

FORCES AND ENERGY CHANGES IN THE LEG DURING WALKING

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The study of locomotion yields information not only concerning the mechanics of progression but also concerning the part that muscles play in this intricate activity. Early attempts at a quantitative analysis of human locomotion were made by the Webers, Marey and Otto Fischer, followed in recent years by Bernstein (1927, 1935) and by Fenn (1929, 1930). The present paper is concerned with a detailed analysis of the dynamics of the human leg in walking, providing data concerning muscle function.

In his fundamental investigations of the kinematics of walking, Fischer (1901) was hindered by the fact that he could not determine the point of application of the force exerted by the ground on the foot. The path of this point during the time the foot is on the ground was determined by Elftman and Manter (1934) from cinematic records of pressure distribution in the foot, obtained by a method described by Elftman (1934). For the purposes of the present research, however, a new apparatus has been devised (Elftman, 1938), which gives not only the point of application of this force, but also the magnitude of the force in three components. In addition to solving Fischer's difficulty, this obviates the necessity of dealing with the entire body when only one portion, such as the leg, is of immediate interest.

The point of application of this force, as it passes forward during the course of the step, is shown in figure 1. The force itself is plotted in two components, one in the plane of progression, the other lateral, in the horizontal plane. Only the component in the plane of progression will be considered in the present discussion. It is apparent from the diagram that it has two maxima and that it is at first directed upward and backward against the foot, later upward and forward. This reaction of the platform is plotted in figure 6 in two components, one vertical and the other horizontal, in the plane of progression.

In addition to this knowledge concerning the external force exerted by the platform, it is necessary to know the disposition of the leg in space. This is shown in figure 2 for the left leg during the double step under con-

sideration. The original information was obtained from cinematic records taken at the rate of 92 exposures per second as the subject walked behind a rectangular grid. The timing was obtained by including a vibrating reed

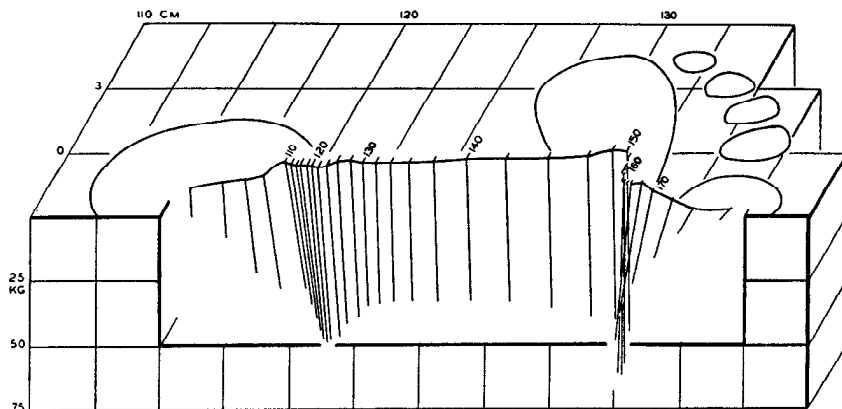


Fig. 1. Reaction of the platform on the foot and its point of application. The reaction is shown in two components, one in the plane of progression, the other lateral, in the horizontal plane.

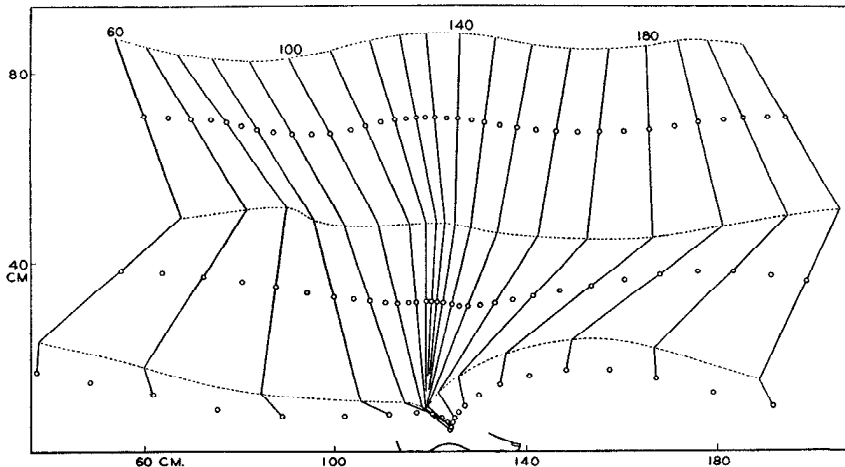


Fig. 2. Position of the left leg during a double step. The centers of gravity of the thigh, shank and foot are indicated by circles. Time interval between successive positions of the leg, 0.08 sec. The phases are numbered in hundredths of a second, the time of application of the left foot to the platform being taken as 100.

of known period in the photographic field. These records were projected on a large sheet of paper and the positions of the axes of the hip-, knee- and upper ankle-joints determined. After plotting these positions against

time, values were read off at intervals of 0.02 sec. and these were used in all further calculations. The phases are numbered according to the time at which they occurred in hundredths of a second, taking the time of establishment of contact by the foot with the platform as 100. In figure 2 the positions of the axes of the limb are shown at intervals of 0.08 sec. The circles indicate the positions of the centers of gravity of the thigh, shank and foot. They have been determined by applying the proportions

