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Forensic Engineering Analysis of Commercial Vehicle Air Brake Systems Performance

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Abstract

Braking systems for heavy commercial vehicles differ greatly from the design for light-duty motor vehicles. For example, 49 CFR 571.121 and 49 CFR 393.52 require loaded buses, single unit commercial vehicles, and vehicle-trailer combinations equipped with air brake systems to generate sufficient braking force to meet specific stopping distance, stopping acceleration rate, and brake force-to-weight percentage performance criteria. The combination of unique design, mechanical complexity, and maintenance issues characteristic to air brake systems also pose difficulty in the analysis of air brake system performance. Air brake system performance presents a difficult problem for the forensic engineer with limited familiarity regarding air brake system functions and the elements affecting brake performance. This paper provides insight into the evolution of air brake system standards and the applicable performance criteria for heavy commercial vehicles. The methods presented allow the forensic engineer to mathematically analyze and determine the effects of brake size, mismatched components, brake adjustment, and system air pressure on the overall braking force and stopping capabilities of air brake equipped commercial vehicles.

Keywords

Forensic engineering, air brakes, pneumatic brakes, commercial vehicles, commercial vehicle brakes, braking performance, s-cam brakes, air brake standards

Background

Analyzing single and multiple vehicle crashes involving commercial vehicles often requires the expertise of a knowledgeable forensic engineer. Commercial vehicle collisions oftentimes require investigations into potential pneumatic braking system failures. Several completed and ongoing studies attempt to quantify the frequency of braking defects present on commercial vehicles operating on public roadways. In 2001, the National Highway Traffic Safety Administration (NHTSA) and the Federal Motor Carrier Safety Administration (FMCSA) initiated the Large Truck Crash Causation Study (LTCCS). Results of the study estimate that deficient braking systems played a part in 26% of all heavy vehicle crashes[1]. The Fatal Accident Complaint Team of the Michigan State Police Motor Carrier Enforcement Division found that during inspections of 407 heavy vehicles following crashes, 32.7% of the involved heavy vehicles had one or more braking system deficiencies[2].

Pneumatic braking systems were originally developed for use by the locomotive industry. The fundamental design principles for pneumatic braking systems on most modern commercial vehicles stem from the original design principles used for locomotive brakes. George Lane was the first to develop and deploy pneumatic brakes for on-road heavy vehicles. Lane worked as a logging truck driver in the northwestern United States and observed the need for better, more reliable braking systems on the logging trucks in operation at the time. As a result, the original "Lane" braking system for commercial vehicle use was introduced in 1919. The Lane braking system consisted of an accumulator attached to the engine's combustion chamber, allowing compressed gas developed during the engine's compression stroke to pass through a one-way check valve and into a holding reservoir. The compressed gas was stored in a holding reservoir until brake application. The Lane brake was designed to function on only the rear axle of a heavy vehicle[3].

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The Lane brake system had the following major design flaws, which limited universal adoption for use on heavy vehicles:

• Decrease of the engine's effective compression ratio due to the accumulator valve capturing a portion of the engine's cylinder gases during the compression stroke.

• Introduction of contaminants from the engine into the braking system.

By 1924, Westinghouse developed an engine-driven air compressor to operate a commercial vehicle's pneumatic braking system in lieu of an accumulator valve. The engine-driven compressor heralded the coming of the modern pneumatic braking system. Following the advent of the engine-driven compressor, foot-operated brake valves (treadle valves) and pressure regulators were deployed on commercial vehicles, ensuring the braking system operated within normalized pressures.

Commercial Vehicle Braking Performance Standards

Following the rapid developments of the commercial vehicle pneumatic braking system, the U.S. government initiated braking system type and performance regulations. Government-mandated stopping distance performance regulations for commercial vehicles were first issued in 1933. The regulation required a pneumatically braked commercial vehicle to stop from 20 mph within 50 feet.

The 1950s through '70s saw the introduction of numerous regulations for commercial vehicle braking systems. However, none was more widespread and influential than the major legislative effort of the Federal Motor Vehicle Safety Standard (FMVSS) 121 (49 CFR 571.121) in the 1970s. FMVSS 121 was issued in 1971, but implementation was delayed — and the regulations were amended until 1975. FMVSS 121 required newly manufactured commercial vehicles to be equipped with many of the following safety features found on modern vehicles:

• Anti-lock braking systems (ABS)

• Brakes on all axles, including front axle brakes

• Spring-actuated parking brakes

• Dual circuit braking systems

The original version of FMVSS 121 required commercial vehicles to stop from 60 mph in 217 feet (0.55 g). This stopping distance was amended first to 245 feet (0.49 g), then to 258 feet (0.47 g), and then to a 277 foot (0.43 g) stopping distance with the implementation of the law in 1975. To meet these early FMVSS 121 stopping distance criteria, commercial vehicles were designed with front brakes that generated significantly more torque than previous designs. The "overpowered" front brakes were prone to locking when the ABS system malfunctioned or failed, which occurred with regularity during the infancy of pneumatic ABS. When the front wheels stop rotating and lock, the vehicle loses directional stability and functional steering. Due to the "overpowered" front brake issue and others, the stopping distance requirement in FMVSS 121 was again amended in 1978 to 293 feet (0.41 g)[4]. In 1978, Paccar and the American Trucking Association successfully sued NHTSA to repeal the requirement for ABS brakes and the 293-foot stopping distance requirement.

The update of FMVSS 121 in 1995 re-established stopping distance requirements. The new requirement mandated that most truck tractors stop from 60 mph within 355 feet (0.34 g) while pulling an un-braked semi-trailer at its gross vehicle weight rating (GVWR). Unloaded tractors were mandated to stop within 335 feet (0.36 g)[5]. It would take until 1997 for a new legislative effort to again require ABS braking systems on pneumatically braked commercial vehicles.

In 2009, FMVSS 121 was again updated and beginning in 2011, most newly manufactured tractors were required to stop from 60 mph within 250 feet (0.48 g) while pulling an un-braked semi-trailer at GVWR. FMVSS 121 requires unloaded tractors to stop within 235 feet (0.51 g). It should be noted that even under the updated FMVSS 121, vehicles are not required to stop as quickly as mandated by the 1971 version of FMVSS 121[6]. **Figure 1** depicts the stopping distance requirements of FMVSS 121 for commercial vehicles manufactured after 2011/2013.

In-service vehicles are governed by Federal Motor Carrier Safety Regulations (FMCSR) — specifically FMCSR 393 (49 CFR 393) — with regard to the braking system. A common area of confusion in collision investigation and litigation involving commercial vehicles surrounds which braking performance regulation applies to the vehicle in question. The simple answer is FMVSS only applies to newly manufactured vehicles, not in-service vehicles. If, as manufactured, the vehicle in question

TABLE II - STOPPING DISTANCE IN FEET

Vehicle speed in miles per hour	Service brake						Emergency brake	
	PFC 0.9	PFC 0.9	PFC 0.9	PFC 0.9	PFC 0.9	PFC 0.9	PFC 0.9	PFC 0.9
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
30	70	78	65	78	84	61	170	186
35	96	106	89	106	114	84	225	250
40	125	138	114	138	149	108	288	325
45	158	175	144	175	189	136	358	409
50	195	216	176	216	233	166	435	504
55	236	261	212	261	281	199	520	608
60	280	310	250	310	335	235	613	720

NOTE:

(1) Loaded and Unloaded Buses.

(2) Loaded Single-Unit Trucks.

(3) Loaded Tractors with Two Axles; or with Three Axles and a GVWR of 70,000 lbs. or less; or with Four or More Axles and a GVWR of 85,000 lbs. or less. Tested with an Unbraked Control Trailer.

(4) Loaded Tractors with Three Axles and a GVWR greater than 70,000 lbs.; or with Four or More Axles and a GVWR greater than 85,000 lbs. Tested with an Unbraked Control Trailer.

(5) Unloaded Single-Unit Trucks.

(6) Unloaded Tractors (Bobtail).

(7) All Vehicles except Tractors, Loaded and Unloaded.

Figure 1
FMVSS 121 table.

does not meet FMVSS regulations, then it would be in violation of FMVSS standards. However, if an in-service vehicle has not been properly maintained and no longer meets FMVSS standards, this is not in violation of FMVSS regulations, but rather a potential violation of the Federal Motor Carrier Safety Regulations.

FMCSR 393 mandates that tractor semi-trailers be capable of generating 43.5% peak braking force (0.435 g) as a percentage of their combination weight, decelerate with a peak rate of at least 14 feet/sec^2 (0.435 g), and stop from 20 mph within 40 feet (0.33 g). These criteria must be met by in-service vehicles in the as-loaded condition. Take note that the deceleration rate and braking force percentage of vehicle/combination weight is the peak value, not the average, whereas the stopping distance is a road test designed to account for overall braking system effectiveness. **Figure 2** depicts the braking performance mandated for in-service vehicles[7].

General Pneumatic Brake System Overview

Pneumatic braking systems have several commonalities with the hydraulic braking system employed in passenger vehicles. However, instead of using an (ideally) incompressible fluid in a hydraulic system, a pneumatic system uses a compressible fluid (air). Pneumatic braking systems, in general, are more complex when compared to hydraulic braking systems. The majority of pneumatic braking systems on heavy vehicles in the United States employ a type of brake system called S-cam drum brakes.

Pneumatic braking systems use compressed air to activate a series of mechanical linkages, which, in turn, press friction material (brake shoe/pad) into a heat sink (brake drum/rotor). Brakes, whether a passenger vehicle equipped with a hydraulic braking system or a heavy vehicle equipped with pneumatic brakes, complete the same function, converting kinetic energy into thermal energy to slow the vehicle. The thermal energy is then dissipated into the atmosphere so that the vehicle braking system's heat sinks can accept more energy.

