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Formalism of gauge-invariant curvatures and constructing the cubic vertices for massive spin-$\frac{3}{2}$ field in AdS_4 space

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Abstract We study the interaction of a massive spin-3/2 field with electromagnetic and gravitational fields in the four dimensional AdS space and construct the corresponding cubic vertices. The construction is based on a generalization of Fradkin–Vasiliev formalism, developed for massless higher spin fields, to massive fermionic higher spin fields. The main ingredients of this formalism are the gauge-invariant curvatures. We build such curvatures for the massive theory under consideration and show how the cubic vertices are written in their terms.

1 Introduction

Till now most of the investigations of consistent cubic interaction vertices for higher spin fields were devoted to bosonic massless fields (see e.g. [1–9]). The results on massive higher spin interactions are not so numerous, although there exists a classification of massive and massless cubic vertices in flat space by Metsaev [10–12] as well as some concrete examples [13–21]. At the same time there exist just a small number of papers devoted to fermionic higher spin interactions [11,22–24]. At least one of the reasons for this is that technically such investigations appear to be much more involved. In this paper, using massive spin $\frac{3}{2}$ as a simple but physically interesting and non-trivial example of massive fermionic higher spin fields, we apply the so-called Fradkin–Vasiliev formalism [25,26] (see also [21,27–32]) to the construction of electromagnetic and gravitational cubic vertices.

Let us briefly recall the basic properties of this formalism. A higher spin particle is described by a set of fields that we collectively denote Φ here and for each field one can construct a gauge invariant object that we will call curvature and denote as $\mathcal{R}$. Moreover, with the help of these curvatures a free Lagrangian can be rewritten in explicitly gauge-invariant form as

$$\mathcal{L}_0 \sim \sum \mathcal{R} \wedge \mathcal{R}.$$

Using these ingredients one can construct two types of non-trivial[1] cubic vertices:

– Abelian vertices that have the form

$$\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \Phi,$$

– non-Abelian ones that look like

$$\mathcal{L} \sim \mathcal{R} \wedge \Phi \wedge \Phi.$$

For the massless bosonic fields Vasiliev [33] has shown that any such non-Abelian vertex can be obtained as a result of a deformation procedure that can be described as follows.

– Construct quadratic in fields deformations for the curvatures and linear corrections to the gauge transformations

$$\Delta\mathcal{R} \sim \Phi \wedge \Phi, \quad \delta_1\Phi \sim \Phi\xi, \tag{1}$$

so that the deformed curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$ transform covariantly,

$$\delta\hat{\mathcal{R}} = \delta_1\mathcal{R} + \delta_0\Delta\mathcal{R} \sim \mathcal{R}\xi. \tag{2}$$

At this step most of the arbitrary parameters are fixed, however, there still remains some ambiguity. The reason is that covariance of the transformations for deformed

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[1] We call a vertex trivial if it can be constructed using gauge-invariant curvatures only, i.e. $\mathcal{L} \sim \mathcal{R} \wedge \mathcal{R} \wedge \mathcal{R}$.

curvatures guarantees that the equations of motion will be gauge-invariant but it does not guarantee that they will be Lagrangian.
- Non-Abelian cubic vertex arises then one put these deformed curvatures into the free Lagrangian and using explicit form of the gauge transformations requires it to be gauge-invariant

$$\mathcal{L} \sim \sum \hat{\mathcal{R}}\hat{\mathcal{R}} \quad \Leftrightarrow \quad \delta\mathcal{L} \sim \sum \mathcal{R}\mathcal{R}\xi = 0.$$

Note, finally, that Vasiliev has shown [33] that any non-trivial cubic vertex for three fields with spins s_1, s_2 and s_3 having up to $s_1+s_2+s_3-2$ derivatives can be constructed as some combination of Abelian and non-Abelian vertices. However, it has been pointed out that the Vasiliev construction [33] was initially formulated only for massless higher spin field theories. In this paper we generalize the Vasiliev approach to massive higher spin theories on the example of cubic electromagnetic and gravitational coupling for a massive spin-$\frac{3}{2}$ field in the AdS_4 space. Let us stress that in this paper we systematically restrict ourselves to cubic terms in the Lagrangians and terms linear in the fields in the gauge transformation laws, so all the Lagrangians we obtain are gauge-invariant in the linear (first non-trivial) approximation only. As is well known for higher spin theories in four or greater dimensions to proceed beyond this linear approximation one has to introduce an infinite number of fields with ever increasing spins and the corresponding gauge algebra becomes infinite dimensional. Thus in what follows all the higher order terms (quartic terms in the Lagrangians and quadratic terms in the gauge transformation laws) will be discarded.

The paper is organized as follows. In Sect. 2 we give all necessary kinematics formulas. Here we introduce two gauge-invariant objects (for the physical field ψ_μ and Stueckelberg one ϕ) and rewrite the free Lagrangian using these objects. Sect. 3 devoted to the electromagnetic interaction for such massive spin-$\frac{3}{2}$ field (for previous results see e.g. [34–36][2]). In Sect. 3.1, as an independent check for our calculations as well as for instructive comparison, we consider this task using straightforward constructive approach. Then in Sect. 3.2 we consider the same task in the Fradkin–Vasiliev formalism. At last in Sect. 4 we construct gravitational cubic vertex in the Fradkin–Vasiliev formalism.

