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Force Analysis of Whole Hand Grasp by Multifingered Robotic Hand

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***Abstract*— Under a whole hand grasp, it may not be possible to generate grasping forces in all directions. Thus, the traditional techniques developed based on fingertip contacts is inadequate. In this paper, we decompose the contact force space into four orthogonal subspaces, each with a clear physical interpretation. Based on linear matrix inequalities (LMI's) representations of grasping constraints, we address and formulate the active force closure and the active grasp feasibility problems as LMI feasibility problems. Combining the effects of both active and passive forces, we propose a new cost index for the whole hand grasping force optimization problem. We further simply the force optimization problem for a whole hand grasp, which is active force closure.**

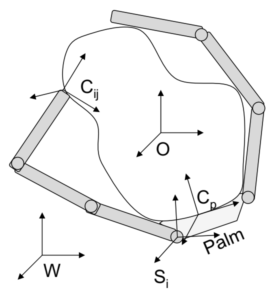

Fig. 1. A k-fingered hand grasping an object with whole hand grasp

I. Introduction

There are numerous literatures on grasping force analysis and optimization developed over the last two decades. The three main problems that have been frequently addressed are: *force closure*, *force feasibility*, and *force optimization* problems. To deal with the nonlinear nature of the friction models, Buss *et al.* [1] observed an important fact that the friction cone constraints are equivalent to positive definiteness of certain symmetric matrices and transformed grasping force optimization problem into a convex optimization problem with linear constraints. In order to eliminate structure constraints and thus reduce corresponding dimension of the optimization problem, Helmke *et al.* [2] refined the semi-definite representation of the friction cone. Han *et al.* [3] transformed the friction cone constraints into Linear Matrix Inequalities (LMIs) and formulated the three main problems as convex optimization problems with LMI constraints. All those, however, rely upon a fundamental assumption that an arbitrary resultant contact force can be actively applied by the hand, which is no longer true in the case of a whole hand grasp.

To deal with the incomplete controllability of contact forces, Yoshikawa [4] introduced the concepts of active and passive contact forces, and classified force closure into passive, active and hybrid closures. He gave conditions for each type of force closure for a constrained mechanism. Based on Yoshikawa's work, Watanabe [5] further defined the direction of active and passive force closure and showed the orthogonality of those two directions. From the physical nature of contacts, Wang and Liu [6] introduced the concepts of active and passive contacts, and proposed a new definition of active and passive contact forces. Based on the minimum norm principle, they proposed a framework for force and closure analysis involving force passivity. The active contact force defined by the styles of contacts, however, still contains the force that can not be generated actively by the hand. By interpreting the geometric structure of contact force space, Bicchi [7] proposed a force distribution method for a whole hand grasp. A geometric control method was provided for the control of passive internal forces. Zhang *et al.* [8] presented a new classification of contact force. They proposed a new force distribution method for a whole hand grasp based on the connectivity and mobility of the grasped object. Omata [9] showed that frictional forces involved in a whole hand grasp are indeterminate because of the limited mobility of the inner links and the palm. He proposed a method to compute the bound of those indeterminate frictional forces. Yu *et al.* [10] described the region of feasible joint torques for a stable whole hand grasp and introduced a method for determination of the optimal whole hand grasp. However, resultant optimal grasping forces sometimes can not be generated by the hand because of its passivity and indetermination. For this reason, most previous research focus on the distribution but not the optimization of contact force for a whole hand grasp.

In this paper, by intersecting two different decompositions of contact force space, we derive four orthogonal subspaces, each with a clear physical meaning. Based on this decomposition and Han's grasp analysis approaches [3], active force closure and active grasp force feasibility problems are formulated as LMI feasibility problems. Considering the roles of both active and passive forces, a new cost index is proposed. The grasping force optimization problem for a whole hand grasp is thus formulated as a convex optimization problem with LMI constraints. This problem can be readily solved using fast convex programming techniques. Finally, several numerical examples show the validity of the problem

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formulations and the performance of solution algorithms.