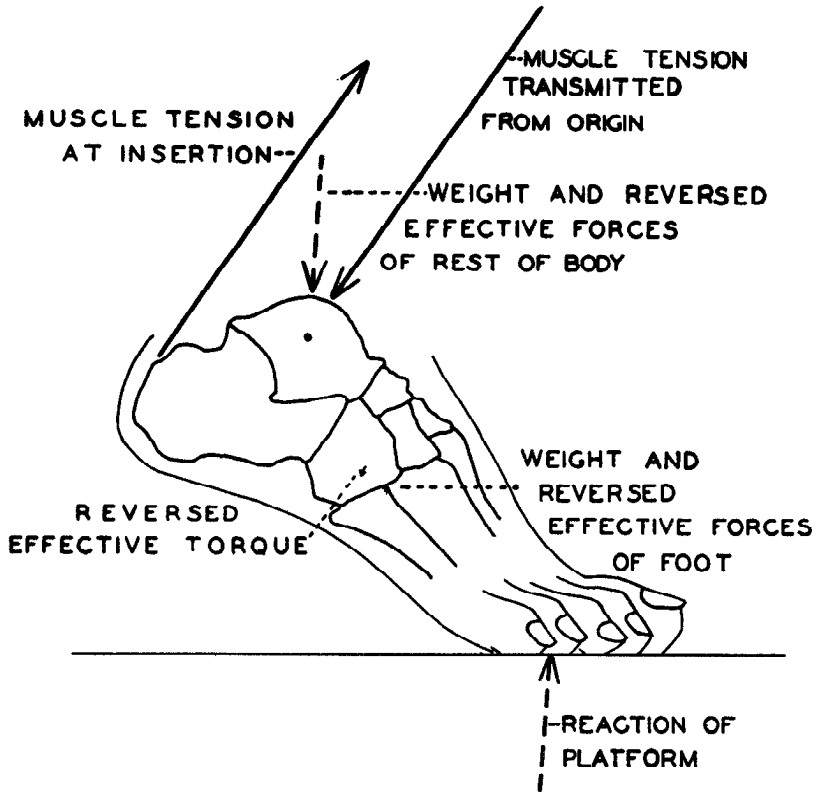


Fig. 3. Free body diagram of foot

given by Fischer (1906) for their positions with respect to the adjacent joints.

Determination of forces and torques. The external force and the position of the leg, as portrayed in figures 1 and 2, constitute the fundamental observed data, from which other information, more essential to our purpose, may be derived. The next step is to determine the forces acting on the foot. These are shown in figure 3, which is a free-body diagram of the foot. According to D'Alembert's principle, the reversed effective forces

must be in equilibrium with all other forces acting on the body. Consequently if all the forces are known with one exception, that exception can be solved. Upon inspection of the forces, we find that the reaction of the platform is known (figs. 1 and 6). The weight of the foot is known to be 1.14 kgm. by applying Fischer's proportions to the total body weight of 63.4 kgm. The reversed effective force is equal to $-ma$, when m is the mass of the foot in gravitational units and a the acceleration of the center of gravity of the foot. The acceleration is obtained by plotting displacement against time and differentiating graphically, using the prism method, obtaining the velocity. The process is then repeated with the velocity to get the acceleration. The reversed effective force can therefore be calculated; it is plotted in figure 5.

The only force left which is not a member of a couple is the one which acts through the ankle-joint, due to the weight and reversed effective forces of the rest of the body. Since this force must be in equilibrium with the forces already known, it is readily calculated. This ankle-joint force is shown in figure 6, plotted, however, as it acts on the shank, which is opposite to its effect on the foot.

With all the forces determined, it is possible to approach the torques. Since the positions of the forces are known, their lever arms about any desired point can be measured and so their moments, or torques, about that point determined. The reversed effective torque is calculated in a manner analogous to that used for reversed effective forces. The moment of-inertia, calculated from Fischer's data, is 0.046 kgm. slug cm^2 for the foot. The angular acceleration is obtained by double differentiation of the angular displacement. The reversed effective torque in walking is quite small; it is plotted in figure 7. The only torque still unknown is that of the muscles, consequently that can be found by solving the equations for equilibrium.

With the forces and torques acting on the foot completely determined, it is possible to repeat the operations with the shank and the thigh. This is possible because the joint force acting on the member above the joint is equal in magnitude but opposite in direction to the force acting on the member lying below the joint.

The uppermost curve of figure 7 shows the torque exerted by the leg muscles on the trunk. It is possible to calculate this torque because the sum of the torques exerted by any muscle must be zero. This is illustrated for a one-joint muscle in figure 4. The muscle under a tension F will exert a force of $+F$ at its origin and $-F$ at its insertion. When the two members AB and BC upon which the muscle acts are considered separately, it is found that the force $-F$ exerted at the insertion is transmitted to AB through the joint B . Since the force on AB at the origin is $+F$ and at the joint $-F$, the two forces acting together constitute a

couple, the torque of which is obtained by multiplying the tension by the perpendicular distance between the two forces. It is obvious from the diagram that the torque on BC is equal in magnitude but opposite in direction to that on AB . The sum of the two torques is consequently zero. The same proof may be elaborated for muscles which traverse more than one joint. Consequently the torque exerted by the leg muscles on the trunk must be such that when it is added to the sum of the torques exerted on the foot, shank and thigh the result is zero.

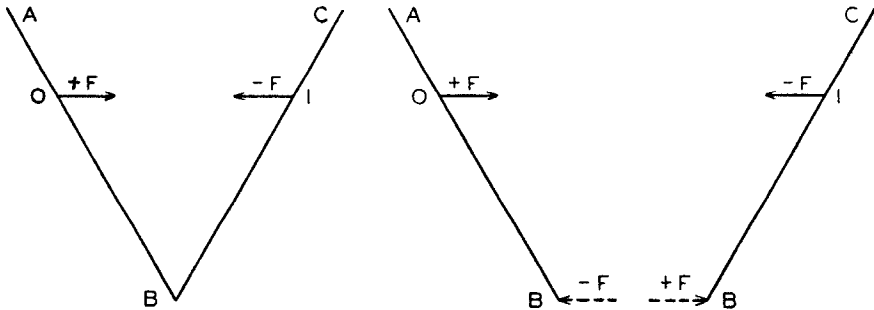


Fig. 4. Diagram of one-joint muscle. The member AB is acted on by the force $+F$ of the muscle at its origin. The force $-F$ exerted by the muscle at its insertion is transmitted to AB through the joint. The two forces constitute a couple.

Significance of the forces and torques. The reversed effective forces vary with changes in the momentum of the member, since

$$\text{reversed effective force} = -ma$$

$$\text{and } -ma = -\frac{dmv}{dt}$$

The reversed effective force is consequently the force exerted by the member on other parts of the system due to a decrease in the momentum of the member. When the momentum of the member is increasing, the reversed effective force is negative. The rate of change of linear momentum of the parts of the leg can consequently be read from figure 5. While the left foot is in contact with the ground, the horizontal momentum of all parts of the leg first decreases and then increases. The vertical momentum varies in a less regular fashion. The time during which the foot is stationary on the ground is indicated by the fact that the reversed effective forces of the foot are zero.

