

# Foreign Official Holdings of U.S Treasuries, Stock Effect and the Economy: A DSGE Approach

John Nana Francois\*  
Job Market Paper

## Abstract

Previous studies focus on quantifying the effect of foreign official holdings of long-term U.S Treasuries (FOHL) on the long-term interest rate. The consensus is that FOHL has a large and negative effect on the long-term interest rate. Long-term interest rate matter in determining aggregate demand, Andrés et al. (2004). However, these studies discount the macroeconomic implications of FOHL on the U.S economy. This paper extends the literature and studies the macroeconomic implications of FOHL shocks through their impact on the long-term interest rate in a dynamic stochastic general equilibrium (DSGE) model. The model treats short and long-term government bonds as imperfect substitutes through endogenous portfolio adjustment frictions(costs). Three main findings emerge from the baseline model: (1) A positive shock to FOHL impacts the long-term interest rate negatively through a stock effect channel– defined as persistent changes in interest rate as a result of movement along the Treasury demand curve. This result is consistent with the empirical literature; (2) The decline in the long-term interest rate creates favorable economic conditions that feed back into the economy and increases consumption, output and inflation through an endogenous term structure implied by the model and; (3) Monetary authority responds to the increase in inflation and output by raising the short-term interest rate. The simultaneous increase in the short-term interest rate and fall in the long-term interest rate causes the term spread to fall. This last result sheds light on the decoupling of interest rates observed between 2004-2006, a phenomenon known as the “Greenspan Conundrum”. The findings from the DSGE model are supported by impulse response functions obtained from a structural near-Vector Autoregression(near-VAR) model.

*JEL Classification:* E43, E52, E58, F21, G12

*Keywords:* Foreign Official Holdings of U.S Treasuries, DSGE, Aggregate Demand, Endogenous Term Structure, Near-Structural VARs

---

\*jfrancois@ku.edu PhD Candidate, University of Kansas, Department of Economics, 1460 Jayhawk Blvd. Lawrence, KS 66044. I am grateful to Matteo Falagiarda for making his codes available for replication and for his technical assistance. I also want to thank Alexander Richter, Lee Smith, Brent Bundick, Chen Sun and Soumya Bhadury for useful comments and conversations. Additionally, I would like to thank conference participants at the 2015 Southern Economic Meeting at New Orleans and participants in the Macroeconomic Research Seminar at the University of Kansas. Lastly, I would like to thank my Advisor, Shu Wu for his continual support and encouragement. All errors are hereby mine.

# 1 Introduction

The U.S bond market plays a vital role on the global economy, in the U.S economy as well as the daily lives of every American. Through debt issuance the government uses the bond market to borrow internationally and domestically in order to carry out key governmental spending on highways, bridges, military spending amongst other government programs. These governmental programs in turn create thousands of new jobs for the unemployed.<sup>1</sup> Furthermore, for several forward looking households in the economy, the bond market plays an essential role in their economic planning decisions including consumption, investment and savings both in the short and long-run.

These points underscore the fact that it is naïve to discount the macroeconomic implications of the actions and interactions of major holders of U.S debt such as foreign official institutions on the U.S economy. It is worth mentioning that excluding the Federal Reserve holdings of long-term government bonds, the share of outstanding long-term U.S Treasuries held by foreign officials increased from 13 percent of outstanding long-term debt in January 1990 to about 50 percent by June 2011.<sup>2</sup> This paper develops a dynamic stochastic general equilibrium model to study and understand the macroeconomic implications of FOHL.

To further elucidate the importance of studying the macroeconomic implications of FOHL, consider figure 1 and figure 2. As shown in figure 1, between July 2004 and July 2006, the 1-year interest rate increased from 1.24 percent to 5.22 percent (approximately 320 percent increase) following the Federal reserve tightening of policy rates, however, the 10-year interest rate only increased from 3.89 percent to 5.09 percent (approximately 34 percent increase).<sup>3</sup> The term spread which is given by the gray line fell during this period and in some cases attained negative values.<sup>4</sup> More importantly, other key factors at the time that included rising energy prices and robust real economic activity that tend to impact long-term interest rates positively in the past made the slow response of long-interest rates to the increase in short-term interest rates more unusual [Rudesbusch et al. \(2006\)](#).

This presented a deviation from the conventional wisdom that long-term interest rates will normally move in the same direction as short-term interest rates after controlling for expectations

---

<sup>1</sup>See for instance [Morrison and Labonte \(2011\)](#) for the examination of the importance to the U.S economy of China's investments in U.S Treasuries.

<sup>2</sup> Foreign officials consists of foreign ministries of finance, foreign central banks such as Bank of Japan, Bank of England, Central Bank of Republic of China and other foreign governmental institutions. The percentage of FOHL debt when the Fed's holdings are included in outstanding debt is about 33 percent. This is still a significant share.

<sup>3</sup>The Federal Funds Target rate was raised from 2 percent to 5.25 percent in 0.25 percent increments at seventeen consecutive meetings.

<sup>4</sup>The spread between the 10-year yield and 1-year yield is given by the 10-year yield minus the 1-year yield.

and other risk factors.<sup>5</sup> The sluggish increase in long-term interest rates while short-term rate increased sharply was referred to as the “Greenspan Conundrum”. Large asset purchases by foreign official institutions have been shown to have significantly contributed to the Conundrum.<sup>6</sup> This point succinctly highlights the significant role of foreign official agents in pricing of assets in the U.S which are important determinants for intertemporal economic decisions of domestic households.

Moreover, in figure 2, monthly long-term bond holding of U.S Treasuries held by foreign official institutions is compared to the Federal Reserve’s holding over the period January 1990 to June 2011. It is clear that FOHL has consistently been higher than the Federal Reserve holdings. The striking observation from the figure is that, even at the time of the quantitative easing (specifically, QE2- from November 2010 to June 2011), FOHL was approximately two times the Fed holdings of long term bonds.

It is important to note that the quantitative easing and FOHL are both forms of large asset purchases of long and medium term bonds. However, while the quantitative easing was specifically used as an unconventional policy tool to lower long-term interest rates at the Zero Lower Bound with a goal of stimulating the economy, not much is known about how FOHL affects the macroeconomy.<sup>7</sup> Moreover, large asset purchases by foreign official institutions even take place in the absence of monetary policy constraint such as Zero Lower Bound, hence they can have unpleasant implications– e.g. the Conundrum – given that monetary policy can be active during such large asset purchases.

This paper draws motivation from the aforementioned examples and examines the macroeconomic implications of large asset purchases by foreign official agents. Specifically, the paper focuses on investigating the impact of FOHL on major macroeconomic variables including consumption, output and inflation through FOHL effect on the long-term interest rate.<sup>8</sup> To see this connection, consider for instance the example of inter-temporal decisions by households in an economy. Inter-temporal decisions on savings, investment and consumption depend not only on the dynamics the of short-term interest rates but also on the long-term interest rate and their derivatives such as the term spread.<sup>9</sup> That is, all other things being equal, low interest rates,

---

<sup>5</sup>See for instance, the Expectation Hypothesis Theorem.

<sup>6</sup>See [Bernanke et al. \(2004\)](#); [Warnock and Warnock \(2009\)](#); [Sierra \(2010\)](#); [Bertaut et al. \(2011\)](#); [Beltran et al \(2013\)](#) and [Kaminska and Zinna \(2014\)](#), [Kohn \(2015\)](#) for example. An exception to this finding is [Rudesbusch et al. \(2006\)](#) who find no effect of foreign official asset purchases of U.S Treasuries on the long-term interest rate.

<sup>7</sup>See for example [Joyce et al. \(2012\)](#), [Gertler and Karadi \(2011\)](#), [Swanson \(2011\)](#), [Falagiarda \(2014\)](#) for evidence of the effectiveness of Quantitative Easing policy in the U.S.

<sup>8</sup>FOHL forms 70 percent of total foreign official holdings, see figure 3.

<sup>9</sup>For example [Andés et al. \(2004\)](#) shows that the long-term interest rate unambiguously affects aggregate demand. Moreover, [Marzo et. al \(2008\)](#) provides an empirical and theoretical support for the feedback channel from the term- structure to the macroeconomy. [Rudesbush et al. \(2007\)](#) instead find that although there is no reverse relationship from the term-premium to the economy structurally, reduced form empirical analysis suggests that falls in the term structure is usually associated with stimulus to real economic activity.

both short and long, create favorable economic conditions that stimulates real economic activity hence increase in consumption and output for example.<sup>10</sup> Consequently, by studying the impact of FOHL on long-term interest rates, one can reasonably study the impact of shocks to FOHL on the macroeconomy via the long-term interest rate.

As mentioned earlier, the key link to studying the implications of shocks to FOHL on the economy is through its impact on the long-term interest rate. Thus, in order to examine the effect of FOHL on the long-term interest rate and its subsequent implications on the macroeconomy, the standard DSGE model employed in this paper is modified to include these three key ingredients: (i) long-term government debt; (ii) endogenous financial cost term; and (iii) non-zero-exogenous foreign official holding of long term bonds.

The endogenous financial cost term allows for imperfect substitution between short and long-term government bonds. Specifically, following [Tobin \(1969,1982\)](#), [Andrés et al.\(2004\)](#) and more recently [Falagiarda \(2015\)](#), the paper introduces an endogenous financial cost term in the form of a portfolio adjustment costs into a DSGE model to allow for imperfect substitution between short and long-term government bonds.<sup>11</sup>

Imperfect substitution exist between short and long-term bonds because households rationalize that they lose liquidity any time they hold long-term bonds relative to holding short term bonds. Hence, households perceive entering the long-term bond market as riskier – longer-term bonds are illiquid relative to the same investment in shorter term bonds. For this reason, households internalize the loss of liquidity by holding additional short-term bonds to compensate themselves of the loss of liquidity anytime they hold long-term bonds. Households therefore self-impose a reserve requirement on their long-term investment in the form of liquidity costs associated with holding them.<sup>12</sup>

The endogenous financial cost term then permits for simultaneous examination of (i) how shocks to FOHL affect the long-term interest through the stock effect channel– defined as persistent changes in price and hence interest rate that result from movements along the Treasury demand curve and include the market reaction due to changes in expectations about future withdraws of supply of Treasuries<sup>13</sup>– and (ii) the shocks implications on the macroeconomy through a feedback channel from the endogenous term-structure implied by the model.

FOHL is modelled so that it is an exogenous time varying share of long-term outstanding

---

<sup>10</sup>Section 3 provides a simple graphical intuition on this example.

<sup>11</sup>For more on portfolio adjustment friction see for instance [Marzo et al. \(2008\)](#), [Falagiarda and Marzo \(2012\)](#), [Harrison \(2011\)](#), [Falagiarda\(2014\)](#). See also [D’Amico and King \(2013\)](#) for empirical evidence of imperfect substitution or segmentation within the Treasury market.

<sup>12</sup>See [Andrés et al. \(2004\)](#)

<sup>13</sup>See [D’Amico and King \(2010\)](#) for more on the stock effect. I elaborate more on the stock effect channel in my model in section 4.3.

government debt thus they evolve independently of bond prices. This modelling stance on FOHL is in line with [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) who show that foreign officials' demand for U.S Treasuries is inelastic. Essentially, a foreign central bank accumulates more dollar reserves in response to receipt of a dollar capital inflow— buying Treasuries regardless of their prices relative to other assets. Moreover, these foreign officials demand of Treasuries are only slightly sensitive to risk-return considerations.<sup>14</sup>

The model is approximated to the first order and solved numerically using *Dynare*. The impulse response functions from shocks to FOHL in the model are then studied. The findings from the model show that FOHL plays an important role in the economy and their actions have expansionary effects on the economy. In particular, in the baseline results, positive shocks from FOHL in the form of large purchases of U.S Treasuries affect the long-term interest rate negatively through the stock effect channel on impact. This negative impact on the long-term interest rate generates a feedback mechanism from the endogenous term structure to the economy which creates favorable economic conditions that stimulates the economy leading to an increase in consumption, output and inflation. Moreover, since monetary authority responds to inflation hawkishly with some degree of policy inertia, short-term interest rate increases. The simultaneous fall in long-term interest rates and increase in short-term interest rate causes the term spread to fall. This last result sheds light on the mechanisms behind the interest rate Conundrum between 2004-2006.

Othe key findings are:

1. The degree of persistence of the FOHL shock demonstrates that the effect of FOHL on the long-term interest rate can range from no impact to a sizeable negative impact on the long-term interest rate. Particularly, on impact when the persistence of FOHL is high, shocks to FOHL have no effect on the long-term interest rate. In contrast, when the persistence is low the shock has a significant and negative impact on the long-term interest rate on impact. This finding in the model is key to understanding the mixed result of the effect of FOHL on the long-term interest rate that exist in the literature. In all cases however, the model predicts a consistent negative effect of the shock on the term spread and the term premium.
2. Given different degrees of persistence of FOHL and imperfect asset substitution, FOHL shocks on impact have similar effect on consumption and output as in the baseline results. However, high (low) degrees of persistence of FOHL and imperfect asset substitutions between the assets causes a longer(faster) return of the term spread to its steady-state generating

---

<sup>14</sup>In contrast, recent surveys of central banks show that most reserve managers do change their reserve portfolios in response to changes in Treasury prices and other macroeconomic variables. Specifically, foreign official institutions optimize their foreign reserve portfolio, hence they are in fact endogenous (See for instance, [Beltran et. al \(2013\)](#); [Borio, Galati and Heath \(2008\)](#); [Pringle and Carver \(2002\)](#)). The focus of this paper is not to study the factors that drive foreign official holding of U.S Treasuries. Thus to keep the model tractable without losing its ability to answer the main questions examined in this paper, I follow studies such as [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Warnock and Warnock \(2009\)](#) by treating FOHL as exogenous.

higher (lower) feedback from the endogenous term structure over time. This yields higher (lower) peak values for consumption and output respectively.

The key assumptions and features in the model are incorporated into a five-variable structural near-VAR model to assess the empirical implication of the model. The empirical results from the near-VAR model complements the core results from the DSGE model.

## 2 Related Literature

There is an extensive empirical literature that employ different empirical models ranging from excess returns regression, term premium regressions, cointegrated vector autoregression models to no-arbitrage models to estimate the impact of FOHL on the long-term interest rate. The general consensus is that foreign official holdings have a significant and negative impact on the long term interest rate (Bernanke et al. (2004); Warnock and Warnock (2009); Sierra (2010); Bernanke et al. (2004); Bertaut et al. (2011); Beltran et al. (2013) and Kaminska and Zinna (2014)).<sup>15</sup> An equally important but exception to this finding is Rudesbusch et al. (2006). Employing an affine no-arbitrage macro-finance model, they find no effect of foreign official asset purchases of U.S Treasuries on the long-term interest rate.

In a literature that has predominantly focused on examining the empirical effects of FOHL on the long-term interest rate, the primary contribution of this paper is to examine the macroeconomic effects of FOHL on the U.S economy in the context of a DSGE model.<sup>16</sup> This is achieved by studying the effect of shocks to FOHL on the long-term interest rate through the stock effect channel and their consequent effects on the economy through a feedback mechanisms from the model implied endogenous term-structure. Both the stock effect and feedback mechanism is facilitated by the introduction of portfolio adjustment frictions (costs).

In addition, the paper is able to shed light on an explicit transmission channel– stock effect channel– of how FOHL impacts the long-term interest rate in a context of a DSGE model. In the baseline result, FOHL impacts the long-term interest rate negatively. This result is captured through the stock effect channel generated by the introduction of portfolio adjustment costs.<sup>17</sup>

---

<sup>15</sup>See also Bernanke (2005), who argues that unconventional movements of the long term rates is as a result of a global savings glut (GSG) hypothesis. The GSG hypothesis explains that increased capital inflows from countries in which desired savings greatly exceeded desired investment including Asia emerging markets and commodity exporters were an important reason that US longer term interest rates during this period were lower than expected.