Notations and conventions
We work in four-dimensional Anti-de Sitter space with cosmological constant Λ. The AdS covariant derivatives D_μ is normalized so that[3]

$$[D_\mu, D_\nu]\xi^a = \lambda^2\left(e_{[\mu}{}^a\xi_{\nu]} + \frac{1}{2}\Gamma_{\mu\nu}\xi^a\right), \quad \lambda^2 = -\frac{\Lambda}{3}. \quad (3)$$

In (3) $e_\mu{}^a$ is a background (non-dynamical) tetrad linking world and local indices and has the standard definition, $e_\mu{}^a e_\nu{}^b g^{ab} = g_{\mu\nu}$, where $g_{\mu\nu}$ and g^{ab} are, respectively, curved world AdS metrics and flat local one. Wherever it is convenient local indices are converted into the world ones by $e_\mu{}^a$ and its inverse $e^\mu{}_a$, in particular the matrix γ_μ with world index is understood as $\gamma_\mu = e_\mu{}^a\gamma^a$. Also we introduce the following notation for antisymmetric combinations of $e_\mu{}^a$:

$$\left\{{}^{\mu\nu}_{ab}\right\} = e^{[\mu}{}_a e^{\nu]}{}_b, \quad \left\{{}^{\mu\nu\alpha}_{abc}\right\} = e^{[\mu}{}_a e^\nu{}_b e^{\alpha]}{}_c.$$

In the expression (3) $\Gamma^{ab} = \frac{1}{2}\gamma^{[a}\gamma^{b]}$ is an antisymmetric combination of two gamma matrices and is a particular case of the more general definition which we will use,

$$\Gamma^{a_1a_2\dots a_n} = \frac{1}{n!}\gamma^{[a_1}\dots\gamma^{a_n]}$$

where $n = 2, 3, 4$ for four-dimensional space. Let us present the main properties of such Γ-matrices:

$$\begin{aligned}
\Gamma^{ab_1\dots b_n} &= -g^{a[b_1}\Gamma^{b_2\dots b_n]} + \gamma^a\Gamma^{b_1\dots b_n}, \\
\Gamma^{ab_1\dots b_n} &= g^{a[b_1}\Gamma^{b_2\dots b_n]} + (-1)^n\Gamma^{b_1\dots b_n}\gamma^a, \\
\gamma^a\Gamma^{b_1\dots b_n} &= 2g^{a[b_1}\Gamma^{b_2\dots b_n]} + (-1)^n\Gamma^{b_1\dots b_n}\gamma^a, \\
\gamma_a\Gamma^{ab_1\dots b_n} &= (d-n)\Gamma^{b_1\dots b_n}, \\
\Gamma^{ab_1\dots b_n}\gamma_a &= (-1)^n(d-n)\Gamma^{b_1\dots b_n}.
\end{aligned} \quad (4)$$

The identities (4) are used in the paper to verify the gauge invariance of the Lagrangians.

All fermionic fields are anticommuting and Majorana. We also use the Majorana representation of the γ-matrices with hermitian conjugation defined as follows:

$$\begin{aligned}
(\gamma^a)^\dagger &= \gamma^0\gamma^a\gamma^0 \\
(\Gamma^{a_1a_2\dots a_n})^\dagger &= \gamma^0\Gamma^{a_n\dots a_2a_1}\gamma^0 = -\gamma^0\Gamma^{a_1a_2\dots a_n}\gamma^0.
\end{aligned}$$

In this case $\gamma^0\gamma^a$ and $\gamma^0\Gamma^{ab}$ are symmetric in the spinor indices, while γ^0, $\gamma^0\Gamma^{abc}$, and $\gamma^0\Gamma^{abcd}$ are antisymmetric.

2 Kinematic of massive spin-3/2 field

For a gauge-invariant description of a massive spin-3/2 field besides the master vector-spinor field ψ_μ one has also introduced Stueckelberg spinor field ϕ. The free Lagrangian is

[2] Some aspects of higher spin equations of motion in an external electromagnetic field have been studied in [37]; however, these equations are non-Lagrangian.

[3] We use Greek letters $\mu, \nu, \dots$ for world indices and Latin letters $a, b, \dots$ for local ones. Summation over any repeated indices is implied. For indices in (square) round brackets we use the convention of complete (anti)-symmetrization without normalization factor. Spinor indices of (tensor)-spinor fields are omitted.

known and has the form

$$\mathcal{L}_0 = -\frac{i}{2}\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\psi}_\mu \Gamma^{abc} D_\nu \psi_\alpha + \frac{i}{2} e^\mu{}_a \bar{\phi}\gamma^a D_\mu \phi + 3ime^\mu{}_a \bar{\psi}_\mu \gamma^a \phi - \frac{3M}{2}\left\{{}^{\mu\nu}_{ab}\right\}\bar{\psi}_\mu \Gamma^{ab}\psi_\nu - M\bar{\phi}\phi. \tag{5}$$

This Lagrangian is invariant under the following gauge transformations:

$$\delta_0 \psi_\mu = D_\mu \xi + \frac{iM}{2}\gamma_\mu \xi$$
$$\delta_0 \phi = 3m\xi$$

where $M^2 = m^2 + \lambda^2 = m^2 - \Lambda/3$ and m is a mass parameter. Note that such a description works in de Sitter space as well, provided $m^2 > \Lambda/3$, $m^2 = \Lambda/3$ being the boundary of the unitarity region.

Using the explicit form of gauge transformations one can construct two gauge-invariant objects (curvatures):

$$\Psi_{\mu\nu} = D_{[\mu}\psi_{\nu]} + \frac{m}{6}\Gamma_{\mu\nu}\phi + \frac{iM}{2}\gamma_{[\mu}\psi_{\nu]},$$
$$\Phi_\mu = D_\mu \phi - 3m\psi_\mu + \frac{iM}{2}\gamma_\mu \phi, \tag{6}$$

which satisfy the Bianchi identities

$$D_{[\mu}\Psi_{\nu\alpha]} = \frac{m}{6}\Gamma_{[\mu\nu}\Phi_{\alpha]} - \frac{iM}{2}\gamma_{[\mu}\Psi_{\nu\alpha]},$$
$$D_{[\mu}\Phi_{\nu]} = -3m\Psi_{\mu\nu} - \frac{iM}{2}\gamma_{[\mu}\Phi_{\nu]}. \tag{7}$$

Note that the curvatures (6) are closely related with the Lagrangian equations of motion. It can be seen from the total variation of Lagrangian (5) that can be written as follows:

$$\delta\mathcal{L}_0 = -\frac{i}{2}\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abc}\delta\psi_\alpha - ie^\mu{}_a \bar{\Phi}_\mu \gamma^a \delta\phi. \tag{8}$$