Modern pneumatic brakes consist of two braking systems: service brakes and parking brakes. Service brakes simply provide stopping power while the vehicle is in service. The parking, or spring brakes, ensure that a vehicle does not move while parked, and will not release until the system has built enough pressure to operate the service brakes. Additionally, spring brakes activate to slow a vehicle when absent sufficient air pressure to operate

Type of motor vehicle	Service brake systems			Emergency brake systems
	Braking force as a percentage of gross vehicle or combination weight	Deceleration in feet per second per second	Application and braking distance in feet from initial speed at 20 mph	Application and braking distance in feet from initial speed of 20 mph
A. Passenger-carrying vehicles:				
(1) Vehicles with a seating capacity of 10 persons or less, including driver, and built on a passenger car chassis	65.2	21	20	54
(2) Vehicles with a seating capacity of more than 10 persons, including driver, and built on a passenger car chassis; vehicles built on a truck or bus chassis and having a manufacturer's GVWR of 10,000 pounds or less	52.8	17	25	66
(3) All other passenger-carrying vehicles	43.5	14	35	85
B. Property-carrying vehicles:				
(1) Single unit vehicles having a manufacturer's GVWR of 10,000 pounds or less	52.8	17	25	66
(2) Single unit vehicles having a manufacturer's GVWR of more than 10,000 pounds, except truck tractors. Combinations of a 2-axle towing vehicle and trailer having a GVWR of 3,000 pounds or less. All combinations of 2 or less vehicles in drive-away or tow-away operation	43.5	14	35	85
(3) All other property-carrying vehicles and combinations of property-carrying vehicles	43.5	14	40	90

Figure 2
FMCSR table.

the service brakes. Spring brakes are operated by evacuating air from the spring brake chamber, which is facilitated through a push/pull button on the dash at the driver's position in the cab.

Generally, S-cam drum brake function can be described in the following manner:

• The driver presses the treadle valve (foot brake) to apply the service brakes.

• Valves are opened, allowing compressed air to flow into the braking circuit from the supply circuit.

• Compressed air pressurizes the brake chambers at each axle, energizing the brakes.

• The brake pushrod extends from the brake chamber and applies a force to the brake slack adjustor

• In response to the force from the brake pushrod, the slack adjustor rotates and applies torque to the S-cam.

• The S-cam rotates, forcing the brake shoes to ride up the S-cam and displace outward.

• The friction lining on the brake shoes are forced against the inner surface of the brake drum, generating friction.

• The friction generated between the brake shoe lining and the brake drum surface converts the vehicle's kinetic energy into thermal energy.

• The brake drum acts as a heat sink and radiates heat to the atmospheres.

• Upon release of the treadle valve, air is evacuated from the brake chambers and the pushrods retract.

• As the pushrods retract, the brake shoes move away from the drum and cease to generate friction.

Braking System Failures

During the century following the invention of the commercial vehicle pneumatic braking system, pneumatic brakes have been refined with greater efficiency and reliability. Still, braking system deficiencies are found on a regular basis during inspections or following a collision event. Ineffective pre-trip inspections and a lack of proper maintenance lead to many braking deficiencies overlooked prior to a potentially catastrophic event.

The factors involved in partial or complete braking system failure are often not readily evident without a technical inspection of the braking system. Following are some of the most common areas where deficiencies are found within a commercial vehicle's braking system:

• Excessive pushrod stroke (out of adjustment)

• Thermal failures in the drum

• Fluid contamination between the drum and brake shoe friction material

• Air leaks or low pressure

• Non-functioning valves

• Worn drums and/or shoes

• Improperly matched brake components

• S-cam rollover (i.e., beyond operational limits)

• ABS failures

Excessive pushrod travel, or "stroke," is the most commonly cited brake system deficiency found during roadside inspection of heavy vehicles[8]. Excessive pushrod travel during brake application results in an "out-of-adjustment" brake. Pushrod travel is simply the change in distance between the fully retracted (no braking) position of the pushrod and its fully extended (full braking air pressure applied) position. The travel of the pushrod is commonly referred to as "pushrod stroke." Pushrod stroke is determined by measuring the distance of an arbitrary point on the pushrod (usually the clevis pin connection to the slack adjuster) from the brake chamber face without brake application. Following brake application with 90 to 100 psi of pressure, the distance from the brake chamber face to the same arbitrary point on the pushrod is again measured. The difference between the two measurements provides the pushrod stroke.

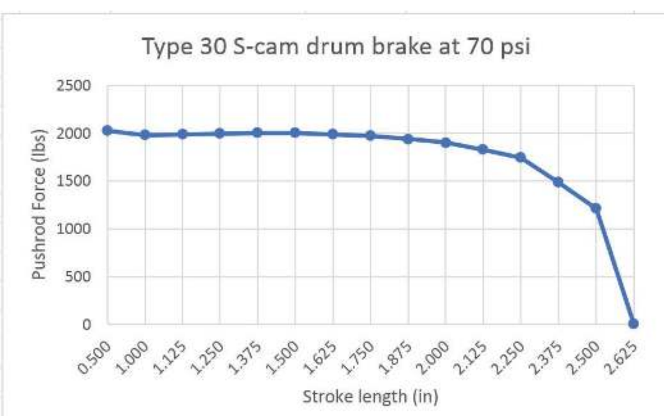

Figure 3
Type 30 S-cam at 70 psi pressure.

Excessive pushrod stroke decreases the available braking force to activate a given brake. The reason excessive pushrod stroke is detrimental to braking force generation lies in the fact that the brake chamber diaphragm can only flex so much before it starts binding on the interior of the brake chamber. Once binding occurs, the applied force decreases rapidly. At the extreme end of the excessive pushrod travel, the pushrod "strokes out" or "bottoms out," such that the diaphragm can no longer move the pushrod to apply further torque to the S-cam. When the diaphragm bottoms out, no additional braking force can be generated, regardless of applied air pressure. Plotting the force applied through the pushrod at a given brake pressure application for increasing pushrod strokes based upon published data[9] generates the graph shown in **Figure 3**. The force curve illustrates that the force generated decreases as the stroke length increases, and finally drops to zero when the brake diaphragm bottoms out.

If the brake shoe does not sufficiently engage the brake drum as a brake strokes-out, the brake will cease to develop braking force. When one brake fails to develop force, the amount of work required at the other brakes to slow the vehicle will increase. Increasing the work required of otherwise fully functioning brakes, even in non-emergency slowing situations, can lead to excessive heat build-up, which, in turn, can produce additional brake failures. Additionally, one non-functioning brake on an axle can lead to imbalance, and potentially decrease the vehicle's linear stability while braking.

To combat excessive pushrod stroke, the Federal Motor Carrier Safety Regulations (FMCSR) were modified to mandate automatic slack adjustors. Even following this 1994 mandate, excessive pushrod travel remains a common braking deficiency issue, although with less frequency.

Automatic slack adjustors are not the panacea for excessive pushrod stroke. On September 7, 2017, the Commercial Vehicle Safety Alliance conducted its annual Brake Safety Day, in which 7,698 commercial motor vehicles were inspected. As a result, 14% of the commercial motor vehicles were deemed "Out-of-Service"[10] due to brake system deficiencies. The "Out-of-Service" designation indicates that, at a minimum, 20% of the vehicle's brakes were defective/out-of-adjustment, or the braking system had other significant safety issue(s).

Braking Performance Analysis

Several available methods provide for the analysis of pneumatic braking system performance. The methods range from simple to complex modeling. Several software suites offer brake analysis packages utilizing a variety of these methods. The methods presented in this study have their foundations in both physical constraints and empirical modeling. The selection of a brake analysis methodology depends upon the information available and the level of precision necessary to assess performance. The models presented are commonly used to determine the rate of deceleration of a heavy vehicle, and the speed of the vehicle at the beginning of observable brake application.

Commercial Vehicle Factor Method Using Skid to Stop

The Commercial Vehicle Factor (CVF) method, also known as the commercial motor vehicle factor (CMV_n)[11] or single adjusted drag factor method[12], uses an empirical percentage of the full drag factor to estimate the deceleration of a vehicle under full locked-wheel brake application. Using a CVF requires knowledge of the coefficient of tire-roadway friction for a passenger vehicle on the surface in question. Once the coefficient of tire-roadway friction for a passenger vehicle is known or determined, a CVF efficiency percentage is applied to approximate an effective drag factor for the heavy vehicle braking on the same roadway surface. The CVF is based upon empirical testing of heavy vehicles on surfaces with known passenger vehicle tire-roadway friction. The CVF is commonly ranged anywhere between 65% to 85%, depending upon the vehicle configuration, condition, tread of the tires, and other factors related to tire design. Multiplying the passenger vehicle tire-roadway coefficient of friction by the proper CVF determines the effective drag factor for a heavy vehicle. Using the adjusted drag factor, kinematic principles are applied to estimate the vehicle's speed at the beginning of observable brake application using a "skid-to-stop" formula, provided the vehicle skids to a complete stop. An example of this type of analysis is shown in **Figure 4**.

COMMERCIAL VEHICLE FACTOR METHOD

$\mu_{pv} := \begin{bmatrix} 0.70 \\ 0.75 \end{bmatrix}$ Passenger vehicle roadway friction

$CVF := 80\%$ Commercial vehicle factor

$D_{stop} := 118.5 \cdot ft$ Skid mark distance

$V_{SkidtoStop} := \begin{bmatrix} \sqrt{2 \cdot g \cdot \mu_{pv_0} \cdot CVF \cdot D_{stop}} \\ \sqrt{2 \cdot g \cdot \mu_{pv_1} \cdot CVF \cdot D_{stop}} \end{bmatrix} = \begin{bmatrix} 44.6 \\ 46.1 \end{bmatrix} mph$ Speed range

Figure 4
Commercial vehicle factor.

Weight Distribution Method

The weight distribution method provides the simplest analysis accounting for non-functioning brakes on a multi-axle vehicle. As with the CVF method, this method cannot account for brakes having partial functionality below wheel lockup. This method expands upon the CVF method with added considerations.

The weight distribution method requires knowing or estimating the weight at each axle end. This is accomplished by either measuring the weight at each wheel or set of duals, or by using general models of weight distribution based upon load and configuration. In this analysis, if a brake at any road wheel position is non-functional, the CVF for that braked wheel position (CVFn) is set to 0%, which results in no braking force at that wheel. The following equations determine the slowing acceleration rate and the vehicle's speed at the beginning of the observable brake application. An example analysis, conducted using the weight distribution method of both a fully braked vehicle and a vehicle with disabled semi-trailer brakes, are contained in **Appendix A** to this paper.

