FORECASTING LAND OCCUPANCY CHANGES THROUGH MARKOVIAN PROBABILITY MATRICES: A CENTRAL CITY EXAMPLE

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Preface

This is Report No. 14 in the series on Urban Development studies, prepared in the Department of Geography for the Centre for Urban and Community Studies, University of Toronto, under a grant from Bell Canada. It represents one of a number of exploratory attempts at forecasting change in urban structure and development based on selected types of statistical techniques. These will act to augment and substantiate intuitive forecasting and speculation as the research program evolves.

In this paper the focus is on the changing patterns and proportions of land occupancy within the city. Given an initial matrix of land use change the probabilities of conversion from one type of use to another are calculated and then utilized as the basis for forecasting future rates of change. The technique employed is Markov-chain analysis, which is essentially a sequential manipulation of probability matrices through discrete time periods. The example selected is the City of Toronto. For each period, the total amount of land use change is calculated and then the resulting aggregate land use structure of the city is derived.

The initial probability tables represent the frequency with which new construction has taken place on urban land occupied by specific uses. In other words, these reflect the probabilities of developers selecting sites occupied by particular uses or activities in their redevelopment location decisions. The distributional character of new construction is described by the author in a previous report in this series (Report No. 5).

This paper derives from a number of related research projects, in part

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The increasing concern with process in urban research has not been matched by available data or operational techniques. Attempts at examining change and subsequently of prediction are most frequently limited by inadequate or inaccurate time series statistics. There is little or no information on the ordered sequence of change which gives rise to process. This paper examines urban land use change given an unusually comprehensive set of data, in terms of the probabilistic framework of Markov-chain analysis. The data relate to the number and frequency of conversion of urban parcels from one land use designation to another represented in the form of transition probability matrices. Each element in the matrix is the probability of a particular sequence of conversion taking place during the study period.

Introduction

The paper is divided in two parts. The first examines the basic formulation of Markov-chain models, and the procedures involved in generating the initial transition probability matrices and land use change structure. The second section employs Markov principles, first to extrapolate the conversion probabilities through four inclusive and successively longer time periods, and second, as a constant operator for periods of equal length. Both models are employed subsequently to provide estimates of future rates of land conversion and terminal land use composition. The models relate to the city of Toronto and to the years 1952-1962 inclusive for the basic data on change. Only those properties considered to have undergone change, defined as physical modification in the building stock, are included. This definition acts to limit the sample size to a reasonable number while avoiding semantic questions regarding the measurement of change.

Urban land use conversion, as defined here, represents any physical change in land occupancy following the initial development of land for urban purposes. The concept of succession as a dynamic process has been explored in a parallel paper,[1] and it will suffice here to say that it represents the outcome of an adjustment mechanism in modifying the physical plant of a city to accommodate new demands for space and location. The analytical framework outlined in this paper utilizes regularities defined in this process as the basis for forecasting future changes in urban land occupancy. The limitations of forecasting through extrapolation are recognized, but are given only passing attention in the discussion.

The Markov Model

A Markov-chain is one of a set of Markovian stochastic processes, which describes the probabilistic relationship between the attributes of a variable and the position of this variable in a time sequence. The logical mathematical basis of such models is sufficiently well documented elsewhere that only a brief introduction is necessary here. A Markovian process is one in which it is possible to assign to a variable X_t, which occupies a particular condition or state i at time t, a probability that this variable will occupy state j at time t + 1. This probability is denoted

$P_{ij}^{t,t+1}$. such that

$$P_{ij}^{t,t+1} = P_r(X_{t+1} = j/X_t = i). \qquad (1)$$

which states the conditional probability of X being in state j given that it was previously in state i. The formal extension of (1) above is

$$Pr\ (X_o = i_o,\ X_1 = i_1,\ X_2 = i_2, \ldots\ldots X_t = i_t) =$$

$$(P_{t-1,t}\quad P_{t-2,t-1}\quad P_{t-3,t-2} \ldots\ldots P_{o,1}) \qquad (2)$$

