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Forced vibrations of a current-carrying nanowire in a longitudinal magnetic field accounting for both surface energy and size effects

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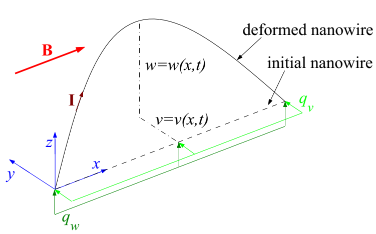
HIGHLIGHTS

- Vibrations of magnetically affected nanowires carrying electric current are of concern.
- Both surface and nonlocality effects are considered in deriving governing equations.
- Strong equations are solved in their weakly discretized form by Galerkin method.
- The effects of magnetic field and electric current on displacements are studied.
- The roles of small-scale parameter and surface effect on deflections are addressed.

GRAPHICAL ABSTRACT

Forced transverse vibrations of current-carrying nanowires in the presence of a longitudinal magnetic field are aimed to be investigated by considering both nonlocality and surface effects.



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ABSTRACT

Forced vibrations of current-carrying nanowires in the presence of a longitudinal magnetic field are of interest. By considering the surface energy and size effects, the coupled equations of motion describing transverse motions of the nanostructure are derived. By employing Galerkin and Newmark-β approaches, the deflections of the nanowire subjected to transverse dynamic loads are evaluated. The effects of the magnetic field, electric current, pre-tension force, frequency of the applied load, surface and size effects on the maximum transverse displacements are discussed. The obtained results display that for the frequency of the applied load lower than the nanowire's fundamental frequency, by increasing the magnetic field or electric current, the maximum transverse displacements would increase. However, for exciting frequencies greater than that of the nanowire, maximum transverse displacements would increase or decrease with the magnetic field strength or electric current. Additionally, the pre-tension force results in decreasing of the maximum transverse displacements. Such a reduction is more apparent for higher values of the magnetic field strength and electric current. The present study would be useful in the design of the micro- and nano-electro-mechanical systems expected to be one of the most wanted technologies in the near future.

1. Introduction

In applied mechanics, nanowires are classified as one-dimensional structures whose length-to-width ratio is commonly greater than 20 and their transverse dimensions constrained to ten nanometers. Due to high Young's modulus of nanowires [1–3], their potential applications in mechanically enhanced composites [4–6] as well as resonators and actuators [7,8] have been extensively examined. Additionally, nanowires are commonly located in vicinity of each other within bundles. Such a fact provides them as tribological additives to enhance friction behavior and stability of nano-electro-mechanical systems (NEMs) made of them. On the other hand, due to their small volumes

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and high surface-to-volume ratio, such NEMs are of great interest for detecting nano-objects with high sensitivity [9–11]. In the near future, nanowires can be employed to link small components into tremendously small circuits. By means of nanotechnology, such components can be built out of chemical compounds. For the later application, the mechanism of vibrations of current-carrying nanowires is aimed to be realized in some detail.

When an electric current passes through a magnetically affected deformed nanowire, a magnetic force would exert on each element of the nanowire. Such a force can be evaluated via Lorentz's formula. The magnitude of the applied force is proportional to the magnetic field strength and the magnitude of electric current. For a straight nanowire subjected to a longitudinal magnetic field, it can be easily shown that the exerted force on the nanowire is equal to zero. However, for a nanowire with an initial deflection, the vector of the electric current would be no longer parallel to the magnetic field vector. Thereby, in such a case, a transverse magnetic force is applied on the nanowire which is a function of the slope of the deformed nanowire as well. It implies that under certain circumferences, the internal stiffness of the nanowire can approach zero and the current-carrying nanostructure would be dynamically unstable. Capturing such extreme conditions is another important goal of the present work since it is expected that the magnetically affected nanowire should transfer safely and effectively electric currents from one place to another one. Herein, the exerted nonlocal magnetic force on the current-carrying nanowire is evaluated, and then through using a string model, the governing equations of the nanostructure are derived via appropriate continuum-based models.

Nanowires also exhibit other unusual electrical properties due to their size. In contrast to single-walled carbon nanotubes whose the electrons can freely travel from one electrode to the other, nanowire conductivity is strongly affected by edge effects. The edge effects originate from surface atoms which are not fully bonded to neighboring atoms. Such unbonded atoms are often a source of deficiency, and may cause the nanowires to conduct electricity more weakly than the bulk material. As the dimensions of a nanowire reduce, the ratio of the surface atoms to the total ones increases and the edge effects become more highlighted. In this work, efficiency of the nanowire as well as its capability in carrying electric current is not of concern. However, theoretical investigations on the role of the edge effects on such consequences are so rare, and the need for further theoretical and experimental studies on such an interesting field is highly demanded. As it will be explained, the effects of the surface and its related energies are considered in the proposed continuum-based model via a surface elasticity model.

To date, large deflections [12], free vibrations [13–16], buckling analysis [17–21] of nanowires, their dynamic behavior in a longitudinal magnetic field [22–24], and their longitudinal and transverse vibrations and instabilities in a three-dimensional magnetic field [25] have been studied. However, vibrations of current-carrying nanowires in the presence of a magnetic field have not been investigated. Given the potential applications of such nanodevices in NEMs and importance of the subject, this work is devoted to study forced vibrations of nanowires transferring electric currents in the presence of a longitudinal magnetic field.

Because of the high ratios of the surface-to-volume of nanowires, the influence of the surface layer on the overall dynamic behavior of the nanostructure becomes highlighted. Therefore, surface energy should be appropriately taken into account in the total strain energy of the nanowire. To this end, the Gurtin–Murdoch model [26,27] has been frequently exploited. According to this model, the surface of a solid structure is modeled as a two-dimensional layer of zero thickness in which being in contact with the inside bulk material without slippage. By defining mass per unit area, residual surface stress as well as non-classical constitutive equations for this layer, its kinetic and strain energies can be evaluated. Thereby, the effects of the surface are appropriately incorporated into the governing equations. Further, the above-mentioned energies for the bulk zone are identical to those of the classical continuum theory. It should be noted that the Gurtin–Murdoch theory does not give us any information regarding inter-atomic bonds and long-range interactions. To consider such effects, nonlocal continuum theory of Eringen [28–30] is employed. In comparison to the classical version of the continuum mechanics, in this advanced theory, the stress dependency of each point of the nano-scaled medium to the stresses of its neighboring points is taken into account through a factor, called small-scale parameter. In the present work, both surface elasticity theory of Gurtin-Murdoch and nonlocal elasticity theory of Eringen are considered in modeling the problem at hand.