Braking force at each brake position/axle end:

$$F_n=\mu \times CVF_n \times w_n \quad (1)$$

Where,
F_n=braking force at brake n (lbs)
μ=passenger vehicle tire-roadway coefficient of friction
CVF_n=commercial vehicle factor at brake n
w_n=weight at n axle end (lbs)

Effective braking acceleration rate of heavy vehicle:

$$\mu_{cmv}=(\textstyle\sum_{n}^{i=1} F_n)\div W \quad (2)$$

Where,
μ_{cmv}=drag factor of commercial vehicle
W=total weight (lbs)

Velocity at start of full brake application:

$$v=\sqrt{2\times D\times\mu_{cmv}\times g} \tag{3}$$

Where,
v=velocity at start of full brake application
D=distance of full braking marks (ft)

Heusser Method[13]

In 1991, Heusser published the first practical pneumatic braking analysis method considering the air pressure at the brake chamber and the measured brake stroke. Heusser obtained data from brake dynamometer tests performed by NHTSA, and developed his analysis method based upon applying regression analysis to the data, as well as obtaining test data from brake manufacturers.

The Heusser analysis uses a brake force design calculation modified to fit empirical data. The Heusser method calculates the force applied by the brakes at the tire-road interface for each of n brake positions using the following Equation 4.

Attempted braking force from each n brake:

$$Bforce_n=\left[\frac{2\times Pforce\times SL\times 0.35\times DRad}{CamRad\times TRad}\right]\times 0.6 \tag{4}$$

Where,
Pforce=force of pushrod (lbs)
SL=slack adjustor length (in)
DRad=brake drum radius (in)
CamRad=S-cam radius (in)
TRad=loaded radius of tire (in)

All variables, with the exception of the pushrod force, are directly measured on the vehicle. The ideal pushrod force is calculated by multiplying the air pressure at the brake chamber by the surface area in square-inches of the brake chamber diaphragm. However, direct measurement of pushrod force reveals losses in the system that cannot be accounted for by this idealized equation.

Pushrod force tables have been generated from testing by brake manufacturers and other researchers. Heusser's paper provided pushrod force tables, and some data can be found from other sources. These tables have two independent variables: air pressure and pushrod stroke. Once air pressure at the brake chamber and pushrod stroke are determined, pushrod force is extracted from the tables and entered into Equation 4 to solve for the brake force at each of n brake positions.

When using the Heusser method, it is important to ensure that the calculated attempted brake force does not exceed the maximum force to fully lock the tire(s) at the brake position. The maximum force for each brake position is calculated using Equation 1 of the weight distribution method. The smaller value between the calculated attempted brake force and calculated maximum brake force must be used in the determination of the vehicle drag factor or the analysis is invalid. This check is necessary because a brake cannot generate more force than when it is fully locked. An example using the Heusser analysis method is presented in **Appendix B** to this paper.

Bartlett/Heusser Method[14]

In 2007, the Heusser method was modified by Bartlett[14] to account for the effects of ABS braking within a pneumatic brake force analysis. Bartlett's method introduces modifications to the Heusser braking force equation when the attempted brake force (Equation 4) is greater than the force required to lock the wheel (Equation 1), and the vehicle is equipped with anti-lock brakes. The modification involves reducing the brake application pressure between 8 psi to 20 psi below the pressure required to lock the wheel(s) at the brake location and then recalculating the braking force with this lower application pressure. Here Bartlett suggests that reducing the brake application pressure by 8 psi below what is required to fully lock the wheel(s) at any braking position represents what occurs during the pressure cycling of full ABS braking on a modern pneumatic braking system. Reducing the brake application pressure by 20 psi at a brake position for the analysis is recommended to compensate for the slower cycling rate of previous generations of ABS system.

In a 2004 SAE paper[15], Bartlett rearranged and graphed Heusser's tabulated pushrod force data with brake application pressure as the independent variable on the horizontal axis and pushrod force as the dependent variable on the vertical axis for a fixed brake stroke. This rearrangement of the tabulated pushrod force data for stroke produced a mostly linear data correlation as depicted by **Figure 5**, as opposed to the traditional Heusser method depicted in **Figure 3**. **Figure 5** depicts pushrod force versus pressure for a Type 30 S-cam brake with a stroke of 1.000 inches, 2.000 inches and 2.375 inches.

Rearranging the brake force tables in this manner as put forth by Bartlett allows for a linear regression analysis to determine the slope (mL) and y-intercept (bL) of the pushrod force of a brake at varying pressures with a given stroke. Using the equation generated by the linear

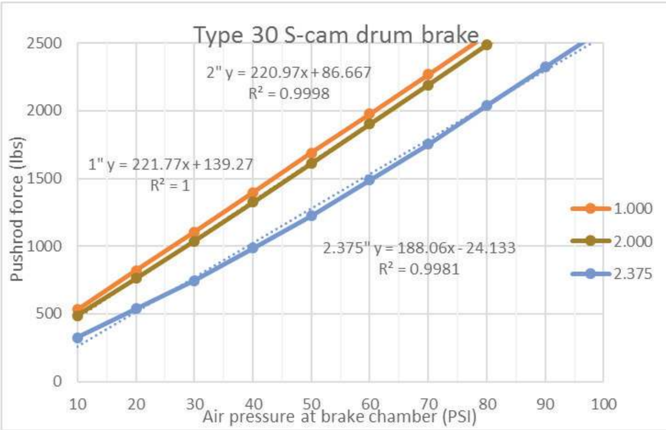

Figure 5
Type 30 S-cam drum brake.

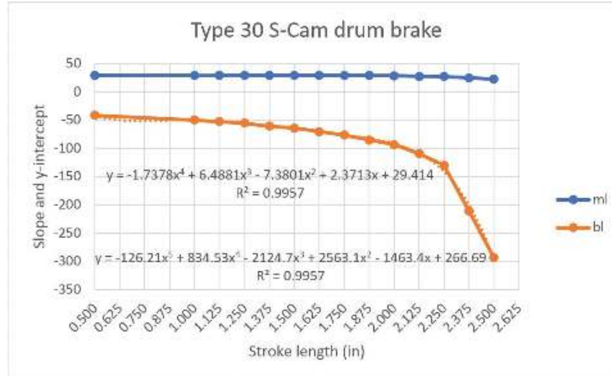

Figure 6
Type 30 S-cam regression analysis.

regression analysis eliminates the arduous task of interpolation and iteration required for the Heusser method, resulting in fewer steps where errors can be introduced when determining the brake force to lock.

Additionally, Bartlett advocates using polynomial regression analysis on both the calculated slope and y-intercepts of the linear regressions. After completing a polynomial regression analysis, the resulting equations result in pushrod force expressed as only a function of pushrod stroke and applied air pressure. This eliminates the need for interpolation for stroke lengths, as depicted in **Figure 6**.

Using the equations from the regression analysis of the brake tables, the brake force equation is rewritten to solve for brake application pressure required to lock the tire(s) at the n brake position. This is accomplished by setting the attempted brake force equal to the maximum brake force at each n brake position and incorporating the results of the regression analysis. Bartlett's brake application pressure to lock is presented as Equation 5.

<u>Brake application pressure to lock each n brake:</u>

$$P_L = \frac{W \times f_r \times CamRad \times TRad}{2 \times SL \times 0.6 \times 0.35 \times DRad \times m_L} - b_L \div m_L \qquad (5)$$

Where,
P_L= Brake pressure to lock brake (psi)
W=weight at wheel end (lbs)
f_r=μ*CVF
CamRad=S-cam radius (in)
TRad=loaded radius of tire (in)
SL=slack adjustor length (in)
DRad=brake drum radius (in)
m_L=the slope of the linear regression (lbs/psi)
b_L=the Y-intercept of the linear regression (lbs)

If the heavy vehicle is generating more attempted brake force than the maximum available brake force (force to lock the wheel), then the brake application pressure at lockup is calculated. Then, the reduction of brake application pressure (8 psi to 20 psi depending upon ABS brake system vintage) for ABS cycling during ABS locked wheel braking is applied. Using this resultant brake application pressure and the regression analysis in **Figure 5**, the pushrod force is calculated. This pushrod force is then used to calculate the brake force during full ABS braking using Equation 4. The summation of brake force at each wheel is then used to determine the slowing acceleration rate of the commercial vehicle during full ABS locked braking by dividing the total brake force by the total weight of the commercial vehicle combination. The Bartlett method produces an accurate and reliable means to analyze the braking capabilities of a pneumatically braked vehicle equipped with anti-lock brakes while accounting for brake system deficiencies.

A complete work-through example analysis using the Bartlett method is presented in **Appendix C**. The example in **Appendix C** covers both full activation of the anti-lock braking system on a tractor and semi-trailer, and the situation where only two lightly loaded axles produce lock up, and a limiting brake application pressure for the entire braking system is determined.

Performance Analysis Steps

When dealing with commercial vehicle braking systems, the forensic engineer must determine several variables. The most accurate means to gather analysis variables results from direct inspection of the braking system following the collision or incident event. However, oftentimes the involvement in a case occurs after the vehicle has been altered or is no longer available. When direct inspection and measurement of the braking system cannot occur, reliable sources of data and sound engineering judgement must be used to determine the variables needed for analysis.

The following items should be collected at a minimum during a direct inspection of the braking system to complete a proper braking performance analysis:

- Weight at each axle end
- Pushrod stroke at each brake
- Brake chamber size/type
- Slack adjustor length
- Brake drum diameter
- Tire rolling radius

Additional braking system information such as ABS configuration, placement of sensors and modulators, etc., may become beneficial to obtain during an inspection depending upon the scope of the analysis and particulars of the incident.