<sup>16</sup>See Favilukis et al. (2012) and Favilukis et al. (2014) for other macroeconomic outcomes (housing price and wealth effects) of large Asset Purchases by Foreign Officials.

<sup>17</sup>Kohn (2015) in a consumption based asset pricing model examines one transmission mechanism by assuming that foreign official purchases of U.S debt directly funds domestic consumption growth. Consumption growth is central in the pricing of these asset via the stochastic discount factor in their model. Hence, foreign official purchases affect the yields on long-term bonds through consumption growth. His model abstracts away from monetary policy. This paper instead models monetary policy explicitly and studies the transmission mechanism of FOHL through

Although the channel and results are not new, the approach employed in this paper to study the effect of FOHL shocks on the long-term interest rate is different from the empirical methodologies usually employed in existing studies. Hence, the baseline results from the model serve as a robustness check for the results in literature.<sup>18</sup>

Moreover, the flexibility of the model allows for a deeper understanding and a panoptic view of how certain characteristics of foreign official institutions, such as the persistence of their holdings of U.S Treasuries can impact the effect of FOHL on the long-term interest rate – a feature not readily observed in the studies above. The effects of such characteristics are studied through sensitivity analysis on the parameter that governs the degree of persistence of FOHL. The results from the sensitivity analysis lends another important contribution to the literature by unifying, in one framework, the contrasting effects of FOHL on the long-term interest rate found in the literature.

Particularly, a low persistence of FOHL in the model shows that FOHL shocks can have a decently large and negative effect on the long-term interest rate. This is consistent with most of the results in the literature (Bernanke et al. (2004); Warnock and Warnock (2009); Sierra (2010); Bernanke et al. (2004); Bertaut et al. (2011); Beltran et al. (2013) and Kaminska and Zinna (2014)). However, high persistence of FOHL shows that shocks to FOHL have no effect on the long-term interest rate, a result similar to those found in Rudesbusch et al. (2006).

Lastly, unlike the other studies, this paper models monetary policy explicitly in a form of a reaction function and thus provides a different perspective on the interest rate Conundrum observed between 2004-2006 in the U.S. Specifically, monetary policy responds to the expansionary effects of FOHL by increasing short-term interest rates hawkishly.<sup>19</sup> Meanwhile, long-term interest rate are down and respond sluggishly to the increase in short-term interest rate due to the persistent negative stock effect of FOHL. This leads to a decoupling of long-term rate from short-term interest rate causing the term spread to fall a result consistent with the *Greenspan Conundrum*.

### 3 Graphical Intuition

Before introducing the full model, a simple graphical exposition is employed to explain the mechanism through which large asset purchases by other agents apart from households, in this case foreign official agents can affect the economy. To do this, three basic economic relationships are employed: (i) demand and supply of long-term Treasuries; (ii) the inverse relationship between

---

the stock effect channel generated by imperfect substitution between long and short term bonds.

<sup>18</sup>See Warnock and Warnock (2009) and Beltran et al. (2013).

<sup>19</sup>It is important to note that, during this period (the mid 2000s) monetary policy was “active” – it responded to increases in inflation and output by increasing policy rate.

bond prices and their interest rates and (iii) the negative relationship between output and the real interest rate (*Long-term interest rate augmented- IS curve*).

Furthermore, for illustration purposes the following simplifying assumptions are made. The supply of long-term bonds,  $S^L$  is assumed to be inelastic so that foreign official purchases only affect the composition of outstanding supply. It is also assumed that inflation expectations are “well-anchored” – they can be taken as fixed and exogenous so that changes in the nominal and real interest rates are one in the same thing.

Figure A: Simple Graphical Exposition of FOHL and the Economy

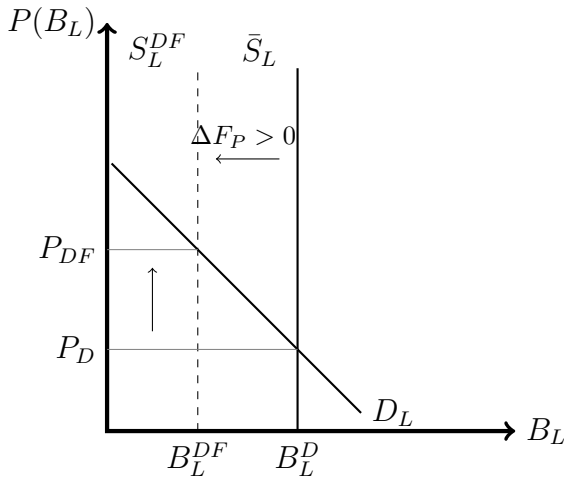


Figure A.1: Long-Term Bond Market with Exogenous Foreign Official Purchases ( $\Delta F_P$ )

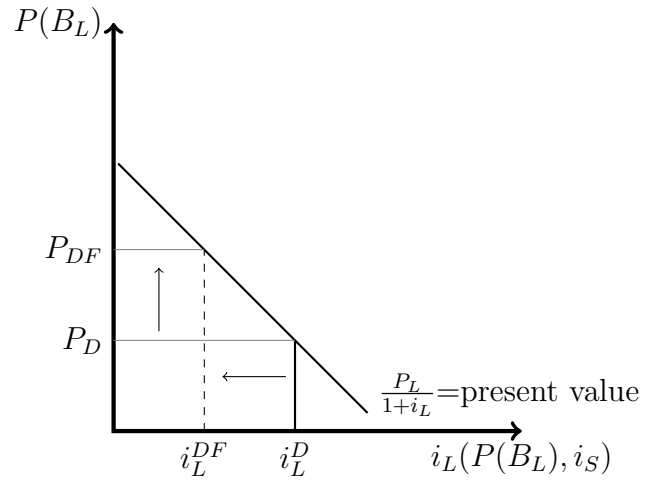


Figure A.2: Bond Price/Nominal Long-Term Interest Rate Negative Relationship

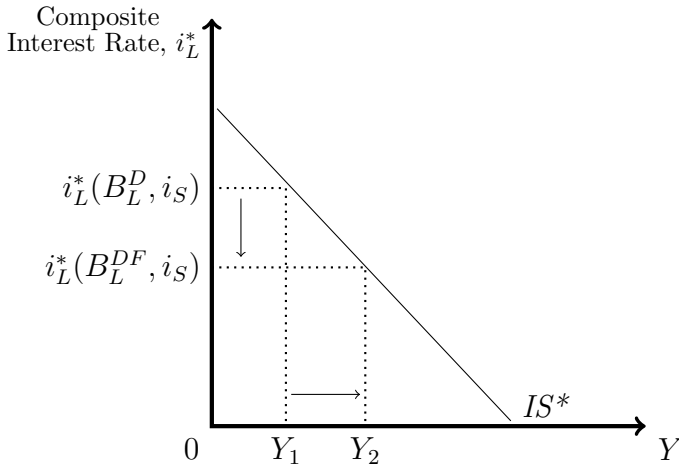


Figure A.3: Long-Term Interest Rate Augmented IS curve

Consider [figure A](#) below. [Figure A.1](#) shows the demand and supply of long-term government bonds. Households demand for long-term bonds are negatively related to bond prices and given by  $D_L$ . Without foreign official purchases,  $\Delta F_P = 0$ , supply of long term bonds available to



households is  $\bar{S}_L$  and the corresponding equilibrium price and quantity are  $P_D$  and  $B_L^D$  respectively. Now given that exogenous foreign officials increase their holdings by purchasing a positive amount of long-term bonds,  $\Delta F_P > 0$  at any given price, relative supply of long-term bonds available to households falls to  $S_L^{DF}$ . With demand high and supply of these assets low, the equilibrium price of long-term bonds increases from  $P_D$  to  $P_{DF}$  while equilibrium quantity for households falls to  $B_L^{DF}$ . Invoking the negative relationship between the price of bonds and interest rates depicted in [figure A.2](#), the increase in price of long-term bonds to  $P_{DF}$ , will in turn cause the nominal long-term interest to fall from  $i_L^D$  to  $i_L^{DF}$ .

It is important to note that in standard DSGE models, long-term interest rates and hence relative bond supply do not play any explicit role in the determination of aggregate demand. Specifically, there is only one interest rate, the short-term rate and its expected path implicitly determines the long-term leaving no room for a separate the role for long-term interest rate and hence supply (quantity) of bonds. However, as shown in [figures A.1](#) and [A.2](#), relative supply of long-term bonds can impact the long-term interest rate independent of short-term interest rates. This implies that long-term interest rates are not simply a function of short-term interest rates but also a function of their relative quantity supplied. Hence without loss of generality, consider a representative interest rate of the economy,  $i_L(B_L, i_S)$ , a function of long-term bond supply and short-term interest rate,  $i_S$ . Call this representative interest rate a composite interest rate (CIR).

The CIR suggests that both long-term and short term interest rates matter in the determination of aggregate demand (e.g. [Andrés et al.\(2004\)](#) shows that long-term interest rates unambiguously play a role in aggregate demand).<sup>20</sup> Now employing the negative relationship between aggregate demand and the real interest rate, [figure A.3](#) shows the long-term interest rate augmented IS curve,  $IS^*$ .

The transmission mechanism of large asset purchases by foreign officials is as follows: In [figure 1.A](#), a large asset purchase by foreign officials reduces outstanding supply of long-term bonds available to households from  $\bar{S}_L$  to  $S_L^{DF}$ . This bids up the price of long-term bonds from  $P_D$  to  $P_{DF}$ . Given the negative relationship between bond prices and interest rate, the increase in price decreases the interest rates on long-term bonds from  $i_L^D$  to  $i_L^{DF}$  as shown in [figure 2.A](#). Holding inflation expectation and the short-term interest rate constant, the CIR falls from  $i_L^*(B_L^D, i_S)$  to  $i_L^*(B_L^{DF}, i_S)$  following the large asset purchase by foreign officials. The decrease in  $i_L^*$  stimulates the economy and leads to an increase in aggregate demand from  $Y_1$  to  $Y_2$  as depicted in [figure A.3](#).<sup>21</sup>

<sup>20</sup>In this case we abstract from the short-term interest rate by assuming it to be constant. The full model the role of short-term interest rate.

<sup>21</sup> Notice that since supply of bonds are negatively related to interest rate we have  $\frac{\partial i_L}{\partial B_L} < 0$  as demonstrated in [figure A.1](#) and [A.2](#) above. Moreover, from standard arbitrage conditions long-term interest rates are positively related to the short-term interest rate hence  $\frac{\partial i_L}{\partial i_S} > 0$ . The interaction effect of changes in bond supply and the

## 4 A DSGE Model with Foreign Official Holding of U.S Treasuries

This section presents the full model. The model comprises a representative agent who populates the economy and supplies labor inputs for firms; a monopolistically competitive firm that hires the labor to produce differentiated goods; a final good firm who purchases the intermediate goods to produce final goods; a government sector that conducts both monetary policy– by targeting inflation and the output gap with some degree of monetary policy inertia to stabilize economic fluctuation– and fiscal policy by levying lump-sum taxes on households as well as issuing both short and long-term debt to generate revenue for government spending. Lastly, there is a foreign official agent whose demand for long-term government bonds is an exogenous evolving share of outstanding long-term government bonds.

### 4.1 Households

There is a representative agent who lives infinitely. The agent gains utility by choosing consumption bundle  $C_t$ , real money holdings  $M_t/P_t$  and labor hours  $N_t$  according to the utility function

$$u\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{(C_t - \theta C_{t-1})^{1-\gamma}}{1-\gamma} + \frac{\vartheta}{1-\eta} \left(\frac{M_t}{P_t}\right)^{1-\eta} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \quad (1)$$

where  $\chi > 0$ ,  $\vartheta > 0$ ;  $\gamma > 0$  is the coefficient of risk aversion,  $\eta > 0$  is the elasticity of money demand;  $\theta > 0$  is the habit formation parameter and  $\varphi \geq 0$  is the inverse of the Frisch elasticity of labor supply. The representative household thus maximizes her life-time utility

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, \frac{M_t}{P_t}, N_t) \quad (2)$$

with  $\beta \in (0, 1)$  as the discount factor. Since there is a continuum of consumption goods available for purchase,  $C_t$  corresponds to a Dixit-Stiglitz aggregate of consumption;

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (3)$$

where  $i \in (0, 1)$  represent the continuum of differentiated final goods and  $\varepsilon > 1$  governs the elasticity of substitution between different final goods.

---

substitute price, short-term interest rate,  $\frac{\partial^2 i_L}{\partial B_L \partial i_S}$ , on the long-term interest rate can be ambiguous and it depends on the relative magnitude of each effect as well as inflation expectations. For simplicity we assume this effect to be zero in the graphical exposition. The full model captures this interactive effect.

The household's budget constraint which incorporates the secondary market for bond trading as Ljungqvist and Sargent (2004) is

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + \rho_t) + \frac{M_t}{P_t} + T_t \leq \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} N_t - C_t - \frac{D_t}{P_t} \quad (4)$$

The household agent allocates wealth between money holding,  $M_t$  and two zero-coupon bonds which differ in maturity, these bonds are purchased at their nominal prices. The bonds are short term bonds and long term bonds denoted  $B_t$  and  $B_{L,t}^H$  respectively.  $B_t$  yields  $R_t$  and  $B_{L,t}^H$  yields  $R_{L,t}$ . The budget constraint of households reveals an active secondary market as proposed by Ljunquist and Sargent (2004).

Particularly, the right hand side of the household budget constraint shows that long term bonds  $B_{L,t-1}^H$  are priced with short-term rates, that is, the agent carries over long term bonds purchased at time  $t-1$  and sells it on the secondary market at the rate  $1/R_t$ . However, at time  $t-1$ , an agent who buys long-term bonds and intends to sell them in period  $t$  faces price uncertainty as  $R_t$  is not known at time  $t-1$ .<sup>22</sup> This formulation of the budget constraint to incorporate secondary market allows for a straightforward modelling of assets of different maturities. Moreover, this helps to capture the active participation of foreign central banks on the secondary market.

In line with Andrés et al. (2004), Falagiarda and Marzo (2012), Harrison (2012) and Falargiada (2014), the paper assumes that intratemporal trading between bonds of different maturities is costly to agents thus they pay a cost whenever they shift the portfolio allocation between short and long-term bonds, the endogenous cost function is then modelled as :

$$\rho_t = \frac{\phi_L}{2} \left( \kappa_L \frac{B_t}{B_{L,t}^H} - 1 \right)^2 Y_t \quad (5)$$

where  $\phi_L > 0$  and  $\kappa_L = B_{L,t}/B_t$  is the inverse of steady state household holding of short-term to long-term bonds. This implies that  $\rho_t$  is zero at steady state. The financial friction term allows for imperfect substitutability between long and short term bonds.

