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Forced convection analysis for generalized Burgers nanofluid flow over a stretching sheet

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This article reports the two-dimensional forced convective flow of a generalized Burgers fluid over a linearly stretched sheet under the impacts of nano-sized material particles. Utilizing appropriate similarity transformations the coupled nonlinear partial differential equations are converted into a set of coupled nonlinear ordinary differential equations. The analytic results are carried out through the homotopy analysis method (HAM) to investigate the impact of various pertinent parameters for the velocity, temperature and concentration fields. The obtained results are presented in tabular form as well as graphically and discussed in detail. The presented results show that the rate of heat transfer at the wall and rate of nanoparticle volume fraction diminish with each increment of the thermophoresis parameter. While incremented values of the Brownian motion parameter lead to a quite opposite effect on the rates of heat transfer and nanoparticle volume fraction at the wall. [http://dx.doi.org/10.1063/1.4935043]

I. INTRODUCTION

The forced convective analysis of boundary layer flows involving non-Newtonian fluids induced by a stretching surface has been the focus of extensive attempts during the past few decades due to their industrial and technological applications e.g., biological, chemical, food, and pharmaceutical industries. Several fluids such as drilling muds, shampoos, ketchup, granular suspension, apple sauce, paper pulp, slurries, paints, certain oils, polymer solutions, and clay coating are the non-Newtonian fluids. It seems quite difficult to propose a single model which incorporates the various characteristics like shear-thinning, shear-thickening,[1] viscoelasticity and viscoplasticity of non-Newtonian fluids as accomplished for the Newtonian fluid. Thus numerous model constitutive equations have been proposed. Furthermore, because of the behavior of non-Newtonian fluids the governing equations become more complex to handle as extra non-linear terms appear in the equation of motion. Particularly, reaction of numerous viscoelastic fluids can be caught sensibly well by the rate type fluid models. The fluid model under thought is a subclass of the rate-type fluid that is known as the generalized Burgers fluid. Consequently, a thermodynamic framework has been put into place to develop one-dimensional model due to Burgers[2] to the frame indifferent three dimensional form by Rajagopal and Srinivasa.[3] The Burgers model has been successfully used to describe the response of asphalt and asphalt concrete[4] as well as used to model the geological structures like Olivine rocks.[5] In spite of diverse applications, the Burgers model has not been given due attention. This model has been inspected by a few researchers.[6–11]

In recent years, the importance of flow and heat transfer in the field of nanofluids have been simulated significantly because of their potential applications in industrial processes such as in heating or cooling processes, chemical processes, power generation and so forth. Nevertheless, the applications of nanofluids are largely due to enhanced thermal conductivity. Choi[12] was the first who proposed the term nanofluid to describe the pure fluid with suspended nanoparticles. He found

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that adding metallic particles in nanosize to a base fluid can dramatically enhanced the thermal conductivity of the base fluid and improve the heat transfer performance of these fluid. A model for convective transport in nanofluids considering of the Brownian diffusion and thermophoresis was developed by Buongiorno.[13] He established an explanation for abnormal convective heat transfer enhancement observed in the nanofluids. He also showed that Brownian diffusion and thermophoresis were the most important nanoparticle/base-fluid slip mechanisms. Although the nanofluids have been studied by many researchers; however, we mention here some examples. Kuznetsov and Nield[14] investigated the natural convective boundary-layer flow of a nanofluid past a vertical plate. They showed that their similarity solution was achieved, which depended on the Lewis number Le , the buoyancy-ratio number Nr , the Brownian motion number N_b and the thermophoresis number N_t . Makinde *et al.*[15] analyzed the combined effects of buoyancy force, convective heating, Brownian motion, thermophoresis and magnetic field on stagnation point flow and heat transfer of a nanofluid flow towards a stretching sheet. Rahman *et al.*[16] discussed the boundary layer flow of a nanofluid past a permeable exponentially shrinking/stretching surface with second order slip using Buongiorno's model. Pal *et al.*[17] analyzed flow and heat transfer of nanofluids towards a stagnation point over a stretching/shrinking surface in a porous medium with thermal radiation. The numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles was investigated by Nadeem *et al.*[18] Khan *et al.*[19] analyzed three-dimensional flow of an Oldroyd-B nanofluid towards a stretching sheet with heat generation/absorption. The steady flow of Burgers nanofluid over a stretching surface with heat generation/absorption was studied by khan and khan.[20] Khan *et al.*[21] discussed the flow and heat transfer to sisko nanofluid over a nonlinear stretching sheet. Haroun *et al.*[22] reported the heat and mass transfer of nanofluid through an impulsively vertical stretching surface using the spectral relaxation method. Haroun *et al.*[23] examined the effect of unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using the spectral relaxation method.