The following general analytical steps provide the braking force and deceleration rate of a pneumatically braked vehicle:

1. Determine force to lock wheel(s) at each axle end (Equation 1).
2. Obtain pushrod force from tables[9,13] or regression analysis (**Figure 4** and **Figure 6**), based upon application pressure, brake chamber type/size, and pushrod stroke.
3. Calculate attempted brake force (Equation 4).
4. Determine if attempted brake force is greater than the force required to cause the wheel(s) on the axle ends to lock (Equation 4 ≥ Equation 1).
5. If vehicle/vehicle combination is not equipped with an ABS system, or attempted brake force is less than force to lock wheel(s) at axle end (Equation 4 < Equation 1), then calculate deceleration rate using the lower of the attempted brake force versus the force to lock the wheel(s) (Equation 2).
6. If vehicle/vehicle combination is equipped with a functioning anti-lock braking system and the attempted brake force is greater than the force to lock the wheel(s) at axle end (Equation 4 ≥ Equation 1), then calculate the brake application pressure to lock the wheel(s) at each axle end (Equation 5).

6a. Alternatively, if the pushrod force tables are used instead of the regression analysis, then Equation 6 determines the pushrod force to lock the brake at an axle end.

<u>Pushrod force to lock each n brake:</u>

$$PR_L = \frac{W \times f_r \times CamRad \times TRad}{2 \times SL \times 0.6 \times 0.35 \times DRad} \tag{6}$$

Where,
PR_L= Pushrod force to lock wheel (lbs)
W=weight at wheel end (lbs)
f_r=μ*CVF
CamRad=S-cam radius (in)
TRad=loaded radius of tire (in)
SL=slack adjustor length (in)
DRad=brake drum radius (in)

Interpolation can then be used to determine brake application pressure, which will lock the brake at an axle end (P_L) by using Equation 7.

<u>Brake application pressure to lock each n brake:</u>

$$P_L = \frac{\Delta P}{PF_{L+10} - PF_{L+10}} \times (PF_L - PF_{L-10}) + PSI_{-10} \tag{7}$$

Where,
P_L =Pressure to lock by interpolation (PSI)
PSI_{-10}=Air pressure at data point in pushrod force tables below pushrod force required to lock n brake
ΔP=Difference in pressure between to two data points in table (typically 10)
PF_{L+10}=Pushrod force from tables at data point greater than calculated pushrod force to lock
PF_{L-10}=Pushrod force from tables at data point less than calculated pushrod force to lock

7. Reduce the brake application pressure by 8 psi for faster cycling modern ABS systems and up to

20 psi for slower cycling ABS systems generally seen on older vehicles — from the brake application pressure calculated in Step 6, and recalculate pushrod force from tables[9,13] or regression analysis with new, lower brake application pressure (PFABS) (Fig. 4 and 6)

7a. If using the tables instead of regression analysis, interpolate the pushrod force under full ABS braking (Equation 8).

Pushrod force during full ABS braking at each n brake:

$$PF_{ABS}= \frac{PF_{L+10} - PF_{L+10}}{\Delta P} \times(PSI_{ABS} - PSI_{-10})+PF_{L-10} \qquad (8)$$

Where,
PF_{ABS}=Pushrod force during full ABS braking (lbs)
PSI_{ABS}=8 psi to 20 psi subtracted from pressure to lock
PSI_{-10}= Air pressure at data point in pushrod force tables below calculated pushed force
ΔP=Difference in pressure between to two data points in table (typically 10)
PF_{L+10}=Pushrod force from tables at data point greater than calculated pushrod force
PF_{L-10}=Pushrod force from tables at data point less than calculated pushrod force

8. Calculate brake force using newly calculated pushrod force (PFABS) (Equation 4).

9. Determine deceleration rate using the calculated ABS brake force (Equation 2).

Findings and Final Observations

The braking system of modern commercial vehicles is complex, presenting many different areas where defects can occur. A thorough technical understanding of pneumatic brakes is necessary for the forensic engineer to accomplish a proper inspection and analysis of the braking system. Thorough post-crash inspection of braking components often represents an important step in the scope of a forensic engineering investigation. Oftentimes, without the thorough inspection of the braking system, factors related to speed and avoidance may be mistakenly identified or missed altogether.

A proper commercial vehicle's braking system performance analysis may be crucial to determining the speed, deceleration or elements related to vehicle loss of control for a commercial vehicle leading up to a collision or incident event. The methods presented in this paper, which are generally accepted and widely used when assessing pneumatic braking system performance, produce reliable results when performed correctly.

Future work should investigate and publish data regarding the brake application pressure drop during full ABS braking. To-date, no publicly published papers measure and report more exacting data regarding the air pressure drop during ABS cycling. Such research is anticipated to provide greater understanding to the forensic engineering community with increased precision when analyzing full ABS locked wheel braking events.

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Appendix A

WEIGHT DISTRIBUTION METHOD:
WEIGHTS, LOADS:

$i := 1..2$

Left side weight by axle:

$$W_L := \begin{bmatrix} 5300 \\ 3650 \\ 3500 \\ 2900 \\ 2500 \end{bmatrix} \cdot lb$$

Right side weight by axle:

$$W_R := \begin{bmatrix} 5650 \\ 3400 \\ 3200 \\ 3650 \\ 3300 \end{bmatrix} \cdot lb$$

$$Weight1 := \sum_{i=1}^{5} W_{L_i} + \sum_{i=1}^{5} W_{R_i} = 37050\ lb$$

$$\mu_{pv} := \begin{bmatrix} 0.70 \\ 0.75 \end{bmatrix}$$ Passenger vehicle roadway friction

$CVF := 80\%$ Commercial vehicle factor

Force generated by left side brakes (all brakes functioning) Force generated by right side brakes

$$F_{nLf} := \begin{bmatrix} \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_1} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_2} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_3} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_4} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_5} \end{bmatrix} = \begin{bmatrix} 3074 \\ 2117 \\ 2030 \\ 1682 \\ 1450 \end{bmatrix} lb$$

$$F_{nRf} := \begin{bmatrix} \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_1} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_2} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_3} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_4} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_5} \end{bmatrix} = \begin{bmatrix} 3277 \\ 1972 \\ 1856 \\ 2117 \\ 1914 \end{bmatrix} lb$$

Force generated by left side brakes (trailer brakes disabled) Force generated by right side brakes

$$F_{nL} := \begin{bmatrix} \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_1} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_2} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{L_3} \\ \text{mean}\left(\mu_{pv} \cdot 0\right) \cdot W_{L_4} \\ \text{mean}\left(\mu_{pv} \cdot 0\right) \cdot W_{L_5} \end{bmatrix} = \begin{bmatrix} 3074 \\ 2117 \\ 2030 \\ 0 \\ 0 \end{bmatrix} lb$$

$$F_{nR} := \begin{bmatrix} \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_1} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_2} \\ \text{mean}\left(\mu_{pv} \cdot CVF\right) \cdot W_{R_3} \\ \text{mean}\left(\mu_{pv} \cdot 0\right) \cdot W_{R_4} \\ \text{mean}\left(\mu_{pv} \cdot 0\right) \cdot W_{R_5} \end{bmatrix} = \begin{bmatrix} 3277 \\ 1972 \\ 1856 \\ 0 \\ 0 \end{bmatrix} lb$$

$$\mu_{cmvf} := \frac{\left(\sum F_{nLf} + \sum F_{nRf}\right)}{Weight1} = 0.58$$

Drag factor of commercial vehicle with all brakes functioning

Appendix A

$$\iota_{cmv} := \frac{\left(\sum F_{nL} + \sum F_{nR}\right)}{Weight1} = 0.387$$

Drag factor of heavy vehicle with trailer brakes disabled

$$D_{stop} := 118.5 \cdot ft$$

Skid mark distance

$$V_{SkidtoStop} := \begin{bmatrix} \sqrt{2 \cdot g \cdot \mu_{cmv} \cdot D_{stop}} \\ \sqrt{2 \cdot g \cdot \mu_{cmvf} \cdot D_{stop}} \end{bmatrix} = \begin{bmatrix} 37.0 \\ 45.3 \end{bmatrix} mph$$

Speed range by weight distribution method

Appendix B

HEUSSER METHOD
WEIGHTS, LOADS:

$i := 1..5$

Left side weight/slack adj(in)/stroke(in) by axle:

$$W_L := \begin{bmatrix} 5300 \\ 3650 \\ 3500 \\ 2900 \\ 2500 \end{bmatrix} \cdot lb \qquad SA_L := \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \\ 6.0 \end{bmatrix} \cdot in$$

Right side weight/slack adj(in)/stroke(in) by axle:

$$W_R := \begin{bmatrix} 5650 \\ 3400 \\ 3200 \\ 3650 \\ 3300 \end{bmatrix} \cdot lb \quad SA_R := \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \end{bmatrix} \cdot in$$

$$Weight1 := \sum_{i=1}^{5} W_{L_i} + \sum_{i=1}^{5} W_{R_i} = 37050\ lb$$

$$ST_L := \begin{bmatrix} 1.25 \\ 1.375 \\ 1.625 \\ 1.875 \\ 1.5 \end{bmatrix}$$

Measured brake stroke

$$ST_R := \begin{bmatrix} 1.375 \\ 1.375 \\ 1.75 \\ 1.75 \\ 1.875 \end{bmatrix}$$

Left brake lining coeff. by axle:

$$L_L := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \end{bmatrix}$$

(SAE 910126)

Right brake lining coeff. by axle:

$$L_R := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \end{bmatrix}$$

Left Drum/Tire rolling radius (in) by axle

$$D_L := \begin{bmatrix} 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \end{bmatrix} \cdot in \quad T_L := \begin{bmatrix} 20.25 \\ 21 \\ 21 \\ 21 \\ 21 \end{bmatrix} \cdot in$$

Right Drum/Tire rolling radius (in) by axle:

$$D_R := \begin{bmatrix} 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \end{bmatrix} \cdot in \quad T_R := \begin{bmatrix} 20.25 \\ 21 \\ 21 \\ 21 \\ 21 \end{bmatrix} \cdot in$$

$Rc := 0.5 \cdot in$ S-Cam Radius (SAE 910128)

$Cf := 0.6$ Chamber Factor (SAE 910128)

$\mu_{pv} := \begin{bmatrix} 0.71 \\ 0.77 \end{bmatrix}$ Passenger vehicle roadway friction