Expanded statements of these propositions, and proofs of the theorems, are available in a number of basic texts including those by Kemeny and Snell[2], Bharucha-Reid[3], Bailey[4], Karlin[5], and the recent translation of Dynkin.[6] Applications of Markov-chain models are becoming more numerous in geography and planning[7], as the advantages of a probabilistic approach are documented.[8] Relevant applications here include those of Clark[9] on the movement of rental housing areas in American cities, Marble[10] on urban travel behavior, Harris[11] on the suburban land development process, Golledge and Brown[12] on the market decision process, and Cowan[13] and associates on the changing supply of office space in London.

The matrix P_{ij} is the Markovian transition probability matrix. Most commonly the constituent probability elements in these matrices p_{ij} are independent of the time dimension t, and thus are referred to as discrete time or stationary transition probabilities. The matrices are of the form

$$P_{ij} = \begin{vmatrix} p_{11} & p_{12} & p_{13} & p_{14} & \cdots \\ p_{21} & p_{22} & p_{23} & p_{24} & \cdots \\ p_{31} & p_{32} & p_{33} & p_{34} & \cdots \\ p_{41} & p_{42} & p_{43} & p_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{vmatrix}$$

In this study, the first row of the probability distribution is the probability of a property X shifting from land use type i = 1 to type or condition j during one time interval, and as a result of modifications to the physical inventory. Clearly, as transition states, the probabilities satisfy the conditions

$$P_{ij} \geq 0 \quad i,\ j = 0,1,2\ldots m \tag{3}$$

$$\sum_{j=1}^{m} p_{ij} = 1.0 \quad i = 0,1,2,\ldots m \tag{4}$$

and i = j = m

The condition expressed in (4) indicates that a change is considered to have occurred during a given time period even if the state or land use type remains unchanged. This follows logically from the initial statement that only those properties undergoing change are included in the analysis. Usually after one iteration (3) becomes $P_{ij} > 0$.

Derivation of subsequent distributions depends on the particular mode of matrix analysis employed. Given an initial vector of land use states X_t, and an initial transition matrix P_{ij} the operation

$$\begin{aligned} X_{t+1} &= X_t * P_{ij} \\ X_{t+2} &= X_t * [P_{ij} * P_{ij}] \\ X_{t+3} &= X_t * [P_{ij} * P_{ij}^2] \\ &\vdots \end{aligned} \tag{5}$$

provides estimates of a new state vector at the end of each time period and in which each period is of successively longer length. For periods of similar length of course the matrix P_{ij} remains stationary.

Land Use Change Model

The estimation of land use conversion occurs through the same matrix multiplication operation. The transition matrix for the entire city area P_{ij} is a single matrix representing the summation of individual matrices of similar order for each of k areas. Each element in the matrix, furthermore, is taken as the weighted average of two probabilities for two approximately equal subdivisions of the study period, weighted according to the length of the period. These matrices were not utilized independently because of instability in the coefficients and sample size problems. The initial P_{ijk} matrices are described in some detail in another paper.[14] No attempt is made to weight probabilities by the total sample size of changed properties in each period, although this would be a desirable and simple addition.

The components of both models include the following input matrices and vectors:

$P_{ij}^{t,t+1}$ = transition probability matrix for the initial study period (1952-1962)

T_i^{t+1} = vector for the aggregate land use structure for entire city (1962)

$TC_i^{t,t+1}$ = total land use change vector for the initial study period. (1952-1962)

T_i^t = vector for the aggregate land use structure for the entire city at the beginning of the period (1952)

PC_i^t = vector of percentages - total change as a proportion of total land use acreage in each of i types.

The initial iteration simply replicates the observed patterns of change. Direct row-wise multiplication of the total change vector TC_i and the probability matrix P_{ij} spreads the change over the terminal land use types j.