The objective of the present study is to examine the flow and heat transfer characteristics to generalized Burgers nanofluid over a linear stretching sheet. By means of similarity reduction, a set of three non-linear coupled ordinary differential equations with linear boundary conditions are acquired. This specific sort of non-linear coupled ordinary differential equations are solved analytically by utilizing the homotopy analysis method (HAM).[24] The effects of the governing physical parameters on the velocity, temperature and nanoparticle volume fraction profiles are displayed graphically and discussed in details.

II. GOVERNING EQUATIONS

The conservation equations of mass, momentum, energy and nanoparticles describing the flow of nanofluid in the vectorial form can be written as

$$\operatorname{div}\mathbf{V} = 0, \tag{1}$$

$$\rho\,(\mathbf{V}\cdot\boldsymbol{\nabla}) = -\boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot\mathbf{S}, \tag{2}$$

$$(\mathbf{V}\cdot\boldsymbol{\nabla})\,T = \alpha\boldsymbol{\nabla}^2 T + \tau\left(D_B\boldsymbol{\nabla}C\cdot\boldsymbol{\nabla}T + \frac{D_T}{T_\infty}\boldsymbol{\nabla}T\cdot\boldsymbol{\nabla}T\right), \tag{3}$$

$$(\mathbf{V}\cdot\boldsymbol{\nabla})\,C = D_B\boldsymbol{\nabla}^2 C + \frac{D_T}{T_\infty}\boldsymbol{\nabla}^2 T, \tag{4}$$

where $\mathbf{V}$ is the velocity vector, T the temperature of the fluid, C the concentration of the fluid, ρ the fluid density, p the pressure, α the thermal diffusivity, T_∞ the ambient fluid temperature, $\tau = \frac{\rho c_p}{(\rho c_p)_f}$ the ratio of effective heat capacity of the nanoparticle material to the heat capacity of the fluid with c_p the specific heat of fluid at constant temperature, D_B the Brownian diffusion coefficient and D_T the thermophoresis diffusion coefficient. Moreover, the extra-stress tensors for an incompressible generalized Burgers fluid is related to the fluid motion satisfies the following constitutive equation:

$$\left(1+\lambda_1\frac{D}{Dt}+\lambda_2\frac{D^2}{Dt^2}\right)\mathbf{S}=\mu\left(1+\lambda_3\frac{D}{Dt}+\lambda_4\frac{D^2}{Dt^2}\right)\mathbf{A}_1, \tag{5}$$

where $\mathbf{A}_1=(\nabla\mathbf{V})+(\nabla\mathbf{V})^{\mathrm{T}}$ is the first Rivlin-Ericksen tensor, μ the dynamic viscosity, λ_1 and λ_3 ($<\lambda_1$) the relaxation and retardation times, respectively, λ_2 ($<\lambda_1\lambda_3$) and λ_4 ($\lambda_1\lambda_3-\lambda_2+\lambda_4>-2\sqrt{\lambda_1\lambda_3\lambda_4}$) the material parameters of the generalized Burgers fluid and $\frac{D}{Dt}$ denotes the upper convected derivative given as

$$\frac{Da_i}{Dt}=\frac{\partial a_i}{\partial t}+u_r a_{i,\,r}-u_{i,\,r}a_i. \tag{6}$$

For a two-dimensional flow in Cartesian coordinates, we seek the velocity, temperature, concentration and stress fields of the form

$$\mathbf{V}=[u\,(x,y)\,,v\,(x,y)\,,0],\ T=T(x,y),\ C=C(x,y),\ \mathbf{S}=\mathbf{S}(x,y). \tag{7}$$

Now substituting Eq. (7) in Eqs. (1) to (4), having in mind Eqs. (5) and (6) a lengthy but straight forward calculations yield the following governing equations of motion for the steady flow of a generalized Burgers nanofluid

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$\begin{aligned}
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)\\
&+\lambda_2\left[\begin{array}{c}
u^3\dfrac{\partial^3 u}{\partial x^3}+v^3\dfrac{\partial^3 u}{\partial y^3}+u^2\left(\dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x^2}+2\dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial x\partial y}\right)\\
+3v^2\left(\dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}\right)+3uv\left(u\dfrac{\partial^3 u}{\partial x^2\partial y}+v\dfrac{\partial^3 u}{\partial x\partial y^2}\right)\\
+2uv\left(\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}-\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x\partial y}\right)
\end{array}\right]\\
&=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\begin{array}{c}
\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\lambda_3\left(u\dfrac{\partial^3 u}{\partial x\partial y^2}+v\dfrac{\partial^3 u}{\partial y^3}-\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2}-\dfrac{\partial u}{\partial x}\dfrac{\partial^2 v}{\partial y^2}\right)\\
+\lambda_4\left(\begin{array}{c}
u^2\dfrac{\partial^4 u}{\partial x^2\partial y^2}+2uv\dfrac{\partial^4 u}{\partial x\partial y^3}+v\dfrac{\partial u}{\partial y}\dfrac{\partial^3 u}{\partial x\partial y^3}-u\dfrac{\partial u}{\partial y}\dfrac{\partial^4 u}{\partial x^2\partial y^2}\\
-2u\dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial x\partial y^2}+u\dfrac{\partial v}{\partial x}\dfrac{\partial^3 u}{\partial y^3}+v^2\dfrac{\partial^4 u}{\partial y^4}+3v\dfrac{\partial v}{\partial y}\dfrac{\partial^3 u}{\partial y^3}-v\dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3}\\
-u\dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial^2 u}{\partial y^2}+(\dfrac{\partial u}{\partial x})^2\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial v}{\partial x}\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial y^2}-u\dfrac{\partial^2 u}{\partial x\partial y}\dfrac{\partial^2 v}{\partial y^2}-v\dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3}
\end{array}\right)
\end{array}\right],
\end{aligned} \tag{9}$$