Moreover, using these curvatures the free Lagrangian (5) can be rewritten in explicitly gauge-invariant form. The most general ansatz looks like

$$\mathcal{L}_0 = c_1 \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abcd}\Psi_{\nu\alpha} + ic_2\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abc}\Phi_\alpha + c_3\left\{{}^{\mu\nu}_{ab}\right\}\bar{\Phi}_\mu \Gamma^{ab}\Phi_\nu. \tag{9}$$

The requirement to reproduce the original Lagrangian (5) partially fixes the parameters

$$3c_3 = -8c_1, \quad 6c_2 m = \frac{1}{2} - 16c_1 M. \tag{10}$$

The remaining freedom in the parameters is related with the identity

$$\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} D_\mu[\bar{\Psi}_{\nu\alpha}\Gamma^{abcd}\Phi_\beta] = 0.$$

Using the Bianchi identities for the curvatures (7) we obtain

$$-3m\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abcd}\Psi_{\nu\alpha} + 8iM\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abc}\Phi_\alpha + 8m\left\{{}^{\mu\nu}_{ab}\right\}\bar{\Phi}_\mu\Gamma^{ab}\Phi_\nu = 0.$$

As will be seen in the next section sometimes this ambiguity may be important in order to reproduce the most general cubic vertex, so we will not fix it here.

3 Electromagnetic interaction

3.1 Constructive approach

We prefer to work with Majorana fermions; thus for the description of electromagnetic interactions we will use a pair of them, ${\psi_\mu}^i$, ϕ^i, where $i = 1, 2$ is the SO(2) index. Let us switch to the minimal electromagnetic interaction by the standard rule,

$$D_\mu {\psi_\nu}^i \Rightarrow D_\mu {\psi_\nu}^i + e_0 \varepsilon^{ij} A_\mu {\psi_\nu}^j,$$

and similarly for ϕ^i. The Lagrangian (5) with the above replacement for the covariant derivative is not invariant under the free gauge transformations, and the variation of the Lagrangian has the form

$$\delta_0\mathcal{L}_0 = -\frac{ie_0}{2}\varepsilon^{ij}\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\psi}_\mu{}^i \Gamma^{abc}F_{\nu\alpha}\xi^j = -3ie_0\varepsilon^{ij}e^\mu{}_a \bar{\psi}_\mu{}^i \Gamma^{abc}F^{bc}\xi^j. \tag{11}$$

To compensate for this non-invariance let us introduce non-minimal interactions:

$$\mathcal{L}_1 = \varepsilon^{ij}\Big[\left\{{}^{\mu\nu}_{ab}\right\}\bar{\psi}_\mu{}^i(b_1 F^{ab} + b_2\Gamma^{abcd}F^{cd}){\psi_\nu}^j + ie^\mu{}_a\bar{\psi}_\mu{}^i(b_3 F^{ab}\gamma^b + b_4\Gamma^{abc}F^{bc})\phi^j + b_5\bar{\phi}^i(\Gamma F)\phi^j\Big], \tag{12}$$

where $(\Gamma F) = \Gamma^{ab}F_{ab}$, which produce the following variations:

$$\begin{aligned}\delta_0\mathcal{L}_1 = {} & 4b_1\varepsilon^{ij}\bar{\psi}_\mu{}^i\eta^j(DF)^\mu - ib_3\varepsilon^{ij}\bar{\phi}^i\gamma_\mu\eta^j(DF)^\mu \\ & + 2\varepsilon^{ij}\left\{{}^{\mu\nu}_{ab}\right\}D_\mu{\psi_\nu}^i(b_1F^{ab} + b_2\Gamma^{abcd}F^{cd})\eta^j \\ & + i\varepsilon^{ij}e^\mu{}_a\bar{\psi}_\mu{}^i((2Mb_1 + 3mb_3)F^{ab}\gamma^b \\ & + (2Mb_2 + 3mb_4)\Gamma^{abc}F^{bc})\eta^j \\ & - i\varepsilon^{ij}e^\mu{}_a D_\mu\bar{\phi}^i(b_3F^{ab}\gamma^b - b_4\Gamma^{abc}F^{bc})\eta^j \\ & + \left(\frac{M(b_3 + 2b_4)}{2} + 6mb_5\right)\varepsilon^{ij}\bar{\phi}^i(\Gamma F)\eta^j\end{aligned}$$

where we have $(DF)^\mu = D_\nu F^{\nu\mu}$ as well as the corresponding corrections to the gauge transformations:

$$\delta_1{\psi_\mu}^i = i\alpha_1\varepsilon^{ij}(\Gamma F)\gamma_\mu\xi^i,$$
$$\delta\phi^i = \alpha_2\varepsilon^{ij}(\Gamma F)\xi^j,$$
$$\delta_1 A_\mu = \beta_1\varepsilon^{ij}(\bar{\psi}_\mu{}^i\xi^j) + i\beta_2\varepsilon^{ij}(\bar{\phi}^i\gamma_\mu\xi^j), \tag{13}$$

which in turn give us

$$\begin{aligned}\delta_1\mathcal{L}_0 &= \beta_1\varepsilon^{ij}\bar{\psi}_\mu{}^i\eta^j(DF)^\mu + i\beta_2\varepsilon^{ij}\bar{\phi}^i\gamma_\mu\eta^j(DF)^\mu \\ &\quad -6\alpha_1\varepsilon^{ij}\left\{{}^{\mu\nu}_{ab}\right\}D_\mu\bar{\psi}_\nu{}^i(2F^{ab}+\Gamma^{abcd}F^{cd})\eta^j \\ &\quad -3i\varepsilon^{ij}e^\mu{}_a\bar{\psi}_\mu{}^i[(4M\alpha_1-2m\alpha_2)F^{ab}\gamma^b \\ &\quad -(2M\alpha_1+m\alpha_2)\Gamma^{abc}F^{bc})]\eta^j \\ &\quad -i\alpha_2\varepsilon^{ij}e^\mu{}_aD_\mu\bar{\phi}^i(2F^{ab}\gamma^b+\Gamma^{abc}F^{bc})\eta^j \\ &\quad -2M\alpha_2\varepsilon^{ij}\bar{\phi}^i(\Gamma F)\eta^j.\end{aligned}$$

From these explicit expressions one can see that gauge invariance can be restored and that gives the solution for arbitrary coefficients:

$$b_1=6\alpha_1,\quad b_2=3\alpha_1,\quad b_3=-2\alpha_2,\quad b_4=\alpha_2$$
$$b_5=\frac{M\alpha_2}{3m},\quad \beta_1=-24\alpha_1,\quad \beta_2=-2\alpha_2,$$

provided the following important relation holds:

$$4M\alpha_1+2m\alpha_2=e_0. \tag{14}$$

The Lagrangian (12) together with the correction, stipulated by the minimal switch to the interaction in the free Lagrangian, is the final cubic electromagnetic interaction vertex for the spin-3/2 field. As we see, this vertex contains two arbitrary parameters.