The result (note that this is not matrix multiplication)

$$TC_i^{t,t+1} * P_{ij}^{t,t+1} = ALC_{ij}^{t,t+1} \qquad (6)$$

is an intermediate conversion matrix showing total land use change from each existing type i to each terminal use type j. Summing the rows of ALC_{ij} gives the total land area converted out XX_i, which of course is equal to TC_j, while summing the columns provides estimates of new land occupance ZZ_j. The differences between the row summations (conversion out) and column summations (new construction) represent net change in the aggregate land use structure. The summation procedure is as follows:

$$\sum_j^m ALC_{ij} = XX_i = TC_i \qquad (7)$$

$$\sum_j^m ALC_{ij} = ZZ_j \qquad (8)$$

then $$T_i^t - XX_i + ZZ_j = T_i^{t+1} \qquad (9)$$

produces the new aggregate land use structure at the end of the period.

The Extrapolation Models

As noted earlier, two versions of the above procedure are employed. The first is dependent on the derivation of transition matrices for successive time periods describing the conversion process over one, two, three, and four stage intervals. Although the four are derived independently and may be treated as such, the fact that each matrix is inclusive of the preceding permits direct comparison of the evolving structure of city over the entire length of the period.

Direct Markovian sequential applications on the initial transition matrix P_{ij} provides new matrices for successively longer time periods each adding an interval of similar length. These matrices are derived as follows

$$P_{ij}^{t,t+2} = P_{ij}^{t,t+1} * P_{ji}^{t,t+1} \quad \text{add second time period} \quad (10)$$

$$P_{ij}^{t,t+3} = P_{ij}^{t,t+2} * P_{ij}^{t,t+1} \quad \text{add third time period} \quad (11)$$

and so on.

Following the calibration of the transition probability matrices for each period, it remains to calculate the resulting impact on the land use structure. Given that the model is from this point a simple extrapolation procedure, the assumption must be made that the overall rates of land use change will remain constant, although the structure of transition probabilities is considered to follow a Markovian process over time. This assumption may take either of two forms: 1) that the actual proportions of use in each category i remain the same, even as the aggregate land use composition evolves through time, or 2) that the total volume of land use change in area terms remains constant, and therefore the relative proportions of conversion change. At anything but a high level of aggregation, for example at the census tract level, these two assumptions would produce drastically different results. As the present model, however, has been formulated for one transition matrix encompassing the entire city, and with only ten land use subdivisions, the results are not likely to differ sharply. Nevertheless, it was decided for operational reasons to assume that the actual percentage of each use zone undergoing change remains the same, that is a constant rate of conversion for the city as a whole, but producing differing anticipated volumes of change within any respective land use category over successively longer time periods.

Given this assumption, the estimated future land use composition at each terminal point is derived on the basis of the vector of percentage land use change

$$PC_i = TC_i^{t,t+1} / T_i^{t} \qquad (12)$$

then $$PC_i * 2 = TC_i^{t,t+2} \qquad (13)$$

and so on.

This vector is then employed to derive a vector of total land area change for each period, which in turn is distributed directly over the rows of the transition matrices, as in the initial example. The difference between the row and column summations of the resulting intermediate conversion matrices, represents the cumulative net change in the total inventory of land uses during that period and all preceding periods. The model assumes and the transition matrices assure that not all land in a given category of use will be converted into another use. That is

$TC_i^{t} < T_i^{t-1}$ for each period.

The first iteration follows:

$$TC_i^{t,t+2} * P_{ij}^{t,t+2} = ALC_{ij}^{t,t+2} \qquad (14)$$

$$\sum_{j}^{m} ALC_{ij}^{t,t+2} = XX_i^{t+2} \qquad (15)$$

$$\sum_{i}^{m} ALC_{ij}^{t,t+2} = ZZ_j^{t+2} \qquad (16)$$

$$T_i^{t+1} - XX_i^{t+2} + ZZ_j^{t+2} = T_i^{t+2} \quad (1972) \qquad (17)$$

The second iteration

$$TC_i^{t,t+3} * P_{ij}^{t,t+3} = ALC_{ij}^{t,t+3} \quad \text{produces} \qquad (18)$$

$$T_i^{t+1} - XX_i^{t+3} + ZZ_j^{t+3} = T_i^{t+3} \quad (1982) \qquad (19)$$

The process is then replicated for subsequently longer time intervals in exactly the same manner. The problem is in fact one of the redistribution of land among competing uses, as the total area of the central city and thus the area available for conversion is held constant.