$$\begin{aligned}
&u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+2uv\frac{\partial^2 v}{\partial x\partial y}\right)\\
&+\lambda_2\left[\begin{array}{c}
u^3\dfrac{\partial^3 v}{\partial x^3}+v^3\dfrac{\partial^3 v}{\partial y^3}+2uv\left(\dfrac{\partial^2 v}{\partial x^2}\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial y^2}-\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x\partial y}-\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial x\partial y}\right)\\
+u^2\left(\dfrac{\partial^2 v}{\partial x^2}\dfrac{\partial v}{\partial x}-3\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial x^2}+3v\dfrac{\partial^3 v}{\partial x^2\partial y}-\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial x\partial y}+2\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x\partial y}\right)\\
+v^2\left(2\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial y^2}-\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial y^2}+2u\dfrac{\partial^3 v}{\partial x\partial y^2}+2\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x\partial y}+u\dfrac{\partial^3 v}{\partial x\partial y^2}\right)\\
+2uv\left(\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}-\dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x\partial y}\right)
\end{array}\right]
\end{aligned}$$

$$
= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \lambda_3\left(\begin{array}{c} u\frac{\partial^3 v}{\partial x\partial y^2} + u\frac{\partial^3 v}{\partial x^3} + v\frac{\partial^3 v}{\partial x^2\partial y} + v^3\frac{\partial^3 v}{\partial y^3} \\ -\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\end{array}\right)\right]
$$

$$
+\nu\lambda_4\left[\begin{array}{c} u\frac{\partial u}{\partial x}\frac{\partial^3 v}{\partial x^3} + u^2\frac{\partial^4 v}{\partial x^4} + v\frac{\partial u}{\partial y}\frac{\partial^3 v}{\partial x^3} + uv\frac{\partial^4 v}{\partial y\partial x^3} \\ -u\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^3} - u\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial x^3} + u\frac{\partial u}{\partial x}\frac{\partial^3 v}{\partial x\partial y^2} + u^2\frac{\partial^4 v}{\partial x^2\partial y^2} \\ +v\frac{\partial u}{\partial y}\frac{\partial^3 v}{\partial x\partial y^2} + uv\frac{\partial^4 v}{\partial x\partial y^3} - u\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x\partial y^2} - u\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial x\partial y^2} \\ +u\frac{\partial v}{\partial x}\frac{\partial^3 v}{\partial x^2\partial y} + uv\frac{\partial^4 v}{\partial x^3\partial y} + v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^2\partial y} + v^2\frac{\partial^4 v}{\partial x^2\partial y^2} \\ -v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^2\partial y} - v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^2\partial y} - v\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial x^2\partial y} + u\frac{\partial v}{\partial x}\frac{\partial^3 v}{\partial y^3} \\ +uv\frac{\partial^4 v}{\partial x\partial y^3} + v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial y^3} + v^2\frac{\partial^4 v}{\partial y^4} - v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial y^3}\end{array}\right]
$$

$$
+\nu\lambda_4\left[\begin{array}{c} -v\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial y^3} + u\frac{\partial^2 v}{\partial x\partial y}\frac{\partial^2 v}{\partial x^2} + u\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^3} + v\frac{\partial^2 v}{\partial y^2}\frac{\partial^2 v}{\partial x^2} \\ +v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x^2\partial y} - (\frac{\partial v}{\partial y})^2\frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2} + u\frac{\partial^2 v}{\partial x\partial y}\frac{\partial^2 v}{\partial y^2} \\ +u\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x\partial y^2} + v(\frac{\partial^2 v}{\partial y^2})^2 + v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial y^3} - 2(\frac{\partial v}{\partial y})^2\frac{\partial^2 v}{\partial y^2} \\ +u\frac{\partial^2 v}{\partial x^2}\frac{\partial^2 u}{\partial x^2} + u\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial x^3} + v\frac{\partial^2 v}{\partial x\partial y}\frac{\partial^2 u}{\partial x^2} + v\frac{\partial v}{\partial x}\frac{\partial^3 u}{\partial x^2\partial y} \\ -\frac{\partial v}{\partial y}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial v}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + u\frac{\partial^2 v}{\partial x\partial y}\frac{\partial^2 v}{\partial y^2} + u\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial x\partial y^2} \\ +v(\frac{\partial^2 v}{\partial y^2})^2 + v\frac{\partial v}{\partial y}\frac{\partial^3 v}{\partial y^3} - \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}\end{array}\right] \tag{10}
$$

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)
$$

$$
+\tau\left[D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right)\right], \tag{11}
$$

$$
u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{12}
$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity.