A few comments are in order.

– Calculating the commutator of two gauge transformations we obtain

$$[\delta(\xi_1),\delta(\xi_2)]A_\mu = -8\varepsilon^{ij}(6\alpha_1{}^2+\alpha_2{}^2)(\xi_2{}^i\gamma^\nu\xi_1{}^j)F_{\mu\nu},$$

and it means that for non-zero electric charge e_0 any such model must be a component of some (spontaneously broken) supergravity.
– From the supergravity point of view the remaining freedom in the parameters α_1 and α_2 is clear: in general our vector field is a superposition of the graviphoton (i.e. a vector field from the supermultiplet $(\frac{3}{2},1)$) and the goldstino's superpartner (i.e. a vector field from the supermultiplet $(1,\frac{1}{2})$ (see e.g. [38,39] and references therein).
– From the expression for the parameter b_5 one can see that we have an ambiguity between flat and massless limits. Indeed, in the flat case $b_5=\frac{\alpha_2}{3}$ and does not depend on the mass any more, while for the non-zero cosmological term this parameter is singular in the massless limit.
– The most simple case—$\alpha_2=0$, i.e. the vector field is just the graviphoton. In this case the electric charge $e_0=\frac{2}{3}M\alpha_1$ becomes zero at the boundary of the unitarity region.

As we have already noted in the Introduction the Lagrangians obtained in this paper are gauge-invariant in the linear approximation only. Indeed, the introduction of cubic terms in the Lagrangian and linear terms in the gauge transformation laws generate quadratic variations of the form $\delta_1\mathcal{L}_1$. To compensate for these variations one has to introduce quartic terms and/or additional fields and so on. In supergravities (i.e. theories containing spins not higher than 2 and 3/2) such a program can indeed be realized leading to models with spontaneous supersymmetry breaking (see e.g. [38,39] and references therein). But for the theories with spins greater than 2 such a program will require the introduction of an infinite number of fields with ever increasing spins. The basic aim of our paper is to construct first a nonvanishing term in the interacting Lagrangian. It means that we consider only those terms in the Lagrangian and gauge transformations which are required for the realization of this aim. The higher order terms in the gauge transformations do not affect the form of the above cubic interaction vertex, and therefore we do not study these higher order terms here.

3.2 Formulation in terms of curvatures

Now we reformulate the results of Sect. 3.1 on the base of Fradkin–Vasiliev formalism. Following the general procedure we begin with a deformation of the curvatures so that they transform covariantly; see (1), (2). In the case under consideration $\mathcal{R}=\{\Psi_{\mu\nu}{}^i,\Phi_\mu{}^i,F_{\mu\nu}\}$. For the fermionic curvatures the deformations correspond simply to the minimal substitution:

$$\begin{aligned}\Delta\Psi_{\mu\nu}{}^i &= e_0\varepsilon^{ij}A_{[\mu}\psi_{\nu]}{}^j,\\ \Delta\Phi_\mu{}^i &= e_0\varepsilon^{ij}A_\mu\phi^j,\end{aligned} \tag{15}$$

while the transformations for the deformed curvatures look like

$$\delta\hat{\Psi}_{\mu\nu}{}^i = e_0\varepsilon^{ij}F_{\mu\nu}\xi^j,\quad \delta\hat{\Phi}_\mu{}^i=0 \tag{16}$$

where $\hat{\Psi}_{\mu\nu}{}^i=\Psi_{\mu\nu}{}^i+\Delta\Psi_{\mu\nu}{}^i$, $\hat{\Phi}_\mu{}^i=\Phi_\mu{}^i+\Delta\Phi_\mu{}^i$, and $\Psi_{\mu\nu}{}^i$, $\Phi_\mu{}^i$ are given by (6).

The most general deformation for the electromagnetic fields strength quadratic in the fermionic fields can be written as follows:

$$\Delta F_{\mu\nu}=\varepsilon^{ij}[a_1\bar{\psi}_{[\mu}{}^i\psi_{\nu]}{}^j+ia_2\bar{\psi}_{[\mu}{}^i\gamma_{\nu]}\phi^j+a_3\bar{\phi}^i\Gamma_{\mu\nu}\phi^j]. \tag{17}$$

The corresponding corrections to the gauge transformation have the form

$$\delta_1A_\mu=\varepsilon^{ij}[2a_1\bar{\psi}_\mu{}^i\xi^j-ia_2\bar{\phi}^i\gamma_\mu\xi^j]. \tag{18}$$

The deformed curvatures will transform covariantly

$$\delta_1\hat{F}_{\mu\nu}=\varepsilon^{ij}[2a_1\bar{\Psi}^i_{\mu\nu}\xi^j-ia_2\bar{\Phi}_{[\mu}{}^i\gamma_{\nu]}\xi^j], \tag{19}$$

provided the following relation holds:

$$3ma_3 = -\frac{ma_1}{6} - Ma_2. \tag{20}$$

Here $\hat{F}_{\mu\nu} = F_{\mu\nu} + \Delta F_{\mu\nu}$.