II. Grasp Model and Constraints of a Whole Hand Grasp

Consider a k-fingered hand grasping an object using a whole hand grasp as shown in Fig. 1, with a total of n contact points. Denote by n_i the number of contact points on the links of the i^{th} finger, and n_p the number of contact points on the palm, we have

$$n = n_p + \sum_{i=1}^{k} n_i$$

Note that contact points on the same inner link have the same kinematics property, so contact forces applied at such contacts also have the same properties.

Assumption *1*.To simplify the grasp model and make the problem well defined, we make the following assumptions:

(a1). Without loss of generality, we consider the case that there is at most one contact point on each link of a finger. Moreover, there is at most one contact point on the palm.

(a2). The contact model is point contact with friction (PCWF). Coulomb friction coefficients at all contact points are the same, and is denoted by μ.

Following the notations in [11][12], attach a body frame O to the mass center of the object, and a spatial frame W to the palm of the hand. For the i^{th} finger, attach a base frame S_i to the base of the finger and contact frames C_{ij} to the j^{th} contact point on this finger. Similarly, attach a contact frame C_p to the contact point on the palm.

Static balance of all forces exerted on the object implies that

$$Gx = G_p x_p + G_f x_f = -\omega_0, \tag{1}$$

where $\omega_0 \in \mathbb{R}^6$ is the external object wrench, $G = [G_p, G_f] = [G_p, G_{f_1}, \cdots, G_{f_k}] \in \mathbb{R}^{6\times 3n}$ the grasp map, with $G_{f_i} = [G_{c_{i1}}, \cdots, G_{c_{in_i}}] \in \mathbb{R}^{6\times 3n_i}$ and $G_p \in \mathbb{R}^{6\times 3}$ being, respectively, the grasp map of the i^{th} finger and the palm; $x = [x_p^T, x_{f_1}^T, \cdots, x_{f_k}^T]^T := [x_p^T, x_f^T]^T \in \mathbb{R}^{3\cdot n}$ the contact forces of the hand, with $x_{f_i} = [x_{c_{i1}}^T, \cdots, x_{c_{in_i}}^T]^T$ and $x_p^T \in \mathbb{R}^3$ being, respectively, the contact forces of the i^{th} finger and the palm.

The physics of contact imposes a nonlinear quadratic constraint on all finger forces. For example, under the PCWF model, the contact force x_p is constrained to the friction cone

$$\mathcal{FC}_p = \{x_p \in \mathbb{R}^3 | x_{p,1}^2 + x_{p,2}^2 < \mu^2 x_{p,3}^2 \ , \ x_{p,3} > 0\} \tag{2}$$

Where $x_{p,1}$ and $x_{p,2}$ are the tangential components, and $x_{p,3}$ the normal component of the contact force on the palm. A similar constraint holds for $x_{c_{ij}} \in \mathbb{R}^3$, with the corresponding friction cone $\mathcal{FC}_{c_{ij}}$. Collectively, we let

$$\mathcal{FC}_f = \mathcal{FC}_{c_{11}} \times \cdots \times \mathcal{FC}_{c_{kn_k}}$$

and

$$\mathcal{FC} = \mathcal{FC}_p \times \mathcal{FC}_f$$

See [1], [3], [13] for a detailed model of grasping statics and friction cone constraints.

By refining the results of Buss, Hashimoto and Moore [1],Helmke, Hueper and Moore [2] showed that the friction cone constraint (2) is equivalent to the positive definiteness of the following 2×2 symmetric matrices

$$P_p = \begin{bmatrix} \mu x_{p,3} + x_{p,1} & x_{p,2} \\ x_{p,2} & \mu x_{p,3} - x_{p,1} \end{bmatrix} \succ 0, \tag{3}$$

and the totality of the hand constraints $x \in \mathcal{FC}$ is equivalent to

$$P \in \mathbb{R}^{2n\times 2n} = \mathrm{diag}(P_p, P_{f_1}, \cdots, P_{f_k}) \succ 0, \tag{4}$$

where $P_{f_i} = \mathrm{diag}(P_{c_{i1}}, \cdots, P_{c_{in_i}})$.