Using the standard boundary layer approximation, one finally has the following boundary layer equations for a generalized Burgers fluid

$$
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\partial y}\right)
$$

$$
+\lambda_2\left[\begin{array}{c} u^3\frac{\partial^3 u}{\partial x^3} + v^3\frac{\partial^3 u}{\partial y^3} + u^2\left(\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right) \\ +3v^2\left(\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) + 3uv\left(u\frac{\partial^3 u}{\partial x^2\partial y} + v\frac{\partial^3 u}{\partial x\partial y^2}\right) \\ +2uv\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2} + \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y}\right)\end{array}\right] \tag{13}
$$

$$
= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \left[\begin{array}{c} \dfrac{\partial^2 u}{\partial y^2} + \lambda_3 \left(u \dfrac{\partial^3 u}{\partial x \partial y^2} + v \dfrac{\partial^3 u}{\partial y^3} - \dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial y^2} \right) \\ + \lambda_4 \left(\begin{array}{c} u^2 \dfrac{\partial^4 u}{\partial x^2 \partial y^2} + 2uv \dfrac{\partial^4 u}{\partial x \partial y^3} + v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 u}{\partial x \partial y^3} - u \dfrac{\partial u}{\partial y}\dfrac{\partial^4 u}{\partial x^2 \partial y^2} \\ - 2u \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial x \partial y^2} + u \dfrac{\partial v}{\partial x}\dfrac{\partial^3 u}{\partial y^3} + v^2 \dfrac{\partial^4 u}{\partial y^4} + 3v \dfrac{\partial v}{\partial y}\dfrac{\partial^3 u}{\partial y^3} - v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3} \\ - u \dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial^2 u}{\partial y^2} + \left(\dfrac{\partial u}{\partial x}\right)^2 \dfrac{\partial^2 u}{\partial y^2} - u \dfrac{\partial^2 u}{\partial x \partial y}\dfrac{\partial^2 v}{\partial y^2} - v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3} \end{array} \right) \end{array} \right],
$$

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right], \tag{14}
$$

$$
u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{15}
$$

III. PROBLEM FORMULATION

Let us consider the steady, two-dimensional convective boundary layer flow of an incompressible generalized Burgers nanofluid over a linear stretching sheet of constant surface temperature T_w and concentration C_w. The fluid occupies the half space $y > 0$ and flow is induced due to the stretching of the sheet at $y = 0$ along the $x - axis$ with velocity $U_w = ax$, where $a > 0$ is the stretching rate of sheet. The uniform temperature and concentration far away from the surface of the sheet are T_∞ and C_∞, respectively. With these assumptions along with standard boundary layer approximation, the governing equations can be written as

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{16}
$$

$$
\begin{aligned} & u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1 \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) \\ & + \lambda_2 \left[\begin{array}{c} u^3 \dfrac{\partial^3 u}{\partial x^3} + v^3 \dfrac{\partial^3 u}{\partial y^3} + u^2 \left(\dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x^2} + 2\dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial x \partial y} \right) \\ + 3v^2 \left(\dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x \partial y} \right) + 3uv \left(u \dfrac{\partial^3 u}{\partial x^2 \partial y} + v \dfrac{\partial^3 u}{\partial x \partial y^2} \right) \\ + 2uv \left(\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial v}{\partial x}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial x \partial y} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial x \partial y} \right) \end{array} \right] \end{aligned} \tag{17}
$$

$$
= \nu \left[\begin{array}{c} \dfrac{\partial^2 u}{\partial y^2} + \lambda_3 \left(u \dfrac{\partial^3 u}{\partial x \partial y^2} + v \dfrac{\partial^3 u}{\partial y^3} - \dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 v}{\partial y^2} \right) \\ + \lambda_4 \left(\begin{array}{c} u^2 \dfrac{\partial^4 u}{\partial x^2 \partial y^2} + 2uv \dfrac{\partial^4 u}{\partial x \partial y^3} + v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 u}{\partial x \partial y^3} - u \dfrac{\partial u}{\partial y}\dfrac{\partial^4 u}{\partial x^2 \partial y^2} \\ - 2u \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial x \partial y^2} + u \dfrac{\partial v}{\partial x}\dfrac{\partial^3 u}{\partial y^3} + v^2 \dfrac{\partial^4 u}{\partial y^4} + 3v \dfrac{\partial v}{\partial y}\dfrac{\partial^3 u}{\partial y^3} - v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3} \\ - u \dfrac{\partial^2 u}{\partial x^2}\dfrac{\partial^2 u}{\partial y^2} + \left(\dfrac{\partial u}{\partial x}\right)^2 \dfrac{\partial^2 u}{\partial y^2} - u \dfrac{\partial^2 u}{\partial x \partial y}\dfrac{\partial^2 v}{\partial y^2} - v \dfrac{\partial u}{\partial y}\dfrac{\partial^3 v}{\partial y^3} \end{array} \right) \end{array} \right],
$$