This paper deals with the transverse vibrations of a current-carrying nanowire in the presence of a longitudinal magnetic field and externally applied loads. To this end, using nonlocal constitutive equations and Hamilton's principle, the equations of motion of the nanoscale structure are derived accounting for the surface effects. By adopting Galerkin and Newmark-β approaches, the resulting governing equations are solved. In a particular case, the obtained results are compared with those of another work, and a reasonably good agreement is achieved. Subsequently, the roles of the influential factors on the forced vibrations of the nanowire are addressed in some detail. The obtained results would be very helpful in the design of magnetically affected nanowires exploited as electric carriers which is expected to be building blocks of the upcoming NEMs.

2. Description and assumptions of the nanomechanical problem

Consider an elastic nanowire of length l_b with the initial tensile force T_0 whose two ends are prohibited from any transverse motion. The nanowire is subjected to a transverse load denoted by $\mathbf{q}(x,t) = q_v(x,t)\mathbf{e}_y + q_w(x,t)\mathbf{e}_z$ where q_v and q_w are the transverse dynamic loads associated with the y- and z-axes, respectively. The unit base vectors associated with the x-, y-, and z-axes in order are represented by $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ respectively. A constant electric current, $\mathbf{I}$, passes through the nanowire whereas it is acted upon by a longitudinal magnetic field with flux $\mathbf{B} = B_0\mathbf{e}_x$ (see Fig. 1). The coordinate system has been attached to the left end of the nanowire such that the x-axis is coincident with the revolutionary axis of the nanowire and the y-axis towards upward. The density and the cross-sectional area of the nanowire are denoted by ρ_b and A_b, respectively.

The following assumptions are made in studying the problem at hand: (1) the deformation of the nanowire is investigated in the context of the nonlocal-linear elasticity of Eringen; (2) the produced

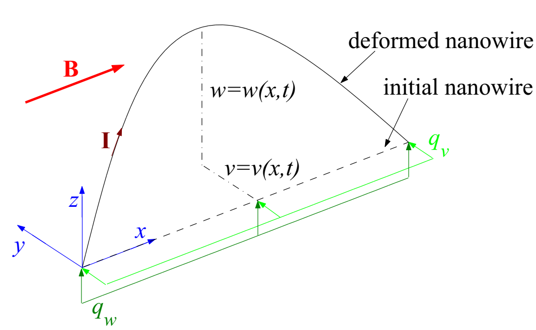

Fig. 1. Schematic representation of a current-carrying nanowire subjected to both a longitudinal magnetic field and laterally distributed loads.

tensile force within the nanowire due to the laterally applied load is negligible in comparison to T_0; (3) the electric current does not alter with time and it is in the same direction of the deformed nanowire. Thereby, it can be expressed by $\mathbf{I} \approx I_0\mathbf{e}_x + I_0 v_{,x}\mathbf{e}_y + I_0 w_{,x}\mathbf{e}_z$ where I_0 is the magnitude of the electric current; (4) the cross-sectional area of the nanowire is very small such that eddy currents can be rationally neglected; (5) the generated magnetic field due to the electric current $\mathbf{I}$ is fairly negligible in comparison to the exerted longitudinal magnetic field; (6) the cross-sectional area of the nanowire is small enough such that the bending rigidity of the nanowire can be neglected; (7) the only externally applied electro-magnetic force on the nanowire is that explained by Lorentz's formula as

$$\mathbf{f}_m = f_{mv}\mathbf{e}_y + f_{mw}\mathbf{e}_z = \mathbf{I} \times \mathbf{B} = B_0 I_0 (w_{,x}\mathbf{e}_y - v_{,x}\mathbf{e}_z). \tag{1}$$

In the following part, the equations of motion of a current-carrying nanowire in the presence of a longitudinal magnetic field are firstly provided in a more general context, called nonlocal-surface energy continuum-based beam. Subsequently, the above-mentioned assumptions are imposed. Thereby, the governing equations would reduce to those of nonlocal lengthy string accounting for the surface energy.

3. Formulations

Based on the Euler–Bernoulli beam theory accounting for the surface energy, the governing equation associated with the transverse vibration of a solid circular nanowire in the z-plane is explained by Ref. [31]:

$$(\rho_b A_b + \rho_0 S_0)\ddot{w} + \frac{\nu_b I_b \rho_0}{r_0}\ddot{w}_{,xx} - M^l_{b,xx} - (T_0 + H_0) w_{,xx} - \left((\lambda_0 + 2\mu_0) I_0 - \frac{\nu_b I_b \tau_0}{r_0}\right) w_{,xxxx} = q_w + f_{mw}, \tag{2}$$

where $w = w(x,t)$ is the transverse displacement along the z-axis, the overdot represents differentiation with respect to time, the parameter after the comma in the subscript denotes the derivative with respect to that parameter, ρ_b and ρ_0 in order are the bulk density and surface density, respectively, $H_0 = S_0\tau_0$, τ_0 is the residual surface stress under unconstrained conditions, $S_0 = \pi r_0$, μ_0 and λ_0 are the surface Lame constants, I_b and I_0 in order are the moment of inertia of the bulk and surface matters, respectively, ν_b is Poisson's ratio of the bulk, M^l_b is the local bending moment, and q_w is the applied dynamic load per unit length of the nanowire.

Based on the nonlocal continuum theory of Eringen, the nonlocal axial stress is related to the local one as Refs. [32–34]:

$$\sigma^{nl}_x - (e_0 a)^2 \sigma^{nl}_{x,xx} = \sigma^l_x = E_b\left(-z w_{,xx} + \frac{T_0}{E_b A_b}\right), \tag{3}$$

where $e_0 a$ is the small-scale parameter and E_b is the modulus of elasticity of the bulk. By premultiplying both sides of Eq. (3) by z and then integrating the resulting expression over the cross-sectional area of the nanowire,

$$M^{nl}_b - (e_0 a)^2 M^{nl}_{b,xx} = M^l_b = -E_b I_b w_{,xx}, \tag{4}$$

where M^{nl}_b represents the nonlocal bending moment within the nanowire. By combining Eqs. (2) and (4) and recalling the sixth assumption, the nonlocal equation of motion of a lengthy current-carrying nanowire subjected to a longitudinal magnetic field accounting for the surface energy is obtained. Therefore,

$$(\rho_b A_b + \rho_0 S_0)(\ddot{w} - (e_0 a)^2 \ddot{w}_{,xx}) - (T_0 + H_0)(w_{,xx} - (e_0 a)^2 w_{,xxxx}) + B_0 I_0 (v_{,x} - (e_0 a)^2 v_{,xxx}) = q_w - (e_0 a)^2 q_{w,xx}. \tag{5}$$