The platform reaction, figure 6, represents the only external force acting on the body while the left foot alone is in contact with the substratum, since air resistance may be neglected at the low velocity of walking. The

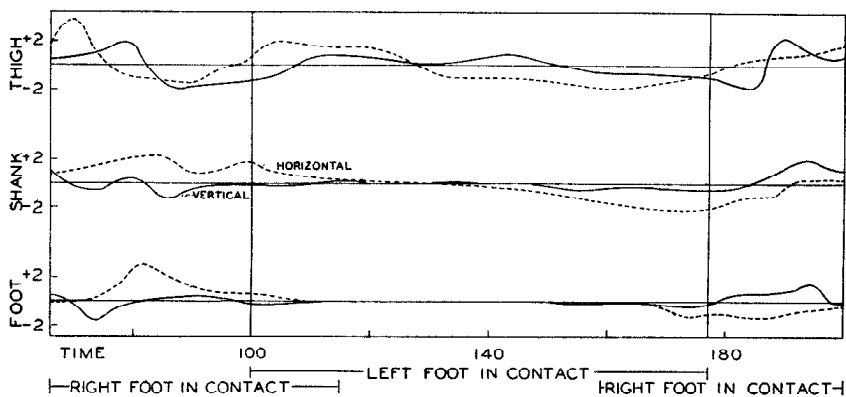


Fig. 5. Reversed effective forces in kgm. Vertical component, continuous line; horizontal component, dotted.

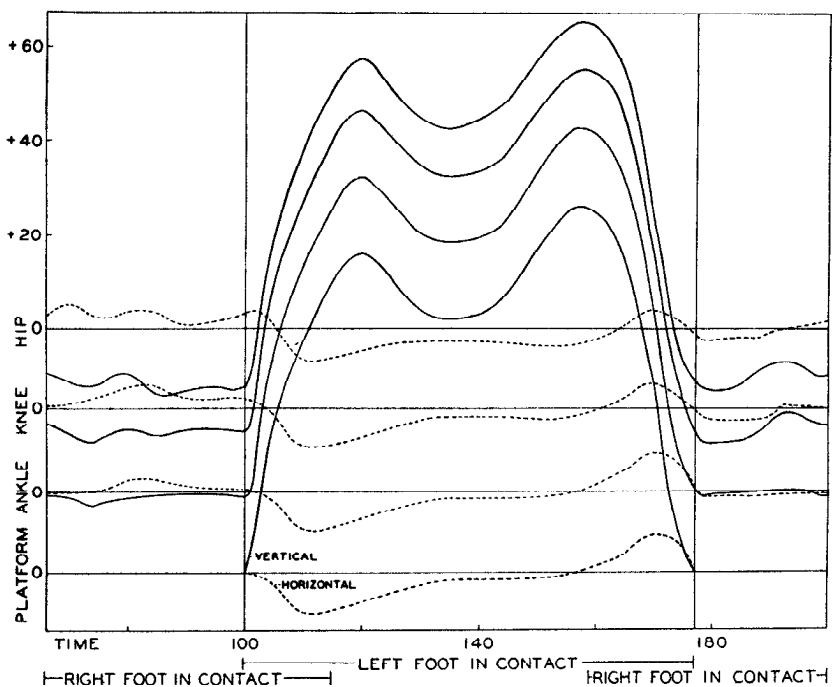


Fig. 6. Platform reaction and the joint forces which are due to the platform reaction, gravity and effective forces. Forces in kgm. The forces plotted are those which act on the member above the joint. The force on the member below the joint is of the same magnitude but opposite in direction. Vertical forces, continuous line; horizontal forces, dotted.

horizontal component of this reaction is at first negative, pushing backwards against the body, and is later positive. This may also be seen in figure 1. The vertical component includes the reaction to gravity as well as the reaction to the effective forces transmitted through the left leg. The reaction to gravity is equal to the weight of the body, but is directed upward, for the interval during which the left foot alone is in contact. By subtracting this value from the total vertical reaction, the portion due to acceleration of the body as a whole may be determined and from this, by integration, changes in the momentum of the body as a whole.

The vertical force exerted by the platform describes a characteristic bimodal curve. Both maxima exceed the value for the reaction to gravity, while the intervening minimum is lower than this value.

The forces acting at the joints, due to the platform reaction, gravity and effective forces, are plotted in figure 6 as they act on the member above the joint. There is, of course, an equal but oppositely directed force which acts on the member below the joint. The force plotted here is not the total force acting through the joint, since it does not include the muscle forces. For purposes of dynamics the effect of the muscle forces can best be obtained from a consideration of the couples by means of their torques. If our purpose were the investigation of pressure in the joints, the procedure would be different.

The joint forces for the interval during which the foot is in contact with the platform vary, in general, in the same way as the platform reaction, since this is the largest component entering into their composition. Each vertical component is decreased, as we ascend from the foot to the thigh, by the weight of the parts of the leg which lie below the joint under consideration as well as by the effective forces of these parts. The joint forces are present while the foot is off the ground, at which time the platform reaction is absent.

The reversed effective torques are shown in figure 7 in dotted lines, when they are of sufficient magnitude to be plotted. A positive reversed effective torque represents the rate at which the angular momentum of the member about its own center of gravity is decreasing. In walking, the reversed effective torques are so small that they could be disregarded during the interval of contact of the foot with the ground. In the present research they have been included in all calculations.

The torques of the reversed effective forces and joint forces are not plotted in figure 7. Since they must be in equilibrium with the muscle torques and reversed effective torques, they must, at each instant, be equal in magnitude to the sum of the muscle and reversed effective torques, but of opposite sign.