Now let us consider the Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + c_1 \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \hat{\bar{\Psi}}_{\mu\nu}{}^i \Gamma^{abcd} \hat{\Psi}_{\alpha\beta}{}^i \\ & + ic_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \hat{\bar{\Psi}}_{\mu\nu}{}^i \Gamma^{abc} \hat{\Phi}_\alpha{}^i \\ & + c_3 \left\{ {}^{\mu\nu}_{ab} \right\} \hat{\bar{\Phi}}_\mu{}^i \Gamma^{ab} \hat{\Phi}_\nu{}^i \end{aligned} \tag{21}$$

where all initial curvatures (including the electromagnetic field strength) are replaced by the deformed ones, and let us require that this Lagrangian be invariant. Non-trivial variations arise only with respect to transformations with the spinor parameter ξ^i. Using the explicit form of the covariant transformations (16), (19) we obtain

$$\begin{aligned}\delta\mathcal{L} = & -\frac{1}{2}\varepsilon^{ij} \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Psi}_{\mu\nu}{}^i (a_1 F^{ab} - 48 e_0 c_1 \Gamma^{abcd} F^{cd}) \xi^j \\ & + i\varepsilon^{ij} e^\mu{}_a \bar{\Phi}_\mu{}^i (a_2 \gamma^b F^{ab} + 6 e_0 c_2 \Gamma^{abc} F^{bc}) \xi^j. \end{aligned} \tag{22}$$

To compensate these terms one needs to introduce non-minimal corrections (note that they have exactly the same form as in the previous subsection):

$$\delta_1 \psi_\mu{}^i = i\alpha_1 \varepsilon^{ij} (\Gamma F) \gamma_\mu \xi^j, \quad \delta_1 \phi^i = \alpha_2 \varepsilon^{ij} (\Gamma F) \xi^j, \tag{23}$$

which in turn produce the following variations:

$$\begin{aligned}\delta_1 \mathcal{L}_0 = & -\alpha_1 \varepsilon^{ij} \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Psi}_{\mu\nu}{}^i (6F^{ab} + 3F^{de}\Gamma^{abde}) \xi^j \\ & - i\alpha_2 \varepsilon^{ij} e^\mu{}_a \bar{\Phi}_\mu{}^i (2\gamma^b F^{ab} + \Gamma^{abc} F^{bc}) \xi^j. \end{aligned} \tag{24}$$

Comparing with (22) we conclude

$$\begin{aligned} & a_1 = -12\alpha_1, \quad a_2 = 2\alpha_2, \\ & \alpha_1 = 8 e_0 c_1, \quad \alpha_2 = 6 e_0 c_2. \end{aligned} \tag{25}$$

Thus we see that the choice of the parameters α_1 and α_2 is related with the choice of parameters c_1 and c_2 in the free Lagrangian. Moreover, if we use the relation

$$6mc_2 = \frac{1}{2} - 16Mc_1,$$

we again obtain

$$4M\alpha_1 + 2m\alpha_2 = e_0.$$

Let us extract the cubic vertex. Using the relations (10), (25) we get

$$\begin{aligned}\mathcal{L}_1 = & 3\alpha_1 \varepsilon^{ij} \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\psi}_\mu{}^i (2F^{ab} + \Gamma^{abcd} F^{cd}) \psi_\nu{}^j \\ & - i\alpha_2 \varepsilon^{ij} e^\mu{}_a \bar{\psi}_\mu{}^i (2\gamma^b F^{ab} - \Gamma^{abc} F^{bc}) \phi^j \\ & + \frac{M\alpha_2}{3m} \varepsilon^{ij} \bar{\phi}^i (\Gamma F) \phi^j \\ & + \frac{ie_0}{2} \varepsilon^{ij} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} A_\mu \bar{\psi}_\nu{}^i \Gamma^{abc} \psi_\alpha{}^j \\ & + \frac{ie_0}{2} \varepsilon^{ij} e^\mu{}_a A_\mu \bar{\phi}^i \gamma^a \phi^j. \end{aligned} \tag{26}$$

Here the last two lines correspond to the minimal interactions, while the other terms are non-minimal corrections.

The Lagrangian $\mathcal{L}_1$ (26) up to the minimal interaction is the same as obtained in Sect. 3.1 for the electromagnetic cubic vertex.

4 Gravitational interaction

4.1 Kinematics for gravity

Let us briefly review basic features of gravity in the AdS_4 space at free level. In the frame formulation the gravitational field is described by dynamical frame $h_\mu{}^a$ and Lorentz connection $\omega_\mu{}^{ab}$ being antisymmetric in local indices. The free Lagrangian in AdS space has the form

$$\begin{aligned}\mathcal{L}_0 = & \frac{1}{2} \left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac} \omega_\nu{}^{bc} - \frac{1}{2} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab} D_\nu h_\alpha{}^c \\ & + \lambda^2 \left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b \end{aligned} \tag{27}$$

and is invariant under the gauge transformations

$$\begin{aligned} \delta_0 \omega_\mu{}^{ab} & = D_\mu \hat{\eta}^{ab} - \lambda^2 e_\mu{}^{[a} \hat{\xi}^{b]}, \\ \delta_0 h_\mu{}^a & = D_\mu \hat{\xi}^a + \hat{\eta}_\mu{}^a. \end{aligned} \tag{28}$$

The gauge-invariant objects (linearized curvature and torsion) have the form

$$\begin{aligned} R_{\mu\nu}{}^{ab} & = D_{[\mu} \omega_{\nu]}{}^{ab} - \lambda^2 e_{[\mu}{}^{[a} h_{\nu]}{}^{b]}, \\ T_{\mu\nu}{}^a & = D_{[\mu} h_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a. \end{aligned} \tag{29}$$

They satisfy the Bianchi identities

$$\begin{aligned} D_{[\mu} R_{\nu\alpha]}{}^{ab} & = \lambda^2 e_{[\mu}{}^{[a} T_{\nu\alpha]}{}^{b]}, \\ D_{[\mu} T_{\nu\alpha]}{}^a & = -R_{[\mu\nu,\alpha]}{}^a. \end{aligned} \tag{30}$$