In the second approach, each ten year period is considered independently in that the initial transition probability matrix remains unchanged. The model generates a new land use structure at the end of each period (T_i), a new vector of total estimated land use change (TC_i) over the subsequent period, which is then in turn applied to the initial P_{ij} matrix. The procedure is much the same as above except that the land use structure is allowed to evolve with each iteration. Otherwise the results would be identical. The procedure is

$$TC_i^{t,t+1} \ * \ P_{ij} = ALC_{ij}^{t,t+1} \qquad (20)$$

as above to obtain T_i^{t+1}

then $$T_i^{t+1} \ * \ PC_i = TC^{t+1,t+2} \qquad (21)$$

repeat statement (20) to produce T_i^{t+2} and so on.

Constraints on the Model

Constraints other than total area on the operation of the model are minimal. In most instances it is necessary to include two sets of constraints: one, a volume constraint limiting the total conversion that may take place in any given local geographic area, either through the limit of size itself or by policy constraints; and two, a competitive constraint which measures the changing differentials between types of use in their attractiveness for conversion. Both constraints clearly become less relevant as the size of area and number of observations increase. In this study, the fact that the analysis refers to the entire city removes the effect of a local volume constraint (although a lower limit on vacant land was necessary) and renders the inclusion of a competitive constraint, in addition to the Markovian process, a meaningless exercise since the market processes of location selection for a given activity seldom operate over an entire city area. Clearly, the location decision-making process of urban activities will be responsive to the changing composition of land occupancy in each subarea of the city but these are not likely to have a substantial impact on the aggregate composition.

More difficult to ignore, however, are constraints on space utilization and location deriving from policy decisions. These decisions include both those reflected in restrictive codes, the most obvious of which are various zoning and building ordinances, and those involving investment decisions such as the type, scale, and location of new transportation, housing, and service facilities, which induce major shifts in the competitive attractiveness of different areas for conversion. Again the high level of aggregation reduces the necessity for the inclusion of such variables, although as a forecasting device it is clear that whatever errors exist are compounded. In order to simplify this assumption and the resulting errors, all public land uses were subsequently deleted both from the initial data and the transition matrices.

Results of the Extraplation Models

The first sets of output from the analysis are the initial and extrapolated transition probability matrices. Table 1 summarizes the original 10 x 10 matrix, again noting that it is based on two weighted time periods, and Table 2 lists the four extrapolated sets of Markov probabilities. The first matrix in Table 2, representing the square of Table 1 describes the

TABLE I LAND USE CHANGE PROBABILITIES TOTAL CITY 1952 - 1962

	01	02	03	04	05	06	07	08	09	10
01	0.130	0.340	0.100	0.040	0.040	0.220	0.030	0.020	0.000	0.080
02	0.020	0.410	0.050	0.040	0.000	0.040	0.000	0.000	0.000	0.440
03	0.000	0.070	0.430	0.050	0.010	0.280	0.140	0.000	0.000	0.020
04	0.020	0.010	0.090	0.300	0.090	0.270	0.050	0.080	0.010	0.080
05	0.000	0.000	0.110	0.070	0.700	0.060	0.000	0.010	0.000	0.050
06	0.080	0.050	0.140	0.080	0.120	0.390	0.040	0.000	0.010	0.090
07	0.010	0.030	0.020	0.120	0.030	0.110	0.380	0.210	0.010	0.080
08	0.010	0.020	0.020	0.030	0.030	0.080	0.180	0.610	0.000	0.020
09	0.010	0.180	0.140	0.040	0.100	0.390	0.030	0.030	0.080	0.000
10	0.250	0.080	0.030	0.030	0.050	0.150	0.220	0.130	0.000	0.060