Another observation by Han, Trinkle and Li [3] shows that constraint (4), by a reordering of the indices for the contact forces, has the form of Linear Matrix Inequalities (LMIs), which is studied extensively in [14]:

$$P(x) \ = \ A_{P,0} + \sum_{l=1}^{3n} A_{P,l} x_l \succ 0 \tag{5}$$

with $A_{P,0} = 0$. The force balance equation (1) can be rearranged as a set of linear constraints:

$$\mathrm{Tr}(B_i P) = \omega_{oi} \ , \ i = 1, \cdots, 6, \tag{6}$$

where $B_i = B_i^T$ are symmetric k-block diagonal matrices with dimension $n\times n$, ω_{oi} the i^{th} component of the external wrench $\omega_o \in \mathbb{R}^6$. We will assume that the B_i's, $i = 1, \cdots, 6$ are linearly independent, and using the scheme of [2] or the standard Gram-Schmidt process to orthonormalize the B_i's.

Denote by τ_{ij} the joint torque of the j^{th} joint of finger i, and $\tau_i = [\tau_{i1}, \cdots, \tau_{iq_i}]^T \in \mathbb{R}^{q_i}$ the joint torques of finger i, where q_i is the number of joints of finger i. The relationship between the joint torques and the contact wrenches are [11]

$$\tau_i = \sum_{j=1}^{n_i} J_{h_{ij}}^T x_{c_{ij}} = \begin{bmatrix} J_{h_{i1}}^T \cdots J_{h_{in_i}}^T \end{bmatrix} \begin{bmatrix} x_{c_{i1}} \\ \vdots \\ x_{c_{in_i}} \end{bmatrix} := J_{h_i}^T x_{f_i} \tag{7}$$

and

$$J_{h_{ij}} = \begin{bmatrix} \hat{J}_{ij} & 0_{3\times(q_i - m)} \end{bmatrix} \in \mathbb{R}^{3\times q_i},$$

where $q_i - m$ is the number of joints which can not generate contact forces at contact point C_{ij}, m is the number of joints between the contact point C_{ij} and the palm, and $\hat{J}_{ij}$ the Jacobian matrix of contact frame C_{ij}. Note that contact force on the palm, x_p, is only the reaction force generated by the palm, so it will not result in any joint torque. In the case that the palm contacts the object, the joint effort of the hand, denoted by $\tau = [\tau_1^T, \cdots, \tau_k^T] \in \mathbb{R}^q$, where $q = \sum_{i=1}^k q_i$, has the following relations with the contact forces

$$\tau = Hx = [0_{q\times 3}, J_h^T]x, \tag{8}$$

where

$$J_h^T \ = \ \mathrm{diag}(J_{h_1}^T, \cdots, J_{h_k}^T).$$

When the palm does not contact the object, $H = J_h^T$. Note that each joint torque τ_{ij} is limited by its upper and lower bound τ_{ij}^U and τ_{ij}^L as,

$$\tau_{ij}^L < \tau_{ij} < \tau_{ij}^U.$$

We formulate the corresponding LMIs in terms of x with the same order as the LMI constraint (5)

$$\begin{aligned} T^L(x) &= \mathrm{diag}(\tau - \tau^L) \\ &= T_0^L + \textstyle\sum_{l=1}^{3n} T_l^L x_l \succeq 0 \\ T^U(x) &= \mathrm{diag}(-\tau + \tau^U) \\ &= T_0^U + \textstyle\sum_{l=1}^{3n} T_l^U x_l \succeq 0 \end{aligned} \quad (9)$$

where τ^L and τ^U are the lower and upper bound of the whole hand's joint efforts. Let $T(x) = \mathrm{diag}(T^L(x), T^U(x))$, then the positive semi-definiteness of both $T^L(x)$ and $T^U(x)$ is equivalent to the positive semi-definiteness of $T(x)$.