Note that on the mass shell for the auxiliary field $\omega_\mu{}^{ab}$ by virtue of (30) we have

$$T_{\mu\nu}{}^a = 0 \quad \Rightarrow \quad R_{[\mu\nu,\alpha]}{}^a = 0, \quad D_{[\mu} R_{\nu\alpha]}{}^{ab} = 0. \tag{31}$$

Finally, the free Lagrangian can be rewritten as follows:

$$\mathcal{L}_0 = c_0 \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd}, \quad c_0 = \frac{1}{32\lambda^2}. \tag{32}$$

4.2 Gravitational coupling for massive spin-3/2

Following the general scheme (1), (2) we begin with the deformation of the curvatures that in the case under consideration are $\mathcal{R} = \{\Psi_{\mu\nu}, \Phi_\mu, R_{\mu\nu}{}^{ab}, T_{\mu\nu}{}^a\}$. As in the case of electromagnetic interactions deformations for spin-3/2 curvatures correspond to the minimal substitution rules, i.e. to the replacement of the covariant derivative $D \to D + \omega$ and background tetrad $e_\mu{}^a \to e_\mu{}^a + h_\mu{}^a$:

$$\Delta\Psi_{\mu\nu} = a_1\left(\omega_{[\mu}{}^{ab}\Gamma_{ab}\psi_{\nu]} + 2Mih_{[\mu}{}^a\gamma_a\psi_{\nu]} - \frac{2m}{3}h_{[\mu}{}^a\Gamma_{\nu]}{}^a\phi\right),$$
$$\Delta\Phi_\mu = a_1(\omega_\mu{}^{ab}\Gamma_{ab}\phi + 2Mih_\mu{}^a\gamma_a\phi). \quad (33)$$

The corrections to the gauge transformations will look like

$$\delta_1\psi_\mu = -a_1\left(\Gamma^{ab}\psi_\mu\hat{\eta}_{ab} + 2iM\gamma^a\psi_\mu\hat{\xi}_a - \frac{2m}{3}\Gamma_\mu{}^a\phi\hat{\xi}_a - \omega_\mu{}^{ab}\Gamma_{ab}\xi - 2iMh_\mu{}^a\gamma_a\xi\right),$$
$$\delta_1\phi = -a_1(\Gamma^{ab}\phi\hat{\eta}_{ab} + 2iM\gamma^a\phi\hat{\xi}_a), \quad (34)$$

while the transformations for the deformed curvatures will have the form

$$\delta\hat{\Psi}_{\mu\nu} = -a_1\left(\Gamma^{ab}\Psi_{\mu\nu}\hat{\eta}_{ab} + 2iM\gamma^a\Psi_{\mu\nu}\hat{\xi}_a + \frac{2m}{3}\Gamma_{[\mu}{}^a\Phi_{\nu]}\hat{\xi}_a - R_{\mu\nu}{}^{ab}\Gamma_{ab}\xi - 2iMT_{\mu\nu}{}^a\gamma_a\xi\right),$$
$$\delta\hat{\Phi}_\mu = -a_1(\Gamma^{ab}\Phi_\mu\eta_{ab} + 2iM\gamma^a\Phi_\mu\hat{\xi}_a). \quad (35)$$

Here $\hat{\Psi}_{\mu\nu} = \Psi_{\mu\nu} + \Delta\Psi_{\mu\nu}$, $\hat{\Phi}_\mu = \Phi_\mu + \Delta\Phi_\mu$ and $\Psi_{\mu\nu}{}^i$, Φ^i are given by (6).

Now let us consider the most general deformations for gravitational curvature and torsion quadratic in the spin-3/2 fields:

$$\Delta R_{\mu\nu}{}^{ab} = b_1\bar{\psi}_{[\mu}\Gamma^{ab}\psi_{\nu]} + ib_2e_{[\mu}{}^{[a}\bar{\psi}_{\nu]}\gamma^{b]}\phi + ib_3\bar{\psi}_{[\mu}\Gamma_{\nu]}{}^{ab}\phi + b_4e_{[\mu}{}^a e_{\nu]}{}^b\bar{\phi}\phi + b_5\bar{\phi}\Gamma_{\mu\nu}{}^{ab}\phi,$$
$$\Delta T_{\mu\nu}{}^a = ib_6\bar{\psi}_{[\mu}\gamma^a\psi_{\nu]} + b_7e_{[\mu}{}^a\bar{\psi}_{\nu]}\phi + b_8\bar{\psi}_{[\mu}\Gamma_{\nu]}{}^a\phi + ib_9\bar{\phi}\Gamma_{\mu\nu}{}^a\phi. \quad (36)$$

Note that there are three possible field redefinitions

$$\omega_\mu{}^{ab} \Rightarrow \omega_\mu{}^{ab} + i\kappa_1\bar{\phi}\Gamma_\mu{}^{ab}\phi,$$
$$h_\mu{}^a \Rightarrow h_\mu{}^a + \kappa_2\bar{\psi}_\mu\gamma^a\phi + \kappa_3e_\mu{}^a\bar{\phi}\phi \quad (37)$$

that shift parameters b_3, b_6 and b_7.

In order that deformed curvatures transform covariantly we have to introduce the following corrections to the gauge transformations:

$$\delta_1\omega_\mu{}^{ab} = 2b_1\bar{\psi}_\mu\Gamma^{ab}\xi - ib_2e_\mu{}^{[a}\bar{\phi}\gamma^{b]}\xi - ib_3\bar{\phi}\Gamma_\mu{}^{ab}\xi,$$
$$\delta_1h_\mu{}^a = 2ib_6\bar{\psi}_\mu\gamma^a\xi + b_7e_\mu{}^a\bar{\phi}\xi + b_8\bar{\phi}\Gamma_\mu{}^a\xi. \quad (38)$$