The muscle torques, figure 7, represent the resultant torque of all muscles acting on the member. When antagonistic muscles are in action simul-

taneously, the algebraic sum of their torques will be equal to this resultant torque. The torques are plotted in the usual fashion, a positive torque tending to produce counter-clockwise acceleration of the parts of the leg as they are oriented in figure 2. If the leg behaved as a conventional compound pendulum, there would be no muscle torques during its swing. This was the basis of Fischer's proof (Fischer, 1904) that the swinging leg does not act as a pendulum. On the basis of figure 7 it is now possible

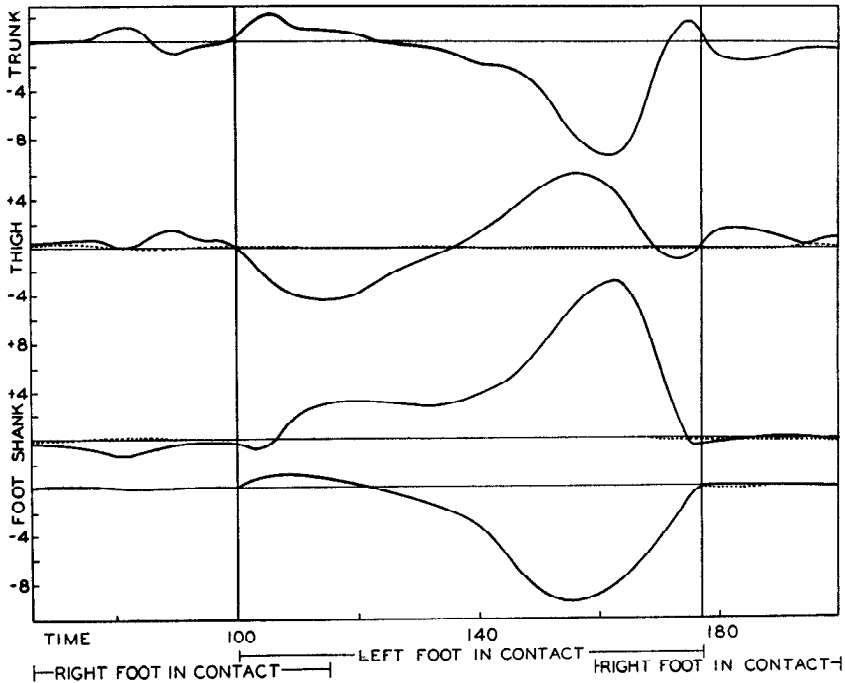


Fig. 7. Muscle and reversed effective torques in kgm. m. The muscle torques represent the resultant of all muscle torques acting on the member. Positive torques tend to produce counter-clockwise rotation. Muscle torque, continuous line; reversed effective torque, dotted.

to extend this conclusion to the body as a whole while the foot is in contact with the ground.

The tension in the muscles can be computed from the torque if the lever arms are known. In general, limiting values for tension can be obtained in this way by considering the range in values for the lever arms of various muscles which might be concerned in the production of the resultant torque. The minimum value thus calculated would be exceeded when antagonistic muscles are simultaneously under tension. The maximum torque about the ankle-joint occurred at phase 156, at which time

it was 950 kgm. cm. With a lever arm of 4.0 cm. at that moment, the triceps surae was exerting a tension of 237.5 kgm., a value 3.7 times the weight of the individual. This is by no means a maximum value for the muscle, since it occurred while walking at a very moderate rate.

Energy transfer. The rate at which energy is transferred, or the rate at which work is being done on or by the various components of the system, can be determined since the forces and torques and the velocities of their points of application are known. The results are plotted in figure 8 in kgm. m. / sec., from which the value in horse-power may be derived, if desired, since 1 h.p. = 76.065 kgm. m. / sec. The area under each curve represents the total energy involved. A positive area measures the amount of work done by the force on other parts of the system, a negative area the energy which it receives from the other parts.

The rates of change of potential energy and of kinetic energy due to changes in linear and angular velocity have all been added together and the negative value of this sum plotted. Consequently when the curve labelled K.E. + P.E. is positive, energy is being released from the member, either to be passed on by the joint forces or into the muscles. From the time the left heel leaves the ground (which is indicated by the resumption of muscle power on the foot), through the early part of the swing, energy is being absorbed to increase the combined kinetic and potential energy of the parts of the leg. During the latter part of the swing, shown in the first part of the graph, this energy is given back to the other parts of the system. When considered with respect to the muscle and joint forces, kinetic and potential energy changes in the leg are of more importance when the leg is swinging than when it is in contact with the ground. While the foot is on the ground it is the changes in kinetic and potential energy of the rest of the body which are of importance, as indicated by the curve for the action of the hip-joint forces on the thigh.

The rate of energy transfer due to the joint forces is plotted separately for the forces at the proximal and distal ends of each member. The work done on the member by these forces is the algebraic sum of the areas under these curves, but by plotting them separately it is possible to follow the transfer of energy from one member to those adjacent to it. The platform reaction does no work on the foot. The uppermost curve in figure 8 gives the rate at which work is done on the rest of the body by forces acting through the hip-joint.

The rate at which the muscles do work on, or receive energy from, each part of the leg is shown in figure 8 in continuous lines. This does not represent solely the energy received by the muscle in stretching or released in contraction, but also energy which is transmitted by the muscle from one point of attachment to another. The value for the power of the leg muscles acting on the trunk is obtained by multiplying the torque of these

muscles, shown in figure 7, by the angular velocity of the trunk. The determination of this torque is equal in accuracy to the determination of the other muscle torques. Difficulty, however, is experienced in measuring