Constraints (4), (6),(8) and (9) comprise the system model for our subsequent analysis of the whole hand grasp problems. A whole hand grasp is said to be valid if it satisfies the above grasping constraints simultaneously.

III. Grasping Force Decomposition

Given a whole hand grasp G and an external wrench ω_o, the resultant contact force can be solved from the force balance constraint (1) as

$$x = -G^{\ddagger}\omega_0 + V_1 z,$$

where $G^{\ddagger} = G^T(GG^T)^{-1}$ is the generalized inverse of the grasp map G. Columns of the matrix V_1 form a basis of the null space $Ker(G)$ of G, and z is a vector of free variables with the same dimension as $Ker(G)$. In this way, the contact force space X can be decomposed into two orthogonal subspaces

$$x \in Ker(G)^{\perp} \oplus Ker(G).$$

$Ker(G)^{\perp}$ is called the object force subspace because it contains grasping forces balancing the external object force. $Ker(G)$ is called the internal force subspace, because it contains all internal forces.

On the other hand, from the relationship between the grasping forces and the joints efforts (8), the grasping forces can also be expressed as

$$x = H^{\ddagger}\tau + V_2 y$$

where $H^{\ddagger} = H^T(HH^T)^{-1}$ is the generalized inverse of H. Columns of the matrix V_2 form a basis of the null space $Ker(H)$ of H, and y is a vector of free variables with the same dimension as $Ker(H)$. Thus, another decomposition of the contact force space can be derived as

$$x \in Ker(H)^{\perp} \oplus Ker(H).$$

Note that contact forces lying in $Ker(H)$ can not be actively generated and controlled by the hand's joint torques. We call such forces *passive forces*, and $Ker(H)$ the subspace of passive forces. On the other hand, contact forces in $Ker(H)^{\perp}$ can be actively generated and controlled by the hand. We call such forces *active forces* and $Ker(H)^{\perp}$ the subspace of active force.

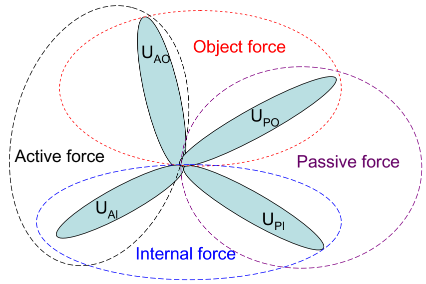


Fig. 2. Decomposition of grasping force into 4 subspaces

From the above two different decompositions, the contact force space X can be decomposed into four subspaces which are orthogonal relative to each other, as shown in Fig. 2,

$$X = U_{AO} \oplus U_{PO} \oplus U_{AI} \oplus U_{PI},$$

where

$$\begin{aligned} U_{AO} &= (Ker(G))^{\perp} \cap (Ker(H))^{\perp}, \\ U_{AI} &= Ker(G) \cap (Ker(H))^{\perp}, \\ U_{PO} &= (Ker(G))^{\perp} \cap Ker(H), \\ U_{PI} &= Ker(G) \cap Ker(H). \end{aligned}$$

Subspace U_{AO} is called the subspace of *active object forces*, subspace U_{AI} is called the subspace of *active internal forces*, subspace U_{PO} is called the subspace of *passive object forces*, and subspace U_{PI} is called the subspace of *passive internal forces*. Thus, a contact force can be written as

$$x = x_{AO} + x_{PO} + x_{AI} + x_{PI}.$$

When an external wrench ω_0 is applied to the object, the passive object force x_{PO} will be generated by the reaction force of the hand to balance ω_0. Then, the object force $x_{AO} + x_{PO}$ is determined as the particular solution to the force balance equation (1)

$$x_{AO} + x_{PO} = -G^{\ddagger}\omega_0 \quad (10)$$

Let V_{AI} and V_{PI} be the two matrices whose columns form a basis of the subspace U_{AI} and U_{PI}, respectively. The contact force has the following form

$$x = -G^{\ddagger}\omega_0 + V_{AI} z_{AI} + V_{PI} z_{PI}, \quad (11)$$

where $z_{AI} \in \mathbb{R}^l$ and $z_{PI} \in \mathbb{R}^r$ are free variables, and l and r are the dimensions of U_{AI} and U_{PI}, respectively.