Similarly, the equation of motion pertinent to the nanowire vibration in the y-plane would be obtained as

$$(\rho_b A_b + \rho_0 S_0)(\ddot{v} - (e_0 a)^2 \ddot{v}_{,xx}) - (T_0 + H_0)(v_{,xx} - (e_0 a)^2 v_{,xxxx}) - B_0 I_0 (w_{,x} - (e_0 a)^2 w_{,xxx}) = q_v - (e_0 a)^2 q_{v,xx}. \tag{6}$$

Eqs. (5) and (6) furnish us regarding transverse vibrations of a lengthy current-carrying nanowire in the presence of a longitudinal magnetic field accounting for both size and surface effects. The boundary and initial conditions of the problem are as

$$v(0,t) = v(l_b,t) = 0, \quad w(0,t) = w(l_b,t) = 0, \tag{7a}$$

$$v(x,0) = v_0(x), \quad w(x,0) = w_0(x), \quad \dot{v}(x,0) = \dot{v}_0(x), \quad \dot{w}(x,0) = \dot{w}_0(x), \tag{7b}$$

where $v_0(x)$ and $w_0(x)$ are the initial transverse displacements of the nanowire, and their overdots represent their initial velocities.

4. A solution to the problem

In order to investigate the problem in a more general context, the following dimensionless parameters are considered:

$$\xi = \frac{x}{l_b}, \quad \tau = \frac{Ct}{l_b}, \quad \overline{T}_0 = \frac{T_0}{E_b A_b}, \quad \overline{H}_0 = \frac{H_0}{E_b A_b}, \quad \overline{v} = \frac{v}{l_b},$$
$$\overline{w} = \frac{w}{l_b}, \quad \overline{f}_0 = \frac{B_0 I_0 l_b}{E_b A_b},$$
$$\overline{m}_0 = \frac{\rho_0 S_0}{\rho_b A_b}, \quad \overline{q}_v = \frac{q_v l_b}{E_b A_b}, \quad \overline{q}_w = \frac{q_w l_b}{E_b A_b}, \quad \overline{v}_0 = \frac{v_0}{l_b},$$
$$\overline{w}_0 = \frac{w_0}{l_b}, \quad \dot{\overline{v}}_0 = \frac{\dot{v}_0}{C}, \quad \dot{\overline{w}}_0 = \frac{\dot{w}_0}{C}, \tag{8}$$

where $C = \sqrt{E_b/\rho_b}$ is the speed of the longitudinal wave in the bulk nanowire, ξ is the dimensionless coordinate pertinent to the x-axis, τ is the dimensionless time parameter, μ is the dimensionless small-scale parameter, $(\overline{v}_0, \overline{w}_0)$ denotes the initial dimensionless position of the nanowire, and $(\dot{\overline{v}}_0, \dot{\overline{w}}_0)$ represents the initial velocity of the nanowire in the dimensionless form. By introducing Eq. (8) to Eqs. (5)–(7), the dimensionless equations of motion of a current-carrying nanowire acted upon by a longitudinal magnetic field are obtained as

$$(1+\overline{m}_0)(\overline{v}_{,\tau\tau} - \mu^2 \overline{v}_{,\tau\tau\xi\xi}) - (\overline{T}_0 + \overline{H}_0)(\overline{v}_{,\xi\xi} - \mu^2 \overline{v}_{,\xi\xi\xi\xi}) - \overline{f}_0(\overline{w}_{,\xi} - \mu^2 \overline{w}_{,\xi\xi\xi}) = \overline{q}_v - \mu^2 \overline{q}_{v,\xi\xi}, \tag{9a}$$

$$(1+\overline{m}_0)(\overline{w}_{,\tau\tau} - \mu^2 \overline{w}_{,\tau\tau\xi\xi}) - (\overline{T}_0 + \overline{H}_0)(\overline{w}_{,\xi\xi} - \mu^2 \overline{w}_{,\xi\xi\xi\xi}) + \overline{f}_0(\overline{v}_{,\xi} - \mu^2 \overline{v}_{,\xi\xi\xi}) = \overline{q}_w - \mu^2 \overline{q}_{w,\xi\xi}, \tag{9b}$$

with the following initial and boundary conditions:

$$\overline{v}(0,\tau) = \overline{v}(1,\tau) = 0, \quad \overline{w}(0,\tau) = \overline{w}(1,\tau) = 0, \tag{10a}$$

$$\overline{v}(\xi,0) = \overline{v}_0(\xi), \quad \overline{w}(\xi,0) = \overline{w}_0(\xi),$$
$$\overline{v}_{,\tau}(\xi,0) = \dot{\overline{v}}_0(\xi), \quad \overline{w}_{,\tau}(\xi,0) = \dot{\overline{w}}_0(x). \tag{10b}$$

In order to solve the set of coupled partial differential equations in Eqs. (9a) and (9b), the Galerkin approach is adopted. To this end, the unknown dimensionless displacements are discretized as

$$\overline{v}(\xi,\tau) = \sum_{i=1}^{NM} \overline{a}_i(\tau)\phi^v_i(\xi), \quad \overline{w}(\xi,\tau) = \sum_{i=1}^{NM} \overline{b}_i(\tau)\phi^w_i(\xi), \tag{11}$$

where $\phi^v_i(\xi)$ and $\phi^w_i(\xi)$ denote the ith admissible mode shapes pertinent to $\overline{v}$ and $\overline{w}$. Also, $\overline{a}_i(\tau)$ and $\overline{b}_i(\tau)$ are the unknown time-dependent parameters associated with the ith vibration mode. Such parameters should be appropriately determined at each time. To this end, both sides of Eqs. (9a) and (9b) in order are premultiplied by $\delta\overline{v}$ and $\delta\overline{w}$ where δ represents the variational sign. After taking the required integration by parts, the following set of ordinary differential equations is derived:

$$\begin{bmatrix} \overline{\mathbf{M}}_b^{vv} & \overline{\mathbf{M}}_b^{vw} \\ \overline{\mathbf{M}}_b^{wv} & \overline{\mathbf{M}}_b^{ww} \end{bmatrix} \begin{Bmatrix} \overline{\mathbf{a}}_{,\tau\tau} \\ \overline{\mathbf{b}}_{,\tau\tau} \end{Bmatrix} + \begin{bmatrix} \overline{\mathbf{K}}_b^{vv} & \overline{\mathbf{K}}_b^{vw} \\ \overline{\mathbf{K}}_b^{wv} & \overline{\mathbf{K}}_b^{ww} \end{bmatrix} \begin{Bmatrix} \overline{\mathbf{a}} \\ \overline{\mathbf{b}} \end{Bmatrix} = \begin{Bmatrix} \overline{\mathbf{f}}_b^{v} \\ \overline{\mathbf{f}}_b^{w} \end{Bmatrix}, \tag{12}$$