There are several motivation for including the transaction cost friction. However following Andrés et al (2004), its argued that households perceive entering the long term bond market as riskier, that is, they are illiquid relative to the same investment in shorter term bonds. Thus as they purchase long-term bonds, they hold additional short-term bonds to compensate themselves of the loss of liquidity. Specifically, households in effect self-impose a reserve requirement on their

---

<sup>22</sup>As explained by Ljungqvist and Sargent (2004) the price  $R_t$  follows from a simple arbitrage arguments, in period  $t$ , these bonds represent identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period  $t$ . See also Falagiarda (2014) for a similar formulation of long-term bonds.

long-term investment.<sup>23</sup>

## 4.2 Optimality Conditions

The first order conditions for the optimizing consumer's problem is given as:

$$C_t : (C_t - \theta C_{t-1})^{-\gamma} - \beta \theta E_t (C_{t+1} - \theta C_t)^{-\gamma} = \lambda_t \quad (6)$$

$$N_t : \chi N_t^\varphi = \lambda_t \left( \frac{W_t}{P_t} \right) \quad (7)$$

$$M_t : \vartheta \left( \frac{M_t}{P_t} \right)^{-\eta} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t \quad (8)$$

$$B_t : E_t \frac{\beta \lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_t} + \frac{\lambda_t \kappa_L \phi_L}{R_{L,t}} \left( \kappa_L \frac{b_t}{b_{L,t}^H} - 1 \right) Y_t \quad (9)$$

$$B_{L,t} : E_t \frac{\beta \lambda_{t+1}}{\pi_{t+1} R_{t+1}} = \frac{\lambda_t}{R_{L,t}} + \frac{\lambda_t \phi_L}{2 R_{L,t}} \left( \kappa_L \frac{b_t}{b_{L,t}^H} - 1 \right)^2 Y_t - \frac{\lambda_t \phi_L \kappa_L b_t}{R_{L,t} b_{L,t}^H} \left( \kappa_L \frac{b_t}{b_{L,t}^H} - 1 \right) Y_t \quad (10)$$

Equation (6) represents the marginal utility of wealth and it depends on the marginal utility of consumption today and the expected marginal utility of consumption tomorrow generated by the presence of habits in consumer preferences. Equation (7) relates real wage to the marginal rate of substitution between labor hours and consumption. Equation (6) and (8) can be combined to obtain an expression for money demand. Finally, equation (9) and (10) are the Euler equations for short and long term bond holdings respectively. As it is standard in the literature, we will show below that those two equations implicitly reveal a term structure relationship linking long and short term rates.<sup>24</sup>

---

<sup>23</sup>Other justifications for including the portfolio friction is the theory of preferred habitat by [Vayanos and Vila \(2009\)](#). Secondly, as in [Falargiada \(2014\)](#), one can rationalize these costs as proxies for the shares of resources devoted to covering information costs or costs of managing bond portfolio.

<sup>24</sup>Similar results can be found in [Andrés et al \(2004\)](#), [Marzo et al \(2008\)](#) and [Falargiada \(2014\)](#)

### 4.3 Stock Effect Channel and Feedback Mechanism

To gain insight of the channel through which foreign official holdings affect the long term rate and hence the term spread in this model, I combine the log-linearized first order conditions of short and long term bond holdings, equations (9) and (10).<sup>25</sup> This yields:

$$\tilde{R}_{L,t} = \tilde{R}_t + \eta_1 E_t \tilde{R}_{t+1} + \eta_2 E_t (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) - \eta_3 (\tilde{b}_t - \tilde{b}_{L,t}^H) \quad (11)$$

or

$$\text{Term Spread} = \tilde{R}_{L,t} - \tilde{R}_t = \eta_1 E_t \tilde{R}_{t+1} + \eta_2 E_t (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) - \underbrace{\eta_3 (\tilde{b}_t - \tilde{b}_{L,t}^H)}_{\text{Stock Effect}} \quad (12)$$

where  $\eta_3 = \phi_L (1 + \frac{\kappa_L}{R}) > 0$  and  $\eta_1$  and  $\eta_2$  are convolutions of steady state values and structural parameters.<sup>26</sup> The stock effect is captured in the last term of equation (12), this is due to the imperfect substitutability between the bonds in this model. Equation (12) is consistent with Tobin's argument that relative supply of different assets affects the spreads of these assets. Notice that,  $\phi_L$  the portfolio cost parameter governs the degree to which relative bond holding movements along the Treasury demand curve affects the long-term rates. If  $\phi_L$  is equal to zero (i.e.  $\eta_3 = 0$ ), equation (11) reduces to a form of expectation hypothesis and the stock effect is absent.

From equation (11), long-term rates depends positively on long term bond supply  $\tilde{b}_{L,t}^H$ . Short term bond supply on the other hand affects the long-term rate negatively. Hence persistent increase in long-term bond holdings by foreign official institutions reduces the relative supply of long-term bonds available to households. The long-term rate then falls given its positive relationship with long-term bond supply  $\tilde{b}_{L,t}^H$ .

An important feature which is central to studying the effects of shocks to foreign official holdings of long term bonds is the feedback channel from the model implied term structure to the macroeconomy. To see this feature, the log-linearized for equation for consumption and the first order condition of long-term bonds are combined to yield:

$$\tilde{C}_t = \eta_4 E_t \tilde{C}_{t+1} - \eta_5 \tilde{R}_{t+1} - \eta_5 \tilde{R}_{L,t} + \dots + \eta_7 E_t \tilde{\pi}_{t+1} \quad (13)$$

Equation (13) shows that both long-term rate and short-term rate are equally important in impacting current consumption. Moreover, in the case where policy rates are at the Zero Lower Bound (ZLB), it is clear that long-term rates play a much more direct role in impacting aggregate demand.<sup>27</sup> However, in this paper where focus is on the pre-ZLB period – active monetary policy,

<sup>25</sup>Throughout the model, variables with tildes represent deviations from their respective steady state.

<sup>26</sup>  $\eta_1 = \frac{\beta \tilde{R}_L}{\tilde{\pi} \tilde{R}}$  and  $\eta_2 = \frac{\beta \tilde{R}}{\tilde{\pi}} - \frac{\beta \tilde{R}_L}{\tilde{\pi}}$ .

<sup>27</sup>I refer to aggregate demand here because in a simple case where one assumes away government expenditure and investment, the market clearing condition will be  $Y_t = C_t$  in which case equation 13 become  $\tilde{Y}_t = \eta_4 E_t \tilde{Y}_t -$

it is important to note that when long-term rates are not in tandem with short-term rates, as in the case of the Greenspan Conundrum, the two rates can have conflicting effects on current consumption.<sup>28</sup> Hence, the impact of a derivative of the two rates— in this case the simple spread between the long-term rates and the short-term rate— on current consumption needs to be considered. To see this, equation (13) can be rewritten such that:

$$\tilde{C}_t = \eta_4 E_t \tilde{C}_{t+1} - \eta_{5,6} \tilde{R}_t - \eta_6 \underbrace{(\tilde{R}_{L,t} - \tilde{R}_t)}_{\text{Term Spread}} + \dots + \eta_7 E_t \tilde{\pi}_{t+1} \quad (14)$$

Equation (14) shows that apart from future consumption, expected inflations and short-term rates, falls in the term spread affects consumption positively and through the resource constraint and other general equilibrium forces can increase aggregate output and affect all the macroeconomic variable present in the model. The latter point can be elucidated as follows.

Suppose the economy is hit by a positive shock that initially stimulates consumption and hence increases inflation and output. Now since the monetary authority responds actively to inflation, it will increase policy rates. From equation (14), the increase in policy rates will decrease consumption. However, if long-term rates responds sluggishly (i.e.  $\tilde{R}_{L,t} < \tilde{R}_t$ ) to the increases in policy rates due to other factors such as the persistence of stock effect, then from equation (14), this can result in falls in the term spread. The fall in the term spread in turn can further lead to stimulating consumption.<sup>29</sup> In conclusion, monetary policy goals can be stifled by deviations of the long-term rates from short-term rates. This is the channel that shocks to FOHL affects the real economy in the model.

#### 4.4 Production of Intermediate Goods

Intermediate goods producing firm  $i$  has access to a constant returns to scale technology,  $Y_t(i) = AN_t(i)$  where  $Y_t(i)$  is output of the intermediate firm  $i$  and  $N_t(i)$  is the amount of labor the firm hires. The firm thus minimizes its total cost subject to the production technology.

#### 4.5 Price setting

A final goods producing firm purchases intermediate inputs at nominal price  $P_t(i)$  and produces the final composite good using the following constant returns to scale  $Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  where

---

$\eta_5 \tilde{R}_{t+1} - \eta_5 \tilde{R}_{L,t} + \dots + \eta_7 E_t \tilde{\pi}_{t+1}$ . See Falagiarda (2014) for the treatment ZLB in a similar context.

<sup>28</sup> This reiterates the importance of including not only short-term into the analysis of aggregate demand but also a separate role for long-term rates needs to be accounted for by including long-term bonds.

<sup>29</sup> Rudesbush et al. (2007) for instance finds that reduced form empirical analysis suggests that falls in the term structure is usually associated with stimulus to real economic activity.

$\varepsilon > 0$  is the elasticity of substitution between goods. Profit-maximization by the final goods producing firm yields a demand for each intermediate good given by

$$Y_t(i) = \frac{P_t(i)^{-\varepsilon}}{P_t} Y_t$$

Monopolistically competitive intermediate goods producing firm  $i$  chooses price  $P_t(i)$  to maximize the expected present value of profits:

$$E_t \sum_{j=0}^{\infty} \beta^j Q_{t+j} \frac{D_{t+j}(i)}{P_{t+j}}$$

where  $Q_{t+j} = \frac{\lambda_{t+j}}{\lambda_t}$  is the household's stochastic discount factor,  $D_t(i)$  are nominal profits for firm  $i$  and  $P_t$  is the nominal aggregate price level in the economy. Real profit are therefore given by,

$$\frac{D_{t+j}(i)}{P_{t+j}} = \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \Psi_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - \frac{\psi}{2} \left( \frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1 \right)^2 Y_t$$

where  $\psi \geq 0$  governs adjustment costs,  $\Psi_t(i)$  is real marginal cost. Price adjustments are introduced through [Rotemberg \(1982\)](#) quadratic costs of adjustment reflecting the negative effect that price changes can have on firm-customer relationship. In a symmetric equilibrium, all firms make identical decisions and hence the first order condition is given as follows:

$$0 = (1 - \varepsilon) + \varepsilon \Psi_t - \psi \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} + \psi E_t \left[ Q_{t+1} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right] \quad (15)$$

#### 4.6 Foreign Official Holdings

As explained earlier, to keep the model simple and tractable without losing its ability to answer the main questions examined in this paper, FOHL are modelled as an exogenous time varying share of long-term outstanding government debt.<sup>30</sup> Particularly, I assume that long-term foreign official holding denoted by  $B_{L,t}^F$  is a share  $x_t$  of outstanding long term debt and  $x_{F,t}$  evolves exogenously according to an AR(1) process. Hence,

$$B_{L,t}^F = x_{F,t} B_{L,t} \quad (16)$$

$$\log \left( \frac{x_{F,t}}{x_F} \right) = \rho_x \log \left( \frac{x_{F,t-1}}{x_F} \right) + \varepsilon_t^{x_F} \quad (17)$$

<sup>30</sup> For example [Warnock and Warnock \(2009\)](#) assume foreign official holdings as exogenous when estimating its effect on interest rates. See also [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) for empirical evidence.

where  $x_F = B_L^F/B_L$  is the steady state values of  $x_t$ .

#### 4.7 Demand for Long-Term Bonds

Households and foreign officials demand outstanding long term bonds so that.

$$B_{L,t} = B_{L,t}^H + B_{L,t}^F \quad (18)$$

where  $B_{L,t}$  are outstanding government long-term bonds.

#### 4.8 The Government

Government expenditure is financed by seigniorage revenues, issuance of long-term, Lump sum taxes and short-term bonds. Thus the government budget constraint is given as

$$\frac{B_t}{P_t R_t} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{M_t}{P_t} + T_t = \frac{B_{t-1}}{P_t} + \frac{B_{L,t}}{P_t R_t} + G_t + \frac{M_{t-1}}{P_t} \quad (19)$$

Furthermore, I model the issuance of new long term bonds to follow an AR(1) process so that shocks to foreign official demand for long term bonds only affects the composition of outstanding government debt (See for instance [Marzo et al.\(2008\)](#) and [Falagiarda \(2014\)](#)).

$$\log\left(\frac{B_{L,t}}{B}\right) = \rho_{bL} \log\left(\frac{B_{L,t-1}}{B_L}\right) + \varepsilon_t^{bL} \quad (20)$$

Government expenditure  $G_t$  is set according to the AR(1) process:

$$\log\left(\frac{G_t}{G}\right) = \phi_G \log\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^G \quad (21)$$

where  $\phi_G \in (0, 1)$  and  $\varepsilon_t^G$  is an i.i.d shock with zero mean and standard deviation  $\sigma_G$

Lump sum taxes  $T_t$  is a function of the total government liabilities:

$$T_t = \zeta_0 + \zeta_1 \left(\frac{b_{t-1}}{\pi_t} - \frac{b}{\pi}\right) + \zeta_2 \left(\frac{b_{L,t-1}}{R_t \pi_t} - \frac{b_L}{R\pi}\right) \quad (22)$$

where  $\zeta_0$  is the steady-state level of  $T_t$ , and  $\zeta_1, \zeta_2$  have been set equal so taxes respond equally to short and long-term debt.



Lastly, the central bank conducts monetary policy with a short-term interest rate feedback rule in the form specified by [Taylor \(1993\)](#) augmented to include interest rate smoothing:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_R)\rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + (1 - \rho_R)\rho_Y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_t^R \quad (23)$$

hence  $R_t$  inflation and output through  $\rho_\pi$  and  $\rho_Y$  respectively with an interest rate smoothing component governed by  $\rho_R$ . The exogenous policy shifter in monetary policy,  $\varepsilon_t^R$  is assumed to be a white noise monetary policy disturbance.

## 4.9 Resource Constraint

With the introduction of endogenous financial cost frictions, aggregate output of the economy is not simply allocated to consumption, government expenditure and price adjustment costs but also to a portfolio adjustment cost term which is priced in output. Thus the model is closed by a resource constraint given as:

$$Y_t = C_t + G_t + \frac{B_{L,t}}{P_t R_{L,t}} \rho_t - \frac{\psi}{2} \left( \frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1 \right)^2 Y_t \quad (24)$$

## 5 Results

This section presents the solution process and results of the model outlined in section 3. Simulations are conducted to study the impact of FOHL shocks on key macroeconomic variables using a calibrated version of the model. The model is log-linearized around its steady state and solved using *Dynare*. In what follows, the calibration of key parameters are discussed and then the results of the baseline model is analyzed. Finally, sensitivity analyses are carried out to examine the effects of varying the key parameters of the model, that is, the parameter governing portfolio costs ( $\phi_L$ ) and the persistence parameter for the share of FOHL ( $\rho_x$ ).

### 5.1 Calibration

The baseline model is calibrated at a quarterly frequency to match the behavior U.S data prior to the financial crisis in 2008.<sup>31</sup> A subset of the parameters are chosen based on previous studies

---

<sup>31</sup>See Table 2, 3 and 4 for the model calibration

and are standard in the literature. Specifically, following for instance [Fuhrer \(2000\)](#) the habit formation parameter  $\theta$  is set to 0.7. The discount factor is set at  $1.04^{-1/4}$ , which implies a steady-state annualized real interest rate of 4 percent. The implied steady-state real long-term interest rate is then given by  $R/\beta$ . Preferences over consumption are logarithmic, hence  $\gamma = 1$ . The Frisch labor supply elasticity is set to unity, so  $\varphi = 1$  and  $\chi$  is set such that the steady state share of time spent in employment is  $1/3$ . As mentioned earlier, intermediate goods-producing firms use a constant returns to scale production function. The common technology parameter,  $A$  is set to normalize the deterministic steady state level of output to 1.

The parameter that determines the interest elasticity of real money balances,  $\eta$  is set to 2.6 [[Mankiw and Summers \(1986\)](#), [Lucas\(1988\)](#), [Chari et al. \(2000\)](#)]. For real balances,  $\vartheta$  is set so that the velocity in the deterministic steady state, defined as  $cP/M$  corresponds to a value of 2.4 as in [Davig and Leeper \(2006\)](#). The price elasticity of demand  $\varepsilon$  and the Rotemberg adjustment cost coefficient  $\psi$  are set to 6 and 100 respectively as in [Schmitt-Grohé and Uribe \(2004\)](#) and [Ireland \(2004\)](#) respectively. The parameter value of price elasticity of demand means firms markup the prices of their goods over marginal cost by 20 percent.