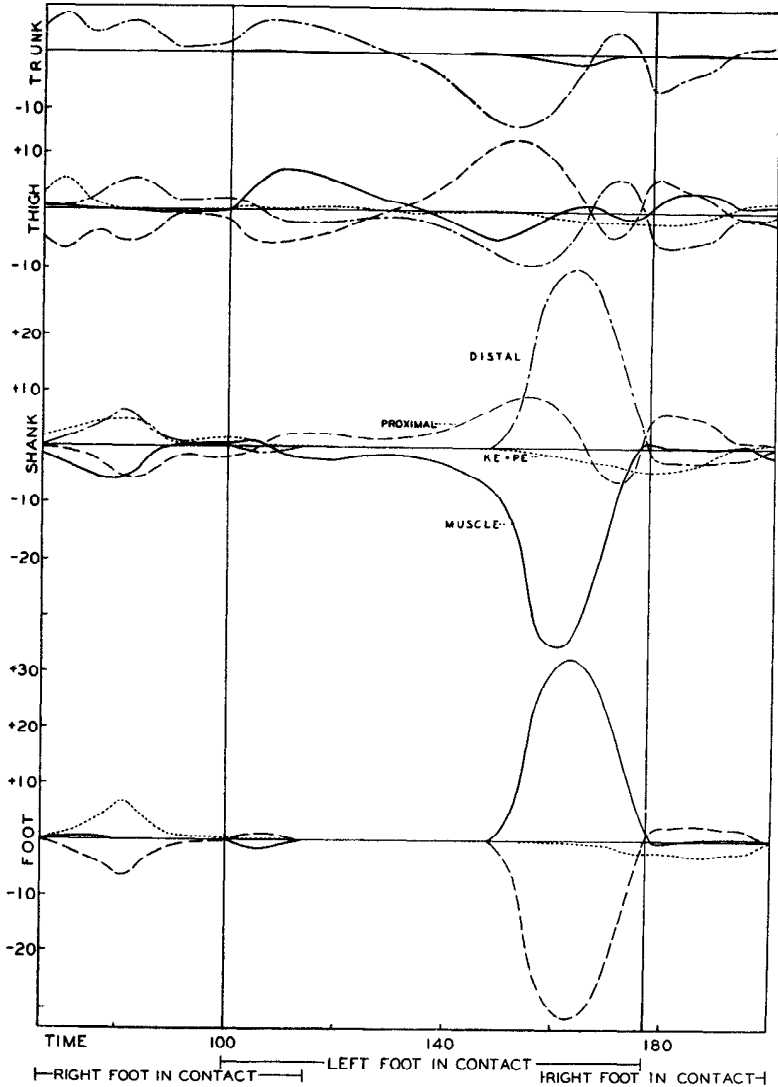


Fig. 8. Rate of doing work (power) in kgm. m./sec. Positive values indicate that energy is being expended, negative that it is being received. The area under each curve measures the total energy involved. Curves for muscle, proximal and distal joint forces, and contribution of energy from changes in combined kinetic and potential energy of member.

pelvic rotation. The values for the power plotted here agree well with those which the author has obtained from unpublished calculations based on Fischer's (1899) data.

Transfer of energy between the leg and the rest of the body. Before considering in detail the transfer of energy within the leg, it is advantageous to consider the leg as a whole. In figure 9 are plotted the combined values for the foot, shank and thigh of rates of energy change due to the action of muscles on the leg and of decrease in kinetic and potential energy of the leg, liberating energy for other purposes. The only joint force now present is the force at the hip-joint, acting on the leg.

In the latter part of the swing of the leg, which occupies the first portion of the graph, the leg is losing combined kinetic and potential energy, and this energy is available for other purposes. Since both the muscle and the hip-joint curves are negative at this time, the energy liberated must

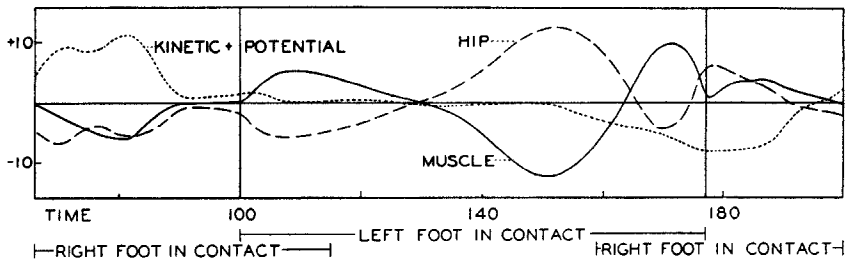


Fig. 9. Power curves for the left leg as a whole, kgm. m/sec. The muscle curve shows the rate at which muscles contribute to, or subtract from, the energy of the leg; the hip curve the rate of energy change due to joint forces at the hip; and the kinetic and potential curve the rate at which energy is contributed by the decrease in combined kinetic and potential energy of the leg.

be absorbed by the muscles and by the rest of the body, in the relative amounts indicated by the curves. The disposition of the energy received by the rest of the body cannot be traced without considering the right leg, which at this time is in contact with the ground. The energy received by the leg muscles is taken up by the muscle tissue, either for storage or dissipation. The other possibility is for these muscles to expend energy on the trunk, but the power of the leg muscles on the trunk is negligible at this time.

After the foot comes into contact with the ground, the combined kinetic and potential energy of the leg continues to liberate energy. The muscles now do work on the leg. The energy released is passed on to the rest of the body, as is indicated by the fact that the hip-joint forces are receiving energy at this time. From the time at which the right foot leaves the ground, the force acting through the hip-joint and the torque of the left

leg muscles on the trunk represent the only external forces, in addition to gravity, which act on the rest of the body. We therefore know that energy absorbed by the rest of the body must be used in increasing the sum of its kinetic and potential energies. That the potential energy of the body is increasing may be inferred from the upward displacement of the hip-joint at this time. From the velocity curves published by Fischer (1899), the velocity of the body as a whole is seen to increase as the foot comes into contact with the ground, decreasing, however, before the center of gravity of the body reaches its maximum height.

It is consequently not surprising that the hip-joint forces begin to do work on the leg, as shown in figure 9, before the hip-joint reaches its maximum elevation. This energy, at first contributed by decrease in kinetic energy of the body, later also by decrease in potential energy, is only used in small part for the increase in combined kinetic and potential energy of the leg. By far the greater part of it is received by the muscles of the leg.

During the last 0.1 sec. of contact of the foot with the ground, it is the muscles of the leg which are releasing energy. This is used chiefly in increasing the kinetic and potential energy of the leg. Some of this energy, however, is transferred to the rest of the body, although towards the end of the period of contact the rest of the body is again contributing energy to the leg.

During the early portion of the swing, the leg is increasing its combined kinetic and potential energy. This energy is contributed by the muscles and by the rest of the body, in proportions which vary as the values in figure 9 indicate.

Transfer of energy within the leg. During the latter portion of the swing, shown at the left in figure 8, the foot is decreasing its combined potential and kinetic energy and to this is added a small amount of work done by muscles acting on the foot. The energy thus made available is transferred to the shank by the joint forces in the ankle, since they receive energy from the foot (dashed line) and contribute it to the shank, where they constitute the distal joint forces (dash-dot line). In addition to receiving energy from the foot, the shank is also contributing energy from its combined kinetic and potential energy. Part of this energy is passed on to the thigh by the knee-joint force, the rest being received by muscles. A similar procedure is taking place in the thigh, although there the energy liberated is almost equal to that which is passed on, through the hip-joint to the trunk, leaving little energy exchange to the muscles.