with the following initial conditions:

$$\begin{Bmatrix} \overline{\mathbf{a}} \\ \overline{\mathbf{b}} \end{Bmatrix}_{\tau=0} = \begin{Bmatrix} \overline{\mathbf{a}}_0 \\ \overline{\mathbf{b}}_0 \end{Bmatrix}; \quad \begin{Bmatrix} \overline{\mathbf{a}}_{,\tau} \\ \overline{\mathbf{b}}_{,\tau} \end{Bmatrix}_{\tau=0} = \begin{Bmatrix} \dot{\overline{\mathbf{a}}}_0 \\ \dot{\overline{\mathbf{b}}}_0 \end{Bmatrix}, \tag{13}$$

where the nonzero vectors and submatrices in Eq. (12) are defined by

$$[\overline{\mathbf{M}}_b^{vv}]_{ij} = (1+\overline{m}_0)\int_0^1 (\phi_i^v\phi_j^v + \mu^2\phi_{i,\xi}^v\phi_{j,\xi}^v)\,\mathrm{d}\xi, \tag{14a}$$

$$[\overline{\mathbf{M}}_b^{ww}]_{ij} = (1+\overline{m}_0)\int_0^1 (\phi_i^w\phi_j^w + \mu^2\phi_{i,\xi}^w\phi_{j,\xi}^w)\,\mathrm{d}\xi, \tag{14b}$$

$$[\overline{\mathbf{K}}_b^{vv}]_{ij} = (\overline{T}_0+\overline{H}_0)\int_0^1 (\phi_{i,\xi}^v\phi_{j,\xi}^v + \mu^2\phi_{i,\xi\xi}^v\phi_{j,\xi\xi}^v)\,\mathrm{d}\xi, \tag{14c}$$

$$[\overline{\mathbf{K}}_b^{vw}]_{ij} = -\overline{f}_0\int_0^1 \phi_i^v(\phi_{j,\xi}^w - \mu^2\phi_{j,\xi\xi\xi}^w)\,\mathrm{d}\xi, \tag{14d}$$

$$[\overline{\mathbf{K}}_b^{wv}]_{ij} = \overline{f}_0\int_0^1 \phi_i^w(\phi_{j,\xi}^v - \mu^2\phi_{j,\xi\xi\xi}^v)\,\mathrm{d}\xi, \tag{14e}$$

$$[\overline{\mathbf{K}}_b^{ww}]_{ij} = (\overline{T}_0+\overline{H}_0)\int_0^1 (\phi_{i,\xi}^w\phi_{j,\xi}^w + \mu^2\phi_{i,\xi\xi}^w\phi_{j,\xi\xi}^w)\,\mathrm{d}\xi, \tag{14f}$$

$$\{\overline{\mathbf{f}}_b^{v}\}_i = \int_0^1 \phi_i^v(\overline{q}_v - \mu^2\overline{q}_{v,\xi\xi})\,\mathrm{d}\xi, \tag{14g}$$

$$\{\overline{\mathbf{f}}_b^{w}\}_i = \int_0^1 \phi_i^w(\overline{q}_w - \mu^2\overline{q}_{w,\xi\xi})\,\mathrm{d}\xi, \tag{14h}$$

$$\{\overline{\mathbf{a}}_0\}_j = 2\int_0^1 \overline{v}_0\ \sin(j\pi\xi)\,\mathrm{d}\xi, \tag{14i}$$

$$\{\overline{\mathbf{b}}_0\}_j = 2\int_0^1 \overline{w}_0\ \sin(j\pi\xi)\,\mathrm{d}\xi, \tag{14j}$$

$$\{\dot{\overline{\mathbf{a}}}_0\}_j = 2\int_0^1 \dot{\overline{v}}_0\ \sin(j\pi\xi)\,\mathrm{d}\xi, \tag{14k}$$

$$\{\dot{\overline{\mathbf{b}}}_0\}_j = 2\int_0^1 \dot{\overline{w}}_0\ \sin(j\pi\xi)\,\mathrm{d}\xi. \tag{14l}$$

For the problem at hand, the following mode shapes are considered:

$$\phi_i^v(\xi) = \sin(i\pi\xi), \quad \phi_i^w(\xi) = \sin(i\pi\xi). \tag{15}$$

These mode shape functions satisfy the boundary conditions in Eq. (10a). By substituting Eq. (15) into Eqs. (14a)–(14l), the elements of the submatrices and vectors can be easily calculated. Thereby, by application of the Newmark-β methodology, the set of second-order ordinary differential equations in Eq. (12) can be solved for the unknown vectors $\overline{\mathbf{a}}$ and $\overline{\mathbf{b}}$ at each time. Thereafter, the transverse displacements of the current-carrying nanowire can be evaluated by using Eq. (11) and their maximum values in the spatial and time domains of the problem are determined.

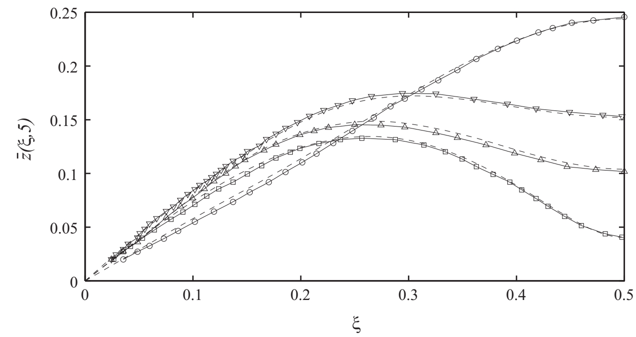

Fig. 2. Comparison of the predicted deflection field of the current-carrying wire immersed in a longitudinal magnetic field by the proposed model with those of Sloss et al. [35] for different levels of the initial tensile force: (○) $\overline{T}_0 = 1.0001$, (△) $\overline{T}_0 = 1.01$, (▽) $\overline{T}_0 = 1.05$, (□) $\overline{T}_0 = 1.1$.