The parameters governing monetary and fiscal rules are calibrated in a standard way. Particularly, the interest rate smoothing parameter  $\rho_R$  is set to 0.75 while  $\rho_\pi$  and  $\rho_Y$  are set to 1.5 and 0.6 respectively. Adopting a passive tax policy rule, the coefficients in the fiscal rule are set to  $\psi_b = \psi_{bl} = 0.15$ . The autoregressive coefficients and standard deviations of the shocks in the model are set to  $\phi_G = \rho_{bL} = 0.9$  while  $\sigma_R = 0.005$  and  $\sigma_G = 0.012$  [[Kim \(2000\)](#), [Andés et al. \(2004\)](#), [Altig et al. \(2011\)](#), [Falagiarda and Marzo \(2012\)](#) and [Zagaglia \(2013\)](#)]. There is one free parameter which is the portfolio adjustment friction  $\phi_L \in [0.005, 0.1]$  which falls between values of [Andés et al \(2004\)](#), [Chen et al. \(2012\)](#) and [Falagiarda \(2014\)](#). It is set to a value of 0.01 in the baseline case. Sensitivity analysis is conducted by perturbing the parameter to analyse its impact on the economy.

Some of the steady-state values of the variables in the model are set to follow previous studies, computed from data or model implied. For the model implied steady state values see Table 6. reports the steady states values of bond holdings by households, foreign official institutions and total bond demand. The steady state values were computed from [Betail-Tyron](#) measures of benchmark consistent positions. The steady-state total of debt to output ratio is 28 percent and this corresponds to a steady-state tax output ratio of 19.5 percent.<sup>32</sup>

The steady-state value for the share of foreign official holding of long term bonds  $x_t$  can be pinned down by equations (15),  $x = B_L^F/B_L$ . The persistence parameter,  $\rho_x$  that governs the AR(1) process for the share of FOHL in outstanding debt ( $x_t$ ) is set to 0.72 and the corresponding

---

<sup>32</sup>The steady debt to output ratio is a little lower than the usual 33 percent value. This is because the analysis abstracts away from the Fed's holdings of debt. This does not change the main results of the model

standard deviation is  $\sigma_x$  is set to 1.53.<sup>33</sup> Sensitivity analysis is conducted on  $\rho_x$  by setting it to high and low values away from the baseline value of 0.72.

## 5.2 Impact of foreign official holding shock

To examine the impact of shocks to foreign official holdings, figure 4 shows the equilibrium models' impulse responses following a positive shock to foreign official purchases of long-term U.S Treasuries (i.e. shock to  $x_t$ ). An average positive shock of  $\sigma_x = 1.53$  to long-term bond holdings by foreign officials reduces the relative supply of long-term bond supply and hence the amount of long-term bonds available to households. Through the stock effect channel shown in equation (12), the reduction of relative supply of long-term bonds available to households then reduces the long-term yield by an average of 13 basis points on impact. The shocks' negative impact on long-term interest rate is consistent with results found in the empirical literature (See for instance, Warnock and Warnock (2009), Bernanke et al. (2004), Beltran et al. (2013), Kaminska and Zinna (2014)).<sup>34</sup>

The shocks effect on the macroeconomy occurs via the feedback mechanism from the endogenous term structure generated by the model. Through the feedback mechanism shown in equation (14), the fall in long-term interest rate creates favorable economic conditions that stimulates consumption hence the increase in consumption. Consequently, through the resource constraint, output increases which in turn increases inflation via equilibrium forces. Monetary policy responds to the increase in output and inflation by increasing the short term rate ( $R_t$ ). Finally, the decoupling of long-term rates from short-term rate reduces the term spread defined as long-term rate minus short-term rate. The simultaneous rise in short-term interest rate, fall in long-term interest rate and fall in the term spread is consistent with the "Greenspan Conundrum", i.e. the decoupling of long-term interest rates from short term interest rates between 2004 to 2006.

It is important to conduct sensitivity analysis to gain insight of the principal mechanisms at work. Specifically, low and high parameter values are assigned to the parameter governing: (i) the persistence of FOHL shock,  $\rho_x$  and (ii) portfolio adjustment cost,  $\phi_L$ . The dynamics of the model following the variations of these parameters is then compared to the baseline model. Furthermore, the impact of FOHL shock on the term-premium is discussed in the sensitivity analysis.<sup>35</sup> The results from the sensitivity analysis is discussed in below.

---

<sup>33</sup>An ARCH-in-Mean estimation is carried out for the AR(1) equation of  $x_t$  to obtain the baseline parameter values for  $\rho_x$  and  $\sigma_x$ .

<sup>34</sup>The quantitative impact of FOHL shock on long-term rates in the model is compared to selected empirical studies in Section 4.

<sup>35</sup>The term premium is computed as the deviation of the long-term interest rates from its expectation hypothesis component and it is given as  $TP_t^{(k)} \equiv R_t^{(k)} - \frac{1}{k} \sum_{j=0}^{k-1} E_t R_{t+j}$  where  $R_t^k$  is the yield of a  $k$  - period zero-coupon bond at time  $t$  proxied by constant maturity bond in the model;  $R_t^1 = R_t$  and  $k = 10$  in the model.

### 5.3 Role of Persistence of Foreign Official Holding Shock, $\rho_x$

To investigate the role of the persistence of the FOHL shock, sensitivity analysis for the parameter governing the AR(1) process for the share of FOHL,  $\rho_x$  is carried out. The parameter is set to a low and high value away from the baseline value of 0.72. The corresponding low and high value for  $\rho_x$  are 0.52 and 0.83 respectively.

Figure 5 plots the impulse response functions when varying  $\rho_x$  to examine the role of the persistence of FOHL shock. The solid blue line is the baseline case, the dotted red line is the low persistence case and the dashed black line is the high persistence case. The mechanisms at work is the same as explained in the baseline case. However, on impact, a higher persistence value associated with FOHL shock (black dashed line) increases consumption, output, and hence inflation higher than the baseline case. This causes monetary authority to raise the short-term rates more aggressively which in turn offsets the negative effect of the shock on long-term interest rate.

This offsetting effect makes it appear that the FOHL shock have little or no impact on the nominal long-term interest rate when it hits. This outcome is in line with results in [Rudesbusch et al. \(2006\)](#) who find an insignificant effect of foreign official purchases on the long-term interest rate. It is however important to note that long-term rates do not respond one-to-one to the aggressive increase in the short-term rates since it takes longer for demand of long-term bonds to return to its steady state– a persistent stock effect. Consequently, there is a persistent delay in the term-spread to return to its steady after it falls. This effect feeds back into the economy inducing higher peak values for consumption, output and inflation relative to baseline case. The opposite effect holds for the case of low low persistence value of  $\rho_x$  (blue line).

### 5.4 Role of Portfolio Adjustment Costs, $\phi_L$

To examine the role of portfolio adjustment cost which reflects the degree of imperfect asset substitutability between short and long-term bonds, the parameter value governing portfolio adjustment costs  $\phi_L$  is varied according to low substitutability (dotted red line,  $\phi_L = 0.005$ ) and high substitutability (dashed green line,  $\phi_L = 0.02$ ) values. These variations in  $\phi_L$  are then compared to the baseline case (solid blue line with  $\phi_L = 0.01$ ). Notice that in the absence of portfolio adjustment costs ( $\phi_L = 0$ )– when short and long term bonds are perfect substitutes, reductions in relative supplies of the two bonds have no impact the interest rates and hence the economy. In this scenario, the stock effect is non-existent.

Figure 6 plots the impulse response functions when varying the parameter that governs imperfect asset substitution,  $\phi_L$ . Given an equal fall in long-term bond supply available to households following a positive shock to FOHL, a higher imperfect asset substitutability generates higher stock effect relative to the baseline value of portfolio adjustment cost. Specifically, in the case of

higher portfolio adjustment cost (the dashed black line), the term spread falls more compared to the low and baseline cases of the portfolio adjustment cost which is given by the dotted red line and solid blue line respectively. Again, through the feedback mechanism from the endogenous term structure explained in equation (13), the peak effect of consumption, output and inflation are higher in the case of high portfolio adjustment costs due to a more severe fall in the term spread. The opposite holds in the case of low portfolio adjustment costs (dashed blue line).

## 5.5 Model’s Effect on Long-Term Interest Rate Compared to Other Studies

While investigating the macroeconomic implications of FOHL shock in the model, the long-term interest rate served as the key link connecting the dots on how FOHL affects the economy. Consequently, the paper directly studies the effect of FOHL on the long-term interest rate through the stock effect channel in the context of a DSGE model with portfolio adjustment costs. The methodology employed in this paper is fundamentally different from those in the existing literature that use a broad spectrum of fully-fledged empirical models to study the impact of FOHL on the long-term interest rates. It is therefore necessary—after acknowledging all conceptual and methodological differences—to compare how well the model performs quantitatively on the impact of FOHL on the long-term rate to other studies.<sup>36</sup>

Table 5 compares the quantitative effect of FOHL on the long-term interest rate and the term spread implied by the model to selected empirical studies. Overall, on impact, the model implied quantitative effect of FOHL on the long-term interest rate (level)—with the maximum and minimum effect of -22 and 0 basis points respectively—is rather low compared to the values from the selected studies. This disparity as explained earlier are due to methodological and conceptual issues which can include the choice of approximation technique selected to numerically solve the model.<sup>37</sup> Notice however that the zero impact effect of the shock on long-term interest rate is consistent with the no effect found in Rudesbusch et al. (2006).

The model implied effect of FOHL on the term premium is however comparable to the empirical studies. Particularly, on impact, the shock’s effect on the term premium ranges from -34 to -54 basis points which compares to a similar result, -46 to -50 basis points— found in Beltran et al (2013) and -51 as in Kohn (2015). Lastly, the stock effect of FOHL in the model is -26 basis points, a value -6 higher than the result in Beltran et al. (2013) and -11 basis point higher than stock effect in Bertaut et al. (2011).<sup>38</sup>

---

<sup>36</sup>A caveat to this comparison is that different methodologies, measures of foreign official holdings (e.g.6-month, 12-month flow measure) or the frequency of the data employed to study the effect of FOHL on the long-term interest rate are likely to lead to very different results. This point is well-emphasized in Beltran et al.(2013). They note that differences in their estimates compared to those from large-scale asset purchases (LSAP) can be attributed to conceptual and methodological issues. I take an agnostic stands on these issues by acknowledging these caveats and compare my results to other studies.

<sup>37</sup>Table 6 compares model generated moments to the empirical moments from data.

<sup>38</sup>The stock effect ( $SE_x$ ) is computed as the impact effect  $I_{\sigma_x}$  divided by one minus the persistence of FOHL

## 6 The Model's Empirical Implication

The goal of this section is to assess the empirical implications of the DSGE model explained above. An important feature of the modelled economy in section 4 is the fact that, FOHL shock impacts the economy through the endogenous term structure of interest rate, in this case captured through the term-spread. Moreover, FOHL do not respond to any asset price or any macroeconomic variable. Essentially, FOHL is assumed to be exogenous in the model. Therefore to assess the empirical implications of the DSGE model, these key features from the model are incorporated into a five variable structural near-VAR model. The effects of FOHL shock on the variables included in the near-VAR are then studied through impulse response functions from the near-VAR. Specifically, the paper does a Monte Carlo integration analysis of a combination of a near-VAR for the lag coefficients and a structural VAR for the covariance matrix.

### 6.1 The near-VAR model

Based on assumptions and implications of the DSGE model discussed above, quarterly data from the period 1986:1 to 2007:4 is used to estimate the near-VAR model below<sup>39</sup>:

$$\begin{bmatrix} y_t \\ \pi_t \\ R_t \\ spd_t \\ x_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) & A_{13}(L) & A_{14}(L) & A_{15}(L) \\ A_{21}(L) & A_{22}(L) & A_{23}(L) & A_{24}(L) & A_{25}(L) \\ A_{31}(L) & A_{32}(L) & A_{33}(L) & A_{34}(L) & A_{35}(L) \\ A_{41}(L) & A_{42}(L) & A_{43}(L) & A_{44}(L) & A_{45}(L) \\ 0 & 0 & 0 & 0 & A_{55}(L) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ R_{t-1} \\ spd_{t-1} \\ x_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{bmatrix}}_{\mathbf{e}_t} \quad (25)$$

The estimated model is partitioned into two blocks. The first block includes the following four variables<sup>40</sup>: the cyclical component of real gross domestic product,  $y_t$ , which is obtained by applying the Hodrick-Prescott filter; the rate of inflation,  $\pi_t$ , computed from the GDP deflator; the effective Federal Funds rate,  $R_t$ ; the term spread,  $spd_t$ , computed as the 10-year interest rate minus the 3-month interest rate. The second block includes one variable, the 3-month average of 3-month flow measure of FOHL,  $x_t$ .<sup>41</sup> As in the DSGE model, the feedback mechanism from the

---

and normalized by 3 to achieve its monthly effect since the model is calibrated quarterly,  $SE_x = \frac{1}{3} \left( \frac{I_{\sigma_x}}{1-\rho_x} \right)$ .

<sup>39</sup>The start date for the data is due to data availability and the end data is to avoid the nonlinearities posed by Zero Lower Bound and Quantitative Easing after the global financial crisis in 2008.

<sup>40</sup>This specification is standard in the structural vectorautoregression literature. See Marzo et al. (2008) for further discussion. In Marzo et al. (2008) the term structure is captured by including the 1-year, 5-year and 10-year interest rate. I instead summarize the term structure by including the term spread. This is discussed more in the identification section.

<sup>41</sup>The data on real GDP, GDP deflator, Federal Funds rate, 10-year and 3-month interest rate were obtained from the FRED database available at: <https://research.stlouisfed.org/fred2/>. Data concerning foreign official holding of long-term U.S. Treasuries was obtained from the Bertaut-Tyron dataset available at: <http://www.federalreserve.gov/Pubs/ifdp/2007/910/default.htm>.

term structure to the economy is capture by including the lag terms of the term spread in the output and inflation equation. Furthermore, the lags of FOHL,  $(x_t)$  is included in all the other equations, however, since FOHL does not respond to any macroeconomic variable, zero restrictions are placed on all the coefficients of the macroeconomic variables in the FOHL equation of the near-VAR model.

Due to the zero restrictions on the lag variables in the last equation, a Seemingly Unrelated Regression (SUR) is employed to estimate the system in (25). One can obtain consistent estimators from using OLS in the presence of a near-VAR, however, as explained in Zellner (1962) there are potential efficiency gains in using SUR. As far as the lag length selection goes, the Shwartz Information Criteria suggests one lag for the estimated VARs.