$CVF := 80\%$ Commercial vehicle factor

Appendix B

$$FL_{max} := \begin{bmatrix} \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_1} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_2} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_3} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_4} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_5} \end{bmatrix} = \begin{bmatrix} 3138 \\ 2161 \\ 2072 \\ 1717 \\ 1480 \end{bmatrix} lb$$

Maximum braking force prior to tire(s) lockup on left side

$$FR_{max} := \begin{bmatrix} \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_1} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_2} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_3} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_4} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_5} \end{bmatrix} = \begin{bmatrix} 3345 \\ 2013 \\ 1894 \\ 2161 \\ 1954 \end{bmatrix} lb$$

Maximum braking force prior to tire(s) lockup on right side

$$PF_{L40} := \begin{bmatrix} 818 \\ 1112 \\ 1100 \\ 1064 \\ 1110 \end{bmatrix} \cdot lb$$

Pushrod force at 40psi at measured pushrod stroke for each brake from tables

$$PF_{R40} := \begin{bmatrix} 822 \\ 1112 \\ 1088 \\ 1088 \\ 1064 \end{bmatrix} \cdot lb$$

Brake Force equation (calculated for 40 psi brake application)

$$BF_{L40} := \begin{bmatrix} \dfrac{2 \cdot PF_{L40_1} \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}}{Rc \cdot T_{L_1}} \\ \dfrac{2 \cdot PF_{L40_2} \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}}{Rc \cdot T_{L_2}} \\ \dfrac{2 \cdot PF_{L40_3} \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}}{Rc \cdot T_{L_3}} \\ \dfrac{2 \cdot PF_{L40_4} \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}}{Rc \cdot T_{L_4}} \\ \dfrac{2 \cdot PF_{L40_5} \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}}{Rc \cdot T_{L_5}} \end{bmatrix} = \begin{bmatrix} 1540 \\ 2018 \\ 1997 \\ 1931 \\ 2198 \end{bmatrix} lb$$

$$BF_{R40} := \begin{bmatrix} \dfrac{2 \cdot PF_{R40_1} \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}}{Rc \cdot T_{R_1}} \\ \dfrac{2 \cdot PF_{R40_2} \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}}{Rc \cdot T_{R_2}} \\ \dfrac{2 \cdot PF_{R40_3} \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}}{Rc \cdot T_{R_3}} \\ \dfrac{2 \cdot PF_{R40_4} \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}}{Rc \cdot T_{R_4}} \\ \dfrac{2 \cdot PF_{R40_5} \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}}{Rc \cdot T_{R_5}} \end{bmatrix} = \begin{bmatrix} 1547 \\ 2018 \\ 1975 \\ 1975 \\ 1931 \end{bmatrix} lb$$

Appendix B

Use smaller of maximum brake force and brake force at application pressure

$$BF_{L_i} := min\left(FL_{max_i}, BF_{L40_i}\right) = \begin{bmatrix} 1540 \\ 2018 \\ 1997 \\ 1717 \\ 1480 \end{bmatrix} lb \qquad BF_{R_i} := min\left(FR_{max_i}, BF_{R40_i}\right) = \begin{bmatrix} 1547 \\ 2013 \\ 1894 \\ 1975 \\ 1931 \end{bmatrix} lb$$

Calculated deceleration determination:

$$Decel_{40} := \frac{\left(\sum BF_L + \sum BF_R\right)}{\left(\sum W_L + \sum W_R\right)} = 0.49$$

$D_{stop} := 118.5 \cdot ft$ Skid mark distance

$V := \sqrt{2 \cdot g \cdot Decel_{40} \cdot D_{stop}} = 41.6\ mph$ **Speed at beginning of skid mark with 40psi brake application**

Appendix C

$i := 1..5$

BARTLETT/HEUSSER METHOD
WEIGHTS, LOADS:

Left side weight/slack adj(in)/stroke(in) by axle:

$$W_L := \begin{bmatrix} 5300 \\ 3650 \\ 3500 \\ 2900 \\ 2500 \end{bmatrix} \cdot lb \qquad SA_L := \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \\ 6.0 \end{bmatrix} \cdot in$$

Right side weight/slack adj(in)/stroke(in) by axle:

$$W_R := \begin{bmatrix} 5650 \\ 3400 \\ 3200 \\ 3650 \\ 3300 \end{bmatrix} \cdot lb \qquad SA_R := \begin{bmatrix} 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \\ 5.5 \end{bmatrix} \cdot in$$

$$Weight1 := \sum_{i=1}^{5} W_{L_i} + \sum_{i=1}^{5} W_{R_i} = 37050\ lb$$

Measured brake stroke

$$ST_L := \begin{bmatrix} 1.25 \\ 1.375 \\ 1.625 \\ 1.875 \\ 1.5 \end{bmatrix} \qquad ST_R := \begin{bmatrix} 1.375 \\ 1.375 \\ 1.75 \\ 1.75 \\ 1.875 \end{bmatrix}$$

Left brake lining coeff. by axle:

$$L_L := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \end{bmatrix}$$

(SAE 910126)

Right brake lining coeff. by axle:

$$L_R := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \end{bmatrix}$$

Left Drum/Tire rolling radius (in) by axle

$$D_L := \begin{bmatrix} 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \end{bmatrix} \cdot in \qquad T_L := \begin{bmatrix} 20.25 \\ 21 \\ 21 \\ 21 \\ 21 \end{bmatrix} \cdot in$$

Right Drum/Tire rolling radius (in) by axle:

$$D_R := \begin{bmatrix} 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \\ 8.25 \end{bmatrix} \cdot in \qquad T_R := \begin{bmatrix} 20.25 \\ 21 \\ 21 \\ 21 \\ 21 \end{bmatrix} \cdot in$$

$Rc := 0.5 \cdot in$ S-Cam Radius (SAE 910128)

$Cf := 0.6$ Chamber Factor (SAE 910128)

$\mu_{pv} := \begin{bmatrix} 0.71 \\ 0.77 \end{bmatrix}$ Passenger vehicle roadway friction

$CVF := 80\%$ Commercial vehicle factor

Appendix C

Maximum braking force prior to tire(s) lockup

$$FL_{max} := \begin{bmatrix} \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_1} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_2} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_3} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_4} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{L_5} \end{bmatrix} = \begin{bmatrix} 3138 \\ 2161 \\ 2072 \\ 1717 \\ 1480 \end{bmatrix} lb \quad FR_{max} := \begin{bmatrix} \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_1} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_2} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_3} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_4} \\ \text{mean}(\mu_{pv}) \cdot CVF \cdot W_{R_5} \end{bmatrix} = \begin{bmatrix} 3345 \\ 2013 \\ 1894 \\ 2161 \\ 1954 \end{bmatrix} lb$$

$$F_{limit} := \begin{bmatrix} FL_{max_5} \\ FR_{max_3} \end{bmatrix} = \begin{bmatrix} 1480 \\ 1894 \end{bmatrix} lb$$

Limiting force before other tractor tires lock and leave skid marks (5L and 3R)

BRAKING ANALYSIS USING:
Results of Regression Analysis

$$\begin{bmatrix} M_{L20L} \\ M_{L30L} \end{bmatrix} := \begin{bmatrix} 0.2645 \cdot ST_L^{\ 4} - 5.2403 \cdot ST_L^{\ 3} + 16.578 \cdot ST_L^{\ 2} - 16.864 \cdot ST_L + 24.211 \\ -1.7378 \cdot ST_L^{\ 4} + 6.4881 \cdot ST_L^{\ 3} - 7.3801 \cdot ST_L^{\ 2} + 2.3713 \cdot ST_L + 29.414 \end{bmatrix} \cdot \frac{lb}{psi}$$

$$\begin{bmatrix} B_{L20L} \\ B_{L30L} \end{bmatrix} := \begin{bmatrix} 19.206 \cdot ST_L^{\ 4} - 100.91 \cdot ST_L^{\ 3} + 158.54 \cdot ST_L^{\ 2} - 106.73 \cdot ST_L - 54.181 \\ -126.21 \cdot ST_L^{\ 5} + 834.53 \cdot ST_L^{\ 4} - 2124.7 \cdot ST_L^{\ 3} + 2563.1 \cdot ST_L^{\ 2} - 1463.4 \cdot ST_L + 266.69 \end{bmatrix} \cdot lb$$

$$\begin{bmatrix} M_{L20R} \\ M_{L30R} \end{bmatrix} := \begin{bmatrix} 0.2645 \cdot ST_R^{\ 4} - 5.2403 \cdot ST_R^{\ 3} + 16.578 \cdot ST_R^{\ 2} - 16.864 \cdot ST_R + 24.211 \\ -1.7378 \cdot ST_R^{\ 4} + 6.4881 \cdot ST_R^{\ 3} - 7.3801 \cdot ST_R^{\ 2} + 2.3713 \cdot ST_R + 29.414 \end{bmatrix} \cdot \frac{lb}{psi}$$

$$\begin{bmatrix} B_{L20R} \\ B_{L30R} \end{bmatrix} := \begin{bmatrix} 19.206 \cdot ST_R^{\ 4} - 100.91 \cdot ST_R^{\ 3} + 158.54 \cdot ST_R^{\ 2} - 106.73 \cdot ST_R - 54.181 \\ -126.21 \cdot ST_R^{\ 5} + 834.53 \cdot ST_R^{\ 4} - 2124.7 \cdot ST_R^{\ 3} + 2563.1 \cdot ST_R^{\ 2} - 1463.4 \cdot ST_R + 266.69 \end{bmatrix} \cdot lb$$

Appendix C

$$PL_L := \begin{bmatrix} \dfrac{FL_{max_1} \cdot Rc \cdot T_{L_1}}{2 \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1} \, M_{L20L_1}} - \dfrac{B_{L20L_1}}{M_{L20L_1}} \\ \dfrac{FL_{max_2} \cdot Rc \cdot T_{L_2}}{2 \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2} \, M_{L30L_2}} - \dfrac{B_{L30L_2}}{M_{L30L_2}} \\ \dfrac{FL_{max_3} \cdot Rc \cdot T_{L_3}}{2 \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3} \, M_{L30L_3}} - \dfrac{B_{L30L_3}}{M_{L30L_3}} \\ \dfrac{FL_{max_4} \cdot Rc \cdot T_{L_4}}{2 \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4} \, M_{L30L_4}} - \dfrac{B_{L30L_4}}{M_{L30L_4}} \\ \dfrac{FL_{max_5} \cdot Rc \cdot T_{L_5}}{2 \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5} \, M_{L30L_5}} - \dfrac{B_{L30L_5}}{M_{L30L_5}} \end{bmatrix} = \begin{bmatrix} 90 \\ 43 \\ 41 \\ 35 \\ 28 \end{bmatrix} psi$$