5. Results and discussion

5.1. A comparison study

Due to the lack of both theoretical and experimental data for the problem at hand, the obtained results of the proposed model are justified with those of a magnetically affected macro-scale wire that carries an electric current. For this purpose, the predicted displacements of the proposed model are compared with those of Sloss et al. [35]. In Ref. [35], an analytical model was proposed to study the vibration control of current-carrying wires immersed in a longitudinal magnetic field. The explicit expressions of natural frequencies and transverse displacements of fixed-wires were obtained, and their instabilities were discussed. Now, consider a current-carrying wire subjected to a longitudinal magnetic field with $\overline{f}_0 = -2\pi$ and other given properties in Ref. [35]. The total transverse displacement is defined by $z(x,t) = \sqrt{v^2(x,t)+w^2(x,t)}$. The initial shape of the wire is given by $\overline{w}_0(\xi) = \xi(1-\xi)$ and $\overline{v}_0(\xi) = 0$. Without exerting any externally applied load on the wire, it starts to vibrate from its initial position. In the numerical analysis of the proposed model, we set $NM = 31$ and $\mu = \tau_0 = 0$. At the time $\tau = 5$, the effect of the initial tensile force on the total transverse displacement field of the wire is of concern. For four levels of the initial tensile force (i.e., $\overline{T}_0 = 1.0001$, 1.01, 1.05, and 1.1), such plots are depicted in Fig. 2. The plotted results of the present work and Sloss et al. [35] are presented by the dashed and solid lines, respectively. As it is seen in Fig. 2, there is a reasonably good agreement between the predicted results by the proposed model and those of Sloss et al. [35] through the entire domain of the wire.

5.2. Numerical studies

Consider an aluminum nanowire with the following geometry and physical properties: $\rho_b = 2700\ \mathrm{kg/m^3}$, $E_b = 70$ GPa, $\rho_0 = 5.46 \times 10^{-7}\ \mathrm{kg/m^2}$, $\tau_0 = 0.9108$ N/m, $r_0 = 2$ nm, $l_b = 1000$ nm, and $e_0a = 1$ nm. The nanowire is initially at rest (i.e., $\overline{v}_0 = \overline{w}_0 = 0$ and $\dot{\overline{v}}_0 = \dot{\overline{w}}_0 = 0$) and is subjected to an initial tensile force with $\overline{T}_0 = 0.001$. A harmonic transversely distributed load of the form: $\overline{q}_w = 2 \times 10^{-6} \sin(\overline{\varpi}\tau)$ causes vibrations within the nanostructure where $\overline{\varpi} = 0.7\varpi$ and ϖ represent the dimensionless fundamental frequency of the current-carrying nanowire immersed in a longitudinal magnetic field. The exerted longitudinal magnetic field and the electric current passes through the nanowire are such that $\overline{f}_0 = 0.005$. In the following parts, the influences of the pretension force, magnetic field as well as electric current, frequency of the applied load, and small-scale effect parameter on the transverse displacements of the nanowire are studied.

5.2.1. Effect of pre-tensioning on the dynamic response of the nanowire

In Fig. 3(a)–(c), the time history plots of the transverse displacements of the mid-span point of the current-carrying nanowire due to the externally applied load are provided. The plotted results are given for three levels of the pre-tension force (i.e., $\overline{T}_0 = 0.001$, 0.002, and 0.003) and for two cases: with and without considering the surface energy effect. As it is seen in these figures, both transverse displacements would reduce as the influence of the pre-tension force becomes highlighted. According to Eq. (14), by increasing the pre-tension force, all elements of the stiffness matrices associated with both lateral directions would increase. Thereby, it could be anticipated that the generated transverse displacements would decrease as the pre-tensioned force grows. On the other hand, the length of time interval associated with two adjacent peaks of the plotted displacements becomes shorten as the pre-tensioned force increases. The main reason of this fact is that by increasing the pre-tension force, the fundamental frequency of the current-carrying nanowire would also increase. Since, $\overline{\varpi} = 0.7\varpi$, therefore, the frequency of the applied load would increase as well. It implies that the period of the dynamic displacements would decrease. The obtained results show that the pre-tensioning could be exploited as an efficient approach for decreasing the dynamic displacements of the current-carrying nanowire due to the applied magnetic field and externally applied loads.

According to Fig. 3(a)–(c), the transverse displacements without considering the surface energy effect are generally greater than those obtained by considering the surface energy effect. Further, the period of the dynamic displacement in the case of without considering the surface effect is greater than that for the case of with consideration of the surface effect. The main reason of this fact is that the surface effect helps to the flexural stiffness of the nanowire by increasing the total tensile force within the nanowire (see Eqs. (14c) and (14f)). As it is seen in Fig. 3(a)–(c), by increasing the initial tensile force, the discrepancies between the predicted transverse displacements by the proposed model for the above-mentioned two cases would reduce. It is chiefly related to this fact that the ratio of the axial force due to the surface effect to the total tensile force would lessen; therefore, the effect of the surface energy becomes vanished.

Hereby it is emphasized that the given discussion in the previous part is only valid for those nanowires which are made of materials with positive residual surface stress. For nanowires with negative residual surface stress, the bending stiffness would reduce with the surface effect (see Eqs. (14c) and (14f)). Therefore, the periods of dynamic displacements would magnify with respect to the case of without consideration of the surface effect. In such a case, it is also anticipated that the discrepancies between the transverse displacements of two cases would lessen as the influence of the initial tensile force becomes highlighted.

5.2.2. Effect of longitudinal magnetic field and electric current on the dynamic response of the nanowire

The predicted dynamic displacements of the midspan point of the nanowire are provided in Fig. 4(a)–(c) for three levels of $\overline{f}_0$ (i.e., 0.05, 0.053, and 0.056). According to Eq. (8), this parameter is proportional to the product of the magnetic field strength and the electric current. Thereby, by studying the effect of this parameter on the dynamic response, the effects of both longitudinal magnetic field and electric current on the vibrations of the nanowire can be displayed. A close scrutiny of the plotted results in Fig. 4(a)–(c) reveals that both transverse displacements would grow as $\overline{f}_0$ increases. Furthermore, the influence of $\overline{f}_0$ on the variation of $\overline{v}$ is more obvious with respect to $\overline{w}$. Actually, the transverse displacement associated with the direction of the applied load (namely w) is highly affected by the exerted load; thereby, the influence of the parameter $\overline{f}_0$ on $\overline{w}$ becomes lesser. As it is obvious from Fig. 4(a)–(c), the discrepancy between the times associated with two adjacent peak points would generally increase with $\overline{f}_0$. It is mainly related to the reduction of the fundamental frequency of the nanowire due to the increasing of $\overline{f}_0$ (for example, the proposed model predicts that the dimensionless fundamental frequency of the nanostructure would be 0.1528, 0.1288, and 0.0971 for $\overline{f}_0 = 0.05$, 0.053, and 0.056, respectively). It means that the frequency of the applied load would also reduce since the ratio of the applied load frequency to that of the nanowire has