## 6.2 Identification

To summarize the identification strategy, let  $\mathbf{e}_t$  denote the  $5 \times 1$  vector that collects the reduced form near-VAR residuals ( $e_{it}$ ) and let  $\boldsymbol{\varepsilon}_t$  denote the  $5 \times 1$  vector that collects the strutural shocks ( $\varepsilon_{it}$ ) for  $i = \{1, 2, 3, 4, 5\}$ . The structural shocks are therefore related to the reduced form residuals through the following equations:

$$\varepsilon_{1t} = e_{1t} \tag{26}$$

$$\varepsilon_{2t} + b_{21}\varepsilon_{1t} = e_{2t} \tag{27}$$

$$\varepsilon_{3t} + b_{32}\varepsilon_{2t} + b_{31}\varepsilon_{1t} = e_{3t} \tag{28}$$

$$\varepsilon_{4t} + b_{43}\varepsilon_{3t} + b_{42}\varepsilon_{2t} + b_{41}\varepsilon_{1t} + b_{45}\varepsilon_{5t} = e_{4t} \tag{29}$$

$$\varepsilon_{5t} = e_{5t} \tag{30}$$

Equation (26) - (30) can be written compactly as:

$$\mathbf{B}\boldsymbol{\varepsilon}_t = \mathbf{e}_t, \quad \text{where} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ b_{21} & 1 & 0 & 0 & 0 \\ b_{31} & b_{32} & 1 & 0 & 0 \\ b_{41} & b_{42} & b_{43} & 1 & b_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

The explanation for the ordering of the shocks in the  $\mathbf{B}$  matrix reveals a combination of two identification strategies:

1. The first block, equations (26) – (29) is ordered such that, inflation only responds to output shock, the policy rate shock responds contemporaneously to output and inflation while the term spread which is given by long-term rate minus the short-term rate respond to output,

inflation and the monetary policy rate shocks. Specifically, the shocks  $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t})$  are ordered in a Choleski fashion which is consistent in the literature. In addition, the term spread shock responds to FOHL shock, in a way capturing the stock effect channel. In essence the term spread shock is impacted by all the shock variables in the VAR model. This is representative of the DSGE model discussed in section 4.<sup>42</sup>

2. The second is a single equation, equation (30) which describes the exogeneity of FOHL assumed in the DSGE model and imposes the restriction that the FOHL shock is not correlated to any other shock in the model. Hence, zero restrictions are imposed on all the coefficients of the other shocks as shown in the last row of  $\mathbf{B}$ .

As mentioned earlier, with the assumption that FOHL is invariant to the other shocks in the near-VAR, the structural covariance system in equation (26)-(30) is overidentified. Hence to obtain the impulse response functions with their corresponding confidence intervals, the paper employs a Monte Carlo integration and Gibbs sampling for the overidentified structural covariance model.<sup>43</sup>

### 6.3 Impulse Response Functions from near-VAR

To analyse the effects of FOHL shock on the variables in the near-VAR model, the impulse response functions which traces out the path of the variables in periods  $t = 0, 1, 2, \dots$  in response to FOHL are considered. Specifically, median responses are reported alongside the error bands in response to a one time structural disturbance in period  $t = 0$  to FOHL. In light of Sims and Zha (1999) the 16th and 84th percentiles are reported for the confidence bands.

Figure 8 shows that in response to a shock to FOHL, the term spread declines. Note that, the term spread comprises two components, the expectation components and the term-premium component. Unfortunately, both components are not readily observable and hence separating these two effects and quantifying them can be a difficult task.<sup>44</sup> However, with the identification strategy for restrictions on the lag coefficients and the structural shocks in the system of equations in (25) and (31) it is reasonable to infer that the first few quarters that the term spread declines can be attributed to fall in the term-premium component. This reveals that on FOHL shocks lowers a component of the long-term interest rate, term premium. This gradual decline in the term spread feeds back into the economy and in turn increases real output. This result is consistent with Rudesbush et al. (2007) who find that falls in the term premium is usually associated with stimulus to real economic activity.

---

<sup>42</sup>In particular, see equation 12.

<sup>43</sup>The Gibbs sampler is a particular technique recently adopted to tackle instances in where it is impossible to make direct draws based on random Normals (See Doan (2010)). The MONTENEARSVAR.RPF provides an example for the implementation of MCMC analysis of a combination of a near VAR for the lag coefficients and a structural VAR for the covariance matrix.

<sup>44</sup>For a comprehensive treatment of decomposition of the term spread see Rosenberg and Maurer (2008).



As output rises, inflation also increases. Monetary authority respond to the increase in inflation and output by raising policy rates. However, even with the increase in policy rates, the term spread assumes only a slight upward trajectory and still remains negative. This highlight an interest rate conundrum similar to the case in DSGE model explained above. Specifically, this implicitly reveals that the long-term interest rate is not purely determined by the current and future path of short-term interest rate, hence breaking down the Expectation Hypothesis Theorem.

In summary, the results from the impulse response functions generated from the near-VAR above provide empirical evidence in support of the hypothesis that shocks to FOHL have expansionary macroeconomic effects on the U.S economy. This complements the core findings from the DSGE model in section 3.

## 7 Conclusion

This paper has investigated the macroeconomic implications of FOHL on key economic variables including consumption, output and inflation through its impact on the long-term interest rate. Employing a DSGE model that treats short and long-term bonds as imperfect substitutes through portfolio adjustment costs, the paper finds that shocks to FOHL have expansionary macroeconomic effects on the U.S economy – FOHL shocks increase consumption, output and inflation. This result is complemented by empirical impulse response from a structural near-VAR model.

Although the primary contribution of this paper is to study the macroeconomic implications of FOHL shocks in a DSGE model, the core results helps draw the following broad conclusions and policy implication:

1. The core results show that it is naïve to discount the macroeconomic effects of the actions of major holders of U.S debt such as foreign official agents. In the context of this paper, these effects are expansionary– FOHL shocks increase consumption, output and inflation. This result is captured through the negative stock effect channel of FOHL shocks on the long-term interest rate which feeds back in an expansionary fashion into the economy from the endogenous term-structure. Both the stock effect and feedback from the endogenous term-structure are generated by the introduction of portfolio adjustment cost. This emphasizes the fact that unlike previous studies that focus on quantifying the impact of FOHL on the long-term interest rate, it important to extend studies to understand the macroeconomic implications FOHL shocks.
2. The characteristics of private agents and foreign official institutions are crucial to understanding the degree of the impact of FOHL on the macro economy. For instance, given the mechanisms at work in the model–stock effect channel and feedback mechanism from the en-

dogenous term structure— if households do not treat short and long-term bonds as imperfect substitutes, the model shows that FOHL will have no impact on the economy. Meanwhile, with some degree of imperfect asset substitutability, a high (low) degree of persistence of FOHL shocks can lead to a high (moderate) expansionary macroeconomic effects. Moreover, a low persistence of FOHL shows that on impact, shock to FOHL can have a decently large and negative effect on the long-term interest rate while a high persistence has no effect. This result contributes to the literature by unifying the mixed results in the existing literature.

3. Lastly, since monetary authority responds to inflation and output in the model, short-term interest rate increases due to the expansionary effect of FOHL shock. However, FOHL shocks have a negative effect on long-term interest rates and hence there is a simultaneous fall in long-term interest rates and increase in short-term interest rate. This causes the term spread to fall similar to the Conundrum experienced between 2004-2006. This last results prompts attention to the fact that, monetary policy going forward must somehow understand and take note of the actions of other economic agents particularly foreign official agents when making policy decisions as their actions can generate unpleasant macro-implications.

## 7.1 Extensions

This paper may be extended in at least two ways: (1) treating FOHL as an endogenous variable—FOHL respond to changes in the bond prices and other macro-factors; and (2) examining the role of FOHL in the face of the quantitative easing at the Zero Lower Bound of interest rates. In Case(1), although this paper treats FOHL as an exogenous evolving share of outstanding long-term U.S Treasuries and is able to study FOHL implications on the macroeconomy, an extension of this will be to examine this same question but in the context of endogenous FOHL. This will be in line with empirical studies that treat foreign officials as endogenous (See for instance, [Beltran et al \(2013\)](#); [Sierra \(2010\)](#)). In Case (2), following the global financial crisis in 2008, policy rates have been constrained at the Zero Lower Bound (ZLB) in the U.S until recently. Unprecedented large asset purchases by the Federal Reserve (i.e. Quantitative Easing) was employed as unconventional monetary policy tool at the ZLB to help stimulate the economy. It will be equally important to study the separate role of FOHL at the ZLB in the presence of Quantitative Easing. This will complement studies such as [Eggertsson and Woodford \(2003\)](#); [Gertler and Karadi \(2011\)](#); and [Falagiarda \(2014\)](#).

## Appendix 1.A

### Calibration of baseline values for $\rho_x$ and $\sigma_x$

An Autoregressive Conditionally Heteroskedastic ARCH(1)-in-mean model is used in estimating the parameters of the process for the share of foreign official holdings of long-term U.S Treasuries. The parameters  $\rho_x$  and the standard  $\sigma_x$  are estimated as follows:

$$foi_{L,t} = \beta_x + \rho_x foi_{L,t-1} + \alpha \sigma_{t,x}^2 + \varepsilon_t^x \quad (1.1A)$$

where  $foi_{L,t}$  is 3-month foreign official inflows of long term bonds official inflows computed from Bertaut-Tyron measures of foreign official holdings. The error term  $\varepsilon_t^x$  is modelled such that it follows an ARCH(1) process:

$$(\varepsilon_t^x)^2 = \alpha_0^x + \alpha_1^x (\varepsilon_{t-1}^x)^2 + v_t^x \quad (1.2A)$$

Table 1. below reports the estimation results.

Table 1: Estimation results : ARCH-in-mean

Variable	Coefficient	(Std. Err.)
Equation 1.1A : $foi_{L,t}$		
$foi_{L,t-1}$	0.726***	(0.076)
$\sigma^2$	31.260***	(4.874)
Intercept	-72.794	(0.000)
Equation 1.2A: ARCH		
$\varepsilon_t^2$	0.005***	(0.001)
Intercept	2.340***	(0.365)
Significance levels : † : 10% * : 5% *** : 1%		

## Appendix 1.B: The Steady-State and Implied Parameters

Steady state values of the economic variables in the model are defined such that, for any time period  $t$ ,  $X_t = X_{t+1} = X$ . Hence, at steady-state, the variable  $X_t$  is time invariant so the time subscripts are dropped. Below are the equations defining steadystate values of the economic variables that have closed form solutions.

FOC Consumption:

$$\lambda = (C - \theta C)^\gamma (1 - \beta\theta) \quad (1.1B)$$

FOC Short Term Bond:

$$R = \frac{\pi}{\beta} \quad (1.2B)$$

FOC Long Term Bond

$$R_L = \frac{\pi R}{\beta} \quad (1.3B)$$

FOC Labor

$$\chi = \lambda \frac{w}{N^\varphi} \quad (1.4B)$$

Velocity of Money Definition:

$$m = \frac{C}{vel.} \quad (1.5B)$$

FOC Labor:

$$\vartheta = \lambda \left(1 - \frac{1}{R}\right) m^\eta \quad (1.6B)$$

Firm Pricing:

$$\Psi = \frac{\varepsilon - 1}{\varepsilon} \quad (1.7B)$$

Constant Technology:

$$A = \frac{Y}{N} \quad (1.8B)$$

Marginal Cost:

$$w = A\Psi \quad (1.9B)$$

Government Budget Constraint:

$$T = \frac{b}{\pi} + \frac{b_L}{R_L \pi} + G + \frac{m}{\pi} - m - \frac{b}{R} - \frac{b}{R_L} \quad (1.10B)$$

## Appendix 1.C: Full Log-Linearized Model

The dynamic economic problem presented in the paper takes on a system of non-linear difference equations. Since there are no closed form solutions, I employ a first order Taylor expansion to approximate the nonlinear model around the neighborhood of its steady-state and solve it numerically. Particularly, for a smooth arbitrary function  $h(x_t)$ , the function is approximated linearly as:

$$h(x_t) = h(x) + h'(x)(x_t - x)$$

Below is the full log linearized model:

FOC Consumption:

$$(\beta\theta\gamma(C\tilde{c}_{t+1} - \theta C\tilde{c}_t) - \gamma(C\tilde{c}_t - \theta C\tilde{c}_{t-1}))(C - \theta C)^{-\gamma-1} = \lambda\tilde{\lambda}_t \quad (1.1C)$$

FOC Real Money Balances:

$$\tilde{m}_t = \frac{1}{\eta} \left( \frac{\pi}{\pi - \beta} \tilde{\lambda}_t - \frac{\beta}{\pi - \beta} E_t(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) \right) \quad (1.2C)$$

FOC Labor:

$$\tilde{w}_t = \varphi\tilde{n}_t - \tilde{\lambda}_t \quad (1.3C)$$

FOC Short Term Bond:

$$E_t \frac{\beta}{\pi} (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) = \frac{\tilde{\lambda}_t}{R} - \frac{\tilde{R}_t}{R} - \frac{\kappa_L \phi_L}{R_L} (\tilde{b}_t^h - \tilde{b}_{L,t}^h) \quad (1.4C)$$

FOC Long Term Bond:

$$E_t \frac{\beta}{\pi R} (\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1} - \tilde{R}_{t+1}) = \frac{\tilde{\lambda}_t}{R_L} - \frac{\tilde{R}_{L,t}}{R_L} - \frac{\phi_L}{R_L} (\tilde{b}_t^h - \tilde{b}_{L,t}^h) \quad (1.5C)$$

Household Budget Constraint:

$$\begin{aligned} \frac{b^h}{R} \tilde{b}_t^h - \frac{b^h}{R} \tilde{R}_t + \frac{b_L^h}{R_L} \tilde{b}_{L,t}^h - \frac{b_L^h}{R_L} \tilde{R}_{L,t} + m\tilde{m}_t &= \frac{b^h}{\pi} \tilde{b}_{t-1}^h - \frac{b^h}{\pi} \tilde{\pi}_t + \\ \frac{b_L^h}{\pi R} \tilde{b}_{L,t-1}^h - \frac{b_L^h}{\pi R} \tilde{R}_t - \frac{b_L^h}{\pi R} \tilde{\pi}_t + \frac{m}{\pi} \tilde{m}_{t-1} - \frac{m}{\pi} \tilde{\pi}_t + Y\tilde{y}_t - C\tilde{c}_t & \end{aligned} \quad (1.6C)$$

Production Technology:

$$\tilde{y}_t = \tilde{n}_t \quad (1.7C)$$

Supply of Long-term bonds available to households:

$$b_L^h \tilde{b}_{L,t}^h = b_L \tilde{b}_{L,t} - x b_L \tilde{b}_{L,t} - x \tilde{x}_t b_L \quad (1.8C)$$

Government Budget Constraint

$$\frac{b}{R}(\tilde{b}_t - \tilde{R}_t) + \frac{b_L}{R_L}(\tilde{b}_{L,t} - \tilde{R}_{L,t}) + m \tilde{m}_t = \frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}_t) + \frac{b_L}{\pi}(\tilde{b}_{L,t-1} - \tilde{\pi}_t - \tilde{R}_t) + \frac{m}{\pi}(\tilde{m}_{t-1} - \tilde{\pi}_t) + G \tilde{g}_t - T \tilde{t}_t \quad (1.10C)$$

Monetary Policy Rule:

$$\tilde{R} = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) \rho_\pi \tilde{\pi}_t + (1 - \rho_R) \rho_Y \tilde{y}_t + \varepsilon_t^R \quad (1.11C)$$

Tax Rule:

$$\tilde{T}_t T = \zeta_1 \frac{b}{\pi}(\tilde{b}_{t-1} - \tilde{\pi}) + \zeta_1 \frac{b_L}{R \tilde{\pi}}(\tilde{b}_{L,t-1} - \tilde{\pi}_t) \quad (1.12C)$$

Firm Pricing:

$$\tilde{\pi}_t = E_t \beta \tilde{\pi}_{t+1} + \frac{\varepsilon - 1}{\psi} \tilde{\Psi}_t \quad (1.13C)$$

AR(1) process for share of FOHL:

$$\tilde{x}_{F,t} = \rho_x \tilde{x}_{F,t-1} + \varepsilon_t^x \quad (1.14C)$$

AR(1) process for Long-Term Bond Supply:

$$\tilde{b}_{L,t} = \rho_{bL} \tilde{b}_{L,t-1} + \varepsilon_t^l \quad (1.16D)$$