Key:

01	Low Density Residential
02	High Density Residential
03	Office Commercial
04	General Commercial
05	Auto Commercial
06	Parking
07	Warehousing
08	Industry
09	Transportation
10	Vacant

TABLE 2 : EXTRAPOLATED CONVERSION PROBABILITY MATRICES, 1972 - 2002

MATRIX AFTER TEN YEARS

0.063	0.210	0.115	0.063	0.070	0.186	0.061	0.035	0.003	0.195
0.125	0.216	0.066	0.048	0.032	0.127	0.108	0.061	0.001	0.216
0.031	0.079	0.237	0.080	0.055	0.265	0.131	0.036	0.005	0.081
0.052	0.045	0.122	0.135	0.133	0.249	0.090	0.095	0.007	0.071
0.019	0.016	0.141	0.082	0.507	0.123	0.034	0.025	0.001	0.051
0.067	0.088	0.151	0.084	0.149	0.261	0.077	0.030	0.006	0.087
0.039	0.048	0.056	0.104	0.068	0.161	0.214	0.229	0.007	0.075
0.022	0.036	0.044	0.060	0.059	0.121	0.190	0.415	0.003	0.049
0.038	0.123	0.152	0.072	0.132	0.254	0.057	0.031	0.011	0.129
0.065	0.142	0.080	0.071	0.079	0.180	0.139	0.141	0.004	0.098

MATRIX AFTER TWENTY YEARS

0.078	0.144	0.114	0.070	0.093	0.188	0.101	0.067	0.003	0.142
0.087	0.164	0.087	0.065	0.063	0.169	0.120	0.095	0.003	0.146
0.051	0.086	0.166	0.085	0.090	0.236	0.123	0.068	0.005	0.091
0.050	0.070	0.129	0.096	0.148	0.220	0.103	0.100	0.005	0.080
0.028	0.036	0.147	0.085	0.384	0.159	0.058	0.041	0.002	0.061
0.056	0.094	0.142	0.083	0.156	0.220	0.092	0.055	0.005	0.097
0.044	0.064	0.082	0.090	0.096	0.177	0.160	0.204	0.005	0.078
0.033	0.051	0.066	0.073	0.083	0.148	0.173	0.306	0.004	0.064
0.063	0.102	0.139	0.077	0.143	0.215	0.092	0.056	0.005	0.108
0.054	0.111	0.097	0.078	0.101	0.179	0.124	0.136	0.004	0.115

MATRIX AFTER THIRTY YEARS

0.067	0.120	0.115	0.076	0.111	0.192	0.111	0.089	0.004	0.117
0.068	0.130	0.100	0.074	0.089	0.181	0.120	0.110	0.004	0.125
0.054	0.090	0.138	0.084	0.114	0.216	0.118	0.088	0.005	0.094
0.049	0.079	0.127	0.086	0.153	0.204	0.108	0.103	0.005	0.087
0.035	0.051	0.144	0.085	0.304	0.176	0.075	0.057	0.003	0.070
0.054	0.092	0.133	0.082	0.156	0.204	0.101	0.075	0.004	0.097
0.046	0.073	0.097	0.084	0.115	0.182	0.139	0.177	0.005	0.083
0.039	0.062	0.083	0.078	0.103	0.165	0.155	0.238	0.004	0.073
0.058	0.098	0.131	0.080	0.147	0.203	0.103	0.077	0.004	0.100
0.057	0.097	0.105	0.079	0.116	0.184	0.123	0.132	0.004	0.102