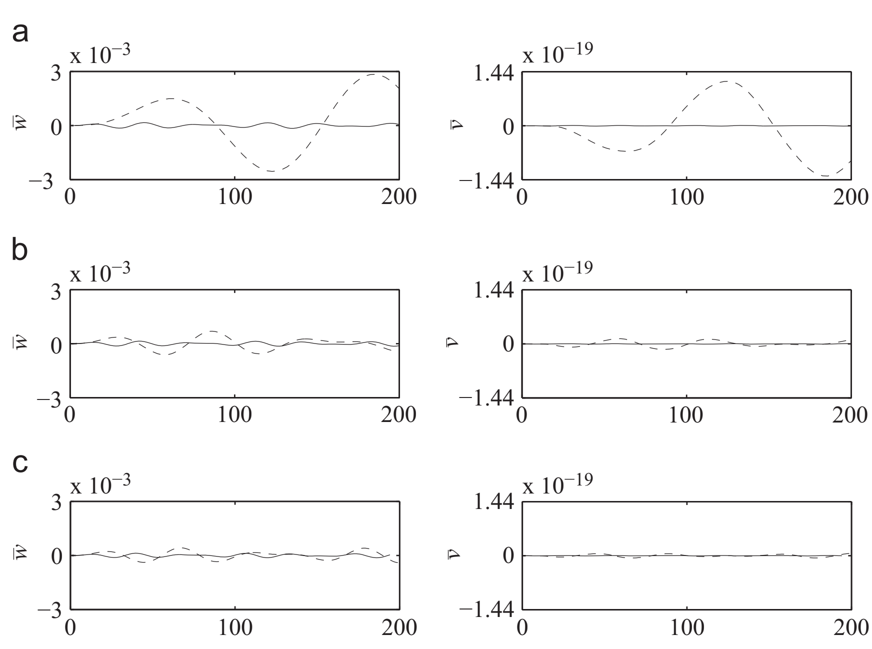

Fig. 3. Time history plots of transverse displacements of the nanowire for different levels of the initially tension force: (a) $\overline{T}_0 = 0.001$, (b) $\overline{T}_0 = 0.002$, (c) $\overline{T}_0 = 0.003$; (—) with surface energy; (– –) without surface energy.

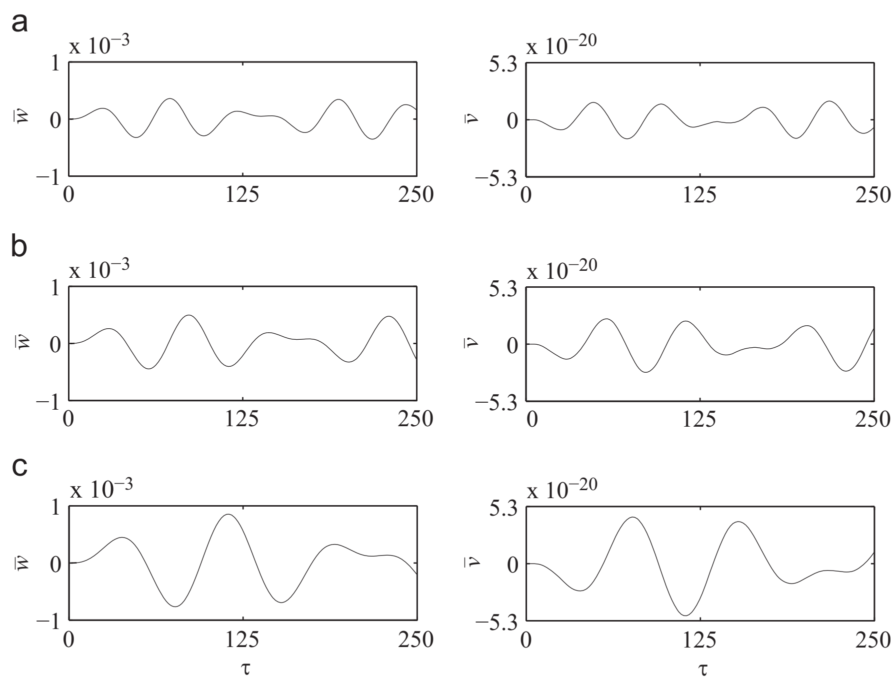

Fig. 4. Time history plots of transverse displacements of the nanowire for different levels of magnetic field as well as electric current: (a) $\overline{f}_0 = 0.05$, (b) $\overline{f}_0 = 0.053$, (c) $\overline{f}_0 = 0.056$.

been kept fixed; thereby, the discrepancy between the times of two adjacent peaks would magnify as $\overline{f}_0$ grows.

5.2.3. Effect of the frequency of the applied load on the maximum transverse displacements of the nanowire

An interesting parametric study has been conducted to determine the role of the frequency of the applied load on the maximum dynamic displacements. In Fig. 5, the plots of the maximum dimensionless transverse displacements of the nanowire in terms of dimensionless frequency of the applied load are provided. The results are provided for three levels of the magnetic field as well as electric current (i.e., $\overline{f}_0 = 0.03$, 0.05, and 0.08). When the frequency of the applied load is lower than the fundamental frequency of the nanowire, both transverse displacements would increase by an increase of the frequency of the applied load. Further, increasing the magnetic field strength or electric current would result in an increase of the transverse displacements. When the frequency of the applied load is greater than that of the nanowire, by increasing the frequency of the applied load up to a certain level, the maximum transverse displacement associated with the direction of the applied load would generally decrease. For such a range of the applied load frequency, maximum value of displacement would decrease or increase as a function of the applied frequency according to the magnitude of the magnetic field or electric current. This matter also holds true for another component of displacement. Such evidence guides us to use an envelope curve to determine the maximum transverse displacement of a nanowire because of a harmonic load. This curve is constructed by connecting the peak points of maximum displacement (for various levels of $\overline{f}_0$) in terms of the frequency of the applied load. The envelope curve would be very useful for practical purposes, particularly when the nanowire may experience various longitudinal magnetic fields or electric currents during its service life.