Brake application pressure to lock for left side

$$PL_R := \begin{bmatrix} \dfrac{FR_{max_1} \cdot Rc \cdot T_{R_1}}{2 \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1} \, M_{L20R_1}} - \dfrac{B_{L20R_1}}{M_{L20R_1}} \\ \dfrac{FR_{max_2} \cdot Rc \cdot T_{R_2}}{2 \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2} \, M_{L30R_2}} - \dfrac{B_{L30R_2}}{M_{L30R_2}} \\ \dfrac{FR_{max_3} \cdot Rc \cdot T_{R_3}}{2 \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3} \, M_{L30R_3}} - \dfrac{B_{L30R_3}}{M_{L30R_3}} \\ \dfrac{FR_{max_4} \cdot Rc \cdot T_{R_4}}{2 \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4} \, M_{L30R_4}} - \dfrac{B_{L30R_4}}{M_{L30R_4}} \\ \dfrac{FR_{max_5} \cdot Rc \cdot T_{R_5}}{2 \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5} \, M_{L30R_5}} - \dfrac{B_{L30R_5}}{M_{L30R_5}} \end{bmatrix} = \begin{bmatrix} 95 \\ 40 \\ 38 \\ 43 \\ 40 \end{bmatrix} psi$$

Brake application pressure to lock for right side

$$P_{ABS} := \begin{bmatrix} 8 \\ 20 \end{bmatrix} \cdot psi$$

Brake application pressure reduction for ABS cycling during full ABS brake application

$$P_{Labs} := PL_L - P_{ABS_1} = \begin{bmatrix} 82 \\ 35 \\ 33 \\ 27 \\ 20 \end{bmatrix} psi$$

Adjusted brake application pressure

$$P_{Rabs} := PL_R - P_{ABS_1} = \begin{bmatrix} 87 \\ 32 \\ 30 \\ 35 \\ 32 \end{bmatrix} psi$$

Appendix C

Pushrod Force to Lock

$$PF_L := \begin{bmatrix} M_{L20L_1} \cdot PL_{L_1} + B_{L20L_1} \\ M_{L30L_2} \cdot PL_{L_2} + B_{L30L_2} \\ M_{L30L_3} \cdot PL_{L_3} + B_{L30L_3} \\ M_{L30L_4} \cdot PL_{L_4} + B_{L30L_4} \\ M_{L30L_5} \cdot PL_{L_5} + B_{L30L_5} \end{bmatrix} = \begin{bmatrix} 1667 \\ 1191 \\ 1142 \\ 946 \\ 747 \end{bmatrix} lb \quad PF_R := \begin{bmatrix} M_{L20R_1} \cdot PL_{R_1} + B_{L20R_1} \\ M_{L30R_2} \cdot PL_{R_2} + B_{L30R_2} \\ M_{L30R_3} \cdot PL_{R_3} + B_{L30R_3} \\ M_{L30R_4} \cdot PL_{R_4} + B_{L30R_4} \\ M_{L30R_5} \cdot PL_{R_5} + B_{L30R_5} \end{bmatrix} = \begin{bmatrix} 1777 \\ 1109 \\ 1044 \\ 1191 \\ 1076 \end{bmatrix} lb$$

Pushrod force for full ABS braking

$$PF_{Labs} := \begin{bmatrix} M_{L20L_1} \cdot P_{Labs_1} + B_{L20L_1} \\ M_{L30L_2} \cdot P_{Labs_2} + B_{L30L_2} \\ M_{L30L_3} \cdot P_{Labs_3} + B_{L30L_3} \\ M_{L30L_4} \cdot P_{Labs_4} + B_{L30L_4} \\ M_{L30L_5} \cdot P_{Labs_5} + B_{L30L_5} \end{bmatrix} = \begin{bmatrix} 1511 \\ 956 \\ 906 \\ 712 \\ 512 \end{bmatrix} lb \quad PF_{Rabs} := \begin{bmatrix} M_{L20R_1} \cdot P_{Rabs_1} + B_{L20R_1} \\ M_{L30R_2} \cdot P_{Rabs_2} + B_{L30R_2} \\ M_{L30R_3} \cdot P_{Rabs_3} + B_{L30R_3} \\ M_{L30R_4} \cdot P_{Rabs_4} + B_{L30R_4} \\ M_{L30R_5} \cdot P_{Rabs_5} + B_{L30R_5} \end{bmatrix} = \begin{bmatrix} 1620 \\ 874 \\ 808 \\ 955 \\ 843 \end{bmatrix} lb$$

Braking force during full ABS braking

$$BF_{Labs} := \begin{bmatrix} \dfrac{2 \cdot PF_{Labs_1} \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}}{Rc \cdot T_{L_1}} \\ \dfrac{2 \cdot PF_{Labs_2} \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}}{Rc \cdot T_{L_2}} \\ \dfrac{2 \cdot PF_{Labs_3} \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}}{Rc \cdot T_{L_3}} \\ \dfrac{2 \cdot PF_{Labs_4} \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}}{Rc \cdot T_{L_4}} \\ \dfrac{2 \cdot PF_{Labs_5} \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}}{Rc \cdot T_{L_5}} \end{bmatrix} = \begin{bmatrix} 2845 \\ 1734 \\ 1644 \\ 1293 \\ 1013 \end{bmatrix} lb \quad BF_{Rabs} := \begin{bmatrix} \dfrac{2 \cdot PF_{Rabs_1} \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}}{Rc \cdot T_{R_1}} \\ \dfrac{2 \cdot PF_{Rabs_2} \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}}{Rc \cdot T_{R_2}} \\ \dfrac{2 \cdot PF_{Rabs_3} \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}}{Rc \cdot T_{R_3}} \\ \dfrac{2 \cdot PF_{Rabs_4} \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}}{Rc \cdot T_{R_4}} \\ \dfrac{2 \cdot PF_{Rabs_5} \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}}{Rc \cdot T_{R_5}} \end{bmatrix} = \begin{bmatrix} 3048 \\ 1586 \\ 1467 \\ 1733 \\ 1530 \end{bmatrix} lb$$

<u>Calculated ABS stop-deceleration rate determination using regression analysis</u>:

$$Decel_{absr} := \frac{\left(\sum_{i=1}^{5} \left(BF_{Labs_i} \right) + \sum_{i=1}^{5} \left(BF_{Rabs_i} \right) \right)}{\left(\sum W_L + \sum W_R \right)} = 0.483$$

Appendix C

BRAKING ANALYSIS USING:
Interpolation

Pushrod Force to Lock

$$PFL_L := \begin{bmatrix} \dfrac{FL_{max_1} \cdot Rc \cdot T_{L_1}}{2 \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}} \\ \dfrac{FL_{max_2} \cdot Rc \cdot T_{L_2}}{2 \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}} \\ \dfrac{FL_{max_3} \cdot Rc \cdot T_{L_3}}{2 \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}} \\ \dfrac{FL_{max_4} \cdot Rc \cdot T_{L_4}}{2 \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}} \\ \dfrac{FL_{max_5} \cdot Rc \cdot T_{L_5}}{2 \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}} \end{bmatrix} = \begin{bmatrix} 1667 \\ 1191 \\ 1142 \\ 946 \\ 747 \end{bmatrix} lb$$

$$PFL_R := \begin{bmatrix} \dfrac{FR_{max_1} \cdot Rc \cdot T_{R_1}}{2 \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}} \\ \dfrac{FR_{max_2} \cdot Rc \cdot T_{R_2}}{2 \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}} \\ \dfrac{FR_{max_3} \cdot Rc \cdot T_{R_3}}{2 \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}} \\ \dfrac{FR_{max_4} \cdot Rc \cdot T_{R_4}}{2 \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}} \\ \dfrac{FR_{max_5} \cdot Rc \cdot T_{R_5}}{2 \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}} \end{bmatrix} = \begin{bmatrix} 1777 \\ 1109 \\ 1044 \\ 1191 \\ 1076 \end{bmatrix} lb$$

INTERPOLATION FOR BRAKE APPLICATION PRESSURE ANALYSIS:

$$PL_{limit} := \begin{bmatrix} PFL_{L_5} \\ PFL_{R_3} \end{bmatrix} = \begin{bmatrix} 747 \\ 1044 \end{bmatrix} lb$$

Limiting non-ABS Pushrod Force

$$PFL_{20} := \begin{bmatrix} 363 \\ 533 \\ 525 \\ 504 \\ 532 \end{bmatrix} \cdot lb$$

Pushrod force at 20psi at measured pushrod stroke for each brake from 2008 Rec-Tec tables

$$PFR_{20} := \begin{bmatrix} 364 \\ 533 \\ 516 \\ 516 \\ 504 \end{bmatrix} \cdot lb$$

$$PFL_{30} := \begin{bmatrix} 591 \\ 823 \\ 813 \\ 784 \\ 821 \end{bmatrix} \cdot lb$$

Pushrod force at 30psi at measured pushrod stroke for each brake from 2008 Rec-Tec tables

$$PFR_{30} := \begin{bmatrix} 593 \\ 823 \\ 802 \\ 802 \\ 784 \end{bmatrix} \cdot lb$$

$$PFL_{40} := \begin{bmatrix} 818 \\ 1112 \\ 1100 \\ 1064 \\ 1110 \end{bmatrix} \cdot lb$$

Pushrod force at 40psi at measured pushrod stroke for each brake from 2008 Rec-Tec tables

$$PFR_{40} := \begin{bmatrix} 822 \\ 1112 \\ 1088 \\ 1088 \\ 1064 \end{bmatrix} \cdot lb$$

$$PFL_{50} := \begin{bmatrix} 883 \\ 1409 \\ 1396 \\ 1355 \\ 1407 \end{bmatrix} \cdot lb$$