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right], \tag{18}
$$

$$
u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{19}
$$

with the relevant boundary conditions

$$u = U_w = ax, \quad v = 0, \quad T = T_w, \quad C = C_w \text{ a t } y = 0, \tag{20}$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty. \tag{21}$$

By introducing the following dimensionless quantities, the above problem can be expressed in a simpler form

$$\psi = x\sqrt{a\nu} f(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{22}$$

where $\psi(x, y)$ is the Stokes stream function and is defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

Consequently, the above governing problem reduces to

$$\begin{aligned} &f''' + ff'' - (f')^2 + \beta_1 \left[2ff'f'' - f^2 f'''\right] + \beta_2 \left[f^3 f^{iv} - 2f(f')^2 f'' - 3f^2 f''^2\right] \\ &+\beta_3 \left[(f'')^2 - ff^{iv}\right] + \beta_4 \left[f^2 f^{v} - 2ff'f^{iv} - 2ff''f''' + f'(f'')^2\right] = 0, \end{aligned} \tag{23}$$

$$\theta'' + \Pr f\theta' + \Pr N_b \phi'\theta' + \Pr N_t (\theta')^2 = 0, \tag{24}$$

$$\phi'' + \Pr Le f\phi' + \frac{N_t}{N_b}\theta'' = 0, \tag{25}$$

$$f = 0, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \tag{26}$$

$$f' \to 0, \quad f'' \to 0, \quad f''' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty, \tag{27}$$

where prime denotes differentiation with respect to η. Moreover, $\beta_i \quad (i = 1, 2, 3, 4)$ are the Deborah numbers, Pr the Prandtl number, N_b the Brownian motion parameter, N_t the thermophoresis parameter and Le the Lewis number. These parameters are defined by

$$\begin{aligned} &\beta_{1,3} = \lambda_{1,3} a, \qquad \beta_{2,4} = \lambda_{2,4} a^2, \qquad \Pr = \frac{\nu}{\alpha} \\ &N_b = \frac{\tau D_B (C_w - C_\infty)}{\nu}, \quad N_t = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}, \quad Le = \frac{\alpha}{D_B}. \end{aligned} \tag{28}$$

The parameters of physical interest of the present problem are the local Nusselt number Nu_x giving the rate of heat transfer at the wall and the local Sherwood number Sh_x giving the rate of nanoparticle volume fraction are defined by

$$Nu_x = -\frac{x}{(T - T_\infty)} \left(\frac{\partial T}{\partial y}\right)\bigg|_{y=0}, \; Sh_x = -\frac{x}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial y}\right)\bigg|_{y=0}. \tag{29}$$

In view of Eq. (22), it can be shown that the above physical quantities are putted in the dimensionless form

$$\mathrm{Re}^{-\frac{1}{2}} Nu_x = -\theta'(0), \qquad \mathrm{Re}^{-\frac{1}{2}} Sh_x = -\phi'(0), \tag{30}$$

in which $Re = U_w x/\nu$ is the local Reynolds number.

A. Homotopy solutions

For the analytic solution of the problem consisting of Eqs. (23)-(27) we employ the homotopy analysis method (HAM). The initial approximations and the auxiliary linear operators for the HAM solutions are chosen as

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}, \quad \phi_0(\eta) = e^{-\eta}, \tag{31}$$

$$£_f = f''' - f', \quad £_\theta = \theta'' - \theta', \quad £_\phi = \phi'' - \phi' \tag{32}$$

The above operators satisfy the properties given as follows:

$$\begin{aligned} &£_f[C_1 + C_2 e^{\eta} + C_3 e^{-\eta}] = 0, \quad £_\theta[C_4 e^{\eta} + C_5 e^{-\eta}] = 0, \\ &£_\phi[C_6 e^{\eta} + C_7 e^{-\eta}] = 0, \end{aligned} \tag{33}$$

in which $C_i (i = 1 - 7)$ elucidate the arbitrary constants.

TABLE I. Convergence of the HAM solution for different order of approximation when $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$, $\beta_4 = 0.1$, Pr = 1, $N_b = N_t = 0.1$ and $L_e = 1.1$ are fixed.