After the foot makes contact with the ground, the chief source of energy is muscle action on the thigh. Part of this energy is passed on to the trunk by the hip-joint force, the remainder to the shank through the

knee-joint, although for about 0.05 sec. the knee-joint force transfers energy in the other direction. The energy carried to the shank is absorbed by muscles.

From phase 130 to 150 energy is supplied by the rest of the body through the hip-joint. This energy is partially taken up by the muscles acting on the thigh, partially transferred to the shank to be received by muscles there.

From phase 150 to 165 energy is still being provided by the hip-joint force and is partially taken up by muscles acting on the thigh, except at the close of the interval, at which time the thigh is receiving a small amount of energy from the muscles. The increase in combined kinetic and potential energy of the thigh withdraws a small amount of energy. The remainder of the energy from the hip-joint force is transferred to the shank. The shank also receives a large amount of energy from the foot through the ankle-joint, this energy being released by the muscles acting on the foot. Energy is taken from the shank by the increase of its combined kinetic and potential energy, but by far the larger portion of it is received by muscles.

From phase 165 to 174 energy is being transferred to the rest of the body through the hip-joint from the thigh. Energy is also taken from the thigh by the increase in its combined kinetic and potential energy. The total amount of energy so involved is smaller than it would be if the subject had not been slowing down slightly. The chief source of the energy is by transmission from the shank. The only source of energy for the shank at this time is transmission from the foot through the ankle. Enough energy is transmitted in this way for forwarding to the thigh, for increase of the kinetic and potential energy of the shank and for reception by muscle. The source of the energy from the foot is the action of muscles on the foot.

In the first part of the swing of the leg, energy is supplied from the rest of the body through the hip-joint and from the muscles working on the thigh, this energy being transmitted so as to allow the increase in kinetic and potential energy of the shank and foot without appreciable muscular work on those members.

Résumé of energy changes. Starting with the leg in mid-air, we find that the combined kinetic and potential energies of the foot, shank and thigh are all decreasing, liberating energy for other purposes. Somewhat more than half of this energy is transmitted to the rest of the body, the remainder being received by the muscles, chiefly those acting on the shank.

During the first portion of the period during which the foot is in contact with the ground, the changes in kinetic and potential energy of the leg are small. Muscles acting on the thigh contribute sufficient energy for

transmission of a considerable amount to the trunk and a smaller amount to the shank, from which it is taken by muscles.

In the middle portion of the contact period, energy is given to the leg by the rest of the body, to be received by muscles acting on the thigh and shank.

When the heel starts to rise, muscles acting on the foot liberate energy, which is transferred to the shank, to be received by muscles acting on the shank. The rest of the body contributes energy to the thigh, from which it is partially taken by muscles acting on the thigh, partially transferred to the shank to be received by muscles there. The kinetic and potential energy of the leg is also increasing at this time.

Toward the conclusion of the period of contact, the rest of the body is receiving energy from the leg. This energy, together with that which is necessary for the increase in kinetic and potential energy of the leg, is supplied by muscles acting on the foot.

During the first portion of the swing, the kinetic and potential energies of all portions of the leg are increasing. The energy for this increase is supplied by the rest of the body through the hip-joint and by muscles acting on the thigh.

The reception and release of energy by the leg muscles take place in two complete cycles for each double step. The first cycle starts in the middle of the swing, the second while the foot is on the ground.

DISCUSSION. Considered as a locomotor mechanism, the human body represents a compromise between the principles of physical efficiency and the dictates of sound anatomical structure. A wheel rolling upon a level surface need only supply enough energy to overcome friction in order to progress with a uniform velocity. Even on an undulating surface the wheel may roll on without expending extra energy, by converting kinetic energy into potential as it rises, reconverting it into kinetic as it falls. This implies changes in velocity but not of total energy.

The human mechanism overcomes the problem of external friction by not rolling, but introduces difficulties of a more serious nature. Not only does the combined center of gravity move in such a fashion that no exact conversion of kinetic into potential energy is possible, but the limbs must swing forward and backward with consequent flux of energy.

Muscles not only provide forces which guide the limbs into trajectories impossible for compound pendulums, but they also regulate the energy distribution of the body. When the total kinetic and potential energy of the mechanism increases, the energy is supplied by muscles. When the total energy decreases, it is taken up by muscles. The extent to which the energy received may again be issued is a moot question. If the muscles were perfect accumulators and were able to exert force without the expendi-

ture of chemical energy, then the efficiency of human locomotion would approach that of the rolling wheel, the friction of the joints and resistance to deformation of the tissues supplanting the friction of the ground. It is consequently not the innate mechanical structure of the body which limits locomotor efficiency as much as it is the imperfect qualification of muscle tissue for the functions it is called upon to perform.

SUMMARY

1. By recording the reaction of the ground and its point of application, together with motion pictures of the displacement of the body, it is possible to study the kinetics of the leg without the necessity of considering the entire body.

2. The method of determining the instantaneous value of the forces and torques acting on each part of the leg is described. By this means the torques exerted by the muscles on the foot, shank and thigh, and of the leg muscles on the trunk, are found.

3. The transfer of energy within the leg and between the leg and the rest of the body is followed by means of the activity of the forces and torques.

4. The total kinetic and potential energy of the leg increases as the leg is swung forward, decreasing again from the middle of the swing until the foot is in contact. This increase and decrease in energy is only partially balanced by transfer of energy from or to the rest of the body. The maintenance of energy equilibrium is due to the action of muscles, which alternately receive the excess energy and supply the deficit.

5. The reception and release of energy by the leg muscles in walking take place in two cycles for each double step, the first starting when the leg is in the middle of its swing, the second while the foot is on the ground. The regular alternation of reception and release of energy suggests the possibility of partial storage of energy by the muscles.

REFERENCES

- BERNSTEIN, N. Untersuchungen der Biodynamik des Ganges und des Laufes. Moscow, 1927.
 Untersuchungen ueber die Biodynamik der Lokomotion. Moscow, 1935.
- ELFTMAN, H. Anat. Rec. **59**: 481, 1934.
 Science **88**: 152, 1938.
- ELFTMAN, H. AND J. MANTER. Science **80**: 484, 1934.
- FENN, W. O. This Journal **92**: 583, 1929.
 This Journal **93**: 433, 1930.
- FISCHER, O. Abh. K. Sächs. Ges. Wiss., Math.-Phys. Classe **25**: 1, 1899; **26**: 471, 1901; **28**: 533, 1904.
 Theoretische Grundlagen für eine Mechanik der lebenden Körper. Leipzig, 1906.