Pushrod force at 50psi at measured pushrod stroke for each brake from 2008 Rec-Tec tables

$$PFR_{50} := \begin{bmatrix} 891 \\ 1409 \\ 1382 \\ 1382 \\ 1355 \end{bmatrix} \cdot lb$$

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$PF_{L80} := 1461 \cdot lb$

Pushrod force at 80psi at measured pushrod stroke for front axle brake from 2008 Rec-Tec tables

$PF_{R80} := 1477 \cdot lb$

$PF_{L90100} := \begin{bmatrix} 1659 \\ 1857 \end{bmatrix} \cdot lb$

Pushrod force at 90psi and 100psi at measured pushrod stroke for front axle brake from 2008 Rec-Tec tables

$PF_{R90100} := \begin{bmatrix} 1677 \\ 1877 \end{bmatrix} \cdot lb$

$PF_{L10} := 243 \cdot lb$

Pushrod force at 10psi at measured pushrod stroke for left axle 5 brake from 2008 Rec-Tec tables

$P_{10} := 10 \cdot psi \quad P_{20} := 20 \cdot psi \quad P_{30} := 30 \cdot psi \quad P_{40} := 40 \cdot psi \quad P_{50} := 50 \cdot psi \quad P_{80} := 80 \cdot psi$

$P_{90} := 90 \cdot psi \quad P_{100} := 100 \cdot psi \quad P_{ABS} := 8 \cdot psi$

Brake application pressure reduction for ABS activation

$$PSI_1 := \frac{(P_{30} - P_{20})}{\left(PFL_{30_5} - PFL_{20_5}\right)} \cdot \left(PL_{limit_1} - PFL_{20_5}\right) + P_{20} = 27.5\ psi$$

$$PSI_2 := \frac{(P_{40} - P_{30})}{\left(PFR_{40_3} - PFR_{30_3}\right)} \cdot \left(PL_{limit_2} - PFR_{30_3}\right) + P_{30} = 38.5\ psi$$

Brake pressure to lock limiting brakes non-ABS

Brake application pressure to lock limiting axles with full ABS lockup

$$PSI_{ABSL} := \begin{bmatrix} \frac{(P_{100} - P_{90})}{\left(PF_{L90100_2} - PF_{L90100_1}\right)} \cdot \left(PFL_{L_1} - PF_{L90100_1}\right) + P_{90} - P_{ABS} \\ \frac{(P_{50} - P_{40})}{\left(PFL_{50_2} - PFL_{40_2}\right)} \cdot \left(PFL_{L_2} - PFL_{40_2}\right) + P_{40} - P_{ABS} \\ \frac{(P_{50} - P_{40})}{\left(PFL_{50_3} - PFL_{40_3}\right)} \cdot \left(PFL_{L_3} - PFL_{40_3}\right) + P_{40} - P_{ABS} \\ \frac{(P_{40} - P_{30})}{\left(PFL_{40_4} - PFL_{30_4}\right)} \cdot \left(PFL_{L_4} - PFL_{30_4}\right) + P_{30} - P_{ABS} \\ \frac{(P_{30} - P_{20})}{\left(PFL_{30_5} - PFL_{20_5}\right)} \cdot \left(PFL_{L_5} - PFL_{20_5}\right) + P_{20} - P_{ABS} \end{bmatrix} = \begin{bmatrix} 82 \\ 35 \\ 33 \\ 28 \\ 19 \end{bmatrix} psi$$

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$$PSI_{ABSR} := \begin{bmatrix} \frac{(P_{100}-P_{90})}{\left(PF_{R90100_2}-PF_{R90100_1}\right)}\cdot\left(PFL_{R_1}-PF_{R90100_1}\right)+P_{90}-P_{ABS} \\ \frac{(P_{40}-P_{30})}{\left(PFR_{40_2}-PFR_{30_2}\right)}\cdot\left(PFL_{R_2}-PFR_{30_2}\right)+P_{30}-P_{ABS} \\ \frac{(P_{40}-P_{30})}{\left(PFR_{40_3}-PFR_{30_3}\right)}\cdot\left(PFL_{R_3}-PFR_{30_3}\right)+P_{30}-P_{ABS} \\ \frac{(P_{50}-P_{40})}{\left(PFR_{50_4}-PFR_{40_4}\right)}\cdot\left(PFL_{R_4}-PFR_{40_4}\right)+P_{40}-P_{ABS} \\ \frac{(P_{50}-P_{40})}{\left(PFR_{50_5}-PFR_{40_5}\right)}\cdot\left(PFL_{R_5}-PFR_{40_5}\right)+P_{40}-P_{ABS} \end{bmatrix} = \begin{bmatrix} 87 \\ 32 \\ 30 \\ 35 \\ 32 \end{bmatrix} psi$$

Analysis of brake application pressure only reaches level to lock certain brakes Interpolated Min/Max Pushrod Force (non-ABS)

$$PF_{Lmin} := \begin{bmatrix} \frac{\left(PFL_{30_1}-PFL_{20_1}\right)}{(P_{30}-P_{20})}\cdot\left(PSI_1-P_{20}\right)+PFL_{20_1} \\ \frac{\left(PFL_{30_2}-PFL_{20_2}\right)}{(P_{30}-P_{20})}\cdot\left(PSI_2-P_{20}\right)+PFL_{20_2} \\ \frac{\left(PFL_{30_3}-PFL_{20_3}\right)}{(P_{30}-P_{20})}\cdot\left(PSI_1-P_{20}\right)+PFL_{20_3} \\ \frac{\left(PFL_{30_4}-PFL_{20_4}\right)}{(P_{30}-P_{20})}\cdot\left(PSI_1-P_{20}\right)+PFL_{20_4} \\ \frac{\left(PFL_{30_5}-PFL_{20_5}\right)}{(P_{30}-P_{20})}\cdot\left(PSI_1-P_{20}\right)+PFL_{20_5} \end{bmatrix} = \begin{bmatrix} 533 \\ 1068 \\ 740 \\ 713 \\ 747 \end{bmatrix} lb$$

Appendix C

$$PF_{Rmin} := \begin{bmatrix} \frac{\left(PFR_{30_1} - PFR_{20_1}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_1 - P_{20}\right) + PFR_{20_1} \\ \frac{\left(PFR_{30_2} - PFR_{20_2}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_1 - P_{20}\right) + PFR_{20_2} \\ \frac{\left(PFR_{30_3} - PFR_{20_3}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_1 - P_{20}\right) + PFR_{20_3} \\ \frac{\left(PFR_{30_4} - PFR_{20_4}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_1 - P_{20}\right) + PFR_{20_4} \\ \frac{\left(PFR_{30_5} - PFR_{20_5}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_1 - P_{20}\right) + PFR_{20_5} \end{bmatrix} = \begin{bmatrix} 535 \\ 749 \\ 729 \\ 729 \\ 713 \end{bmatrix} lb$$

$$PF_{Lmax} := \begin{bmatrix} \frac{\left(PFL_{40_1} - PFL_{30_1}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFL_{30_1} \\ \frac{\left(PFL_{40_2} - PFL_{30_2}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFL_{30_2} \\ \frac{\left(PFL_{40_3} - PFL_{30_3}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFL_{30_3} \\ \frac{\left(PFL_{40_4} - PFL_{30_4}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFL_{30_4} \\ \frac{\left(PFL_{40_5} - PFL_{30_5}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFL_{30_5} \end{bmatrix} = \begin{bmatrix} 783 \\ 1067 \\ 1056 \\ 1021 \\ 1065 \end{bmatrix} lb$$

$$PF_{Rmax} := \begin{bmatrix} \frac{\left(PFR_{40_1} - PFR_{30_1}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFR_{30_1} \\ \frac{\left(PFR_{40_2} - PFR_{30_2}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFR_{30_2} \\ \frac{\left(PFR_{40_3} - PFR_{30_3}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFR_{30_3} \\ \frac{\left(PFR_{40_4} - PFR_{30_4}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFR_{30_4} \\ \frac{\left(PFR_{40_5} - PFR_{30_5}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_2 - P_{30}\right) + PFR_{30_5} \end{bmatrix} = \begin{bmatrix} 787 \\ 1067 \\ 1044 \\ 1044 \\ 1021 \end{bmatrix} lb$$

Appendix C

Interpolated Min/Max Pushrod Force (**ABS**)

$$PF_{ABSL} := \begin{bmatrix} \frac{\left(PF_{L90100_1} - PF_{L80}\right)}{\left(P_{90} - P_{80}\right)} \cdot \left(PSI_{ABSL_1} - P_{80}\right) + PF_{L80} \\ \frac{\left(PFL_{40_2} - PFL_{30_2}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSL_2} - P_{30}\right) + PFL_{30_2} \\ \frac{\left(PFL_{40_3} - PFL_{30_3}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSL_3} - P_{30}\right) + PFL_{30_3} \\ \frac{\left(PFL_{30_4} - PFL_{20_4}\right)}{\left(P_{30} - P_{20}\right)} \cdot \left(PSI_{ABSL_4} - P_{20}\right) + PFL_{20_4} \\ \frac{\left(PFL_{20_5} - PF_{L10}\right)}{\left(P_{20} - P_{10}\right)} \cdot \left(PSI_{ABSL_5} - P_{10}\right) + PF_{L10} \end{bmatrix} = \begin{bmatrix} 1509 \\ 957 \\ 911 \\ 722 \\ 516 \end{bmatrix} lb$$

$$PF_{ABSR} := \begin{bmatrix} \frac{\left(PF_{R90100_1} - PF_{R80}\right)}{\left(P_{90} - P_{80}\right)} \cdot \left(PSI_{ABSR_1} - P_{80}\right) + PF_{R80} \\ \frac{\left(PFR_{40_2} - PFR_{30_2}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSR_2} - P_{30}\right) + PFR_{30_2} \\ \frac{\left(PFR_{40_3} - PFR_{30_3}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSR_3} - P_{30}\right) + PFR_{30_3} \\ \frac{\left(PFR_{40_4} - PFR_{30_4}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSR_4} - P_{30}\right) + PFR_{30_4} \\ \frac{\left(PFR_{40_5} - PFR_{30_5}\right)}{\left(P_{40} - P_{30}\right)} \cdot \left(PSI_{ABSR_5} - P_{30}\right) + PFR_{30_5} \end{bmatrix} = \begin{bmatrix} 1617 \\ 878 \\ 815 \\ 959 \\ 852 \end{bmatrix} lb$$