Order of approximation	-f″ (0)	-θ′ (0)	-ϕ′ (0)
1	0.970712	0.484510	-0.074521
5	0.920563	0.535665	0.250488
10	0.910284	0.539382	0.250488
15	0.909913	0.538080	0.272165
20	0.910203	0.537679	0.271603
26	0.910300	0.537604	0.271490
30	0.910313	0.537600	0.271481
35	0.910313	0.537600	0.271481

B. Convergence analysis

The homotopy analysis method gives an awesome flexibility to choose the auxiliary parameters h_f, h_θ and $\hbar_\phi$ regarding adjustment and control of the convergence of series solutions. To determine the appropriate values of h_f, h_θ and $\hbar_\phi$, we used the minimum square error which is given by

$$F_{f,m} = \frac{1}{N+1} \sum_{j=0}^{N} \left[N_f \sum_{i=0}^{m} F_J \left(i\Delta\eta \right) \right]^2 . \tag{34}$$

Table I is computed to examine the numerical values of -$f''(0)$, -$\theta'(0)$ and -$\phi'(0)$ for different order of approximations. From this table, we have seen that our results for velocity, temperature and concentration converge from 30 th order of approximation.

IV. RESULTS AND DISCUSSION

The transformed set of Eqs. (23) to (25) are highly non-linear equations and constitute a two-point boundary value problem. Graphical analyses of the flow, heat transfer and nanoparticle volume fraction transfer characteristics have been carried out to understand the present problem.

Figs. 1(*a*) and 1(*b*) demonstrate the variation of the Deborah numbers β_1 and β_2 on the velocity, temperature and concentration distributions versus the similarity variable η. It is anticipated from these figures that there is a rise in the temperature and concentration distributions increases with the increase in the Deborah numbers β_1 and β_2. This results in enhancement in the thermal and concentration boundary layers thickness. Moreover, the velocity profile and the associated boundary layer thickness diminishe as the Deborah numbers β_1 and β_2 are augmented. It is illustrated through Figs. 2(*a*) and 2(*b*) that the velocity, temperature and concentration distributions possess a reverse behavior when compared with Figs. 1(*a*) and 1(*b*) for increasing values of the Deborah numbers β_3 and β_4.

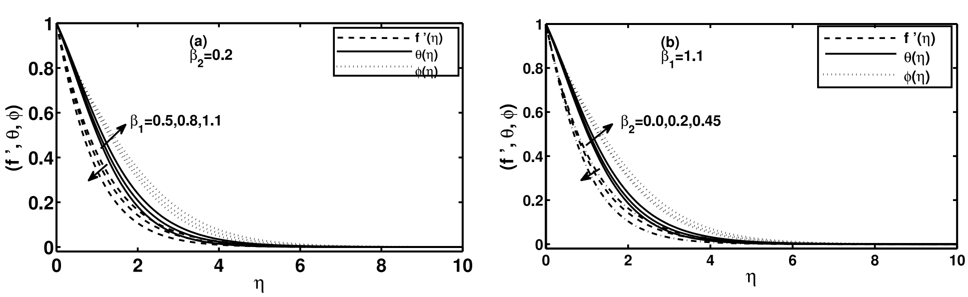

FIG. 1. Diagram of the velocity, temperature and concentration fields for different values of the Deborah numbers β_1 (panel-*a*) and Deborah numbers β_2 (panel- *b*) when $\beta_3 = 0.45$, $\beta_4 = 0.1$, Pr = 1.1 , $N_b = N_t = 0.1$ and $Le = 1.0$ are fixed.

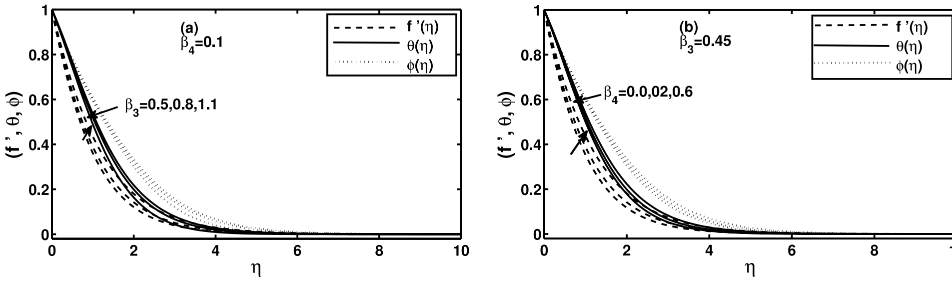

FIG. 2. Diagram of the velocity, temperature and concentration fields for different values of the Deborah numbers β_3 (panel-*a*) and Deborah numbers β_4 (panel- *b*) when $\beta_1 = 0.5$, $\beta_2 = 0.2$, Pr = 1.1 , $N_b = N_t = 0.1$ and $Le = 1.0$ are fixed.

Figs. 3(*a*) and 3(*b*) reveal the variation of the temperature and concentration distributions in response to a change in the Prandtl number Pr. As the value of the Prandtl number Pr increases, the temperature profile, thermal boundary layer thickness, concentration profile and concentration boundary layer thickness diminish. An increase in the Prandtl number Pr means slow rate of thermal diffusion. It is also observed that concentration gradient at the surface increases when the Prandtl number increases. Furthermore, it is also observed that fluids with low Prandtl number decay more slowly when compared to fluids with higher Prandtl number.