AR(1) process for Government Spending:

$$\tilde{g}_t = \phi_G \tilde{g}_{t-1} + \varepsilon_t^g \quad (1.17C)$$

## References

- [1] Andrés, J, López-Salido, J.D., and Nelson, E. (2004). Tobin's Imperfect Asset Substitution In Optimizing General Equilibrium. *Journal of Money, Credit and Banking*, 36(4):665-690.
- [2] Altig, D., Christiano, L. J., Eichenbaum, M., and Lind, J. (2011). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 14(2):225-247.
- [3] Beltran, D. O., Kretschmer, M., Marquez, J., and Thomas, C. P. (2013). Foreign holdings of US Treasuries and US Treasury yields. *Journal of International Money and Finance*, 32, 1120-1143.
- [4] Bernanke, B.S (2005) The Global Savings Glut and the U.S Current Account Deficit, Homer Jones Lecture, April 14, 2005. <http://www.federalreserve.gov/boarddocs/speeches/2005/20020414/default.htm>
- [5] Bernanke, B., Reinhart, V., and Sack, B. (2004). Monetary policy alternatives at the zero bound: An empirical assessment. *Brookings papers on economic activity*, 2004(2), 1-100.
- [6] Bertaut, C., DeMarco, L.P., Kamin, S., Tryon, R., (2011). ABS Inflows to the United States and the Global Financial Crisis. International Finance Discussion Papers 1028. Board of Governors of the Federal Reserve System
- [7] Bertaut, C.C and Tryon, R.W (2007). "Monthly Estimates of U.S. Cross-Border Positions." International Finance Discussion Paper No. 910, Board of Governors of the Federal Reserve system.
- [8] Blinder, A. S. (2010). Quantitative Easing: Entrance and Exit Strategies (Digest Summary). *Federal Reserve Bank of St. Louis Review*, 92(6), 465-479.
- [9] Borio, C., Galati, G. and Heath, A. (2008). FX Reserve Management: Trends and Challenges. BIS Papers 40, Bank for International Settlements.
- [10] Calvo, Guillermo (1983). Staggered Prices in a Utility-Maximization Framework. *Journal of Monetary Economics*, 12:3,383-98.
- [11] Chen, H., Curdia, V., and Ferrero, A. (2012). The macroeconomic effects of large-scale asset purchase programmes. *Economic Journal*, 122(564):F289-F315.
- [12] Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2000). Sticky price models of the business cycle: can the contract multiplier solve the persistence problem?. *Econometrica*, 68(5), 1151-1179.
- [13] Davig, T., and Leeper, E. M. (2006). "Fluctuating Macro Policies and the Fiscal Theory. In NBER *Macroeconomics Annual 2006*, ed. by D. Acemoglu, K. Rogoff and Woodford, pp. 247-316. MIT Press, Cambridge.
- [14] D'Amico, S., and King, T. B. (2013). Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics*, 108(2), 425-448.
- [15] Doan, T.A. (2010). RATS User's Manual Version 8, Estima, 1560 Sherman Avenue, Suite

510, Evanston, IL 60201.

- [16] Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297-308.
- [17] Eggertsson, G. B., and Woodford, M. (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 34(1), 139-235.
- [18] Engle, R. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives*, 157-168.
- [19] Falagiarda, M. (2014). Evaluating quantitative easing: a DSGE approach. *International Journal of Monetary Economics and Finance*, 7(4), 302-327.
- [20] Falagiarda, M. and Marzo, M. (2012). A DSGE Model With Endogenous Term Structure. Quaderni DSE Working Paper 830, Department of Economics, Università di Bologna.
- [21] Favilukis, J., Kohn, D., Ludvigson, S. C., and Van Nieuwerburgh, S. (2012). International capital flows and house prices: Theory and evidence. In *Housing and the Financial Crisis* (pp. 235-299). University of Chicago Press.
- [22] Favilukis, J., Ludvigson, S. C., and Van Nieuwerburgh, S. (2014). Foreign ownership of US safe assets: good or bad? (No. w19917). National Bureau of Economic Research.
- [23] Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, 367-390.
- [24] Gertler, M., and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1), 17-34.
- [25] Greenspan, A. (2005). Testimony of Chairman Alan Greenspan at the Federal Reserve Boards Semiannual Monetary Policy Report to the Congress before the Committee on Banking, Housing, and Urban Affairs, US Senate, February 16, 2005.
- [26] Harrison, R. (2011). Asset Purchase Policies and Portfolio Balance Effects: A DSGE Analysis." In Chadha J. and Holly, S., editors, *Interest Rates, Prices and Liquidity: Lessons From The Financial Crisis*, chapter 5.
- [27] Harrison, R. (2012). Asset Purchases Policy at The Effective Lower Bound For Interest Rates. Bank of England Working Papers 444, Bank of England.
- [28] Ireland, P. N. (2004). Money's role in the monetary business cycle. *Journal of Money, Credit and Banking*, 36(6):969-83.
- [29] Joyce, M., Miles, D., Scott, A., and Vayanos, D. (2012). Quantitative easing and unconventional monetary policy introduction. *The Economic Journal*, 122(564), F271-F288.
- [30] Kaminska, I., and Zinna, G. (2014). Official Demand for US Debt: Implications for US real interest rates (No. 14-66). International Monetary Fund.
- [31] Kim, J. (2000). Constructing and estimating a realistic optimizing model of monetary policy. *Journal of Monetary Economics*, 45(2):329-359.



- [32] King, T. B. (2010). Flow and Stock Effects of Large-Scale Treasury Purchases. In Finance and Economics Discussion Series 201052. Washington: Board of Governors of the Federal Reserve System.
- [33] Kohn, D. (2015). Addicted to Debt: Foreign Purchases of US Treasuries and the Term Premium. Department of Economics Working Papers 2015.1, Universidad Torcuato Di Tella.
- [34] Krishnamurthy, A., and Vissing-Jorgensen, A. (2012). The demand for Treasury debt. *Journal of Political Economy*, 42(2), 215-265.
- [35] Ljungqvist, L. and Sargent, T (2004), *Recursive Macroeconomic Theory*. MIT Press.
- [36] Lucas, R. E. (1988, December). Money demand in the United States: A quantitative review. In *Carnegie-Rochester Conference Series on Public Policy* (Vol. 29, pp. 137-167). North-Holland.
- [37] Mankiw, N. G., and Summers, L. H. (1986). Money demand and the effects of fiscal policies. *Journal of Money, Credit and Banking*, 415-429.
- [38] Marzo, M., Söderström, U, and Zagaglia, P. (2008). The Term Structure of Interest Rates and the Monetary Transmission Mechanism., Unpublished Manuscript.
- [39] Morrison, W. M., and Labonte, M. (2011). China's holdings of US securities: implications for the US economy. *Current Politics and Economics of Northern and Western Asia*, 20(3), 507.
- [40] Pringle, R. and Carver, N (2002). How Countries Manage Reserve Assets. Chapter 1. London: Central Banking Publications.
- [41]
- [42] Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4):517-531. 24.
- [43] Rudesbush, G.D., E.T Swanson, and T. Wu (2006). The Bond Yield “Conundrum” from a Macro-Finance Perspective, *Monetary and Economic Studies*, 24(S1),83-109.
- [44] Rudebusch, G. D., Sack, B. P., and Swanson, E. T. (2007). Macroeconomic Implications of Changes in the Term Premium. *Federal Reserve Bank of St. Louis Review*, July/August 2007, 89(4), pp.241-69.
- [45] Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics*, 26(2):183-209.
- [46] Sierra, J. (2010). International Capital Flows and Bond Risk Premia. Working Papers 10-14, Bank of Canada.
- [47] Sims, C.A. and Zha, T. (1999) Error bands for Impulse Response. *Econometrica* 67: 1113-1156.
- [48] Swanson, E. T. (2011). Let's twist again: a high-frequency event-study analysis of operation twist and its implications for QE2. *Brookings Papers on Economic Activity*, 2011(1), 151-188.
- [49] Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195-214.

- [50] Tobin, J. (1969). A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking*, 1(1):15-29.
- [51] Tobin, J. (1982). Money and Finance in the Macroeconomic process. *Journal of Money, Credit and Banking*, 14(2):171-204.
- [52] Vayanos, D., and Vila, J. L. (2009). A preferred-habitat model of the term structure of interest rates (No. w15487). National Bureau of Economic Research.
- [53] Warnock, F. E., and Warnock, V. C. (2009). International capital flows and US interest rates. *Journal of International Money and Finance*, 28(6), 903-919.
- [54] Zagaglia, P. (2013). Forecasting Long-Term Interest Rates with a General-Equilibrium Model of the Euro Area: What Role for Liquidity Services of Bonds?. *Asia-Pacific Financial Markets*, 20(4), 383-430.
- [55] Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggregation Bias. *Journal of American Statistical Association* 56, 500-509.

Table 2: **Calibrated steady-state values of some variables**

<b>Description</b>	<b>Notation</b>	<b>Value</b>
Output	$Y$	1.00
Taxes-output ratio	$T/Y$	0.195
Labor hours	$N$	1/3
Gross short-term rate, Annual	$R$	1.04
Steady-state inflation rate	$\pi$	1

Table 3: **Baseline Parameter calibration**

<b>Description</b>	<b>Notation</b>	<b>Value</b>
Discount factor	$\beta$	0.995
Habit formation	$\theta$	0.7
Coefficient of risk aversion	$\gamma$	1.0
Elasticity of money demand	$\eta$	2.6
Inverse of elasticity of labor supply	$\varphi$	1
Elasticity of Demand	$\varepsilon$	6.0
Cost of Price Adjustment	$\psi$	100
Portfolio adjustment friction	$\phi_L$	0.01
<i>Monetary policy</i>		
Monetary policy response to output	$\rho_Y$	0.6
Monetary policy response to inflation	$\rho_\pi$	1.5
Monetary policy inertia	$\rho_R$	0.85
<i>Taxation policy</i>		
Steady-state Lump Sum Tax	$\zeta_0$	0.195
Tax response to short-term bonds	$\zeta_1$	0.15
Tax response to short-term bonds	$\zeta_2$	0.15
<i>Autoregressive Coefficients</i>		
Monetary Policy	$\phi_{\phi_R}$	0.85
Government spending	$\phi_G$	0.90
LT bonds shock	$\rho_{bL}$	0.90
<i>Standard Deviations</i>		
Monetary Policy Shock St. Dev.	$\sigma_R$	0.0025
Government Spending Shock St. Dev.	$\sigma_G$	0.012
LT bonds Shock St. Dev.	$\sigma_L$	0.01

Table 4: Calibration of Key Parameter and Steady State Values

Description	Notation	Value
Total LT bonds Outstanding per GDP	$B_{L,t}$	0.220
Total ST bonds Outstanding per GDP	$B_t$	0.056
LT bonds held by households per GDP	$B_{L,t}^H$	0.151
LT bonds held by FOH per GDP	$B_{L,t}^F$	0.068
Share of FOH LT bonds( $x_t$ ) shock Coef.	$\rho_x$	0.72
Magnitude of FOHL shock	$\sigma_x$	1.53

Table 5: Comparison of Model Results to Empirical Estimates of Foreign Official Purchases on Long Term Yield

Studies	On Impact	Methodology
<a href="#">My Model: Long-term Interest Rate</a>	No Effect to -22 bps	Calibrated DSGE model
<a href="#">My Model: Term Premium</a>	-34 to -54 bps	Calibrated DSGE model
Kohn (2015)	-51 bps	CBAPM (Term Premium)
Rudesbusch et. al (2006)	No Effect	ANM-F model
Beltran et al. (2013)	-39 to -62 bps	Excess returns regression
Beltran et al. (2013)	-46 to -50 bps	Term premium regression
Warnock & Warnock (2009)	-68 bps	OLS regression
Stock Effect (Max. Effect)		
<a href="#">My Model</a>	-26 bps	Calibrated DSGE model
Beltran et al. (2013)	-20 bps	Cointegration (Holdings(level))
Bertaut et al. (2011)	-15 bps	Regressions (Holdings(level))

Source: Beltran et al. (2013) and author's computation/compilation

Notes: Consumption Based Asset Pricing Model (CBAPM); Affine no-arbitrage macro-finance (ANM-F)

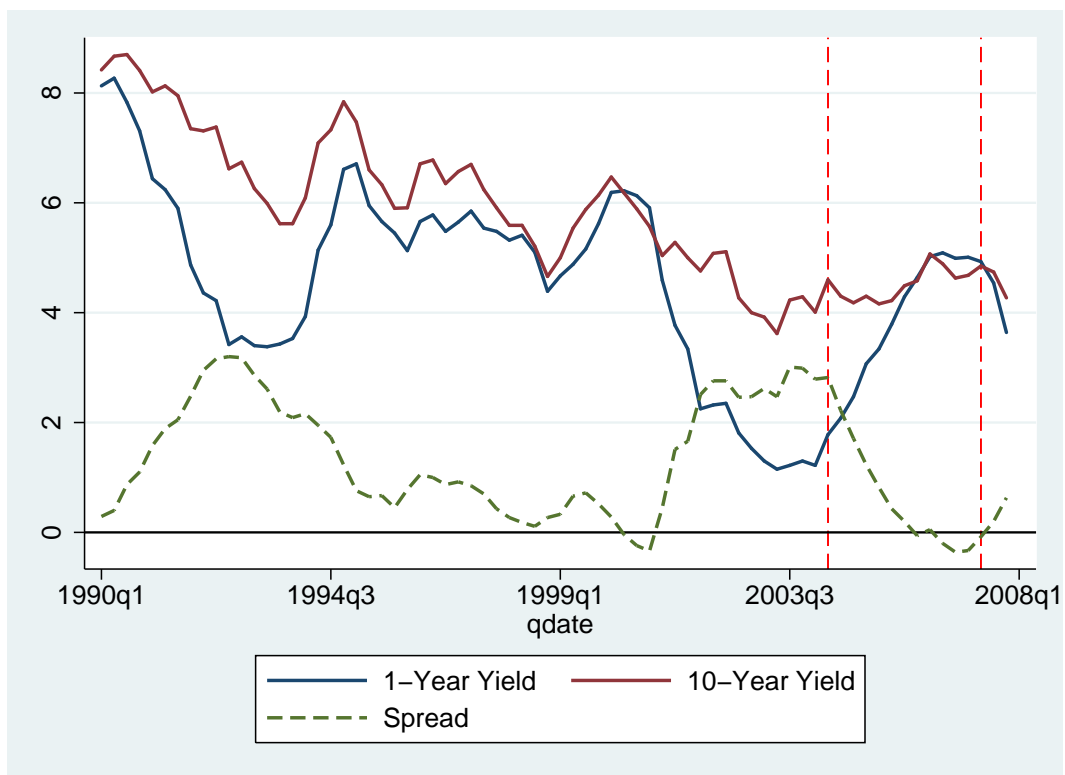
Table 6: Moment Comparison

Std. Dev. of variable	Data	Model	Model	Model
		( $\phi_L = 0.005$ )	( $\phi_L = 0.01$ )	( $\phi_L = 0.02$ )
<u>Macro-variables</u>				
Output	0.46	0.40	0.42	0.44
Consumption	0.72	0.10	0.10	0.11
Inflation	0.32	0.62	0.91	1.10
<u>Financial variables</u>				
Short-term interest rate	0.30	0.50	0.67	0.81
Long-term interest rate	0.12	0.47	0.59	0.71
Term-Spread	0.59	0.32	0.36	0.40

The table compares empirical moments from data and theoretical moments implied by the model. The data is treated similar to the variable in the model and it is filtered using the Hodrick-Prescott filter with a smoothing parameter of  $\lambda = 1600$ . All values are in percent with the exception of the term spread. Theoretical moments from the model with low habits ( $\theta < 0.7$ ) show similar results to baseline model but with a much higher volatility in consumption. Similar results hold for the FOHL persistence parameter ( $\rho_x$ ).