SUMMARY OF FUNDAMENTAL DATA AND METHOD OF CALCULATION. *Constants of the subject.* The total weight of the subject at the time of the experiment was 63.4 kgm. Using the proportions determined by Fischer (1906) the constants of the parts of the body were as follows:

	<i>Foot</i>	<i>Shank</i>	<i>Thigh</i>
Weight in kgm.....	1.14	3.34	7.35
Mass in kgm. slugs (gravitational units).....	0.00116	0.0034	0.0075
Moment of inertia in kgm. slug cm. ²	0.046	0.33	1.19
Length in cm.....	6.4	39.5	40.6
Proportion of distance of center of gravity from lower joint to total length.....		0.58	0.56

The lengths of the shank and thigh are measured between joint axes; the length of the foot given here is the distance from the ankle joint to the center of gravity of the foot.

Explanation of table 1. The fundamental data from which the quantities discussed in this paper are calculated are given in the table for intervals of 0.1 sec. The actual computations were carried out for five times this number of phases. The *x*- and *z*-coordinates are given. The angle ϕ is the angle between the long axis of the member and the vertical axis. Parts of the body are indicated by subscripts: ankle joint, *a*; knee joint, *k*; hip joint, *h*; foot, 7; shank, 5; thigh, 3; trunk, 1. The velocities, or first derivatives with respect to time, are indicated by one dot, e.g., \dot{x} , and the acceleration, or second derivative by two dots, \ddot{x} . The accelerations of the centers of gravity are not given directly, to economize space, since they are multiplied by the mass of the member, which is constant, to give the effective force, denoted by *E*. The force exerted by the platform on the foot is shown in a horizontal component R_x and a vertical component ($R_z + R_g$) which includes components due to gravity and effective forces. The final entry is x_i , the *x* coordinate of the point of application of the force exerted by the platform.

Method of calculation. To illustrate the method of calculation, some of the computations for phase 150 will be presented. Since the joint force, as defined in the text, acting at the proximal, or upper, joint of a member must be in equilibrium with the other external forces acting on the member and the reversed effective force, we may write:

$$\text{proximal joint force} = -(\text{distal force} + \text{force of gravity on member} + \text{reversed effective force of member})$$

For the foot the distal force is the platform reaction. The force is computed in vertical and horizontal, or *z* and *x* components, the term for gravity being present only for the vertical component. For the ankle-joint force, acting on the foot, the *z* component = $-(70.0 - 1.14 - 0) = -68.86$ or -68.9 kgm. The *x* component = $-(-1.3 - 0) = +1.3$ kgm. The ankle joint force acting on the shank is of opposite sign to that acting on the foot, but of the same magnitude. With this force known, the formula given above may be applied to the shank. The *z* component of the knee-joint force, acting on the shank = $-(68.86 - 3.34 - 0.3) = -65.22$ or -65.2 kgm. and the *x* component = $-(-1.3 - 1.24) = +2.5$ kgm. Similarly, the *z* component of the hip-joint force acting on the thigh = $-(65.22 - 7.35 + 0.2) = -58.1$ kgm. and the *x* component = $-(-2.5 - 1.3) = +3.8$ kgm.

The torque of the muscles acting on the foot must be in equilibrium with the torques due to external forces and the reversed effective torque. Taking moments about the ankle joint, the lever arm of the vertical component of the platform reaction is the horizontal distance of the point of application of the force from the ankle-joint, or $131.4 - 119.0 = 12.4$ cm., and the torque is $70 \times 12.4 = +868$ kgm. cm. The

lever arm of the horizontal component is the distance of the floor from the ankle-joint, or -8.5 cm., and the torque is $-(-1.3)(-8.5) = -11$ kgm. cm. The lever arms of the forces acting at the mass center of the foot are $6.4 \cos 52.6^\circ = +5.1$ cm. for the vertical components and $-6.4 \sin 52.6^\circ = -3.9$ cm. for the horizontal component. The torque of gravity is therefore $-1.1 \times 5.1 = -5$; of the vertical reversed effective force, $0 \times 5.1 = 0$; and of the horizontal reversed effective force, $-0(-3.9) = 0$. The reversed effective torque is obtained from the moment of inertia and