As it is seen in Fig. 5, the effect of the magnetic field strength or electric current is more obvious on the transverse displacement which is perpendicular to the applied load. Additionally, the plots of the maximum displacements as a function of the applied load frequency have locally maximum points, in which by an increase of the magnetic field or electric current, their peaks would increase and take place at lower levels of the frequency. As it is explained in the previous part, the fundamental frequency of the nanowire would lessen by an increase of the magnetic field or electric current. Thereby, the resonance of the nanowire due to the externally applied load would occur at lower frequencies of the applied load. It is also worth mentioning that the second peak of the transverse displacement perpendicular to the applied load occurs at the vicinity of the second natural frequency of the current-carrying nanowire. For frequencies of the applied load which are close to the natural frequencies of the nanowire, the effect of the magnetic field flux or electric current on both transverse displacements is more apparent with respect to other frequencies.

5.2.4. Effect of the pre-tension force on the maximum transverse displacements of the nanowire

The effect of the pre-tension force within the current-carrying nanowire on the generated maximum transverse displacements due to the externally applied load and longitudinal magnetic field is of interest. To this end, the plots of the maximum dimensionless displacements as a function of the dimensionless pre-tension force for three levels of $\overline{f}_0 = 0.03$, 0.035, and 0.04 are provided in Fig. 6. According to the plotted results, for all levels of $\overline{f}_0$, the maximum transverse displacements would decrease as the pre-tension force magnifies. Such a fact is also more obvious for those nanowires subjected to higher levels of the magnetic field and electric current. For lower levels of the pre-tension force, the effect of the pre-tension force on the variation of the maximum displacements of the nanowire is more obvious. Additionally, for higher values of the pre-tension force, the effect of the magnetic field or electric current on the maximum transverse displacements becomes less important.

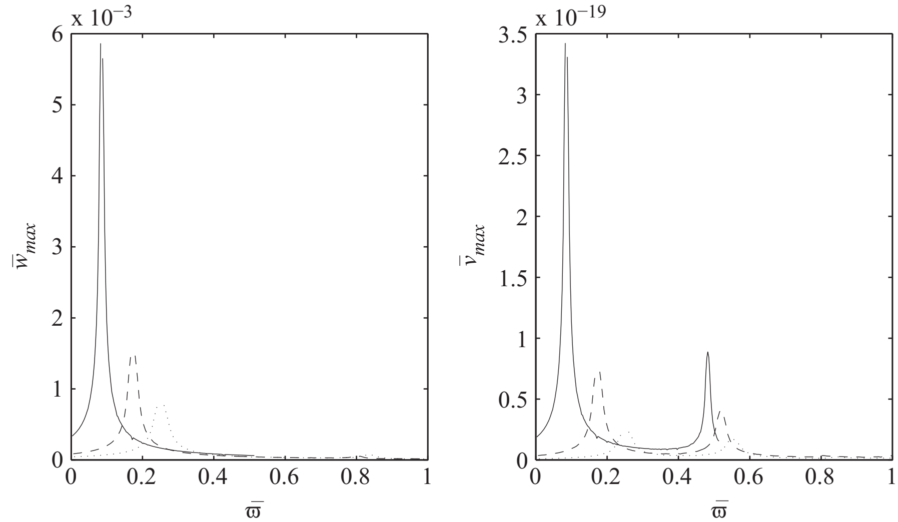

Fig. 5. Maximum transverse displacements as a function of frequency of the applied load for different levels of the magnetic field and electric current: (...) $\bar{f}_0=0.03$, (– –) $\bar{f}_0=0.05$, (—) $\bar{f}_0=0.08$.

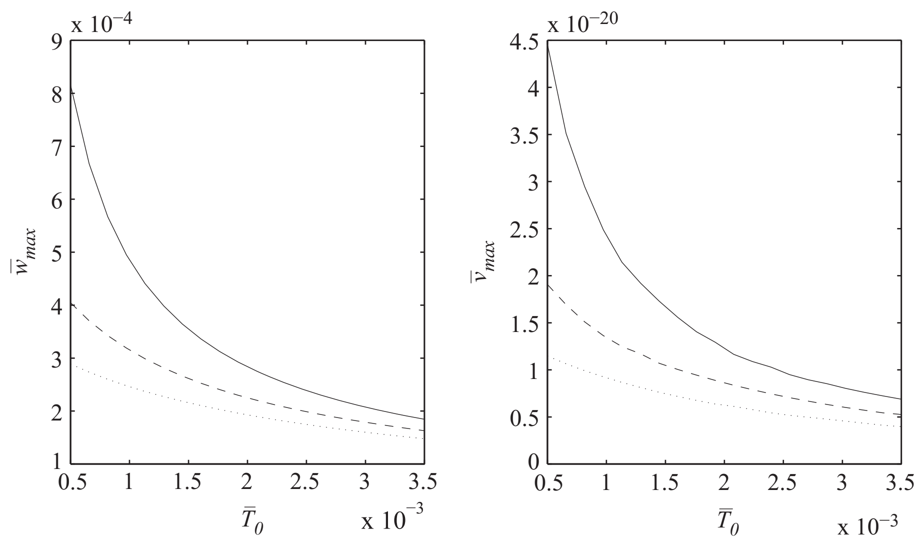

Fig. 6. Maximum transverse displacements as a function of initial tensile force for different levels of the magnetic field and electric current: (...) $\bar{f}_0=0.03$, (– –) $\bar{f}_0=0.035$, (—) $\bar{f}_0=0.04$.

5.2.5. Effect of the longitudinal magnetic field and electric current on the maximum transverse displacements of the nanowire

Equally important is to carefully determine the role of the longitudinal magnetic field as well as electric current on the generated transverse displacements due to a harmonic load. In Fig. 7, the plots of maximum transverse displacements as a function of $\bar{f}_0$ are provided for three levels of the initial tensile force within the nanowire (i.e., $\bar{T}_0=0.0025$, 0.003, and 0.0035). As it is seen in Fig. 7, the maximum transverse displacements of the nanowire would increase by an increase of the magnetic field or electric current. Additionally, such an effect is more obvious for those current-carrying nanowires with lower levels of the initial tensile force and higher levels of magnetic field or electric current. The main reason of this fact is that the transverse stiffness of the nanowire generally decreases as the initial tensile force within the nanowire decreases. On the other hand, by increasing the magnetic field or electric current, the transverse stiffness of the nanowire reduces. Such crucially obtained results guide us to this fact that a combination of these factors (i.e., initial tensile force, strength of the longitudinal magnetic field, and electric current) could endanger stability of the current-carrying nanowires. Such an interesting subject is out of the scope of the present study, and their analytical solutions and the nature of the nanostructure instability will be presented in another work which is underway by the author.