The effects of the Brownian motion parameter N_b on the temperature profile and nanoparticle fraction are shown in Fig. 4(*a*). It is observed that as the Brownian motion parameter N_b increases the thermal boundary layer thickness increases and the temperature gradient at the surface decrease. Physically, the Brownian movement of particles is simply the result of all the impulses of the fluid molecules on the surface of the particles. The fluid molecules have significantly high velocities and these velocities depend on the temperature of the fluid. Actually, the molecular velocities define the temperature of a homogeneous fluid. Furthermore, the molecules of fluids at higher temperatures have higher velocities and vice versa. Therefore, the Brownian movement of particles is more intense at higher temperatures. Molecular collisions with particles are almost random and take place at the molecular time scales, which are of the order of femtoseconds and much shorter than the time scales of the particles. Moreover, as it is seen from the graph that the concentration boundary layer thickness decreases as the Brownian motion parameter N_b increases. The impact of the thermophoresis parameter N_t on the temperature $\theta(\eta)$ and nanoparticle concentration $\phi(\eta)$ distributions is shown in Fig. 4(*b*). This figure reveals that an increase in the thermophoresis parameter N_t results in an increase in the temperature distribution. Physically, when there is a temperature gradient in the flow domain of the particulate system, small particles tend to disperse faster in hotter regions and slower in colder regions. The collective effect of the differential dispersion of the particles is their apparent migration from hotter to colder regions. The result of the migration is the accumulation of particles and higher particle concentrations in the colder regions of the particulate mixture. Therefore, the temperature on the surface of a sheet increases. This is due to fact that the thermophoresis parameter N_t is directly proportional to the heat transfer coefficient associated

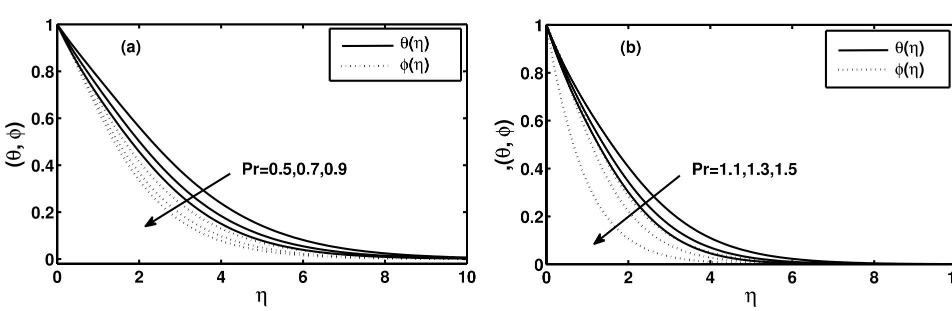

FIG. 3. Diagram of the temperature and concentration fields for different values of the Prandtl number for Pr < 1 (panel-a) and Pr > 1 (panel-b) when $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$, $\beta_4 = 0.1$, $N_b = N_t = 0.1$ and $Le = 1.0$ are fixed.

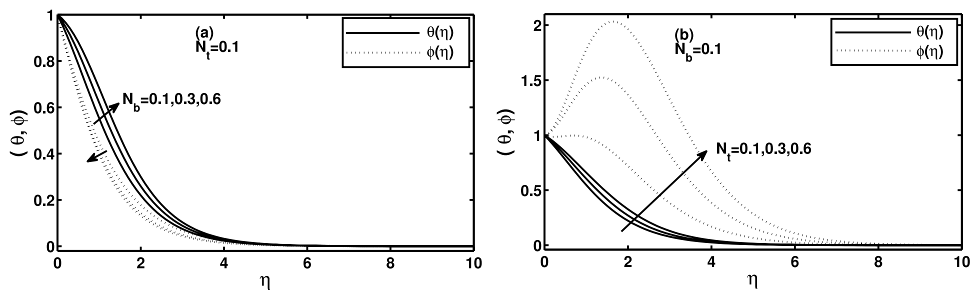

FIG. 4. Diagram of the temperature and concentration fields for different values of the Brownian motion parameter N_b (panel- a) and the thermophoresis parameter N_t (panel- b) when $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$, $\beta_4 = 0.1$, $\mathrm{Pr} = 1.1$ and $Le = 1.0$ are fixed.

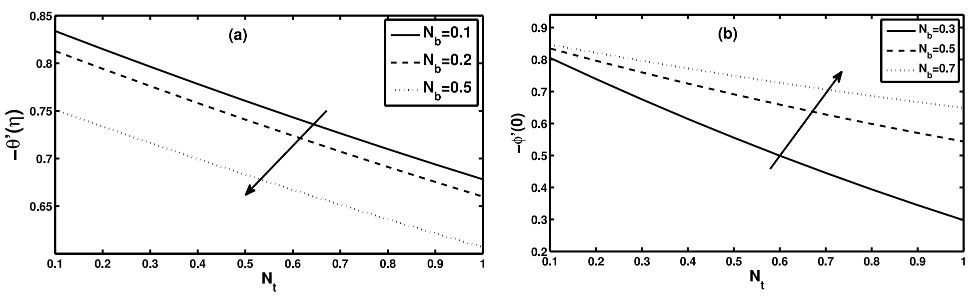

FIG. 5. Diagram of the local Nusselt number (panel- a) and the local Sherwood number (panel- b) for different values of N_b.