IV. Force Analysis of Whole Hand Grasp

It has been shown that a grasp is force closure if and only if the grasp map G has full row rank and there exists an admissible strictly-internal grasp force [3], [11].

Proposition *1:* A grasp is force closure if and only if the following two conditions are satisfied simultaneously,

1). rank(G)=6;

2). there exists admissible internal force x^{int}, s.t. $P(x^{int}) \succ 0, Gx^{int} = 0$.

But for a whole hand grasp, contact forces are constrained by the structure of the hand mechanism, and the hand may not be able to generate the resultant contact forces when an external wrench is applied. If a grasp can actively balance any external wrench, we call it a *active force closure* grasp.

Definition *1:* Active Force Closure Grasp
A grasp is said to be *active force closure* if given any external wrench $\omega_0 \in \mathbb{R}^p$, there exists a contact force $x \in \mathcal{FC}$, which can be actively generated and controlled by the hand, such that $Gx = -\omega_0$.
Because of the existence of the passive internal force in the whole hand grasp, not all admissible internal forces satisfying conditions of Proposition. 1 can be generated actively by the hand. Therefore, a force closure grasp may not also be an active force closure one.

Problem *1:* **Active Force Closure Problem**
Given a grasp, determine whether for any external wrench ω_0, there exists a contact force x, which can be actively controlled and generated by the joint efforts, such that $P(x) \succ 0$ and $Gx = -\omega_0$.
Recall the decomposition of the contact force space in section III, both active and passive object forces, can be determined, as shown in (10), when given any external wrench. However, the passive internal force has to be preloaded and can not be controlled by the hand during the grasp, and only the active internal force is actively controllable by joint efforts. Then, a whole hand grasp is active force closure if and only if the grasp map G has full row rank and there exists an admissible strictly active internal grasp force.

Proposition *2:* A whole hand grasp is active force closure if and only if the following two conditions are satisfied simultaneously,

1). rank(G)=6;
2). there exists active internal force $x_{AI} \in U_{AI}$, s.t. $P(x_{AI}) \succ 0$.

Recall that the admissible active internal force can be written as

$$x_{AI} = V_{AI} z_{AI}, \tag{12}$$

the LMI (5) is equivalent to an LMI in terms of z_{AI} for the active internal forces

$$\tilde{P}(z_{AI}) := P(V_{AI} z_{AI}) = \Sigma_{i=1}^{l} \tilde{A}_i z_{AI_i}. \tag{13}$$

With this formulation, the active force closure problem for a whole hand grasp can be solved by first checking the surjectivity of the grasp map G, then determining whether there exists such a z_{AI} so that (13) holds, which is a standard LMI feasibility problem[3][15].

We are also interested in knowing whether the hand can actively generate grasping forces to make the grasp feasible. This is the active grasp feasibility problem, which can be stated as follows:

Problem *2:* **Active Grasp Feasibility Problem**
Given a grasp G and external wrench ω_0, determine if there exists contact force x, which can be actively applied and controlled by joint efforts τ, and satisfies grasping constraints (4), (6), (8) and (9).

The active grasp feasibility problem can be solved in a similar manner as the force closure problem. First, determine whether there exists an object force $x_0 \in \mathbb{R}^{3n}$ for the force balance equation

$$Gx_0 = \omega_0 \tag{14}$$

Obviously, a simple choice is the particular solution of (14)

$$x_0 = G^{\ddagger} \omega_0 = x_{AO} + x_{PO}$$

where $G^{\ddagger}$ is the generalized inverse of G. This particular solution is deterministic for a given external wrench, because the active object forces x_{AO} can be controlled by joint efforts and the passive object force x_{PO} is the reaction force by the hand mechanism itself. But such x_0 may not be admissible if it does not satisfy force closure constraints. Thus, a term of internal force has to be added such that the grasping force satisfies all the grasp constraints. Note that the joint torques can only generate active internal forces, which lie in the subspace U_{AI}. The admissible resultant contact force for Problem 2 has the form