5.2.6. Effect of the small-scale parameter on the maximum transverse displacements of the nanowire

Another numerical study has been carried out to address the effect of the small-scale parameter on the maximum transverse displacements of the current-carrying nanowire in the presence of a longitudinal magnetic field. The plots of dimensionless transverse displacements of the nanostructure due to a harmonic force as a function of the small-scale effect parameter have been demonstrated in Fig. 8. The obtained results are depicted for a nanowire of length 50 nm with $\bar{f}_0=0.055$ under three levels of the initial tensile force (i.e., $\bar{T}_0=0.0035$, 0.004, and 0.0045). According to the plotted results in Fig. 8, both transverse displacements would lessen as the small-scale parameter increases. A close scrutiny of the obtained results also reveals that variation of the small-scale parameter on the variation of the component of displacement perpendicular to the applied load is more apparent.

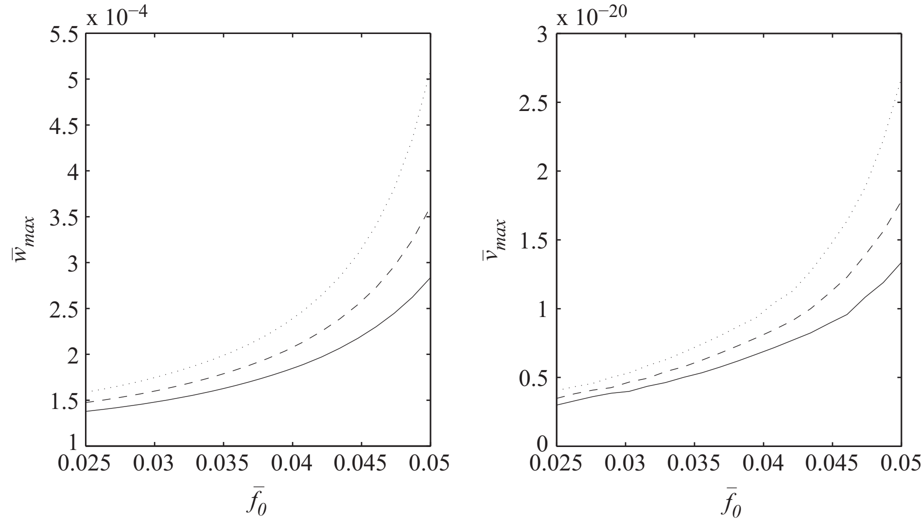

Fig. 7. Maximum transverse displacements as a function of $\overline{f}_0$ for different levels of the initial tensile force: (...) $\overline{T}_0 = 0.0025$, (− −) $\overline{T}_0 = 0.003$, (−) $\overline{T}_0 = 0.0035$.

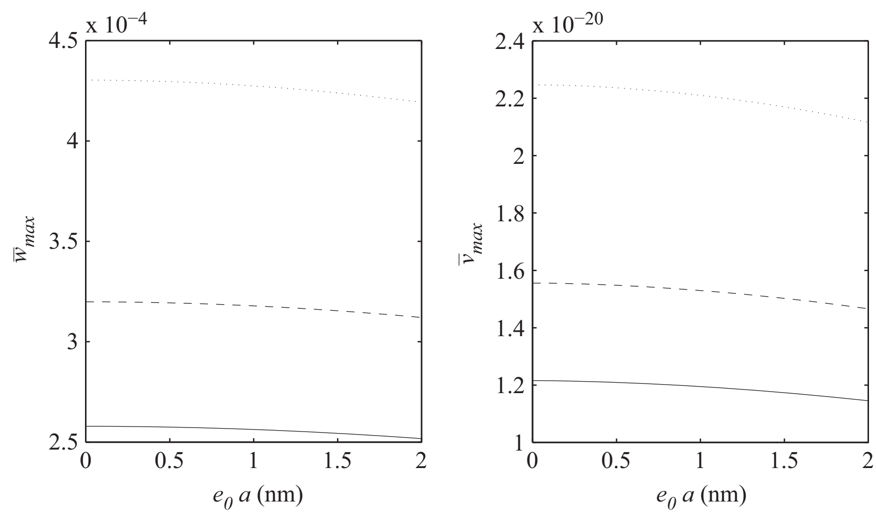

Fig. 8. Maximum transverse displacements as a function of the small-scale parameter for different levels of the initial tensile force: (...) $\overline{T}_0 = 0.0035$, (− −) $\overline{T}_0 = 0.004$, (−) $\overline{T}_0 = 0.0045$; l_b=50 nm, $\overline{f}_0 = 0.055$.

Additionally, by increasing the initial tensile force within the nanowire, the influence of the small-scale parameter on the maximum transverse displacements would slightly decrease.

6. Concluding remarks

Transverse vibrations of lengthy current-carrying nanowires subjected to a longitudinal magnetic field and externally applied loads are investigated. Accounting for both size and surface effects, the equations of motion of the problem are obtained. By employing the Galerkin approach, the set of coupled partial differential equations reduces to a set of second-order ordinary differential equations. Via the Newmark-β method, the time-dependent parameters are then evaluated at each time and the dynamic displacements of the nanostructure are determined. The important findings of this work are as

- The components of the transverse displacements of the nanowire in the presence of both longitudinal magnetic field and electric current within the nanowire are coupled. Such a coupling effect is the main cause of generation of the displacement in a perpendicular direction with respect to the direction of the applied load.
- For nanowires with positive residual surface stress, surface effect has a tendency to reduce transverse displacements since the transverse stiffness of the current-carrying nanowire increases as the effect of surface energy becomes highlighted. Such a role plus to that of the initial tensile force would provide a more stable nanostructure.
- Irrespective of the level of the magnetic field strength or electric current, the pre-tension force would result in a decrease of the maximum transverse displacements. Such a reduction is more obvious for higher levels of the strength of the magnetic field or electric current.
- The influence of magnetic field or electric current on the maximum transverse displacements strongly depends on the load's frequency. For those frequencies which are lower than the fundamental frequency, maximum transverse displacements would increase with the magnetic field strength or electric current. Such a fact is more apparent for lower levels of the initial tensile force. If the frequency of the applied load would be greater than the fundamental frequency, maximum

displacements would increase or decrease as the magnetic field or electric current magnifies.

- Generally, maximum transverse displacements of the current-carrying nanowire would reduce as the small-scale parameter increases. Such a fact becomes less important as the initial tensile force within the nanowire magnifies.

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