the structure before, during and after deployment.

In some of the examples presented, such as the uniaxial transformation structures and the retracting umbrellas, scissor linkages were effective in reducing the degrees of freedom of the folded surfaces. However, in some others, such as in the biaxial transformation structures, rigid single-DOF origami CPs, constrained the scissor linkages.

The simulation methods used in this study, were applied to ideal zero-thickness surfaces. At the upcoming stage of this research, thickness of the material will be taken into consideration, since it may have significant effect on the stiffness and foldability of the studied mechanisms. Furthermore, for the full-scale application of the developed hybrid deployable structures' concepts, additional research is needed. It is anticipated that the hybrid deployable structures proposed in this paper have great potential, as they can find applications as lightweight, demountable, transportable and transformable structures in architecture.

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