TABLE I

TIME	70	80	90	100	110	120	130	140	150	160	170	180	190	200
x_a	66.0	96.7	113.0	118.2	119.0	119.0	119.0	119.0	119.0	120.2	124.4	135.5	153.5	177.5
x_k	84.5	92.8	100.4	109.2	116.2	120.1	122.4	125.0	129.2	137.6	149.9	166.3	184.7	200.9
x_h	62.3	69.6	79.8	90.3	100.8	110.5	117.7	125.8	135.1	144.7	155.0	164.9	173.4	181.9
z_a	16.2	11.0	10.3	9.4	8.6	8.5	8.5	8.5	8.5	10.8	15.0	20.5	23.7	18.7
z_k	51.1	50.3	47.7	47.8	47.9	48.0	47.9	47.5	46.7	46.2	45.0	45.2	47.9	50.5
z_h	85.2	83.6	82.7	83.7	85.5	87.5	88.2	88.1	86.9	86.2	85.3	85.8	86.9	86.4
$\dot{\phi}_7$	+23.0	+56.7	+69.0	+58.5	+52.7	+52.6	+52.6	+52.6	+41.4	+16.8	-9.8	-7.4	+14.8	
$\dot{\phi}_6$	-28.0	+5.7	+18.6	+13.2	+4.0	-1.6	-4.9	-8.7	-15.0	-26.2	-40.2	-51.3	-52.2	-36.3
$\dot{\phi}_5$	+33.1	+33.6	+30.5	+27.7	+22.2	+13.7	+6.7	-1.1	-8.3	-10.0	-7.2	+2.3	+16.1	+27.9
$\dot{\phi}_4$	-6.5	-7.1	-7.2	-6.7	-5.8	-6.1	-7.1	-7.7	-7.8	-7.3	-6.2	-6.2	-6.9	-7.5
x_7	+333	+267	+90	-21	0	0	0	0	+1	+11	+31	+125	+236	+298
x_6	+201	+154	+91	+64	+27	+17	+14	+18	+33	+67	+109	+171	+204	+190
x_5	+109	+81	+96	+102	+78	+58	+50	+63	+78	+99	+121	+142	+130	+110
z_7	-74.5	+7.5	-12.0	-24.0	-4.0	0	0	0	+2.0	+22.0	+37.5	+61.0	-2.0	-73.0
z_6	-29.0	-24.0	-10.0	-0.2	+0.2	0	0	-2.0	-1.6	+10.6	+19.8	+29.7	+21.0	-26.0
z_5	-5.9	-23.0	-6.9	+10.4	+16.0	+5.0	+1.0	-5.0	-13.4	-9.5	-0.3	+13.2	+19.0	+2.0
$\dot{\phi}_7$	+5.20	+4.32	-0.30	-1.96	-0.57	0	0	0	-0.23	-3.40	-5.16	-3.50	+2.35	+4.90
$\dot{\phi}_5$	+5.02	+4.25	0	-1.72	-1.18	-0.68	-0.55	-0.86	-1.36	-2.82	-2.02	-1.20	+1.20	+4.10
$\dot{\phi}_4$	+1.28	-0.40	-0.27	-0.50	-1.86	-1.34	-1.28	-1.50	-0.93	+0.05	+0.80	+1.64	+2.84	+1.18
$\dot{\phi}_1$	-0.15	-0.05	+0.01	+0.17	+0.11	-0.16	-0.14	-0.07	+0.03	+0.15	+0.15	-0.10	-0.14	-0.06
x_a	+302	+252	+91	+28	+2	0	0	0	+2	+28	+59	+160	+221	+266
x_k	+127	+73	+91	+92	+46	+27	+22	+34	+53	+95	+145	+179	+192	+138
x_h	+83	+87	+101	+109	+101	+79	+71	+95	+97	+103	+102	+112	+81	+89
z_a	-90.0	-18.3	-10.0	-15.4	-1.1	0	0	0	+3.0	+39.0	+50.0	+54.0	0	-79.0
z_k	+11.5	-36.0	-10.0	+4.5	+3.6	-0.7	-1.9	-5.4	-10.0	-9.8	-5.0	+14.6	+37.0	+17.0
z_h	-19.6	-19.0	-4.4	+15.0	+29.6	+12.2	+4.3	-6.4	-15.5	-9.2	0	+12.0	+5.0	-6.0
$\dot{\phi}_7$	+18.0	-30.3	-40.0	+12.0	+10.0	0	0	0	-5.0	-22.0	-6.0	+52.0	+9.5	+31.5
$\dot{\phi}_6$	+29.0	-38.0	-25.4	-3.5	+6.3	+5.1	-1.0	-4.4	-6.5	-7.5	+3.5	+17.8	+29.4	+23.4
$\dot{\phi}_4$	-27.6	-8.0	+3.0	-7.0	-2.8	+2.1	-1.4	+0.8	+9.0	+10.6	+10.8	+8.4	+4.0	-13.5
E_{x7}	0	-2.5	-1.2	-0.6	0	0	0	0	0	+0.2	+0.7	+1.1	+1.0	+0.4
E_{x5}	-1.1	-2.1	-0.9	-1.7	-0.5	-0.2	0	+0.3	+1.24	+1.4	+2.15	+1.7	+0.3	-0.4
E_{x3}	-4.1	+1.0	+1.4	-0.8	-1.6	-1.4	+0.5	+1.0	+1.3	+1.8	+1.3	0	-0.8	-1.7
E_{z7}	+0.3	+0.3	-0.4	+0.2	+0.1	0	0	0	0	+0.1	+0.3	-0.3	-0.9	+0.1
E_{z5}	+0.3	-0.4	+0.4	+0.1	0	-0.1	-0.1	0	+0.3	+0.4	+0.4	+0.4	-1.3	-1.1
E_{z3}	-0.7	-1.7	+1.7	+1.2	-0.8	-0.6	-0.2	-0.2	-0.2	+0.6	+0.8	+1.4	-2.3	-0.8
R_x					-8.4	-6.1	-2.5	-1.5	-1.3	+1.6	+7.9			
$R_k + R_h$					+50.0	+68.4	+56.0	+55.6	+70.0	+76.9	+36.0			
x_T					118.5	119.6	121.5	125.4	131.4	131.9	133.3			

angular acceleration and is $-0.046(-5.0) = 0$, since the calculation of the other torques is not carried beyond the decimal point. The sum of the torques is consequently $+852$ kgm. cm.; since the muscle torque must be in equilibrium with this torque, it must have a value of -852 kgm. cm.

Similar computations can then be made for the shank and the thigh, the lever arms of the forces being obtained from the positions of the joints or from the length of the member and its angle of inclination. The proportionate position of the mass center with respect to the joints allows the calculation of the lever arms of forces

acting at the mass center. When these computations are carried out, they yield a value of +755 kgm. cm. for the muscle torque on the shank and of +503 kgm. cm. on the thigh. Since the total torque exerted by any system of muscles must be zero, and the muscles of the leg end either in the leg or on the trunk, the torque which they exert on the trunk is $-(-852 + 755 + 503) = -406$ kgm. cm.

The rate at which the forces and torques are doing work, or their activity, is readily calculated since the velocities of the points of application of the forces and the angular velocities of the members upon which the torques act are known. Taking the thigh as an illustration, the activity of the muscle torque is $+503 (-0.93) = -467$ kgm. cm./sec.; of the distal joint force, with x and z components, $(-2.5 \times 53) + (+65.2) (-10.0) = -785$; and of the proximal joint force, $(+3.8 \times 97) + (-58.1) (-15.5) = +1268$.

In similar fashion the rates at which the potential and kinetic energies of the thigh are decreasing may be obtained from the activity of the force of gravity and of the reversed effective forces and torques. For decrease in potential energy the rate is $-7.35 (-13.4) = +99$ kgm. cm./sec.; for kinetic energy, giving the x and z components of reversed effective force and the reversed effective torque in order, the rate of decrease is $-1.3 \times 78 + 0.2 (-13.4) + (-1.19) (+9.0) (-0.93) = -94$ kgm. cm./sec. The rate of decrease in combined kinetic and potential energy is consequently $99 - 94 = +5$ kgm. cm./sec. The computations for the other parts of the leg are made in comparable fashion.