Appendix C

Brake Force (**non-ABS**)

$$BF_{Lmin} := \begin{bmatrix} \dfrac{2 \cdot PF_{Lmin_1} \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}}{Rc \cdot T_{L_1}} \\ \dfrac{2 \cdot PF_{Lmin_2} \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}}{Rc \cdot T_{L_2}} \\ \dfrac{2 \cdot PF_{Lmin_3} \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}}{Rc \cdot T_{L_3}} \\ \dfrac{2 \cdot PF_{Lmin_4} \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}}{Rc \cdot T_{L_4}} \\ \dfrac{2 \cdot PF_{Lmin_5} \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}}{Rc \cdot T_{L_5}} \end{bmatrix} = \begin{bmatrix} 1003 \\ 1939 \\ 1343 \\ 1294 \\ 1480 \end{bmatrix} lb$$

$$BF_{Rmin} := \begin{bmatrix} \dfrac{2 \cdot PF_{Rmin_1} \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}}{Rc \cdot T_{R_1}} \\ \dfrac{2 \cdot PF_{Rmin_2} \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}}{Rc \cdot T_{R_2}} \\ \dfrac{2 \cdot PF_{Rmin_3} \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}}{Rc \cdot T_{R_3}} \\ \dfrac{2 \cdot PF_{Rmin_4} \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}}{Rc \cdot T_{R_4}} \\ \dfrac{2 \cdot PF_{Rmin_5} \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}}{Rc \cdot T_{R_5}} \end{bmatrix} = \begin{bmatrix} 1006 \\ 1360 \\ 1324 \\ 1324 \\ 1294 \end{bmatrix} lb$$

$$BF_{Lmax} := \begin{bmatrix} \dfrac{2 \cdot PF_{Lmax_1} \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}}{Rc \cdot T_{L_1}} \\ \dfrac{2 \cdot PF_{Lmax_2} \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}}{Rc \cdot T_{L_2}} \\ \dfrac{2 \cdot PF_{Lmax_3} \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}}{Rc \cdot T_{L_3}} \\ \dfrac{2 \cdot PF_{Lmax_4} \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}}{Rc \cdot T_{L_4}} \\ \dfrac{2 \cdot PF_{Lmax_5} \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}}{Rc \cdot T_{L_5}} \end{bmatrix} = \begin{bmatrix} 1474 \\ 1937 \\ 1916 \\ 1853 \\ 2109 \end{bmatrix} lb$$

$$BF_{Rmax} := \begin{bmatrix} \dfrac{2 \cdot PF_{Rmax_1} \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}}{Rc \cdot T_{R_1}} \\ \dfrac{2 \cdot PF_{Rmax_2} \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}}{Rc \cdot T_{R_2}} \\ \dfrac{2 \cdot PF_{Rmax_3} \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}}{Rc \cdot T_{R_3}} \\ \dfrac{2 \cdot PF_{Rmax_4} \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}}{Rc \cdot T_{R_4}} \\ \dfrac{2 \cdot PF_{Rmax_5} \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}}{Rc \cdot T_{R_5}} \end{bmatrix} = \begin{bmatrix} 1480 \\ 1937 \\ 1894 \\ 1894 \\ 1853 \end{bmatrix} lb$$

Appendix C

Brake Force (**ABS**)

$$BF_{ABSL} := \begin{bmatrix} \dfrac{2 \cdot PF_{ABSL_1} \cdot SA_{L_1} \cdot Cf \cdot L_{L_1} \cdot D_{L_1}}{Rc \cdot T_{L_1}} \\ \dfrac{2 \cdot PF_{ABSL_2} \cdot SA_{L_2} \cdot Cf \cdot L_{L_2} \cdot D_{L_2}}{Rc \cdot T_{L_2}} \\ \dfrac{2 \cdot PF_{ABSL_3} \cdot SA_{L_3} \cdot Cf \cdot L_{L_3} \cdot D_{L_3}}{Rc \cdot T_{L_3}} \\ \dfrac{2 \cdot PF_{ABSL_4} \cdot SA_{L_4} \cdot Cf \cdot L_{L_4} \cdot D_{L_4}}{Rc \cdot T_{L_4}} \\ \dfrac{2 \cdot PF_{ABSL_5} \cdot SA_{L_5} \cdot Cf \cdot L_{L_5} \cdot D_{L_5}}{Rc \cdot T_{L_5}} \end{bmatrix} = \begin{bmatrix} 2839 \\ 1737 \\ 1653 \\ 1310 \\ 1022 \end{bmatrix} lb$$

$$BF_{ABSR} := \begin{bmatrix} \dfrac{2 \cdot PF_{ABSR_1} \cdot SA_{R_1} \cdot Cf \cdot L_{R_1} \cdot D_{R_1}}{Rc \cdot T_{R_1}} \\ \dfrac{2 \cdot PF_{ABSR_2} \cdot SA_{R_2} \cdot Cf \cdot L_{R_2} \cdot D_{R_2}}{Rc \cdot T_{R_2}} \\ \dfrac{2 \cdot PF_{ABSR_3} \cdot SA_{R_3} \cdot Cf \cdot L_{R_3} \cdot D_{R_3}}{Rc \cdot T_{R_3}} \\ \dfrac{2 \cdot PF_{ABSR_4} \cdot SA_{R_4} \cdot Cf \cdot L_{R_4} \cdot D_{R_4}}{Rc \cdot T_{R_4}} \\ \dfrac{2 \cdot PF_{ABSR_5} \cdot SA_{R_5} \cdot Cf \cdot L_{R_5} \cdot D_{R_5}}{Rc \cdot T_{R_5}} \end{bmatrix} = \begin{bmatrix} 3044 \\ 1593 \\ 1479 \\ 1740 \\ 1546 \end{bmatrix} lb$$

Use smaller of maximum brake force and brake force at application pressure

$$BFL_{min_i} := min\left(BF_{Lmin_i}, FL_{max_i}\right) = \begin{bmatrix} 1003 \\ 1939 \\ 1343 \\ 1294 \\ 1480 \end{bmatrix} lb \quad BFR_{min_i} := min\left(BF_{Rmin_i}, FR_{max_i}\right) = \begin{bmatrix} 1006 \\ 1360 \\ 1324 \\ 1324 \\ 1294 \end{bmatrix} lb$$

$$BFL_{max_i} := min\left(BF_{Lmax_i}, FL_{max_i}\right) = \begin{bmatrix} 1474 \\ 1937 \\ 1916 \\ 1717 \\ 1480 \end{bmatrix} lb \quad BFR_{max_i} := min\left(BF_{Rmax_i}, FR_{max_i}\right) = \begin{bmatrix} 1480 \\ 1937 \\ 1894 \\ 1894 \\ 1853 \end{bmatrix} lb$$

Calculated Non-ABS deceleration rate min/max determination:

$$Decel_{min} := \frac{\left(\sum BF_{Lmin} + \sum BF_{Rmin}\right)}{\left(\sum W_L + \sum W_R\right)} = 0.361$$

Appendix C

$$Decel_{max} := \frac{\left(\sum BF_{Lmax} + \sum BF_{Rmax}\right)}{\left(\sum W_L + \sum W_R\right)} = 0.495$$

Calculated ABS deceleration rate min/max determination:

$$Decel_{ABS} := \frac{\left(\sum BF_{ABSL} + \sum BF_{ABSR}\right)}{\left(\sum W_L + \sum W_R\right)} = 0.485$$

$D_{stop} := 118.5 \cdot ft$ Skid mark distance

$$V_{nonABS} := \begin{bmatrix} \sqrt{2 \cdot g \cdot Decel_{min} \cdot D_{stop}} \\ \sqrt{2 \cdot g \cdot Decel_{max} \cdot D_{stop}} \end{bmatrix} = \begin{bmatrix} 35.8 \\ 41.9 \end{bmatrix} mph$$

Non-ABS speed range at start of skid mark

$$V_{ABS} := \sqrt{2 \cdot g \cdot Decel_{ABS} \cdot D_{stop}} = 41.5\ mph$$

ABS speed range at start of skid mark

VELOCITY OF HEAVY VEHICLE

$$V1 := \begin{bmatrix} V_{nonABS_1} \\ V_{ABS} \\ V_{nonABS_2} \end{bmatrix} = \begin{bmatrix} 35.8 \\ 41.5 \\ 41.9 \end{bmatrix} mph$$

$$\text{mean}(V1) = 39.7\ mph$$

$$\text{median}(V1) = 41.5\ mph$$

$$\text{stdev}(V1) = 2.8\ mph$$

$$Range\Delta S1 := \begin{bmatrix} \text{mean}(V1) - \text{stdev}(V1) \\ \text{mean}(V1) + \text{stdev}(V1) \end{bmatrix} = \begin{bmatrix} 36.9 \\ 42.5 \end{bmatrix} mph$$

$$Decel_{stop} := \begin{bmatrix} Decel_{min} \\ Decel_{ABS} \\ Decel_{max} \end{bmatrix} = \begin{bmatrix} 0.361 \\ 0.485 \\ 0.495 \end{bmatrix}$$

$$\text{mean}\left(Decel_{stop}\right) = 0.45$$

$$\text{median}\left(Decel_{stop}\right) = 0.48$$

$$\text{stdev}\left(Decel_{stop}\right) = 0.06$$

$$\mu_{Decel} := \begin{bmatrix} \text{mean}\left(Decel_{stop}\right) - \text{stdev}\left(Decel_{stop}\right) \\ \text{mean}\left(Decel_{stop}\right) + \text{stdev}\left(Decel_{stop}\right) \end{bmatrix} = \begin{bmatrix} 0.39 \\ 0.51 \end{bmatrix}$$