MATRIX AFTER FORTY YEARS

0.059	0.105	0.116	0.079	0.123	0.192	0.114	0.101	0.004	0.107
0.061	0.110	0.107	0.077	0.108	0.186	0.120	0.116	0.004	0.111
0.053	0.090	0.127	0.083	0.128	0.204	0.115	0.100	0.005	0.095
0.050	0.082	0.124	0.083	0.153	0.196	0.110	0.106	0.004	0.090
0.040	0.062	0.139	0.084	0.252	0.184	0.087	0.070	0.004	0.077
0.053	0.090	0.128	0.082	0.155	0.198	0.106	0.089	0.004	0.096
0.048	0.077	0.105	0.082	0.127	0.184	0.129	0.157	0.004	0.086
0.043	0.070	0.095	0.080	0.117	0.175	0.141	0.196	0.004	0.080
0.054	0.093	0.126	0.081	0.149	0.197	0.107	0.091	0.004	0.098
0.054	0.092	0.110	0.080	0.127	0.187	0.121	0.129	0.004	0.097

first stage transition process; the second matrix describes the two-stage transition process, and so on. It should also be emphasized that Table 1 is concerned only with change data and not the total land use inventory. Thus, the main diagonal of the matrix represents the proportion of each type of change property which remained in the same general use category despite modification to, or more commonly replacement of, the existing building, and not the proportion of all land which remained unchanged.

A further difficulty of interpretation of these probabilities is that they are independent of the scale of change. In particular, the uses in rows 02 (high-density residential) and 03 (offices) are for obvious reasons of age and density only infrequently replaced and thus the sample size involved is quite small. This explains the unusually high proportion (0.44) of the area of high-density residential properties that were vacant at the end of the study period (column 10). Only a few properties were in fact involved. All other entries in column 10, measuring the proportions of land vacant at any given time, between demolition and new construction, are of the same order of magnitude. Similarly, the proportions along the main diagonal measure stability in the conversion process, but are relative only to each row and not to the proportions for other uses.

Most of the relevant trends are obvious in the behaviour over time of the following set of matrices (Table 2). It is particularly interesting to note the rapid equalization of the divergent probabilities, that were initially apparent in Table 1, along the columns of the new matrices. Rows in the original matrix with a number of zero or near zero elements, primarily the very small land use categories such as transportation services (09) and high-density residential (02), show much more equitable distributions in the first extrapolated matrix of Table 2. This is rather difficult to rationalize as it suggests that differentials in the suitability of given land use types for conversion to other urban uses, are quickly reduced in lengthening the time period by one iteration. Some change in the structure of transition over time is to be expected given that several of the categories included are minor occupiers of urban space, and that the overall composition of land use evolves with each iteration. Exactly how much change one would expect to occur is difficult to estimate from sources and experience outside the Markov model.

The second set of output includes the terminal land use structures for each period estimated from the two models. These are summarized for the entire area of the central city in Tables 3 and 4. In both instances, extrapolation by matrix multiplication allows for shifts in the rate of replacement of existing uses and the rate of expansion through each time period as the composition and availability of urban space changes. In the first case (Table 3) to reiterate, the model is a sequential process in which the initial transition probability matrix P_{ij} is applied to the new or revised land use structure at the end of each period. In the second instance (Table 4), a constant volume and composition of change serve as input to the Markovian matrices of Table 2, each covering successively longer periods of time. These results of the approaches may be compared with those obtained by direct linear extrapolation reported in a previous paper (Bourne,14), and summarized for the twenty year forecasting horizon in Table 5. The discrepancies in the three sets of estimates, reflecting differences between the underlying assumptions in regard to nature of the conversion process, are

TABLE 3: MARKOV-CHAIN FORECASTS OF LAND USE 1972-2002 CITY OF TORONTO, TOTAL ACRES (TRANSITION PROBABILITY MATRIX HELD CONSTANT)*

Use Code	(Actual) 1962	1972	1982	1992	2002
01	7634.09	7490.83	7306.66	7112.29	6918.82
02	377.24	498.20	600.23	692.48	778.45
03	267.38	312.30	352.19	389.83	426.23
04	633.51	646.15	654.63	661.52	667.79
05	215.02	250.52	276.99	300.02	321.64
06	336.69	474.69	577.58	663.95	740.93
07	1012.67	1034.73	1015.53	984.19	951.13
08	1014.39	1064.96	1090.19	1105.19	1115.62
09	14.59	14.53	14.65	14.84	15.05
10	533.66	252.33	150.58	114.95	103.61

*Probability matrix P_{ij} applied to the aggregate and revised land use structure at the end of each ten year period.