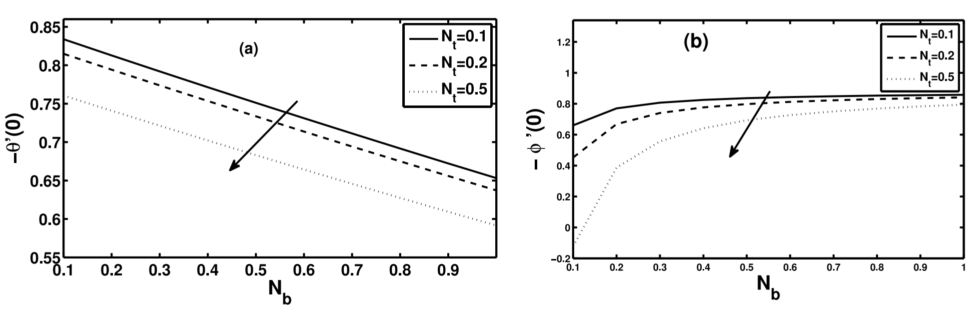

FIG. 6. Diagram of the local Nusselt number (panel- a) and the local Sherwood number (panel- b) for different values of N_t.

with the fluid. Moreover, it is also noticed that as the thermophoresis parameter N_t increases the concentration boundary layer thickness also increases and the concentration gradient at the surface decreases as the thermophoresis parameter N_t increases.

Figs. 5(a) and 5(b) illustrate the variation of local Nusselt number $\theta'(0)$ and the local Sherwood number $\phi'(0)$ in response to change in the Brownian motion parameter N_b. As it can be seen from the graphs, the heat transfer rate on the surface of sheet decreases while the opposite behavior is observed for each incremented value of the Brownian motion parameter N_b. This indicates that an increment in the Brownian motion parameter N_b favor the diffusion of mass. This results in an increase in the concentration gradient on the surface. Figs. 6(a) and 6(b) show the influence of the thermophoresis parameter N_t on the local Nusselt number $\theta'(0)$ and the local Sherwood number $\phi'(0)$. The graphs show that the local Nusselt number and the local Sherwood number both decreases as the thermophoresis parameter N_t increases. Furthermore, we have observed from table II that a similar behavior is obtained for the local Nusselt number $\theta'(0)$ and the local Sherwood number $\phi'(0)$ for increasing values of N_b and N_t.

TABLE II. Variations of the local Nusselt number and local Sherwood number for several sets of physical parameters when Pr, N_b, N_t and Le when $\beta_1 = 0.5$, $\beta_2 = 0.2$, $\beta_3 = 0.45$ and $\beta_4 = 0.1$ are fixed.

Pr	N_t	N_b	Le	$-\theta'(0)$	$-\phi'(0)$
1.0	0.1	0.1	1.0	0.539254	0.222525
1.1				0.570483	0.247336
1.3				0.626940	0.296305
1.1	0.2			0.538991	-0.099143
	0.3			0.508732	-0.418895
	0.4			0.479696	-0.713186
		0.2		0.552397	0.448412
		0.3		0.534892	0.515044
		0.4		0.518096	0.548069
			1.0	0.539254	0.222525
			1.1	0.537600	0.271481
			1.2	0.536122	0.317943

V. CONCLUSIONS

The steady forced convection boundary layer flow of a generalized Burger nanofluid past over a stretching sheet was intended to investigate in this paper. For the nanofluid, we utilize a model proposed by Buongiorno[13] that incorporates the impacts of the Brownian motion and the thermophoresis into the governing equations. The governing nonlinear partial differential equations are transformed into strong nonlinear ordinary differential equations using similarity transformations. The nonlinear ordinary differential equations along with boundary conditions are tackled analytically by the homotopy analysis method. From the presented study, the following conclusions were drawn:

- It was anticipated that the nanofluid temperature $\theta(\eta)$ distribution and thermal boundary layer thickness increases for the Deborah numbers β_1 and β_2 and quite opposite trends were observed for the Deborah numbers β_3 and β_4.
- An increase in the Deborah number β_4 showed an enhancement in the concentration $\phi(\eta)$ and concentration boundary layer thickness.
- The concentration $\phi(\eta)$ and concentration boundary layer thickness were reduced for each incremented values of the Brownian motion parameter N_b.
- The concentration boundary layer thicknes was increased for each incremented values of the thermophoresis parameter N_t.

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