Then the curvature and torsion will transform as follows:

$$\delta\hat{R}_{\mu\nu}{}^{ab} = 2b_1\bar{\Psi}_{\mu\nu}\Gamma^{ab}\xi + ib_2e_{[\mu}{}^{[a}\bar{\Phi}_{\nu]}\gamma^{b]}\xi - ib_3\bar{\Phi}_{[\mu}\Gamma_{\nu]}{}^{ab}\xi,$$
$$\delta\hat{T}_{\mu\nu}{}^a = 2ib_6\bar{\Psi}_{\mu\nu}\gamma^a\xi - b_7e_{[\mu}{}^a\bar{\Phi}_{\nu]}\xi + b_8\bar{\Phi}_{[\mu}\Gamma_{\nu]}{}^a\xi, \quad (39)$$

provided the following restrictions on the arbitrary parameters hold:

$$3mb_2 = b_6(M^2 - m^2) - b_1M \quad 3mb_8 = -b_6M + b_1,$$
$$6mb_4 = \frac{mb_1}{3} + 2b_7(M^2 - m^2) \quad 6mb_9 = -\frac{mb_6}{3} - 2b_3,$$
$$6mb_5 = -\frac{mb_1}{3} - 2b_3M. \quad (40)$$

Here $\hat{R}_{\mu\nu}{}^{ab} = R_{\mu\nu}{}^{ab} + \Delta R_{\mu\nu}{}^{ab}$, $\hat{T}_{\mu\nu}{}^a = T_{\mu\nu}{}^a + \Delta T_{\mu\nu}{}^a$ and $R_{\mu\nu}{}^{ab}$, $T_{\mu\nu}{}^a$ are given by (29).

The general solution to these relations has four free parameters, for example, $b_{1,3,6,7}$. But as we have already noted three of them are related with possible field redefinitions, so we have one non-trivial parameter b_1 only.

Let us consider the following Lagrangian:

$$\mathcal{L} = c_1\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\hat{\bar{\Psi}}_{\mu\nu}\Gamma^{abcd}\hat{\Psi}_{\nu\alpha} + ic_2\left\{{}^{\mu\nu\alpha}_{abc}\right\}\hat{\bar{\Psi}}_{\mu\nu}\Gamma^{abc}\hat{\Phi}_\alpha + c_3\left\{{}^{\mu\nu}_{ab}\right\}\hat{\bar{\Phi}}_\mu\Gamma^{ab}\hat{\Phi}_\nu + c_0\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\hat{R}_{\mu\nu}{}^{ab}\hat{R}_{\alpha\beta}{}^{cd} + ic_4\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abc}\Phi_\alpha h_\beta{}^d + c_5\left\{{}^{\mu\nu\alpha}_{abc}\right\}\bar{\Phi}_\mu\Gamma^{ab}\Phi_\nu h_\alpha{}^c \quad (41)$$

where the first four terms are just the sum of free Lagrangians for the massive spin-$\frac{3}{2}$ case and the massless spin-2 case with the initial curvatures replaced by the deformed ones, while the last two terms are possible Abelian vertices. Note that in dimensions $d > 4$ we would have to introduce one more Abelian vertex:

$$\left\{{}^{\mu\nu\alpha\beta\gamma}_{abcde}\right\}\bar{\Psi}_{\mu\nu}\Gamma^{abcd}\Psi_{\alpha\beta}h_\gamma{}^e.$$

Now we require that this Lagrangian be gauge-invariant. To fix all coefficients it is enough to consider variations that do not vanish on-shell. For ξ-transformations we have

$$\delta\mathcal{L} = (-24c_1a_1 + 4b_1c_0)\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\bar{\Psi}_{\alpha\beta}R_{\mu\nu}{}^{ab}\Gamma^{cd}\xi + i(6c_2a_1 - 8c_0b_3)\left\{{}^{\mu\nu\alpha}_{abc}\right\}R_{\mu\nu}{}^{ad}\bar{\xi}\Gamma^{bcd}\Phi_\alpha + i(6c_2a_1 - 8c_0b_2)\left\{{}^{\mu\nu\alpha}_{abc}\right\}R_{\mu\nu}{}^{ab}\bar{\xi}\gamma^c\Phi_\alpha;$$

the last two terms vanish on-shell and for the second term it can be seen from the identity:

$$0 = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} R_{\mu\nu,\alpha}{}^{a} \bar{\xi} \Gamma^{bcd} \Phi_\beta = 3 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} R_{\mu\nu}{}^{ad} \bar{\xi} \Gamma^{bcd} \Phi_\alpha.$$

The first term does not vanish on-shell so we have to put

$$b_1 c_0 = 6 c_1 a_1. \tag{42}$$

Calculating the variations for the $\hat{\eta}^{ab}$-transformations we obtain

$$\begin{aligned}\delta\mathcal{L} = & -8c_1 a_1 \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abce} \Psi_{\alpha\beta} \hat{\eta}^{de} \\ & -i(12c_2 a_1 - 3c_4) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abd} \Phi_\alpha \hat{\eta}^{cd} \\ & +(8c_3 a_1 - 2c_5) \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Phi}_\mu \Gamma^{ac} \Phi_\nu \hat{\eta}^{bc}.\end{aligned}$$

The first term vanishes in $d = 4$ due to the identity

$$\begin{aligned}0 &= \left\{ {}^{\mu\nu\alpha\beta\gamma}_{abcde} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abcd} \Psi_{\alpha\beta} \hat{\eta}_\gamma{}^{e} \\ &= 4 \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abce} \Psi_{\alpha\beta} \hat{\eta}^{de},\end{aligned}$$

while the last two terms give the restrictions

$$c_4 = 4c_2 a_1, \quad c_5 = 4c_3 a_1. \tag{43}$$

Finally we consider variations under the $\hat{\xi}^a$ transformations

$$\begin{aligned}\delta\mathcal{L} = & -i\left(16c_1 a_1 M + \frac{3}{2} m c_4\right) \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abc} \Psi_{\alpha\beta} \hat{\xi}^d \\ & +(32c_1 a_1 m + 3mc_5) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{ab} \Phi_\alpha \hat{\xi}^c \\ & +(-4Ma_1c_2 + Mc_4) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abcd} \Phi_\alpha \hat{\xi}^d \\ & -i((4a_1c_2 - c_4)m + (4c_3a_1 - c_5)M) \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Phi}_\mu \Gamma^{abc} \Phi_\nu \hat{\xi}^c \\ & +i(2(8a_1c_2 + c_4)m - 4(2c_3a_1 + c_5)M) \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Phi}_\mu \gamma^a \Phi_\nu \hat{\xi}^b.\end{aligned}$$