TABLE 4: MARKOV-CHAIN FORECASTS OF LAND USE 1972-2002 CITY OF TORONTO, TOTAL ACRES (USING SEQUENTIAL TRANSITION PROBABILITY MATRICES)

Use Code	(Actual) 1962	1972	1982	1992	2002
01	7634.09	7416.31	7202.00	6781.86	6420.16
02	377.24	488.41	551.44	593.99	638.16
03	267.38	340.71	434.98	520.95	608.44
04	633.51	660.54	691.29	703.28	720.88
05	215.02	275.89	374.16	479.73	592.93
06	336.69	484.20	638.36	777.83	915.25
07	1012.67	998.46	980.80	934.48	890.92
08	1014.39	1062.37	1103.26	1106.09	1109.55
09	14.59	16.25	17.94	19.02	20.97
10	533.66	296.09	45.02	122.00*	122.00*

*Constraint added to maintain a minimum level of vacant land, reflecting the time required between demolition and new construction, in this instance set at one percent of the developed area.

TABLE 5: COMPARISON OF CONVERSION ESTIMATES BY DIRECT EXTRAPOLATION AND MARKOV MATRICES, 1982 TOTAL ACRES

Land Use	Direct Extrapolation Estimates	Markov Models[a] Constant Probabilities	Sequential Probabilities
01 Low Density Residential	6767.35	7306.66	7202.00
02 High Density Residential	768.01	600.23	551.44
03 Office Commercial	381.22	352.19	434.98
04 General Commercial	628.01	654.63	691.29
05 Auto Commercial	599.71	276.99	374.16
06 Parking	616.04	577.58	638.36
07 Warehousing	1010.63	1015.53	980.80
08 Industry	1065.33	1090.19	1103.26
09 Transportation	8.08	14.65	17.94
10 Vacant	122.01*	150.58	45.02

[a]see text for definitions

*see Table 4.

clearly magnified where the initial rates of conversion are highest either in a positive or negative direction.

The estimated land area changes by the Markov matrices appear to be on the conservative side generally, given the length of time involved, and in particular compared to those derived by direct extrapolation. The estimated rapid decline in low-density uses, and the elimination of remaining scattered parcels of vacant land, is paralleled by a considerable increase in high-density uses. In total acreage, apartments and offices double in size by the 1980's, automobile-oriented acreage and particularly parking space also double in size. Low-density areas are reduced by ten percent in the 1980's and by sixteen percent in 30 years. By the 1980's available vacant land is extrapolated below the minimum necessary in the transition from demolition of the old to new construction. Note that these are net changes in land area, and cannot be related to new construction totals or to the rate of rebuilding. The transition matrices in Table 2 indicate the respective rates of change within each category, while Tables 3 and 4 express only net shifts in land occupancy under different assumptions.

Evaluation of the Model

The Markov model expresses the concept that land use conversion is dependent solely on the transition probabilities in the preceding period. The relative mix of conversions is, however, modified first by the changing composition of the land use inventory, and second by the operation of matrix multiplication itself. Within any given time period of similar length the probabilities of course remain constant. The disadvantages of this approach are obvious. The application of a constant matrix operator limits, in fact predetermines, the shifts in transition probabilities that may occur. In light of the fact that the population rather than a sample of property changes was included in the derivation of the initial matrix and that the number of properties and decision-makers involved is very large, this assumption does not seem to pose a serious conceptual or statistical limitation as it does, for example, in models based on a cumulative learning process.[15]

The utility of the Markov-chain model in forecasting has two dimensions in i.e. example; one, in the changing structure of the transition matrices themselves, the other in the estimated terminal land use composition. In the first instance, the assumption of a Markovian process in operational terms produces estimated transition matrices which are responsive to the evolving composition of land uses in a given geographic area. This avoids the inherent simplicity of direct extrapolation procedures.