$$x = x_0 + V_{AI} z_{AI}. \tag{15}$$

Hence, the force feasibility problem can be solved by checking whether for a given external wrench ω_0, there exists $z \in \mathbb{R}^l$ the following LMIs in terms of z_{AI} hold,

$$\begin{array}{rcl} \tilde{P}(z_{AI}) & := & P(x_0 + V_{AI} z_{AI}) = \tilde{A}_0 + \Sigma_{i=1}^{l} \tilde{A}_i z_{AI_i} \\ \tilde{T}(z_{AI}) & := & T(x_0 + V_{AI} z_{AI}) = \tilde{T}_0 + \Sigma_{i=1}^{l} \tilde{T}_i z_{AI_i}. \end{array} \tag{16}$$

Again, Problem 2 can be solved as an standard LMI feasibility problem in terms of z_{AI}.

V. Grasping Force Optimization for a Whole Hand Grasp

A central issue in the study of dextrous robotic hands is the determination of optimal grasping forces, which balance the external wrench and maintain all grasp constraints simultaneously. This is the *grasp force optimization problem* which can be stated as follows

Problem *3:* **Grasping force optimization problem**
Finding the optimal admissible contact force x, which satisfies grasping constraints (4), (6), (8) and (9), and minimizes some suitable cost function .
One property of a whole hand grasp is that it can balance the external wrench partly using the passive forces generated by the hand mechanism. By doing so, a whole hand needs less joint efforts to grasp the same object much more stably compared with a fingertip grasp. For this reason, we minimize the joint efforts of the hand and formulate the whole hand grasping force optimization problem as

Problem *4:* **Whole Hand Grasping Force Optimization Problem 1**

$$\min \|\tau\|$$

subject to

$$\begin{array}{rcl} P(x) & \succ & 0 \\ \tau & < & \tau^U \\ \tau & > & \tau^L \\ Gx & = & \omega_0 \end{array}$$

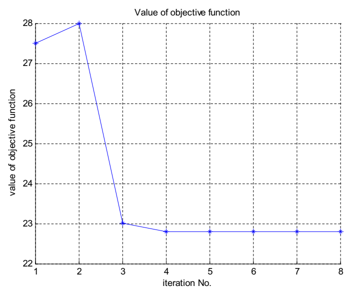


Fig. 3. The convergencye of the cost function of Problem 5

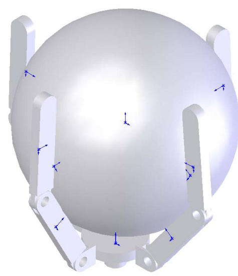
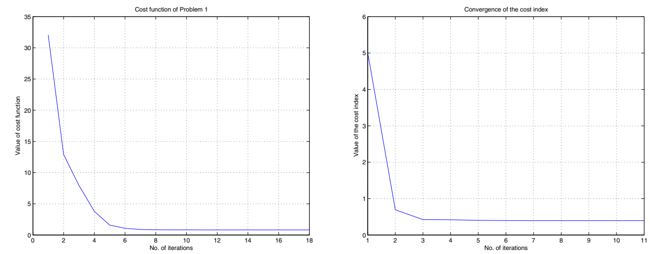
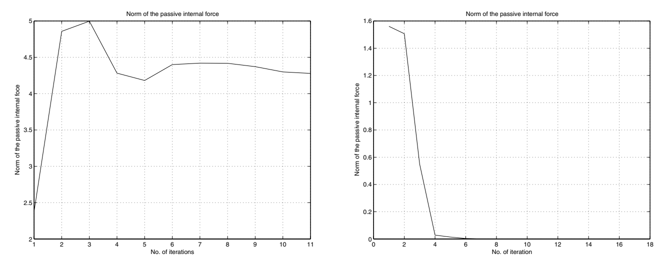


Fig. 4. Example 3: A 2-fingered hand grasp a spherical object



(a) (b)

Fig. 5. (a) Trajectory of the cost index of Problem 4, (b) Trajectory of the cost index of Problem 5.