Taking into account (10), (40), (42), and (43) we obtain simply

$$\delta\mathcal{L} = -\frac{i}{2} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \bar{\Psi}_{\mu\nu} \Gamma^{abc} \Psi_{\alpha\beta} \hat{\xi}^d + 2ia_1 \left\{ {}^{\mu\nu}_{ab} \right\} \bar{\Phi}_\mu \gamma^a \Phi_\nu \hat{\xi}^b.$$

To compensate these variations one can use the following corrections:

$$\delta_1 \psi_\mu \sim \Psi_{\mu\nu} \hat{\xi}^\nu, \quad \delta_1 \phi \sim \Phi_\mu \hat{\xi}^\mu.$$

Thus the requirement that the Lagrangian be gauge-invariant fixes the coefficients $c_{4,5}$ for the Abelian vertices and also relates the coefficients b_1 and c_1 (which is just a manifestation of the universality of the gravitational interaction). Note that by the Metsaev classification [11] in general dimensions $d > 4$ we would have two non-minimal vertices with two derivatives as well as the minimal one. But in $d = 4$ dimensions these two derivative vertices are absent (on-shell and up to possible field redefinitions) as we will now show.

Vertex $\frac{3}{2} - \frac{3}{2} - 2$. For this vertex we obtain

$$\mathcal{L} = (-48c_1 a_1 + 8c_0 b_1) \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} D_\mu \omega_\nu{}^{ab} \bar{\psi}_\alpha \Gamma^{cd} \psi_\beta = 0.$$

This expression vanishes due to the obtained expressions and the restrictions for the arbitrary coefficients.

Vertex $\frac{1}{2} - \frac{1}{2} - 2$. In this case we have

$$\begin{aligned}\mathcal{L} = & (-c_3 a_1 + 8c_0 b_5) \left\{ {}^{\mu\nu}_{ab} \right\} D_\mu \omega_\nu{}^{cd} \bar{\phi} \Gamma^{abcd} \phi \\ & +(8c_3 a_1 - 2c_5) \left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac} D_\nu \bar{\phi} \Gamma^{bc} \phi \\ & +(2c_3 a_1 + 16c_0 b_4) \left\{ {}^{\mu\nu}_{ab} \right\} D_\mu \omega_\nu{}^{ab} \bar{\phi} \phi.\end{aligned}$$

The second term drops out due to the relations among the coefficients, the first term vanishes on-shell for the gravitational field due to the identity

$$\begin{aligned}0 &= \left\{ {}^{\mu\nu\alpha}_{abc} \right\} R_{\mu\nu,\alpha}{}^{d} \bar{\phi} \Gamma^{abcd} \phi \\ &= 2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} (D_\mu \omega_{\nu,\alpha}{}^{d} + \lambda^2 e_\mu{}^{d} h_{\nu,\alpha}) \bar{\phi} \Gamma^{abcd} \phi \\ &= 6 \left\{ {}^{\mu\nu}_{ab} \right\} D_\mu \omega_\nu{}^{cd} \bar{\phi} \Gamma^{abcd} \phi,\end{aligned}$$

while the third term can be removed by the field redefinition

$$h_\mu{}^a \to h_\mu{}^a + \kappa_3 e_\mu{}^a \bar{\phi} \phi.$$

Vertex $\frac{3}{2} - \frac{1}{2} - 2$. For the last possibility we get

$$\begin{aligned}\mathcal{L} = & i(-24c_2 a_1 + 6c_4) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\alpha{}^{da} D_\mu \bar{\psi}_\nu \Gamma^{bcd} \phi \\ & +i(16c_0 b_2 - 12c_2 a_1) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} D_\nu \omega_\alpha{}^{ab} \bar{\psi}_\mu \gamma^c \phi \\ & -i(16c_0 b_3 - 12c_2 a_1) \left\{ {}^{\mu\nu\alpha}_{abc} \right\} D_\nu \omega_\alpha{}^{dc} \bar{\psi}_\mu \Gamma^{abd} \phi.\end{aligned}$$

The first term drops out due to the relations among the coefficients, the last term vanishes on-shell for the gravitational field, and the second term can be removed by the field redefinition

$$h_\mu{}^a \to h_\mu{}^a + \kappa_2 \bar{\psi}_\mu \gamma^a \phi.$$

As a result, the expression (41) is the final form for Lagrangian of coupled spin-2 and spin-3/2 fields in AdS_4 space including the cubic interaction vertex. The parameters c_1, c_2, c_3 are fixed in the free gravitational field Lagrangian (27), the parameter c_0 is given by (32). The parameters c_4, c_5, and the parameters of the gauge transformations are expressed through the single parameter a_1, which is the only free parameter of the theory.

5 Conclusion

In this paper we have constructed the cubic vertices for a massive spin-3/2 field coupled to electromagnetic and gravitational fields in AdS_4 space. The corresponding vertices are gauge-invariant due to the presence of Stueckelberg auxiliary fields and contain a number of free parameters. The results are given by the expressions (12) or (26) for the electromagnetic interaction and (41) for the gravitational interaction.

The construction of the vertices is based on a generalization of the Fradkin–Vasiliev formalism [25,26] where the main building blocks of the Lagrangians are the gauge-invariant curvatures. Although this formalism was known only for massless higher spin theories, we have shown, for the example of a spin-3/2 field, that the gauge invariant curvatures can in principle be constructed for massive higher spin fields as well. Certainly the case of massive fermionic fields appears to be the most technically involved one. Nevertheless it seems worth to apply this formalism to massive fermionic fields with spins higher than 3/2.

As we pointed out, the constructed vertices contain some number of arbitrary parameters (two for the electromagnetic coupling and one for gravitational coupling) which cannot be fixed only by gauge invariance. In principle one can hope that the additional constraints for those parameters can appear from the requirements of causality. The possibility of such constraints was already demonstrated in the papers [17,20].

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