Equally interesting is the tendency to equilibrium in the relative coefficients or rates of conversion of land into and out of each category. It is clear from the first four iterations included here that stability in any meaningful sense has not been achieved, but along most columns it is reasonably close. This suggests that probabilities in site selection by developers converge for each type of terminal activity, regardless of initial use. The extent to which this convergence of probabilities in developer location decisions matches reality holds interesting implications.

The extrapolation of terminal land use based on these iterative transition matrices is, however, open to some question.[16] It may be argued that the size of the sample of observations, over 7,000 properties, and the aggregate nature of the analysis incorporate a strong element of stability into the coefficients of change. Even so, the transition matrix has 100 cells, many of which even at this aggregate level are zero or near zero. A small change in a few cells may result in considerable shifts in the final composition of land use. It seems reasonable, given these limitations, to except as valid a ten or twenty-year forecasting horizon based on the Markovian matrices, but not the longer extrapolations. The latter are included here essentially for reference, and to illustrate what the resulting impact of change on the spatial structure of central areas would be were the present rates of land conversion and redevelopment to continue.

A further and more serious conceptual difficulty is the use of a single matrix to represent the entire complexity of urban land transactions.[17] Obviously this obscures a wealth of relevant relationships which influence the behaviour of site selection and conversion processes. The derivation in preparatory studies of individual transition matrices for each subarea in the city revealed the expected intraurban variability in land conversion rates. On the other hand, the level of generalization employed here affords insights which would not likely be forthcoming from the subarea matrices or the individual decision units. These tend to reflect local and unique occurrences in the land conversion process, and the inadequacies of sample size.

Dependence on a stochastic approach is in itself a statement, as Curry[18] aptly describes, of the inability of existing theory and methodology to sort out all of the relevant variables and interrelationships. Probabilistic models, in view of the complex nature of most socio-economic problems, offer a forecasting technique which complements such traditional and essentially deterministic approaches as multiple regression.[19]

Conclusions

This study has taken as its premise that land use succession, or conversion, the sequential process of adjustment of the existing urban infrastructure to changing conditions, can be adequately described by a set of transition probabilities following a discrete Markov-chain process. The availability of individual parcel time series data facilitated statistical analysis of regularities in this conversion process. Obviously, the level of spatial aggregation employed here acts as a constraint or filter on the results obtained, and these results are relevant only to this particular level of generalization.

Extrapolation of the transition matrices through four successive time periods reveals the extent of convergence in the initial structure of land use change. A marked tendency toward a dynamic equilibrium was apparent but was not reached. Whether such a tendency is realistic is of course another question. Nevertheless, the systematic equalization of rates of future conversion and site selection in several critical land use categories is particularly revealing.

The long-run estimates of land use conversion in the central city are most useful. Although on the conservative side, deflated by the method of calculating proportional change, they indicate a continued rapid expansion of higher-density occupancy with related automobile parking and services. A parallel but accelerating decline in lower-density uses is estimated, primarily following the exhaustion of available vacant parcels. The predicted net impact of these changes is substantial, yet the aggregate spatial structure of the central city in terms of the composition of land occupancy at least, essentially persists.

Markov models in general offer a valuable technique and an operational concept in the evaluation of change processes in complex probabilistic systems, particularly spatial systems.[20] The fact that significant numbers of systems in the real world have probabilistic bases is conducive to the use of such techniques for spatial analysis and spatial forecasting.[21] With the improved data systems for urban areas that are now on the horizon, the simulation of land and property changes as a basis for planning and decision-making becomes more feasible and more relevant. The present formulation obviously needs considerable elaboration and expansion to improve both its practical and theoretical applications. In part, the rigidity of the model could be reduced by incorporating structural and recursive shifts in the composition of conversion, and by adjusting rates of conversion according to forecasts of national investment and growth conditions. Successful elaboration, however, awaits improved time series data.

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