(a) (b)

Fig. 6. (a) Trajectory of the norm of the passive internal force solving from Problem 4, (b) Trajectory of the norm of the passive internal force solving from Problem 5.

With the resultant optimal contact force obtained by solving Problem 4, the hand balances the external wrench mostly with the passive object force generated by the hand mechanism itself. At the same time, the hand may also generate more passive internal force, which causes no joint effort either, to satisfy the friction constraints. However, the passive internal force has to be preloaded and can not be controlled and adjusted when there is a disturbance applied to the object. Therefore, the less the passive internal force get involved, the more controllability the grasp achieves. For this reason, we proposed a new cost index by adding a weighed term of passive internal force, so that the grasp cost the least energy and achieve the most controllability at the same time. With this new cost index, the whole hand grasping force optimization problem is reformulated as

Problem *5:* **Whole Hand Grasping Force Optimization Problem 2**

$$\min W_1\|H(x_0 + V_{AI}z_{AI})\| + W_2\|V_{PI}z_{PI}\|$$

subject to

$$\begin{aligned} \tilde{P}(z) &\succ 0 \\ \tilde{T}(z) &\succ 0 \end{aligned}$$

where $x_0 = -G^{\ddagger}\omega_0$ for a given external wrench ω_0. Note that both the objective function and the LMI constraints are convex [16]. Thus, Problem 5 is a convex optimization problem involving a set of LMI constraints, and it is able to be solved using Interior-point convex programming techniques and SDP programming techniques.[17], [16]

When a grasp is active force closure, an optimal grasping force should be the one which are completely controllable, and, at the same time, minimizes the joint efforts. Then Problem 5 can be further simplified as

Problem *6:* **Grasping Force Optimization for Active Force Closure Whole Hand Grasp**

$$\min \|H(x_0 + V_{AI}z_{AI})\|$$

subject to

$$\begin{aligned} \tilde{P}(z_{AI}) &\succ 0 \\ \tilde{T}(z_{AI}) &\succ 0 \end{aligned}$$

Remark *1:* Since much more contacts are involved in the whole hand grasp comparing with the fingertip grasp, the dimension of the grasping force optimization problem will greatly increase. For a whole hand grasp which is active force closure, however, the dimension of the grasping force optimization problem is greatly reduced from $\dim(V_1)$ to $\dim(V_{AI})$ with the simplified formulation as Problem 6. This will very much reduce the computational complexity and improve the convergence performance of the optimization algorithm.

VI. Numerical Example and Simulation Results

Consider a 4-fingered hand grasping the same spherical object as in Example. 1 with a whole hand grasp, as shown in Fig. 4. Each finger of the hand composes of 2 inner links and has 2 degree of freedoms. Each inner link form one contact with the object and the palm also has one contact point with the object. The local coordinates of the 7 contact points are $\alpha_{o11} = (-\frac{\pi}{4}, 0)^T$,$\alpha_{o12} = (\frac{\pi}{4}, 0)^T$, $\alpha_{o21} = (-\frac{\pi}{4}, \frac{\pi}{2})^T$,$\alpha_{o22} = (\frac{\pi}{4}, \frac{\pi}{2})^T$, $\alpha_{o31} = (-\frac{\pi}{4}, \pi)^T$,$\alpha_{o32} = (\frac{\pi}{4}, \pi)^T$, $\alpha_{o41} = (-\frac{\pi}{4}, \frac{3\pi}{2})^T$,$\alpha_{o42} = (\frac{\pi}{4}, \frac{3\pi}{2})^T$, and $\alpha_{op} = (-\frac{\pi}{2}, 0)^T$. The grasp map

$$G = [G_p\, G_{c_{11}}\, G_{c_{12}}\, G_{c_{21}}\, G_{c_{22}}\, G_{c_{31}}\, G_{c_{32}}\, G_{c_{41}}\, G_{c_{42}}] \in \mathbb{R}^{6\times 27}$$