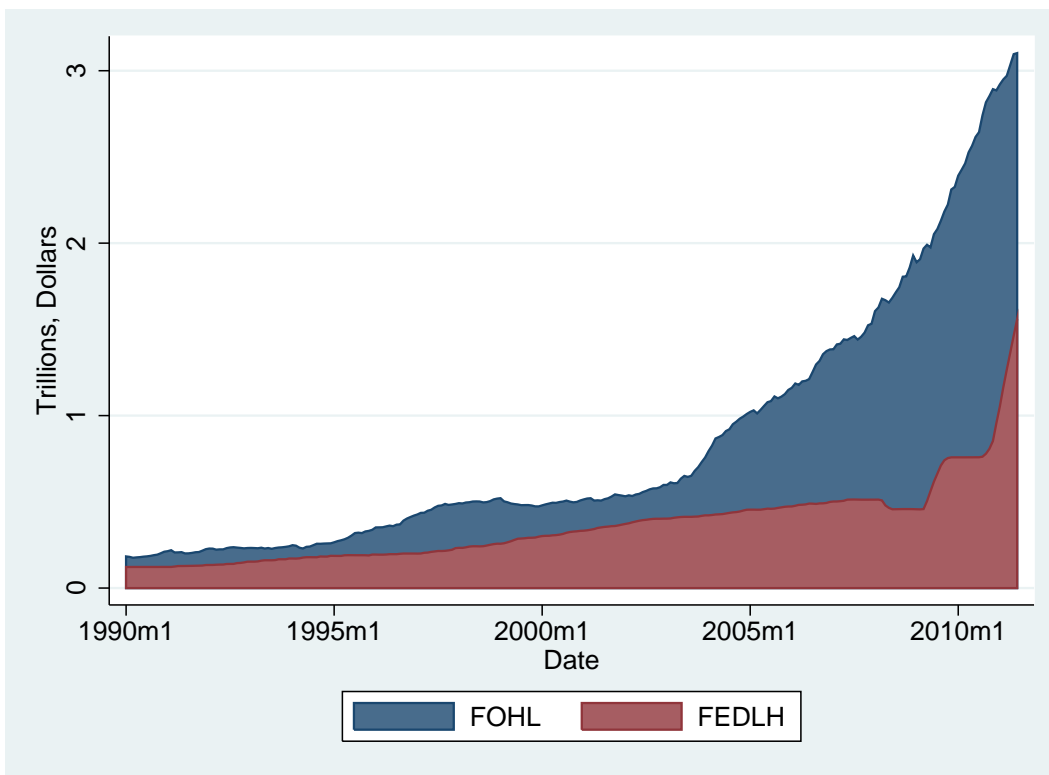
Figure 1: Yields and Spread of 1-year and 10-year Bond, 1990Q1-2007Q12



**Source:** FRED and author's calculation.

**Note:** The Spread is computed as the 10-year yield minus the 1-year yield

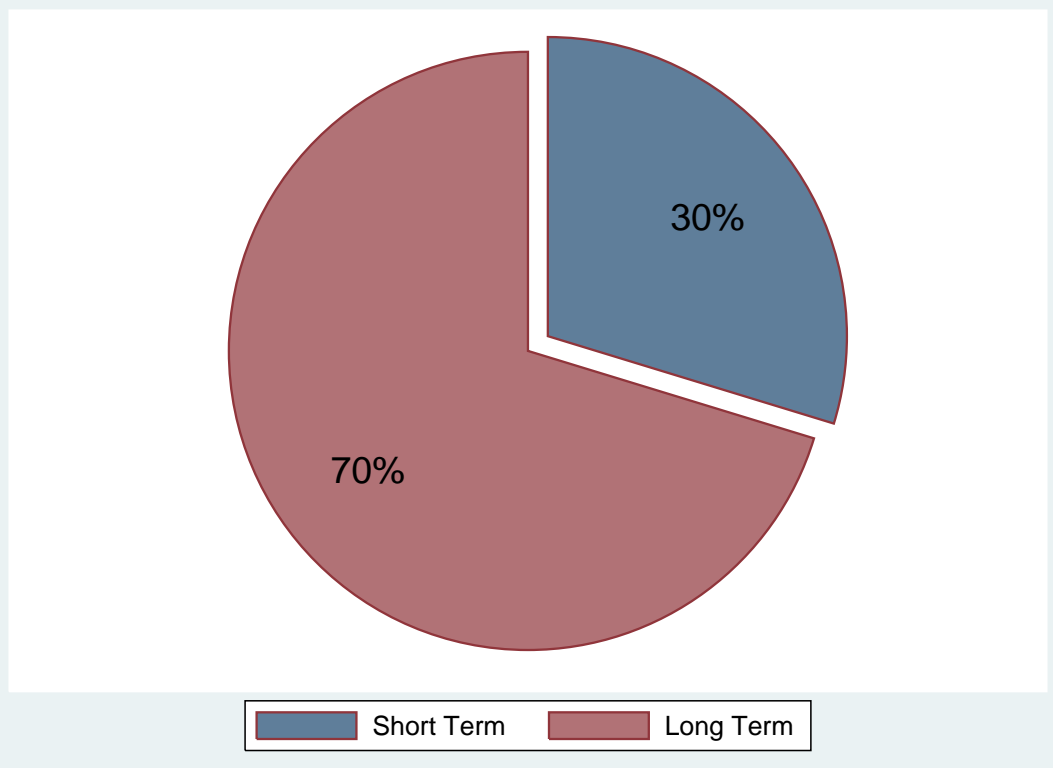
Figure 2: Long Term Bond Holdings, FED vs Foreign Official Institutions



Source: Author's illustration of Bertaut-Tyron Measure of Foreign Official Holdings and FRED



Figure 3: Shares of Short and Long Term Foreign Official Bond Holdings in Total Foreign Official Holding, January 1990 to June 2011



**Source:** Author's illustration of Bertaut-Tyron Measure of Foreign Official Holding (Long-term bond holdings) and Treasury International Capital System (TIC), section A.2 (Short-term bond Holding).

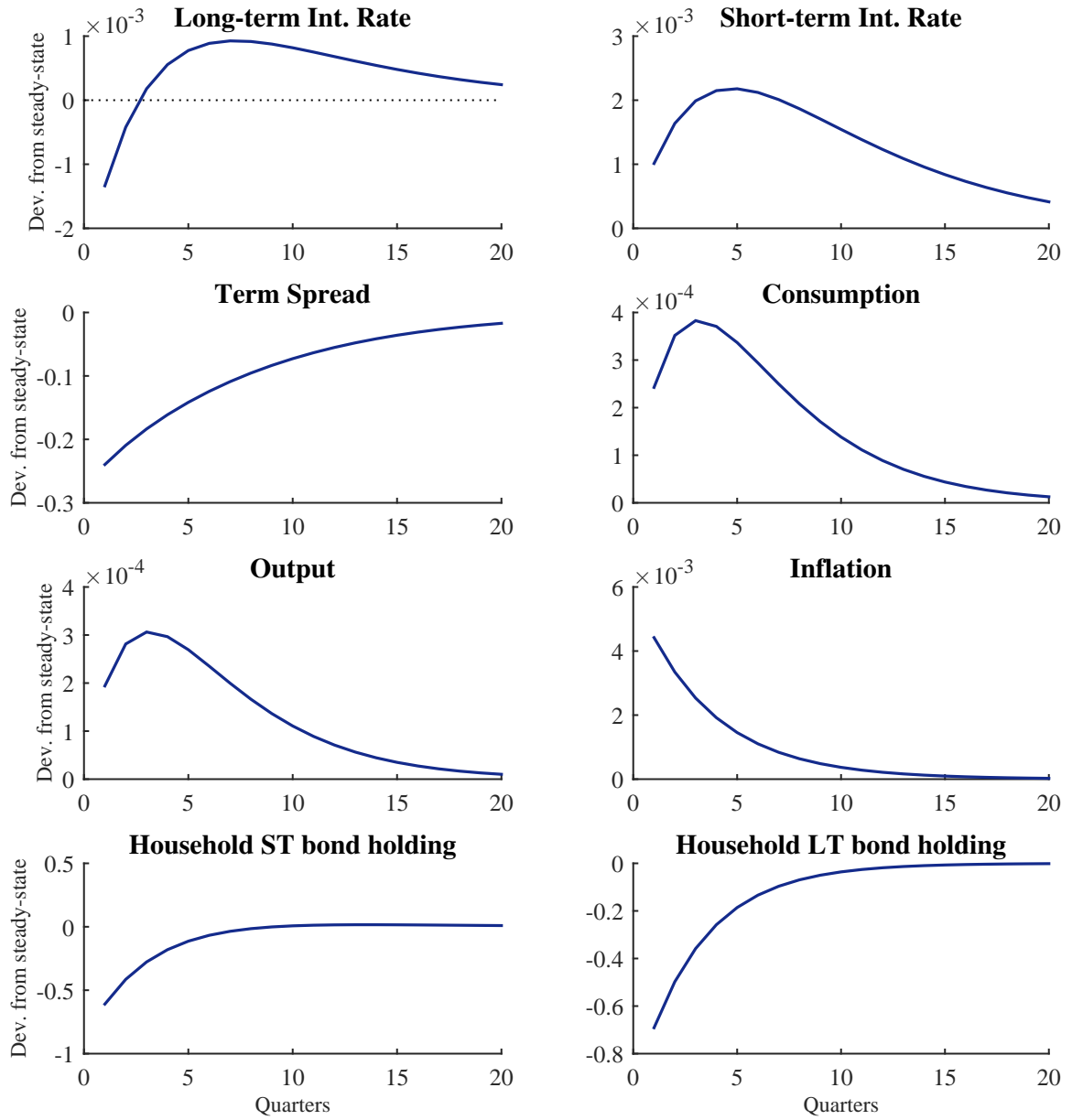


Figure 4: Selected impulse response functions from the equilibrium model following a shock to the share of long term bond holdings of foreign official Institutions ( $x_t$ )

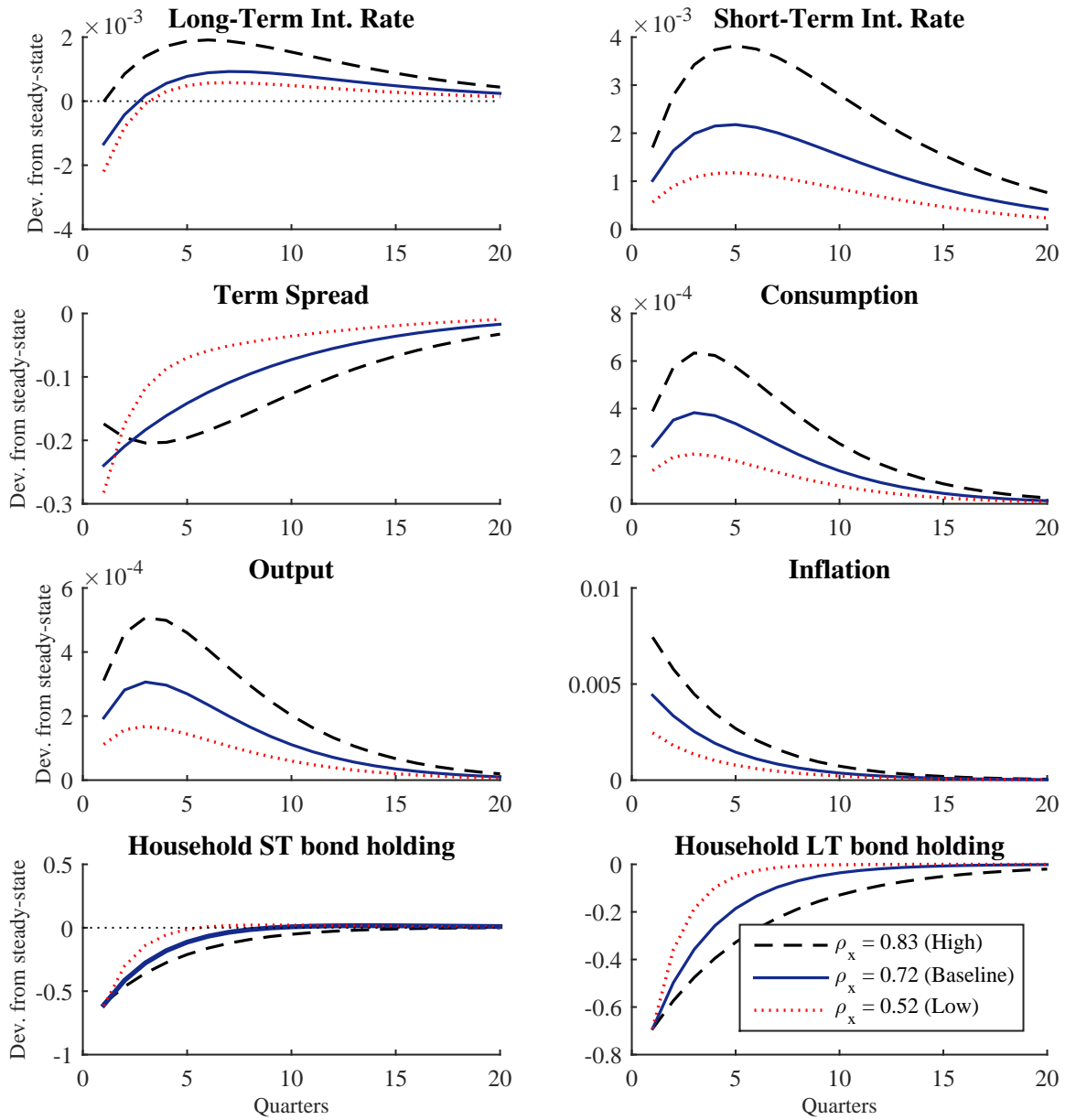


Figure 5: Selected impulse responses to a shock to the share of long term bond holdings of foreign official Institutions ( $x_t$ ) when varying the persistence parameter  $\rho_x$ . The black dashed line represents high persistence, the dotted red line represents low persistence and the solid blue line represents the baseline case.

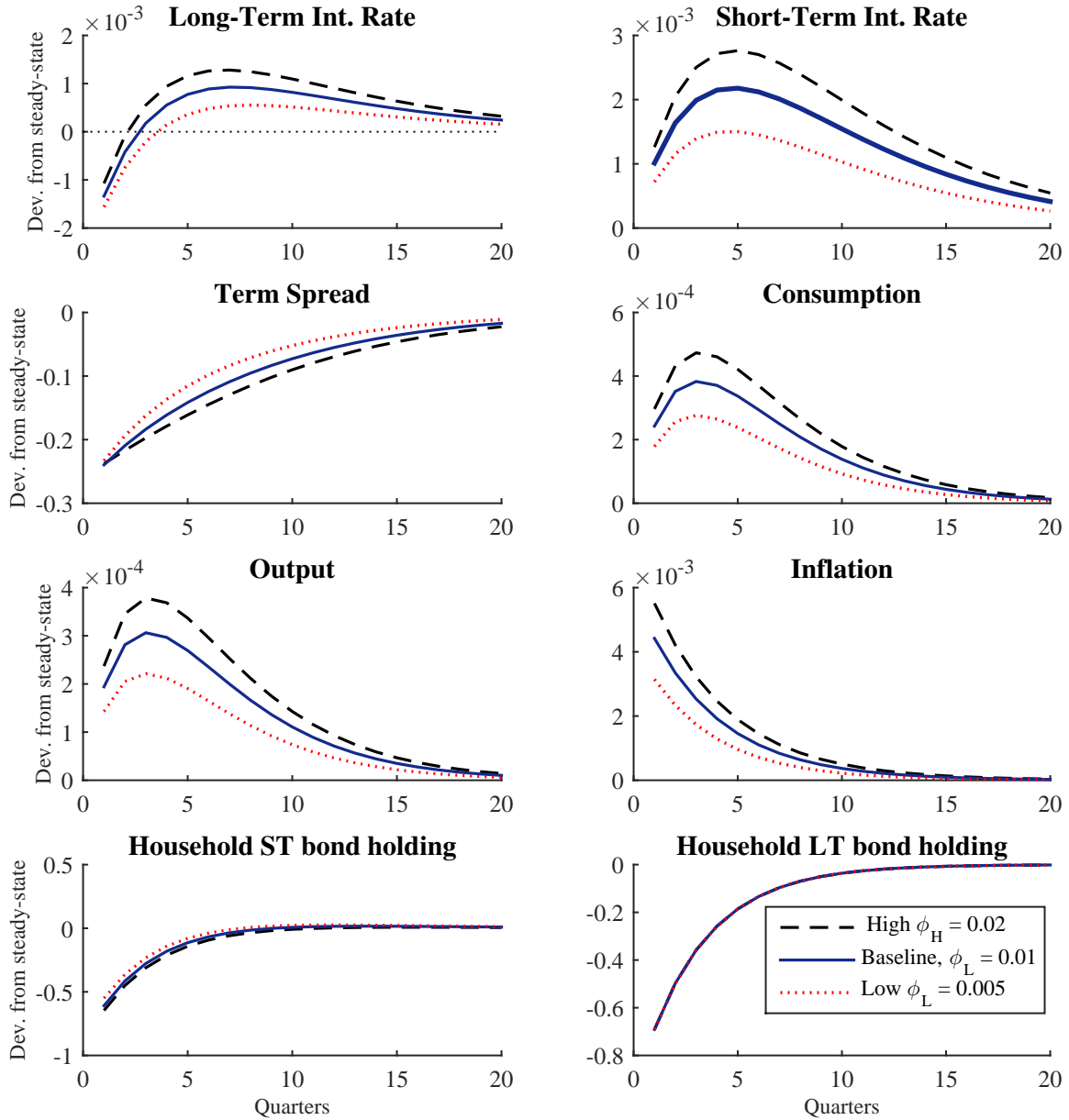


Figure 6: Selected impulse responses to a shock to the share of long-term bond holdings of foreign official Institutions ( $x_t$ ) when varying the portfolio adjustment cost parameter  $\phi_L$ . The black dashed line represents high portfolio adjustment costs, the dotted red line represents low portfolio adjustment costs and the solid blue line represents the baseline case.

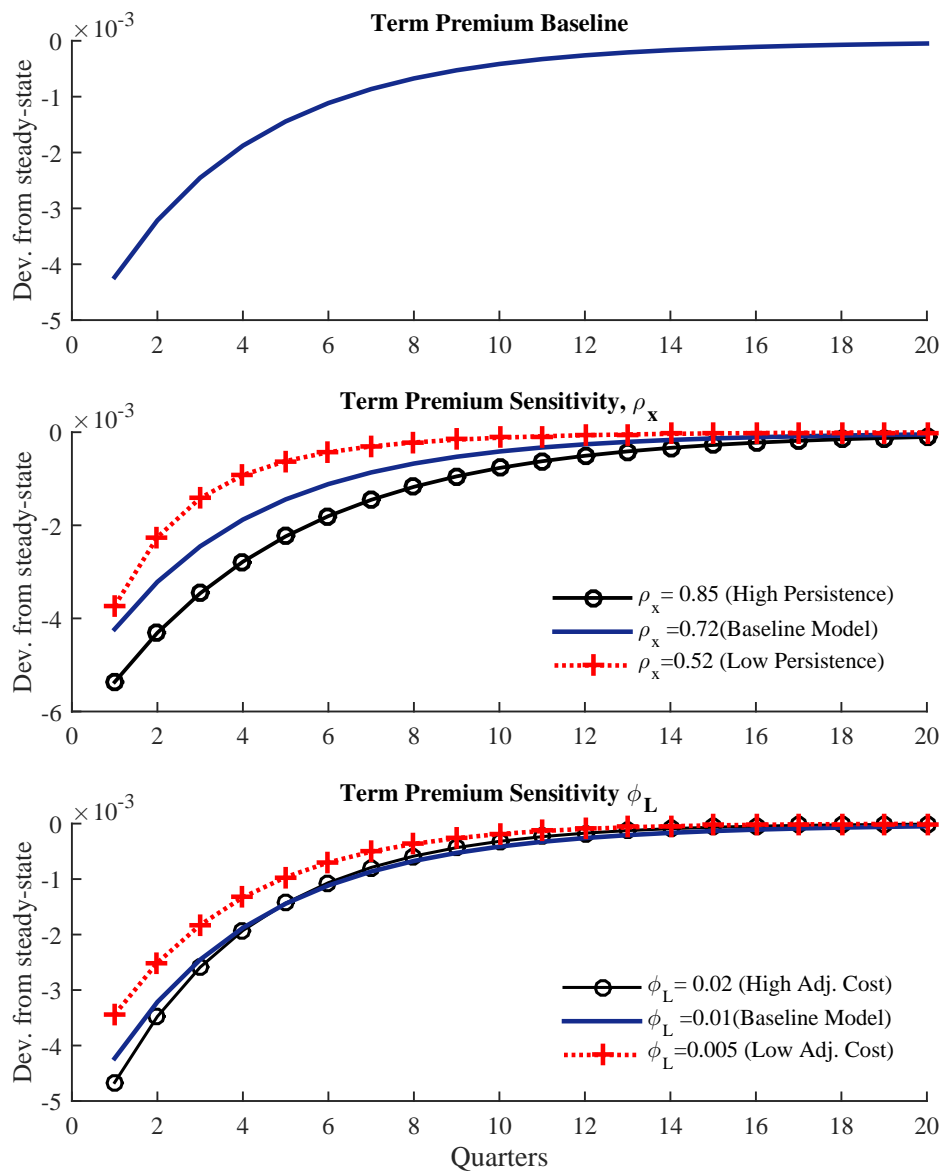
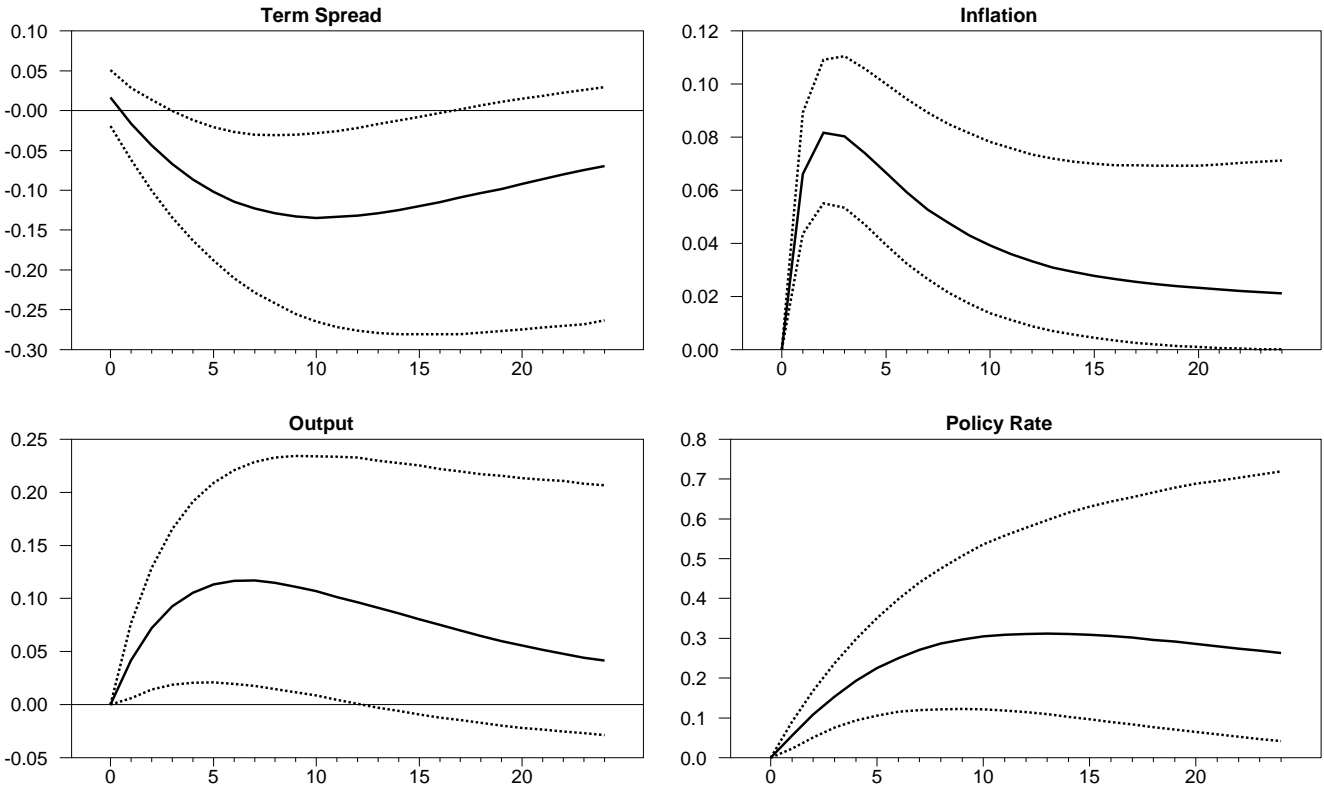


Figure 7: The figure shows the impulse responses of the Term Premium to a positive shock to the share of long-term bond holdings of U.S. Treasuries by foreign officials. The top figure depicts the baseline case; the middle figure show the response of the Term Premium when varying the persistence parameter for the AR(1) process for  $x_t$ , ( $\rho_x$ ) while the bottom figure shows the response of the Term Premium when varying the portfolio adjustment cost parameter,  $\phi_L$ . The thick blue line represents the baseline model. The circled black line and red plus lines represent high and low parameter values respectively.

Figure 8: Median impulse response functions for the term spread, output, inflation and Fedfunds rate are reported following a shock to foreign official holdings of long-term U.S. Treasuries. The corresponding confidence bounds are defined at 68% posterior bands.



Responses to Foreign Official Holding

**Graphical Appendix:** Sensitivity analysis on monetary policy inertia and habit parameter.

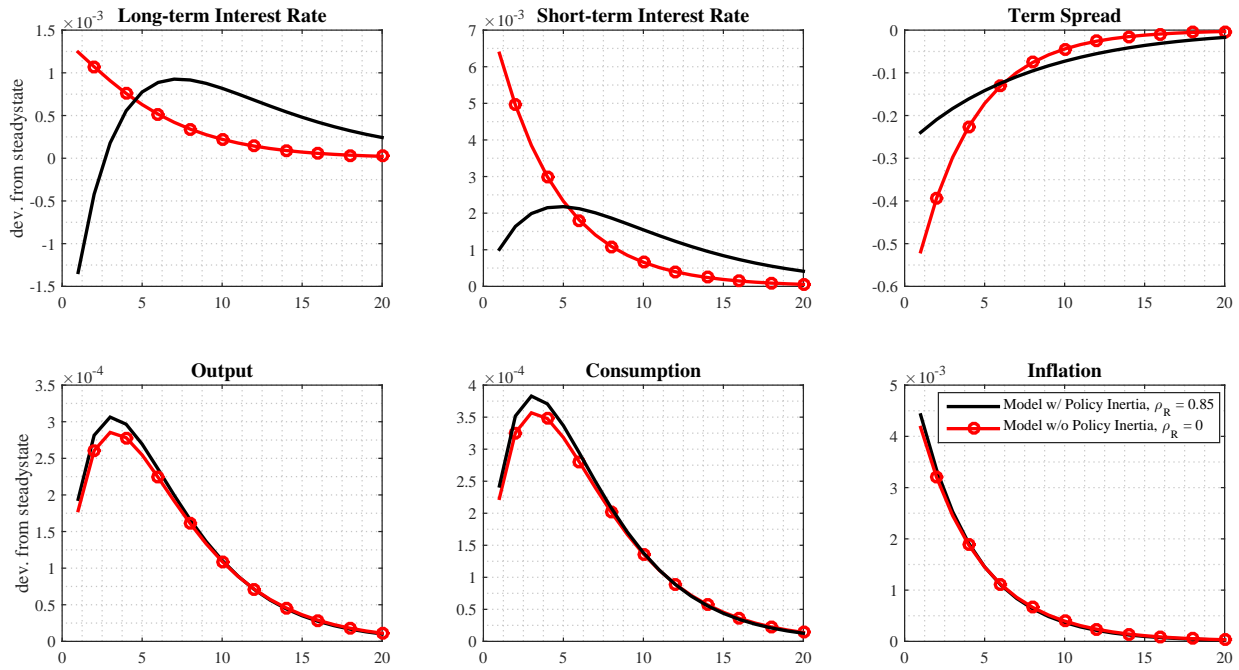


Figure 9: Effect of monetary policy inertia,  $\rho_R$ , following FOHL shock.

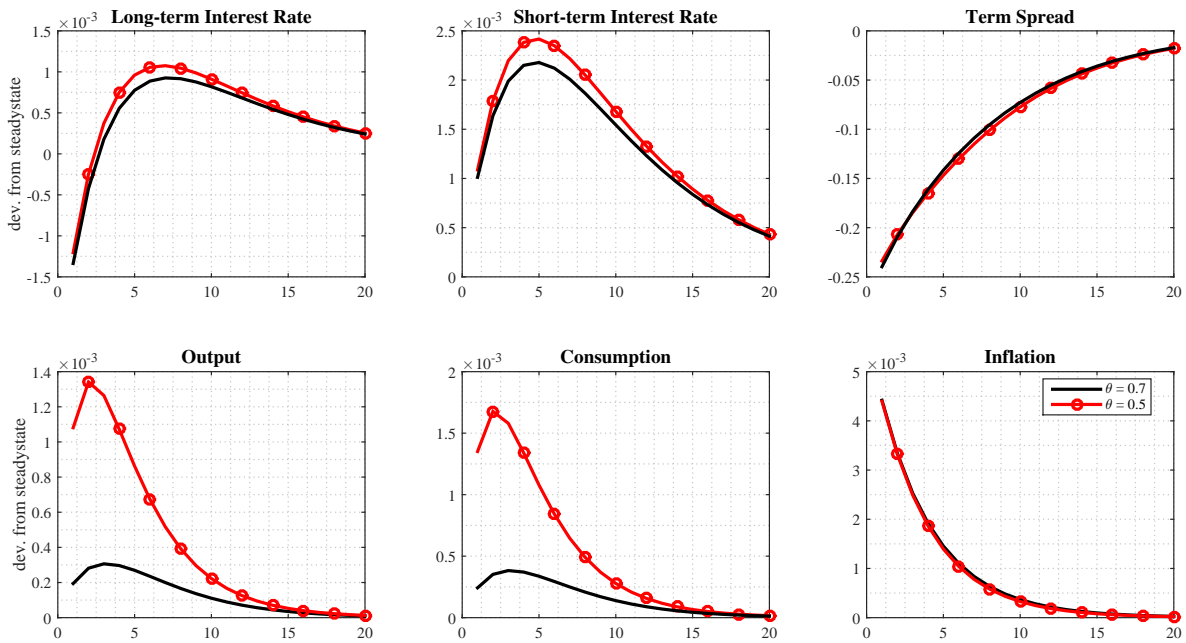


Figure 10: Effect of habits,  $\theta$ , following FOHL shock.