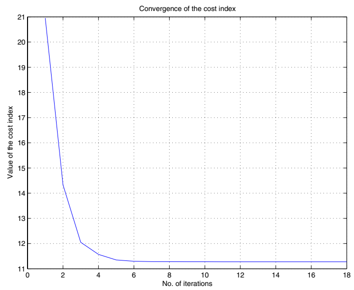


Fig. 7. Trajectory of the cost index of Problem 6

, where

$$G_i = \begin{bmatrix} -\sin u_i \cos v_i & \sin v_i & \cos u_i \cos v_i \\ -\sin u_i \sin v_i & -\cos v_i & \cos u_i \sin v_i \\ \cos u_i & 0 & \sin u_i \\ R \sin v_i & R \sin u_i \cos v_i & 0 \\ -R \cos v_i & R \sin u_i \sin v_i & 0 \\ 0 & -R \cos u_i & 0 \end{bmatrix}$$

for $i \in \{p, c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32}, c_{41}, c_{42}\}$, and

$$H = \begin{bmatrix} O_{8\times 3} & \mathrm{diag}(J_{h_1}^T, J_{h_2}^T, J_{h_3}^T, J_{h_4}^T) \end{bmatrix} \in \mathbb{R}^{8\times 27}$$

where $J_{h_i} \in \mathbb{R}^{6\times 2}$ is the hand jacobian of the i^{th} finger. The bases of the subspace of active internal force U_{AI} and the subspace of passive internal force U_{PI} form two matrices $V_{AI} \in \mathbb{R}^{27\times 8}$ and $V_{PI} \in \mathbb{R}^{27\times 13}$. Hence, the dimension of grasping force optimization Problem 4 and Problem 5 are both equal to 27. By solving Problem 2, it is shown that the grasp is active force closure. Then, Problem 5 is able to be simplified into the optimization Problem 6, whose dimension is reduced to 8. Using the convex optimization techniques, we solve the grasping force optimization with the formulations of Problem 4, Problem 5 and Problem 6, respectively. The weight parameters of the active forces and passive forces in Problem 5 are selected as $W_1 = 1$ and $W_2 = 10$. Fig. 5-(a) and 5-(b) show the convergence of the cost index of Problem 4 and Problem 5, respectively. It is shown that the cost indices we proposed in Problem 4 and 5 convergent to their optimal values after 5 and 9 iterations, respectively. Fig. 6-(a) shows the trajectory of the norm of the passive internal forces of Problem 4. Although it needs less iterations to reach the optimum by solving Problem 4 comparing with Problem 5, it leads to the large passive internal forces. Fig. 6-(b) shows the norm of the passive internal forces by solving Problem 5. The passive internal force goes to zero when we reach the optimal grasping force because the grasp is active force closure.

Fig. 7 shows the trajectory of the cost index of Problem 6. Comparing with Fig. 5-(b), we can see that it only take 4 iterations for the cost index to converge to its optimum, which is much less than the number of iterations of Problem 5. Moreover, the optimal force solved from Problem 6 is the same as the active part of the resultant contact forces solved from Problem 5.

VII. Conclusion

This paper discussed the grasping force analysis and optimization of the whole hand grasp. By intersecting two different decompositions, the space of contact forces was further decomposed into four subspaces, each of which is orthogonal to the others. With this decomposition, *active force closure problem* and *active force feasibility problem* were addressed and formulated into LMI feasibility problems. A new suitable cost index was proposed for the whole hand grasping force optimization. Furthermore, the force optimization problem was reformulated and solved as convex optimization problems involving LMIs. A simplified problem was proposed for those whole hand grasps, which are active force closure, to improve the performance of the optimization. Finally, a numerical example verified the validity of the whole hand grasp analysis problems' formulations and the performance of the solving algorithms.

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